Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
E		2425819

Weather the Storm: Mastering Insurance & Site Selection

Summary

In recent years, the increase in extreme weather events has led to substantial property damage and greater claim pressures on insurance companies. This trend necessitates a reevaluation of coverage scopes by insurance firms to prevent widening the insurance protection gap. In this paper, we employed an **ARIMA** model based on **time series** data to predict probability of extreme-weather events, utilized **EWM-TOPSIS** model to build decision-making and **optimization models** for insurance underwriting. Moreover, we adjusted our model to analyze the **cost and benefit** for property owners in site evaluation. As for high-risk landmarks, we employed **AHP-FCE** model to assess the significance of landmarks and proposed best preservation approaches for community.

For task I: We established an *Insurance Risk-Return Trade-off Model (IRRTM)* to help insurance companies decide whether the benefits will outweigh the risks brought by the extreme-weather events. We selected 7 suitable indicators to evaluate risks and returns, employed EWM to determine the weights and TOPSIS to give a score suggesting corresponding underwriting strategy. Employing Insurance model on California in the United State and New South Wales in Australia, we obtained scores of 55.23 and 44.14 (<70), which suggests insurance companies should not underwrite. Furthermore, for property owners to increase the possibility of obtaining insurance, we suggest they reinforce their property, enhance mutual acknowledgement with insurance companies or increase their deductibles.

For task II: Adjusting IRRTM in task I, we established a *Cost-Benefit Analysis model (CBAM)* to help property owner evaluate sites. We took time value into account and calculated the forward profit of building and growing in certain sites to determine whether a site is worth selecting and provided valuable suggestions.

For task III: Considering some landmarks may fail to be accepted by insurance company due to high risks in IRRTM, we put forward a *Landmark Importance Assessment model (LIAM)* to evaluate whether a landmark is worth especial preservation. In order to easily quantify cultural values, we used the Analytic Hierarchy Process and Fuzzy Comprehensive Evaluation model to obtain weight vectors and evaluation matrixes, and finally scored through comprehensive evaluation. Furthermore, we compared the cost of repair and removal to give suggestion on preservation of landmarks.

For task IV: Having noticed that Yingxian Wooden Pagoda (in Shuozhou, China) achieved a high score of 87.76 in LIAM and after cost analysis in preservation model, we wrote a letter to the Mayor, sharing our key findings and recommendations for better preservation, with exhaustive proposal of plan, timeline and cost estimation.

Finally, we conducted a sensitivity analysis on our significance evaluation model by adjusting the weights corresponding to the seven indicators by ±10%. The results showed that our model possesses robust resistance to perturbations.

Keywords: Extreme-weather event; Property Insurance; EWM-TOPSIS; ARIMA

Contents

1 Introduction

1.1 Problem Background

Climate change is intensifying extreme weather events. The increasing frequency of extreme weather incidents in recent years has led to a surge in substantial compensation incident. A report by S&P Ratings highlighted that losses incurred by insurance companies from "once in a decade" and "once in a century" climate events have been underestimated by 50%. Consequently, property insurance was also impacted. It became more expensive and scarcer, with significant price disparities across regions due to varying weather incidents. The industry must navigate the fine line between offering viable policies in high-risk areas and maintaining financial health amidst rising claims.

In Response, there's a pressing need for models that guide insurers on policy underwriting in high-risk areas, balancing financial stability against the growing risk of climate-related claims. Moreover, the demand for sustainable development practices is paramount, urging a reevaluation of where and how we build to withstand climate threats. Also, for communities, identifying and protecting buildings that may not secure insurance coverage but hold special cultural, historical, economic value or community significance poses a significant challenge.

Figure 1: California's Sand fire fueled by extra-hot temperatures

1.2 Restatement of The Problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- **Task 1:** Construct an insurance model for insurance companies to evaluate underwriting policies in regions facing increased extreme weather events.
- **Task 2:** Adapt this model to guide construction decisions for future property developments in areas susceptible to extreme weather, ensuring resilience.
- **Task 3:** Formulate a preservation model for community leaders to recognize and prioritize the protection of constructions with significant cultural or historical value in disaster-prone areas and possible protective measures to be taken.
- **Task 4:** Compose a letter for community outlining suggested preservation steps for the site, derived from the models and analyses.

1.3 Our Work

For the four questions, we will use the steps shown in the flowchart to solve them.

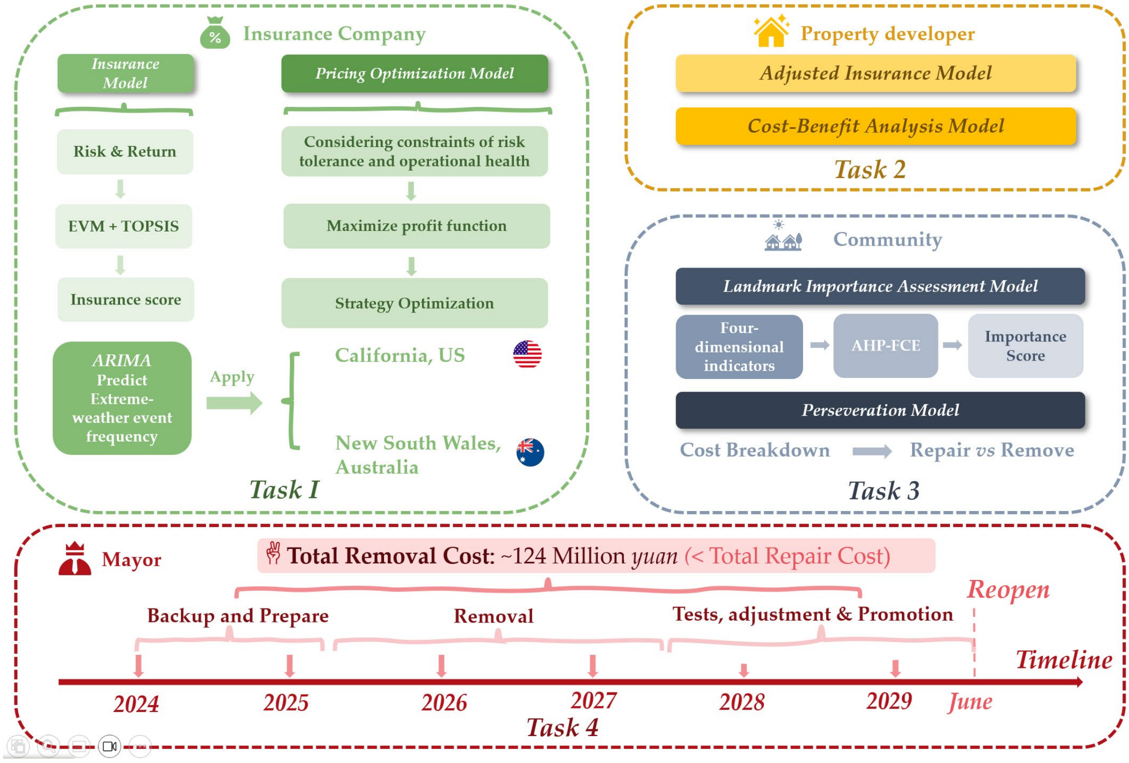

Figure 2: Flow chart

2 Assumptions and Justifications

To simplify the given problems and modify it more appropriate, we make the following basic hypotheses, each of which is properly justified.

