Introduction to Logic

IF2121 Computational Logic 2024/2025

Informatics Engineering Study Program
School of Electrical Engineering and Informatics ITB

Lecturers

Lecturers:

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Vision and Mission STEI ITB

Visi:

Menjadi Institusi pendidikan tinggi, pengembang ilmu pengetahuan Teknik Elektro dan Informatika yang unggul dan terkemuka di Indonesia dan diakui di dunia serta berperan aktif dalam usaha memajukan dan mensejahterakan bangsa.

Misi:

1. Menyelenggarakan pendidikan tinggi dan pendidikan berkelanjutan di bidang teknik Elektro dan Informatika dengan memanfaatkan teknologi komunikasi dan informasi
2. Mengikuti (memelihara) keterkinian (state of the art) serta mengembangkan ilmu pengetahuan Teknik Elektro dan Informatika melalui kegiatan penelitian yang inovatif.
3. Mendiseminasikan ilmu pengetahuan, teknologi dan pandangan/wawasan Teknik Elektro dan Informatika yang dimiliki kepada masyarakat baik melalui lulusannya, kemitraan dengan industri atau lembaga lainnya maupun melalui kegiatan pengabdian pada masyarakat dalam rangka membentuk masyarakat berkearifan teknologi.

Program Educational Objective

- Setiap lulusan IF ITB diharapkan (dalam 3-5 tahun setelah lulus):
  - Lulusan akan memiliki karir yang sukses dan menjadi seorang professional yang produktif di bidangnya
  - Lulusan dapat melanjutkan dan menyelesaikan pendidikan lanjutannya
  - Lulusan dapat berkembang secara professional melalui pembelajaran mandiri dan memiliki peran aktif dan **kepemimpinan** dalam pengembangan alat, teknologi dan metodologi baru.

Student Outcome

Graduates of the program will have an ability to:

1. Analyze a complex computing problem and to apply principles of computing and other relevant disciplines to identify solutions.
2. Design, implement, and evaluate a computing-based solution to meet a given set of computing requirements in the context of the program's discipline.
3. Communicate effectively in a variety of professional contexts.
4. Recognize professional responsibilities and make informed judgments in computing practice based on legal and ethical principles.
5. Function effectively as a member or leader of a team engaged in activities appropriate to the program's discipline.
6. Apply computer science theory and software development fundamentals to produce computing-based solutions.

Learning Outcomes for IF2121

1. Students are able to translate a natural language problem into propositional logic representation.
2. Students are able to translate a natural language problem into relational logic representation.
3. Students are able to prove or draw a conclusion from existing facts/ premises using propositional/ relational proof.
4. Students are able to prove or draw a conclusion from existing facts/ premises using propositional/ relational resolution.
5. Students are able to design and implement a simple program in Prolog to solve simple problem, based on automatic theorem proving.
6. Students are able to evaluate an existing Prolog program, based on the problem to be solved.

Courses

- Credits: 2 credit points
- Courses:
  - Attending classes 1 hour 40 minutes/week (14 weeks):
    - Friday (14.00 – 15.40)
    - attendance is obligatory.
  - Assignments
  - Homework and quiz (individually)
  - Midterm Exam (week 8); Final Exam (week 16)
- No additional exam/ quiz

Courses contents

1. Introduction
2. Concept of logics
3. Propositional Logics (syntax, semantic, proofing)
4. First Order Predicate Logics (syntax, semantic, proofing)
5. Introduction to Proofing Theory
6. Declarative Programming → Prolog
7. Simple Application using Prolog

Grading

- Homework : 2,5%
- Assignments (small + big) : 25%
- Quiz : 17,5%
- Midterm Exam : 27,5%
- Final Exam : 27,5%
- Dishonesty → E

References

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- Stuart J Russell & Peter Norvig, Resources of topics in Artificial Intelligence: A Modern Approach, 4th Edition, Global Edition Paperback, Pearson, 2021, http://aima.cs.berkeley.edu/
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- Patrick Blackburn, Johan Bos, and Kristina Striegnitz, Learn Prolog Now!, last accessed November 2019, can be accessed at http://www.learnprolognow.org/lpnpage.php?pageid=online

Propositional Logic: Syntax & Semantic

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Propositional Logic: Name

- Propositional Calculus
- Statement/sentential calculus
  - Calculus: symbol manipulation system
  - Proposition:
    - declarative sentence that has truth value
    - Possible condition of the world about which we want to say something

Example of Proposition

- Every registered student has registration number.
- It is raining.
- Mary loves Pat.
- Students have a day off.

