

Aljabar Linear dan Geometri

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Latihan Quiz 1 2023.

1. $A = \begin{bmatrix} 1 & 6 & 7 \\ 0 & 2 & -2 \\ 6 & 8 & 9 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 & 8 \\ 4 & 3 & 24 \\ 1 & 12 & -6 \end{bmatrix}$

a) Hitunglah $-2(A^T)^T + \frac{1}{3}B$

b) Hitunglah $\text{tr}(-2(A^T)^T + \frac{1}{3}B)$

a. $-2(A^T)^T + \frac{1}{3}B = 2A + \frac{1}{3}B$

$$= 2 \begin{bmatrix} 1 & 6 & 7 \\ 0 & 2 & -2 \\ 6 & 8 & 9 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} -3 & 1 & 8 \\ 4 & 3 & 24 \\ 1 & 12 & -6 \end{bmatrix} = \begin{bmatrix} 2 & 13 & 14 \\ 0 & 4 & -4 \\ 12 & 16 & 18 \end{bmatrix} + \begin{bmatrix} -1 & 1/3 & 8/3 \\ 4/3 & 1 & 8 \\ 1/3 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -35/3 & -34/3 \\ 4/3 & -3 & 4 \\ -35/3 & -12 & -20 \end{bmatrix}$$

b. $\text{tr}(-2(A^T)^T + \frac{1}{3}B) = -2 + -3 + -20 = -25$

2. $A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$ memiliki balikan jika dan hanya jika $\det k \neq 0$

$$\det(k) = 1 \begin{vmatrix} 1 & k \\ 2 & 1 \end{vmatrix} - k \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 1 - 2k - k(2) = 1 - 4k$$

$1 - 4k \neq 0$, maka $k \neq \frac{1}{4}$

3. pemodelan :

$x = \text{kambing} \rightarrow x + y = 13$, matrix = $\begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix}$, matrix 1 = $\begin{bmatrix} 13 & 1 \\ 32 & 2 \end{bmatrix}$

$y = \text{ayam} \quad 4x + 2y = 32$

a. $|\text{matrix}| = 2 - 4 = -2$

$$x = \frac{\det(\text{matrix}_1)}{\det(\text{matrix})} = \frac{26 - 32}{-2} = 3$$

\therefore ada 3 kambing dan 10 ayam

4. $M = \begin{bmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 \end{bmatrix}$

a. $\dots R_2 \leftarrow R_2 - \frac{1}{2}R_1$ $\dots R_3 \leftarrow R_3 - 2R_2$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 \end{bmatrix} = -1 \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} = -1 \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{5}{3} & 0 & -\frac{2}{3} \end{bmatrix} = (-1) \left(\frac{1}{3} \right) \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{5}{3} & -\frac{2}{3} \end{bmatrix}$$

$$= -\frac{1}{3} \left(\frac{1}{2} \right) = -\frac{1}{6} \text{ (ekspansi kofaktor)}$$

b. karena $|M| \neq 0$, matrix m memiliki balikan.

c. sistem pers. linier homogen memiliki solusi trivial

5. Tentukan matriks A jika diketahui

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

misalkan $(5A^T)^{-1} = a$

$$5A^T = a^{-1} = \frac{1}{\det a} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix} = (5A)^T$$

$$5A = \begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}, \text{ maka } A = \begin{bmatrix} -2/5 & 1 \\ -1/5 & 3/5 \end{bmatrix}$$