▼**Assumption 1:** Ignore capital gains and opportunity cost.

▲**Justification:** These factors have little influence on our decision and ignoring can reduce the complexity of the model and make it more focused on studying the effects of other variables.

▼**Assumption 2:** Risk free rate of return and population growth rate in a certain site remain constant for a certain period of time in the future.

▲**Justification:** The risk-free return rate is usually associated with the yield rate of long-term treasury bond, and the national policy is relatively stable in a certain period. Making static assumptions about population growth rates can make model more simpler without producing significant deviation.

▼**Assumption 3:** People in areas prone to extreme-weather events are willing to buy insurance to minimize property damage.

▲**Justification:** People in high-risk areas are motivated to purchase insurance due to a heightened awareness of risks, the financial security offered in recovering from losses.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
Freq_event	Probability of extreme-weather events	%
SFSI	Soil Foundation Stability Index	-
t_{GHS}	Green house gas emission per capita	ton
Income	Disposable income per capita	$
Loss	Economic loss due to extreme-weather events	million $
Pop	population	million
Ins_den	Insurance density	$
ω	Indicators weight	-

4 Task 1: Insurance company underwriting strategy

4.1 Problem Analysis

In order to evaluating whether an area is suitable for underwriting, we establish an Insurance Risk-Return tradeoff model to take risk and profit in to consideration and give the area a score.

What's more, to determine when to take risk to underwrite, we rigorously calculate the revenue and costs of the insurance company, achieving a break-even point.

Finally, we choose California and New South Wales to Verify the correctness and robustness of the model.

4.2 Insurance Risk-Return Trade-off Model (IRRTM)

After consulting a large number of literature and materials, we have selected three indicators as the measure of *Risk* factor and four indicators as the measure of *Return* factor, as shown in the following figure:

Figure 3: Indicator Selection of Risk-Return Trade-off Model

4.2.1 Risk Factor

Risk is very important. If the risk of suffering from extreme-weather event is too high, the insurance company may pay too much compensation, which negatively affects the insurance company's underwriting choices. The following three indicators affect the risk:

- ***Probability of extreme-weather events***

Extreme climate events refer to significant deviations from the historical norm in the atmospheric, oceanic, and terrestrial systems driven by climate change. The occurrence of extreme events often requires insurance companies to make large payouts. Therefore, the higher the probability of local extreme events, the higher the risk.

- ***Soil foundation stability index SFSI***

The physical properties of soil are crucial for determining whether the area is suitable for construction and agriculture. If the soil in a region is better in terms of constructing more solid houses), then the likelihood of real estate in this area being impacted by extreme weather events (such as floods, mudslides, typhoons, etc.) is smaller, and the risk for insurance companies is also lower. Through research, we proposed *Soil Foundation Stability Index (SFSI)* to measure whether the foundation of houses built on the soil base in this area is solid, which is the average score of three indicators of *rootable soil depth*, *bulk density* and *available water capacity*.

$$SFSI = \frac{score_{dep} + score_{den} + score_{awc}}{3} \tag{1}$$

- ***Rootable soil depth:*** refers to the depth of soil layer where plant roots can expand and absorb nutrient. It indirectly influences building stability. Deeper soil layers offer stable foundations and better drainage, reducing water damage risks. It also resists erosion, especially during extreme weather.
- ***Bulk density:*** reflects the compactness of soil particles, influencing load-bearing capacity. Generally, a higher bulk density implies a more compact soil, potentially with a higher load-bearing capacity. However, if the bulk density is too high, it may reduce the soil's drainage ability.
- ***Available water capacity (AWC):*** reflects the drainage characteristics and potential load-bearing capacity of soil. Overly moist soil may have lower load-bearing capacity, while well-drained soil is more suitable for construction. A high AWC may signal a need for improved drainage to prevent foundation and structural issues.
- ***Carbon dioxide emission per capita:*** represents greenhouse gases. A higher per capita CO_2 emission in a region is more likely to lead to global warming, resulting in an increase in extreme weather conditions and a higher probability of extreme climate events occurring.

Table 2: Score standards of three SFSI indicators

Rootable Soil Depth (cm)	Score_dep	Bulk Density (g/cm^3)	Score_den	AWC (mm)	Score_awc
<50	30’	<1	20’	<75	20′
[50,10)	70’	[1,1.4)	50’	[75,100)	50’
[100,200)	100’	[1.4,1.6)	100’	[150,185)	80’
⩾200	80’	⩾1.6		⩾185	100’

4.2.2 Return Factor

Return is very important as it is a source of profit for insurance companies. The following four indicators may affect their profitability by maximizing revenue and minimizing loss:

- ***Insurance density***

Insurance Density is a key metric in the insurance industry, representing the ratio of premiums collected by insurance companies to the country’s population. Higher insurance density may indicates that a larger portion of the population is insured or that people are generally willing to spend more money on the insurance, leading to increased premium collection an higher revenue for the insurance company.

- ***Disposable income per capita***

Per capita disposable income is a measure of the average amount of money that individuals in a specific region have available for spending and saving after income taxes have been accounted for. It is a vital indicator of consumer purchasing power and the standard of living in a region. With more disposable income, individuals and businesses can afford to spend more on insurance premiums to protect themselves from potential risks. Therefore, they are more likely to purchase insurance policies.

- ***Population***

A larger population can mean a larger customer base for insurance companies. At a stable insurance density rate, if a country has a growing population, then the revenue of local insurance industry must be growing as well. Also, a smaller population size may limit an insurance company’s ability to diversify risk effectively. This is because risk diversification in insurance is predicated on the law of large numbers, which posits that the larger the group of units independently exposed to risk, the more predictable the losses become. Insurance companies can thereby predict risk more accurately and achieve stable returns.

- ***Economic loss due to extreme-weather events***

Insurance companies are often responsible for covering a portion of Economic loss due to extreme-weather events. The greater economic loss due to extreme-weather events, the higher claim payouts the insurance company may risk to pay.

4.3 Determination of Weights Using Entropy Weighting Method

We collected data on various indicators from 266 countries from sources such as World Bank, National Oceanic and Atmospheric Administration (NOAA), and Harmonized World Soil Database v2.0 (HWSD), United Nations Office for Disaster Risk Reduction (UNDRR).

Based on the data, we used objective **entropy weighting method** to determine the weights of each indicator.