- Let's go home.
- When will we go home?

Propositional Logic: Syntax

- Type: simple sentences, compound sentences
- **Simple sentences**: (a.k.a. propositional constants/ logical constants)
  - Express atomic proposition about the world
  - String of letters, digits, and underscore
  - Beginning with lower case letter
  - Examples:

    raINing

    r41_ning

    p

    q
  - Wrong examples:

    Raining

    324567

    raining-or-snowing

Propositional Logic: Syntax (2)

- **Compound sentences**:
  - Formed from simpler sentences and express relationships among the constituent sentences
  - There are six types of compound sentences :
    1. Negations ($\neg p$)
    2. conjunctions ($p \wedge q$)
    3. disjunctions ($p \vee q$)
    4. implications ($p \rightarrow q$)
    5. reductions ($p \leftarrow q$)
    6. equivalences ($p \leftrightarrow q$)
  - Constituent sentences → simple sentence or compound sentence

$((p \vee q) \rightarrow (\neg r))$

Example of compound sentences

Formalization:

p: Today is raining.

q: Students have a day off.

1. Negation (**not**):
   - ¬p: Today is **not** raining.
2. Conjunction (**and**):
   - p ∧ q: Today is raining **and** Students have a day off.
3. Disjunction (or):
   - p ∨ q: Today is raining **or** Students have a day off.

Example of compound sentences (2)

p: Today is raining.

q: Students have a day off.

4. Implications (**if-then, implies, only if, is sufficient for, is necessary for, whenever**,): $p \rightarrow q$

   If Today is raining then students have a day off.

   Today is raining implies students have a day off.

5. Reductions (if, whenever) $(q \leftarrow p)$

   Students have a day off if today is raining.

   Students have a day off whenever today is raining

Example of compound sentences (3)

p: Today is raining.

q: Students have a day off.

6. Equivalences (**if and only if, iff, is necessary and sufficient for, if-then-and conversely**): $p \leftrightarrow q$

Today is raining iff students have a day off

Today is raining, is necessary and sufficient for students have a day off

If today is raining then students have a day off and conversely

Compound Sentence

- Parenthesis is very important → determine the sequence of operations
- Example:

  $p \vee q \rightarrow \neg r$:
  - $((p \vee q) \rightarrow \neg r)$
  - $(p \vee (q \rightarrow \neg r))$

Operator Precedence

- Operator precedence (high to low): ¬, ∧, ∨, → | ←, ↔
- An operand associates with operator of higher precedence
- When an operand is surrounded by two ∧ operators or by two ∨ operators, the operand associates to the left.
- When an operand is surrounded by two ⇒ operators or by two ⇔ operators, the operand associates to the right.

Examples

Precedence: $\neg$, $\wedge$, $\vee$, $\rightarrow$ | $\leftarrow$, $\leftrightarrow$

$p \wedge q \vee r$: how with parenthesis?

$p \vee q \wedge r$: how with parenthesis?

$p \rightarrow q \rightarrow r$: how with parenthesis?

Exercise 1

- Which of these are legal propositional logic sentences?

  p → q → r

  p, r → q

  $a \wedge (b \vee c \vee \neg d) \leftrightarrow \neg\neg z$

  $\neg p(q)$

- Give fully parenthesized expressions for the legal sentences. (If t here is more than one solution, just pick any one).

Exercise 2

- Which of these are legal propositional logic sentences?

(a) $p \wedge \neg p$

(b) $\neg p \vee \neg p$

(c) $\neg(q \vee r) \neg\, q \Rightarrow \neg\neg p$

(d) $(p \vee q) \wedge (r \vee q)$

(e) $p \vee \neg q \wedge \neg p \vee \neg q \Rightarrow p \vee q$

(f) $(p \Rightarrow q) \vdash (q \Leftarrow p)$

(g) $(p \Rightarrow q) \models (q \Leftarrow p)$

(h) $((p \Rightarrow q) \Rightarrow s) \Leftrightarrow (r \Leftarrow t)$

(i) $((p \Leftrightarrow q) \Leftrightarrow s) \Leftrightarrow (r \Leftrightarrow t)$

(j) This $\vee$ is $\neg$ correct.

Exercise 3

- Encode the following English sentences as sentences in Propositional Logic

a. You can go swimming if you know how to swim and the water is not too cold.

b. To take discrete math, you must have taken calculus or a course in computer science.

c. Charles comes if Elsa does and the other way around.

- Propositional constants

p: you can go swimming; q: you know how to swim

r: the water is too cold; v: Charles comes

w: Elsa comes; s: you take discrete math

t: you took calculus; u: you took a course in computer science

Exercise 4

Dari daftar kalimat di bawah ini, tentukan apakah kalimat tersebut dituliskan dalam representasi propositional logic atau tidak, jelaskan dengan singkat alasannya.

a. art-loves-betty ^ art-loves-cathy

b. ¬p ^ ¬ ¬q

c. ((charlesIsWilliamFather ^ dianaIsWilliamMother) → charlesLovesdiana)

d. p ∨ ¬q ∧ ¬p ∨ ¬q ⇒ p ∨ q

e. logic_is_fun ⟵ get_good_grade ∧ not_too_much_work

Propositional Logic: Semantics

- Similar to the semantics of algebra → unconcerned with real-world meaning of variables
  - In algebra: concerns the relationship between variables
  - In logic: concerns the relationship between the truth of simple sentences and the truth of compound sentences
- In logic, variables assignments are necessary → various assignments or all assignments are considered
- Such assignments → interpretation
- Interpretation for propositional logic: assignment of truth value to each simple sentence of the language

Propositional Logic: Semantics (2)

- Example of a language with three propositional logic under interpretation i:
  - p_i = *true*
  - q_i = *false*
  - r_i = *true*
- The same language under interpretation j:
  - p_j = *false*
  - q_j = *false*
  - r_j = *true*
- Both examples are not sentences in propositional logic (metalevel statements) → subscript and '=' symbol

Propositional Logic: Semantics (3)

- Given an interpretation for logical constants of a language, we can draw the interpretation for all compound sentences in that language:
  - Negation
  - Conjunction
  - Disjunction
  - Implication
  - Reduction
  - Equivalence
- What are the rules??
- Interpretation I satisfies a sentence iff it's true under that interpretation

Truth table

- All interpretation for logical constants
- If n is the number of logical constants:
  - Number of interpretation: 2^n
- Make truth table for compound sentences

Evaluation

- Process of determining truth value of propositions: evaluation
- Interpretation i satisfies a proposition iff it is *true* under that interpretation
- Example:
  - we have interpretation *i*
    - p_i = *true*; q_i = *false*; r_i = *true*
  - we have interpretation *j*
    - p_j = *true*; q_j = *true*; r_j = *false*
  - Interpretation i satisfies $(p \vee q) \wedge (\neg q \vee r)$
  - Interpretation j doesn't satisfy $(p \vee q) \wedge (\neg q \vee r)$

Reverse Evaluation

- Given one or more compound sentences, figure out which interpretation satisfy those sentences
- How? → truth table
- Process:
  - Crossing out rows that do not satisfy the sentence
  - Remaining row(s) are all possible interpretation of the sentence

Example Reverse Evaluation

$q \Longrightarrow r$

p	q	r	
1	1	1	
1	1	0	×
1	0	1	
1	0	0	
0	1	1	
0	1	0	×
0	0	1	
0	0	0	

- How about p → qΛr

Example Reverse Evaluation

$q \Longrightarrow r$

p	q	r	
1	1	1	
1	1	0	×
1	0	1	
1	0	0	
0	1	1	
0	1	0	×
0	0	1	
0	0	0	

- How about p → qΛr

Classification of Compound Sentence

- Valid:
  - Iff it is satisfied by every interpretation
  - Example: $p \vee \neg p$
- Satisfiable:
  - Iff it is satisfied by at least one interpretation
  - Example: $\neg p$, $p \rightarrow q$
- Unsatisfiable:
  - Iff is is not satisfied by any interpretation
  - Example: $p \leftrightarrow \neg p$, $p \wedge \neg p$