Probability of extreme-weather events, CO2, economic loss due to extreme-weather are negative indicators. *SFSI, Insurance density, disposable income per capita and population* are positive indicators.

Use min-max normalization to standardize all indicators and Generate new indicator values:

$$Y_{ij} = \frac{X_{ij} - \min(X_i)}{\max(X_i) - \min(X_i)} \quad (Positive\ Indicator) \tag{2}$$

$$Y_{ij} = \frac{\min(X_i) - X_{ij}}{\max(X_i) - \min(X_i)} \quad (Negative\ Indicator) \tag{3}$$

To remove units, we perform dimensionless processing by dividing by the square root of the sum of squares of the new indicator values:

$$s_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{n} y_{ij}^2}} \tag{4}$$

Calculate information entropy:

$$E_j = -\ln(n)^{-1} \sum_{i=1}^{266} s_{ij} \ln s_{ij} \tag{5}$$

Calculate information redundancy:

$$D_j = 1 - E_j \tag{6}$$

Finally, we make normalization process and obtain the weight of each indicator:

$$w_j = \frac{D_j}{\sum_{j=1}^{7} D_j} \tag{7}$$

Finally, we make normalization process and obtain the weight of each indicator:

$$w = (0.2506, 0.0266, 0.1279, 0.1091, 0.2295, 0.0994, 0.1568) \tag{8}$$

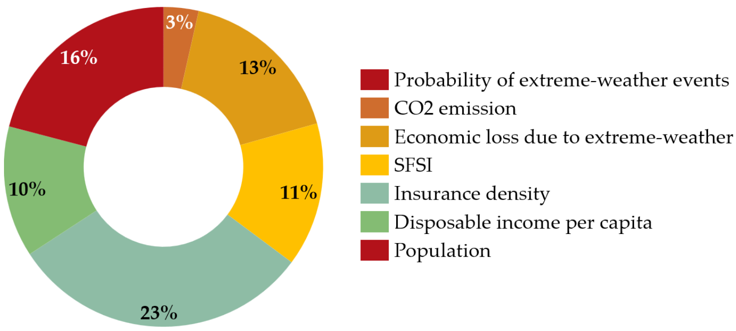

Figure 4: Weights of Indicators

4.4 Scoring Based on TOPSIS

Multiply the standardized matrix by the weight vector to make a weighted matrix:

$$z_{ij} = s_{ij} \times w_j \tag{9}$$

When determining weights, we have already performed min max normalization and dimensionless processing.

$$Z_i^+ = \max\{Z_{ij}\} \quad (j = 1, 2, ..., 266) \tag{10}$$

$$Z_i^- = \min\{Z_{ij}\} \quad (j = 1, 2, ..., 266) \tag{11}$$

The best ideal solution is

$$z^+ = [z_1^+, z_2^+, ..., z_m^+] \tag{12}$$

The worst ideal solution is

$$z^- = [z_1^-, z_2^-, ..., z_m^-] \tag{13}$$

For the i-th region Z_i, we used the calculation method of Euclidean distance for comprehensive evaluation,to obtain its distance from the ideal solutions:

$$d_i^+ = \sqrt{\sum_{j=1}^{m} (z_j^+ - z_{ij})^2} \tag{14}$$

$$d_i^- = \sqrt{\sum_{j=1}^{m} (z_j^- - z_{ij})^2} \tag{15}$$

Finally, normalize to calculate the score of the building:

$$S_i = \frac{d_i^-}{d_i^+ + d_i^-} \times 100 \tag{16}$$

When the final score exceeds 75, it can be underwritten at a low price; If it is 60-75, it is insured at a high price; if it is less than 60, it is not insured.

4.5 Pricing Optimization Model

When the score is very low, insurance companies generally choose not to underwrite. We establish a pricing optimization model to decide when the insurance company can take risks. Assuming that the condition for a company's healthy development is that operating costs are not less than 5% of premium income.

Table 3: Notations used in pricing optimization model

Symbol	Description
R_{tlr}	Risk tolerance(The upper limit of the proportion of acceptable losses to reserves)
Res	Capital reserves
C_{opr}	Operating cost

$$\begin{aligned} & \mathit{Maximize}\ \ \mathrm{z} = \mathrm{Price} \times \mathrm{Pop} - \mathrm{Loss} \times \mathrm{Freq}_{\mathrm{event}} - \mathrm{C}_{\mathrm{opr}} \\ & \text{subject to} \begin{cases} C_{opr} \geqslant \text{ price } \times pop \times 5\% \\ \text{Loss} \times \text{Freq}_{\text{event}} \leqslant R_{\text{tlr}} \times \text{Res} \\ \text{price} \geqslant 0 \end{cases} \end{aligned} \tag{17}$$

If the premium price is sufficient to cover both risk and operating costs, and the risk is within the tolerance range of the insurance company, the insurance company chooses to take risk.

As for property owners, in order to promote insurance companies' willingness to underwrite policies, they can strive in three directions:

1. Lessen the risk of extreme-weather events happen

- ***Promote the awareness of environmental protection*** of the whole community (even the whole nation) and lessen the probability of extreme-weather events, thereby reducing the insurers' expectation of loss.

2. Reduce loss of insurance companies when extreme-weather events happen

- ***Making their house more stable and invulnerable to extreme-weather events.*** Understand the insurer's risk assessment criteria and ensure that the home and its surroundings meet the insurer's requirements.
- ***Voluntarily assume higher deductibles.*** In the event of smaller losses, the property owner will bear those losses on his or her own without claiming from the insurance company. This can entice insurers to overwrite policies.
- ***Joining and supporting community risk management programs***, such as building levees, improving drainage systems, etc., can reduce losses to the community as a whole during extreme weather events, which in turn reduces the risk to insurers.

3. Keep insurance companies well informed of your property's situation

- ***Provide detailed information about the home and environment****, including the house's history, usage, and maintenance records to help insurers more accurately assess risk.*

- ***Maintain good communication with your insurance company to understand their tolerance and preference for specific risks*** *so that you can adjust your home use or maintenance strategy if necessary.*
- ***Select sites with better soils foundation (with higher SFSI index)*** *if they need to obtain new buildings.*

4.6 Application: in California and New South Wales

After some research, we have found that California in the United States and New South Wales in Australia were stricken by more and more severe extreme-weather events in these years. We gathered and cleansed various indicator data for California and New South Wales from various major data sources.

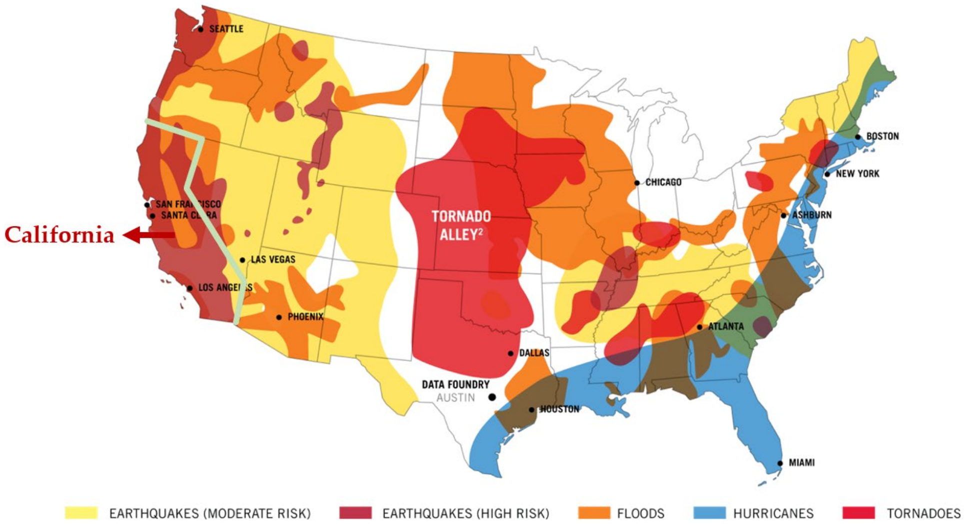

Figure 5: Natural Disaster Distribution of the United States

4.6.1 Prediction of the frequency of extreme-weather events using ARIMA model

We collected information on the duration of extreme weather events in New South Wales, Australia, and California, USA. The ARIMA model was used to analyze and predict the time series of extreme weather days.

We divide this time series plot by order 1 and convert it into a stationary time series. Then, the autocorrelation coefficient (ACF) and partial autocorrelation coefficient (PACF) of the stationary time series were obtained, and the best orders 2 and 1 were analyzed.

Therefore, we chose the ARIMA(2,1,1) model, which means that the second-order autoregressive (AR) and first-order moving average (MA) models are applied after a single difference in the time series.

Among them, the autoregressive model describes the relationship between the current value and the historical value, and uses the historical time data of the variable itself to predict itself.

2nd order autoregressive model AR is:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + u_t \tag{18}$$

In the MR model, if u_t not a white noise, it is generally considered to be a moving average of the Q order. In our model q $= 1$, u_t can be expressed as

$$u_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} \tag{19}$$

where ε_t denotes the white noise sequence.

The prediction resulting forecast chart is as follows:

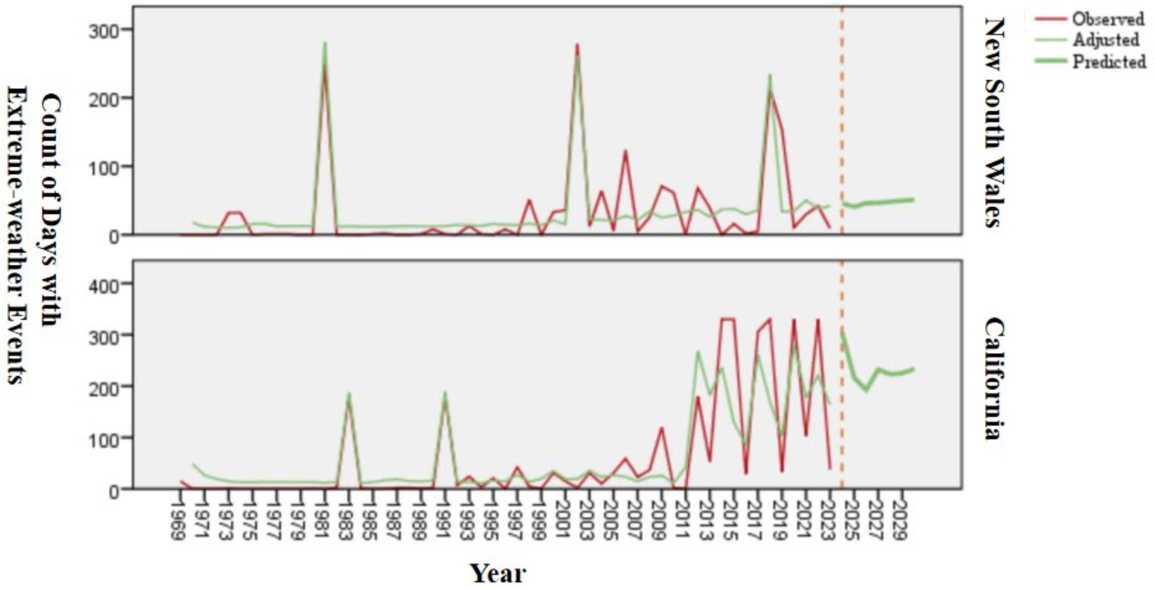

Figure 6: ARIMA forecasting result

The indicator $Freq_{event}$ is calculated based on the predicted number of days d_i in the ith year:

$$Freq_{event} = \frac{\sum_{i=0}^{7} d_i}{365 * 7} \tag{20}$$

4.6.2 Scoring based on TOPSIS

Through model calculations and data queries, we have obtained all seven indicators in California and New South Wales. Their values are shown in the following table:

Table 4: Data of California, United States and New South Wales, Australia

Indicators	Freq_even	SFSI	t_{GHS}	Income	Loss	pop	Ins_den
California,	63.6	86.7	9.3	63219	23401	38.94	10605
New South Wales	12.8	76.7	16.7	55443	65819	8.19	2133

We applied the EWM-TOPSIS model to analyze the data.

According to the results of Section 4.3 Entropy Weighting Method, the weight matrix is:

$$w = (0.2506,\ 0.0266, 0.1279,\ 0.1091,\ 0.2295,\ 0.0994,\ 0.1568) \tag{21}$$

To enhance the scoring system's reflection of the current regional standings in a global context, we have consulted socio-economic data from the United Nations and certain specific areas. This has allowed us to incorporate a set of data that represents he best and worst insurance-bearing scenarios worldwide, serving as a benchmark for our scoring assessment.

$$\begin{bmatrix} 75 & 100 & 44 & 84250 & 79777 & 77.56 & 18742 \\ 0 & 30 & 0 & 800 & 0 & 0.2 & 0 \end{bmatrix} \tag{22}$$

We forward and standardize the matrix composed of indicators to obtain a standardized matrix:

$$Z = \begin{bmatrix} 0.1607 & 0.5561 & 0.5566 & 0.5368 & 0.5803 & 0.4467 & 0.4901 \\ 0.6586 & 0.4920 & 0.4379 & 0.4707 & 0.15531 & 0.093952 & 0.0985 \end{bmatrix} \tag{23}$$

For the i-th region Z_i, use the calculation method of Euclidean distance for comprehensive evaluation, to obtain its distance from the ideal solutions. Then, normalize to calculate the score.

Table 5: Scores of California and New South Wales

California	New South Wales
55.23	44.14

Considering the scores are below 60, it is recommended not to underwrite due to high risk.