Exercise

- For each of the following sentences, indicate whether it is valid, satisfiable, or unsatisfiable

(a) $(p \Rightarrow q) \vee (q \Rightarrow p)$
(b) $p \wedge (p \Rightarrow \neg q) \wedge q$
(c) $(p \Rightarrow (q \wedge r)) \Leftrightarrow (p \Rightarrow q) \wedge (p \Rightarrow r)$
(d) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \wedge q) \Rightarrow r)$
(e) $(p \Rightarrow \mathrm{q}) \wedge (p \Rightarrow \neg q)$
(f) $(\neg p \vee \neg q) \Rightarrow \neg(p \wedge q)$
(g) $((\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p)) \wedge (p \vee q)$
(h) $(\neg p \vee q) \Rightarrow (q \wedge (p \Leftrightarrow q))$
(i) $((\neg r \Rightarrow \neg p \wedge \neg q) \vee s) \Leftrightarrow (p \vee q \Rightarrow r \vee s)$
(j) $(p \wedge (\mathrm{q} \Rightarrow r)) \Rightarrow ((\neg p \vee q) \Rightarrow (p \wedge r))$

Exercise Reverse Evaluation

Pada suatu pulau, hanya terdapat 2 jenis penduduk, yaitu knight dan knave.

Knight selalu mengatakan kebenaran,

Knave selalu mengatakan kebohongan.

Pada suatu saat terdapat 2 penduduk (A dan B) yang tidak diketahui termasuk knight atau knave. A mengatakan: 'we are both knaves'. B tidak mengatakan apapun. Tentukan untuk A dan B, termasuk knight atau knave.

Petunjuk: Ubahlah kalimat di atas ke dalam propositional logic (tuliskan proposisi yang anda gunakan untuk merepresentasikan apa). Gunakan asumsi awal (jenis penduduk untuk A), untuk melakukan pencarian solusi. Gunakan *reverse evaluation* untuk menentukan jenis penduduk untuk A dan B.

Another Exercise of Reverse Evaluation

Terdapat tiga buah kotak, hanya satu kotak yang berisi emas, dan dua kotak lainnya kosong. Terdapat kertas yang ditempelkan pada setiap kotak yang berisi suatu petunjuk. Petunjuk di kotak 1 tertulis: "Emas tidak ada di sini". Petunjuk di kotak 2 tertulis: "Emas tidak ada di sini". Petunjuk di kotak 3 tertulis: "Emas ada di kotak 2".

Hanya ada satu petunjuk yang benar, sedangkan dua petunjuk yang lain salah. Di mana kah emas berada?

Representasikan dalam logika proposisi (hanya memerlukan dua proposisi), dan gunakan tabel kebenaran untuk menjawab pertanyaan tersebut menggunakan *reverse evaluation*.

Exercise 5

- Translasikan kalimat dalam bahasa alami berikut ini ke dalam representasi logika proposisi. Gunakan proposisi yang sudah ditentukan pada slide berikutnya.
  a) Jika hujan, salju, dan hujan es tidak menghambat saya untuk meletakkan surat pada kotak pos anda, maka jika saya bukan seorang tukang pos ataupun sosiopat, berarti saya adalah seorang kawan yang baik.
  b) "A necessary condition of an argument being valid is that it be deductive."
  c) Ang mengatakan bahwa kita harus memaafkan, tapi Ang tidak memaafkan baik Beng maupun Cing, dan Ang menghukum keturunan mereka hingga hari ini.
  d) "If Christina is singing soul and Justin is singing pop are necessary and sufficient conditions for Kelly is singing rock, then neither Beyoncé nor Shakira will sing rap."