5 Task 2: communities and property developers Building strategy

5.1 Adjusted Insurance Model for Site Evaluation

We replaced economic loss by two indicators for Return:

- ***The proportion of property expenditure to disposable income***

The more the property owner spends his disposable income on property, the more importance he may attach to his property, and has a higher willingness to pay for property insurance.

- ***The annual population growth rate***

If a region's population keep growing at a considerable rate, then the region has a great potential for growing market size. Also, a larger consumer base enables insurance company to better predict profits and risks and dynamically adjust the policies.

Using this adjusted model with new indicators, we evaluate whether a specific location is suitable on risk-taking and economic environment for community and property owners to build and grow.

5.2 Cost-Benefit Analysis Model

To evaluate whether an area is profitable to build and grow, we adjust insurance model to measure its risk and financial environment. As for how to get the maximum profit, we establish cost-benefit analysis model to optimize our measures to build on certain sites.

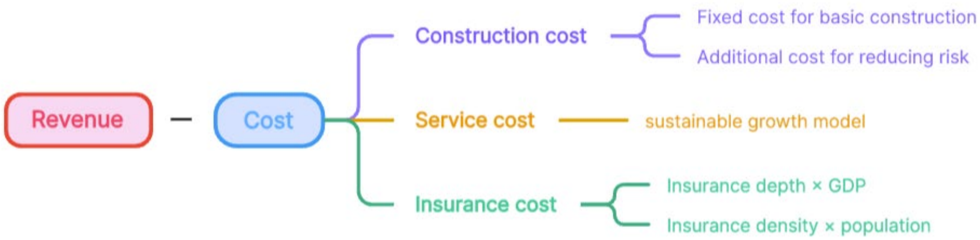

Figure 7: Cost-Benefit Analysis Model

Our benchmark model suggests that the profit from building a house is calculated by subtracting the construction cost, service cost, and insurance cost from the income from selling the house.

$$\pi = Income \ - \ Construction \ cost \ - \ Service \ cost \ - \ Insurance \ cost \tag{24}$$

- **Income** is regarded as the sales revenue of the building that accounts for a relatively large proportion.
- **Construction cost** is divided into fixed basic expenses and extra costs due to carefully construction to cope with possible extreme-weather risk. In Insurance Model, we predicted the probability of extreme weather events occurring in each region over the next five years. The higher the probability, the greater the risk, and the higher the additional cost of building a house. So we set the extra costs as the risk coefficient multiplied by the upper limit of the additional cost.

$$Construction\_\mathrm{cost} = fixed\_\mathrm{cost} + add\_\mathrm{cost} \times Freq\_event \tag{25}$$

- **Service cost:** Since communities and property developers want to offer appropriate services to growing communities and populations, we need to consider the growth rate of population and the time value of money. We assume that the per capita service cost, population growth rate, and risk-free return rate are fixed each year. Construct a discounted model for sustainable growth and calculate the present value of service costs:

$$Service\ Cost = Sev\_p \times pop \times \Sigma \frac{(1+g)^{t-1}}{(1+i)^{t}} \\ = Sev\_p \times pop \times \frac{1}{i-g} \tag{26}$$

➢ **Insurance cost** is calculated by Insurance density or insurance depth:

$$\begin{aligned} Insurance\ Cost &= Insurance\ Density \times pop \\ &= Insurance\ Depth \times GDP \end{aligned} \tag{27}$$

In summary, using above formulas and specific regional indicator data, we can calculated the forward profit of building and growing in certain sites. To maximize the profit π, communities and property developers can raise the house price and choose a place which has litter risk to reduce construction cost.

6 Task 3: Preserve important building strategy

6.1 Landmark Importance Assessment Model

In order to identify buildings in a community that should be preserved and protected due to their cul-tural, historical, economic, or community significance, we established an evaluation model to measure the significance of a building.

The indicators selected are as below:

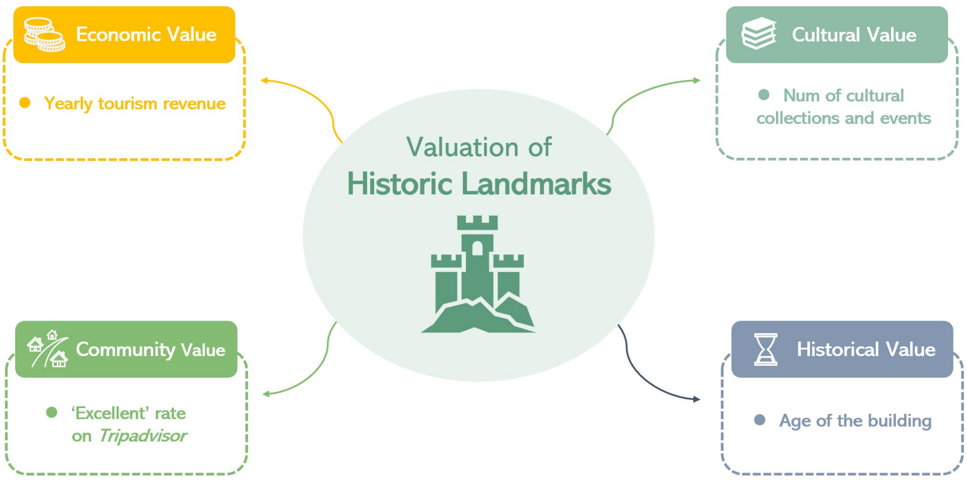

Figure 8: Valuation of Historical sites

- **Cultural value:** ***quantity of cultural heritage and events***

As for historic landmarks, cultural value is undoubtedly one of the most important valuation metrics. We evaluate it from a comprehensive consideration of two aspects: cultural collections and cultural events. A landmark with a huge amount of cultural collections, like British Museum, is more likely to hold vital significance in culture conservation and development. Moreover, if a construction witnessed some crucial cultural events.

- **Historical value:** ***Age of the building***

The age of building is the basis of evaluating its historical value. A building with a longer history experience more historical events and more historic moments or milestones, like wars or the wax and wane of a dynasty, able to tell us more information about history.

- **Community value:** ***'Excellent' rate on Tripadvisor***

Historic sites can contribute to a sense of identity and pride within a community, as they showcase its history and heritage, especially if the sites got very popular outside of this community. They can also attract tourism, promoting economic growth and providing job opportunities for locals. Therefore, we consider that the popularity and acceptance of the construction by tourists around the world is vital. *Tripadvisor* is one of the most visited travel and tourism websites worldwide, concluding tons of traveler's reviews on nearly all well-known historic sites around the world. We suppose that if a site is rated by a more proportion of people as "Excellent", then it's more popular and accepted globally.

- **Economic value:** ***Tourism revenue***

Tourism revenue constitutes the most of economic benefits brought by historic landmarks.

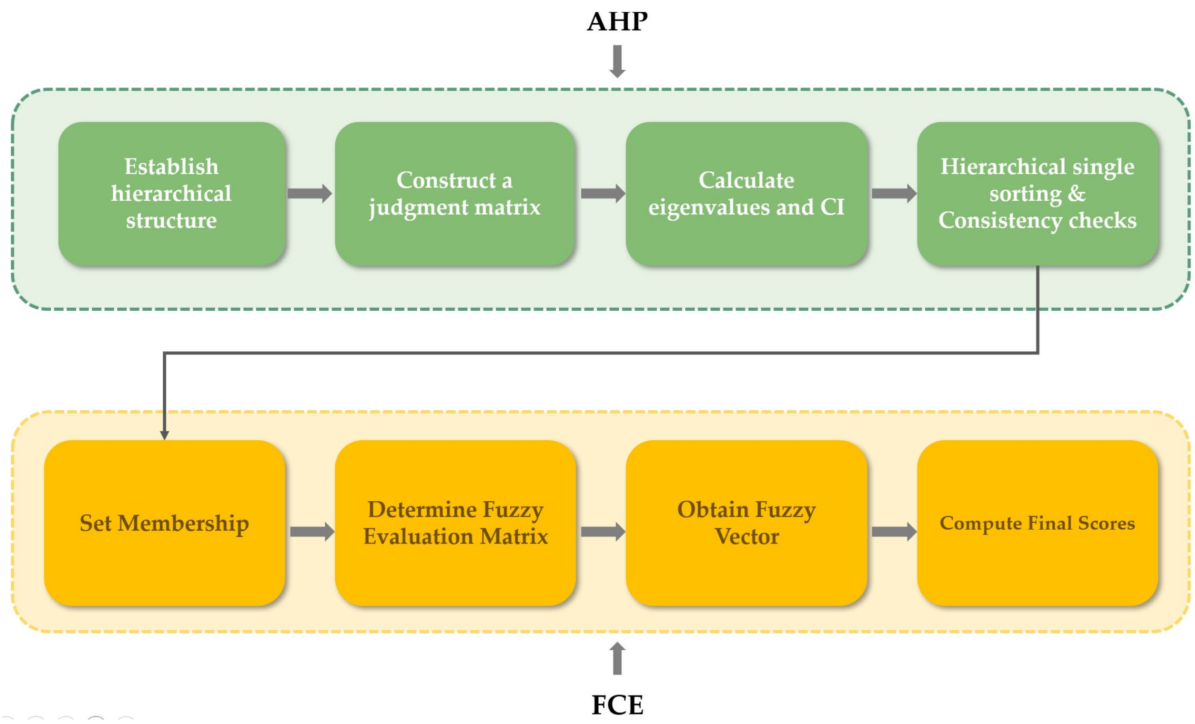

Figure 9: AHP-FCE Process

6.1.1 Analytic Hierarchy Process

Based on the analysis in Section 6.1, we identified four indicators to evaluate the importance of buildings: cultural value, historical value, economic value, and community value. We use analytic hierarchy process to determine the weight of these indicators relative to the importance of building.

- **Construct a judgment matrix**

Firstly, the four indicators were compared in pairs to construct a judgment matrix.

$$A = (a_{ij})_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{28}$$

$$A = \begin{bmatrix} 1 & 2 & 6 & 4 \\ \frac{1}{2} & 1 & 3 & 2 \\ \frac{1}{6} & \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{2} & 3 & 1 \end{bmatrix} \tag{29}$$

- **Calculate the maximum eigenvalues and their corresponding eigenvectors**

Each element A_{ij} is normalized by dividing it by the sum of its respective column. This process yields the relative magnitudes of the criteria under consideration.

$$F_1(x) = \begin{cases} 1 & x \leqslant \alpha_1 \\ \left(\dfrac{\alpha_2 - x}{\alpha_2 - \alpha_1}\right)^2 & \alpha_1 < x \leqslant \alpha_2 \\ 0 & x > \alpha_2 \end{cases} \tag{30}$$

$$A_{ij} = A_{ij} \div \sum_{i,j=1}^{n} A_{ij} \tag{31}$$

Subsequently, the average of these normalized values is computed for each column, which effectively assigns a weight to the corresponding factors. These weights represent the significance of each criterion within the hierarchical structure.

$$W_i = \sum_{i,j=1}^{n} B_{ij} \tag{32}$$

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{(AW)_i}{nW_i} \tag{33}$$

We normalize the resulting eigenvectors to deduce the weights. These normalized weights serve as an approximation of the eigenvector associated with the largest eigenvalue of the judgment matrix, thereby facilitating the calculation of priority vectors in the decision-making framework.

- **Applying consistency checking**

We calculate Consistency index as:

$$\lambda_{\max}=\sum_{i=1}^{n}\frac{(AW)_i}{nW_i} \tag{34}$$

$$CI=\frac{\lambda_{\max}-n}{n-1} \tag{35}$$

Calculate the consistency ratio CR:

Table 6: RI

n	1	2	3	4
RI	0	0	0.58	0.90

Therefore

$$CR=\frac{CI}{RI}=0.0227 \tag{36}$$

CR is less than 0.1, so we consider the judgment matrix to be qualified, taking the eigenvector after normalization as the weight vector.

$$A=(0.5143,\ 0.2572,\ 0.0730,\ 0.1555) \tag{37}$$

6.1.2 Fuzzy Comprehensive Evaluation

- We divided each indicator into three levels: low, medium, high and define the membership degree function:

$$F_1(x)=\begin{cases}1 & x\leqslant\alpha_1\\ \left(\dfrac{\alpha_2-x}{\alpha_2-\alpha_1}\right)^2 & \alpha_1<x\leqslant\alpha_2\\ 0 & x>\alpha_2\end{cases} \tag{38}$$

$$F_2(x)=\begin{cases}\left(\dfrac{\alpha_3-x}{\alpha_3-\alpha_2}\right)^2 & \alpha_2<x<\alpha_3\\ \left(\dfrac{x-\alpha_1}{\alpha_2-\alpha_1}\right)^2 & \alpha_1<x\leqslant\alpha_2\\ 0 & x<\alpha_1 \quad or \quad x>\alpha_3\end{cases} \tag{39}$$

$$F_3(x)=\begin{cases}0 & x\leqslant\alpha_2\\ \left(\dfrac{x-\alpha_2}{\alpha_3-\alpha_2}\right)^2 & \alpha_2<x\leqslant\alpha_3\\ 1 & x>\alpha_3\end{cases} \tag{40}$$

- We Performed single factor evaluation on each of the four indicators to obtain the comprehensive matrix of Yingxian Wooden Pagoda:

$$F=\begin{pmatrix}0 & 0.0625 & 0.5625\\ 0 & 0.2304 & 0.2704\\ 0.1936 & 0.3136 & 0\\ 0 & 0.064 & 0.216\end{pmatrix} \tag{41}$$