Exercise 5 (2)

p: hujan menghambat saya untuk meletakkan surat pada kotak pos anda
q: salju menghambat saya untuk meletakkan surat pada kotak pos anda
r: hujan es menghambat saya untuk meletakkan surat pada kotak pos anda
t: saya seorang tukang pos
s: saya seorang sosiopat
u: saya adalah seorang kawan baik

valid: argument is valid
deductive: argument is deductive
w: kita harus memaafkan
x: ang memaafkan Beng
y: Ang memanafkan Cing
z: Ang menghukum keturuan Ang dan Cing
c: Christine is singing a soul
j: Justine is singing pop
k: Kelly is is singing rock
b: Beyonce is singing pop
sk: Shakira will singing rap

Exercise 6

- Tentukan apakah pernyataan di bawah ini benar atau salah.
  a) 2 + 2 = 4 jika dan hanya jika 1 + 1 =2. Asumsi kalimat tersebut legal, tuliskan semantik kalimat dan evaluasinya.
  b) 1 + 1 = 2 jika dan hanya jika 2+3 = 4. Asumsi kalimat tersebut legal, tuliskan semantik kalimat dan evaluasinya.
  c) 1 + 1 = 3 jika dan hanya jika kucing bisa terbang. Asumsi kalimat tersebut legal, tuliskan semantik kalimat dan evaluasinya.
  d) 0 > 1 jika dan hanya jika 2 > 1. Asumsi kalimat tersebut legal, tuliskan semantik kalimat dan evaluasinya.

Exercise 7

- Terdapat sebuah perusahaan yang memproduksi suatu barang. Barang yang dihasilkan bisa menggunakan beberapa jenis material (aluminium, tembaga, besi); dengan beberapa pilihan warna (merah, hijau, biru, abu2); dan polesan hasil akhir (polos, tambahanlapisan, atau bertekstur). Walaupun terdapat banyak kemungkinan kombinasi, tapi perusahaan hanya memasarkan beberapa jenis kombinasi saja.

Kalimat di bawah ini adalah beberapa batasan yang menggambarkan kombinasi yang diproduksi oleh perusahaan.

(i) aluminium $\vee$ tembaga $\vee$ besi

(ii) aluminium$\rightarrow$ abu2

(iii) tembaga $\wedge$ $\neg$tambahanlapisan $\rightarrow$ merah

(iv) tambahanlapisan $\wedge$ $\neg$tembaga $\rightarrow$ hijau

(v) hijau $\vee$ biru $\leftrightarrow$ $\neg$bertekstur $\wedge$ $\neg$besi

Exercise 7 (2)

a) Misal terdapat seorang pelanggan memesan barang dengan bahan tembaga, warna hijau dan biru, dan polesan akhir polos, tentukan batasan mana saja yang memenuhi pesanan tersebut, dan batasan mana saja yang tidak memenuhi pesanan tersebut. Jelaskan jawaban anda dengan menggunakan evaluasi semantik kalimat proposisi.

b) Jika batasan yang menunjukkan kemungkinan kombinasi barang yang diproduksi perusahaan adalah sebagai berikut, tentukan kombinasi satu jenis material, satu pilihan warna, dan satu polesan akhir yang bisa memenuhi semua batasan berikut. Jelaskan jawaban anda dengan menggunakan evaluasi kalimat proposisi.

(i) aluminium ∨ tembaga ∨ besi

(ii) merah ∨ hijau ∨ biru ∨ abu2

(iii) aluminium → abu2

(iv) tembaga ∧ ¬tambahanlapisan → merah

(v) besi → tambahanlapisan

Catatan: jawaban kemungkinan kombinasi sangat banyak, anda cukup memberikan satu kemungkinan kombinasi saja.

Exercise 8

Pada suatu pulau, hanya terdapat 2 jenis penduduk, yaitu knight dan knave.

- Knight selalu mengatakan kebenaran,
- Knave selalu mengatakan kebohongan.

Pada suatu saat terdapat 2 penduduk (A dan B) yang tidak diketahui termasuk knight atau knave. A mengatakan: ‘at least one of us is a knave’. B tidak mengatakan apapun.

Tentukan untuk A dan B, termasuk knight atau knave.

Ubah ke bentuk logika proposisi dan gunakan *reverse evalvaluation* untuk mendapatkan jawabannya.

Review

- Propositional Logic:
  - Syntax:
    - Simple sentences
    - Compound sentence : relation between simple sentences (6), precedence
  - Semantics:
    - Interpretation → truth value
    - Evaluation → determine truth value under certain interpretation
    - Reverse evaluation → determine which interpretation which satisfies the sentence
    - Classification of compound sentences

THANK YOU