After calculating and normalization, we obtained the Fuzzy Evaluation Matrix of Yingxian Wooden Pagoda:

$$R=\begin{pmatrix}0 & 0.1 & 0.9\\ 0 & 0.4601 & 0.5399\\ 0.3817 & 0.6183 & 0\\ 0 & 0.2286 & 0.7714\end{pmatrix} \tag{42}$$

- After determining the evaluation matrix R and factor weight vector A, we obtained a fuzzy vector by multiplying the composite operators through comprehensive evaluation:

$$B_{1\times 3}=A_{1\times 4}\odot B_{1\times 3}=A_{1\times 4}\odot R_{4\times 3} \tag{43}$$

We assigned different levels and scores as:

$$S=(20,60,100) \tag{44}$$

- So the final score of the landmark is:

$$F=B\times S^{T} \tag{45}$$

Applying data of Yingxian Wooden Pagoda into LIAM, we obtain the fuzzy vector is $B=(0.0279, 0.2505, 0.7217)$ and get the final score $F=87.76$, which indicates that it is very important in cultural, historic, economic and community significance.

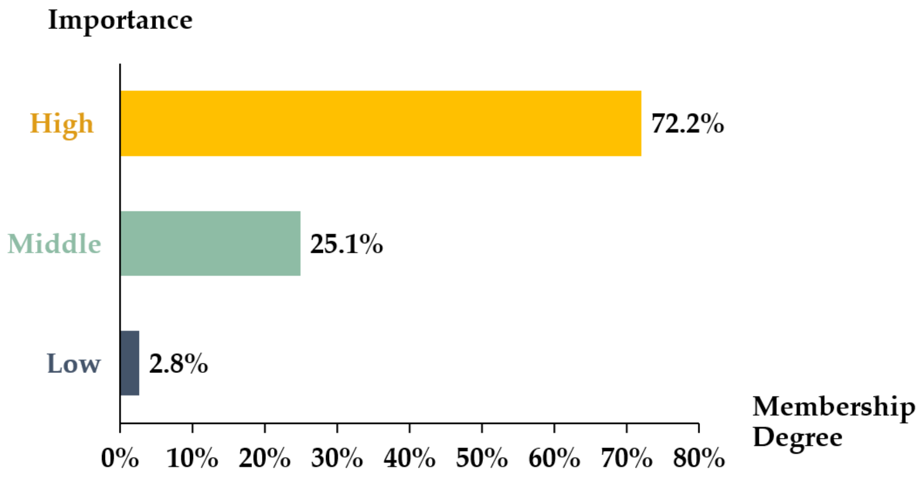

Figure 10: Membership degree of Yingxian Wooden Pigoda

6.2 Preservation Model

There are generally two ways to preserve landmarks, repairs or removal. Some landmarks, facing controllable small risks, may survive well after being repaired or reinforced. However, some may be caught in inevitable and foreseeable huge risks, having to relocate to a safter place. In order to compare the two approaches and select a more effective one, we evaluated the cost of two approaches respectively.

Table 7: Notations used in preservation model

Symbol	Description
R_{be}	Probability of extreme-weather events before removal
R_{af}	Probability of extreme-weather events after removal
L_{amt}	Economic loss due to extreme-weather events
PR	Premium rate
L_{amt_be}	Economic loss due to extreme-weather events before repair
L_{amt_af}	Economic loss due to extreme-weather events after repair
C_{trans}	Transport cost
C_{re}	Cost of repair and reinforcement
L_{cul}	Culture value loss due to removal

- ***Removal Cost***

Due to the high risk of extreme-weather events occurring at the original site, moving to a new address can significantly reduce the risk factor, resulting in a reduction in insurance costs. However, the economic costs of transportation, relocation, and reconstruction, as well as the damage to building integrity, cannot be ignored. So we establish the following model to calculate the cost of relocation:

$$Cost_{removal} = (R_{be} - R_{af}) \times L_{amt\_be} \times PR \ + \ C_{trans} \ + \ C_{re} \ + \ L_{cul} \tag{46}$$

- ***Repair Cost***

Another method is to repair and reinforce in situ, which can reduce the amount of losses caused by extreme weather events by improving waterproof systems, increasing environmental awareness, and strengthening building structures, thereby reducing insurance premiums. At the same time, the cost of repair should also be considered, and the following model should be derived:

$$Cost_{repair} = C_{rep} \ + \ \mathrm{R}_{be} \times (L_{amt\_af} - L_{amt\_be}) L_{be} \times PR \tag{47}$$

After calculating the cost of two approaches, the more economical one will be selected as the best preservation solution.

7 Task 4: A Letter to Mayor of Shuozhou

To: Mayor of *Shuozhou*, Shanxi province in China
From: Team #2425819
Subject: Findings and Proposal for the better preservation of Yingxian Wooden Pagoda

Honorable Mayor of *Shuozhou,*

Driven by huge interests in Yingxian wooden Pagoda of your city, and having noticed some extreme-weather events it experienced, our team has made a comprehensive study to figure out a better approach of preservation. We are glad to share with you our findings, including analysis on its significance and current situation, prediction of future risks and risk mitigation plans.

Significance and current situation: Firstly, we fully recognize the huge value, especially historical and cultural significance of Yingxian Wooden Pagoda through our landmark significant assessment model (LISM). However, despite repeated repairs, the wooden tower has been increasingly deformed and crooked for extreme-weather events, with unignorable risks of collapsing in the event of strong winds or earthquakes.

Predictions: Based on our Insurance Model, we concluded that owing to increasing extreme-weather events in Shanxi, great loss may be caused in the future.

Actions and Plans: Therefore, it's high time that we should take more effective measures to mitigate the risks. Generally, there are two approaches: repairs and reinforcements, or removal. Evaluating the cost of both solutions in a long time period, we found that removal is more economical and more effective. Then considering feasibility, we studied some historical removal exercises, and selected **Yulin, Shaanxi** as possible relocating selection based on comprehensive analysis on geographical distribution of extreme-weather events and proximity. Furthermore, we drafted a 5-year relocating timeline and evaluated the cost. You may see the details from the table as below.

Table 8: Summary of Timeline, Plan and Cost Proposal

Timeline	Plan (Prioritized Tasks, not exhausive)	Cost (Budget) (million yuan)
Apr 2024 – Mar 2025	Make assessment and keep detailed records of sites for backup	**~10**
Apr 2025 - Mar 2027	Start removal according to a certain roadmap and keep track of the process	**~38**
Apr 2027 – Sep 2028	Complete the reconstruction and setup of attractions at new locations	~75
Oct 2028 – Mar 2029	Perform necessary tests and adjustments as well as promotion	~1
Jun 2029	Reopen to the public	/
Total		**~124**

The above is the summary of our study. We sincerely hope that it will provide you with useful information, and welcome to read our paper if you want to know more details. We sincerely wish Yingxian Wooden Pagoda can be free from the risks of extreme-weather events soon.

Yours sincerely,
Team #2425819 of 2024 MCM

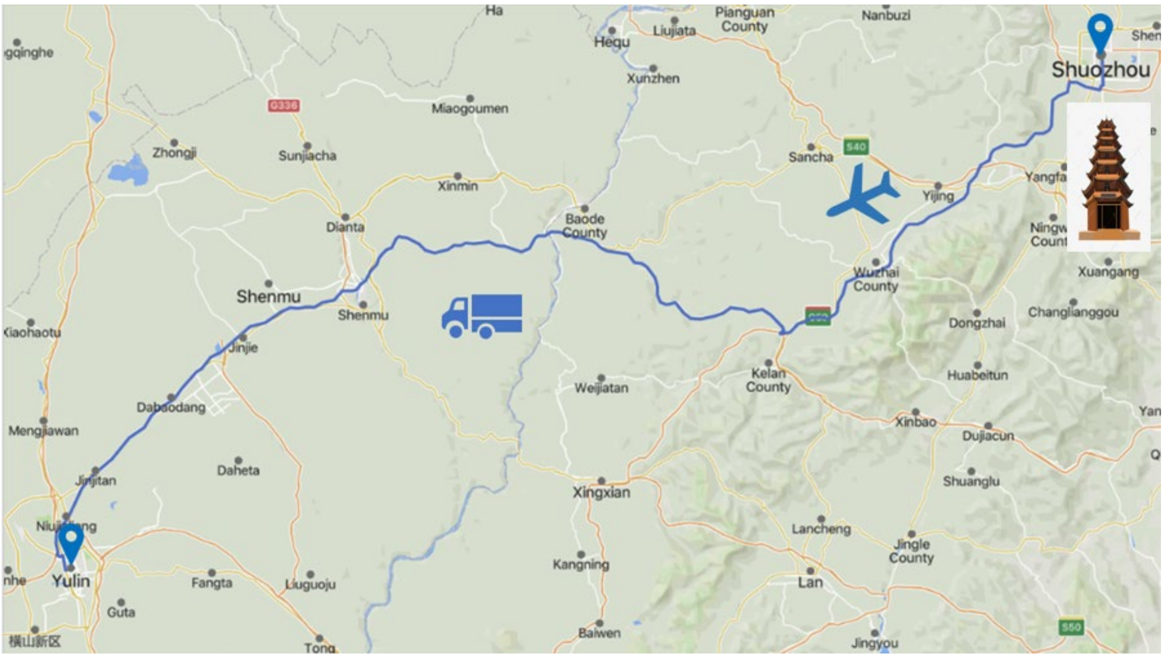

Figure 11: Yingxian wooden Pagoda Removal route (Appendix)

8 Sensitivity Analysis

When assessing the significance of buildings using the TOPSIS method, we employed the Entropy Weighting Method to calculate the weights. Variations in objective factors such as population growth, economic development, global warming, and soil erosion can lead to changes in the distribution characteristics of the data, such as the degree of dispersion, the size of the data set, and the range of the data. Consequently, the computed weights may also change accordingly.

To address this, we analyzed the sensitivity of the score to changes in individual weights. Our approach involved selecting two data sets and controlling the variation of a single weight within a ±0.1 range, to observe changes in the scoring. It was observed that for each specific data set, variations in individual weights had a minimal impact on the model score, indicating the stability of our model. Furthermore, it was evident that the weights of the indices such as Loss, *Freq_event*, and Income had a relatively significant influence on the score assessment.

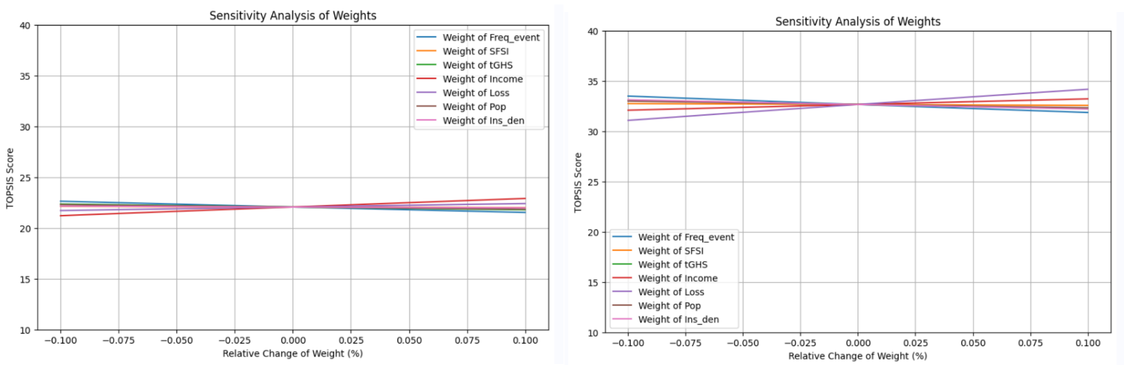

Figure 12: Sensitivity analysis results

9 Model Evaluation

9.1 Strengths

1. Our insurance model uses comprehensive and detailed indicators from both risk and return dimensions.

2. Our cost-benefit analysis model takes into account the time value of money and the cost of services that increases as the population grows.

3. We found suitable and robust indicators for hard-to-quantify cultural values, historical values, and community importance.

4. Our data comes from official authoritative websites such as NOAA, which also makes the accuracy of the model highly credible.

5. The integration of AHP-FCE and EWM-TOPSIS in our model provides a holistic approach to complex multi-criteria decision-making problems.

6. Having undergone sensitivity analysis, our model maintains its validity across a range of varied conditions and assumptions, highlighting its strong foundation and adaptability.

9.2 Weakness and Further Discussion

1. AHP can be subjective, the accuracy and objectivity can be further improved through the subsequent collection of additional questionnaires and expert opinions.

2. The data we used on CO_2 emissions is from 266 countries, which is not detailed enough, which can be improved by continuing to find state and county data.

10 Conclusion

In our study, we tackled the pressing issue of extreme weather events and their impact on insurance underwriting, property development site selection, and the preservation of landmarks. Our analysis, grounded in ARIMA, EWM-TOPSIS, and AHP-FCE models, reveals critical insights for enhancing resilience against these climatic challenges.

Key outcomes include the identification of high-risk areas unsuitable for insurance underwriting, the importance of strategic site selection for minimizing risk in property development, and the prioritization of landmarks for preservation based on their cultural, historical, and economic significance. Our findings advocate for a proactive approach, encouraging insurance companies to adjust underwriting strategies, property developers to consider risk assessments in site selection, and communities to focus on preserving valuable landmarks.

This research underscores the necessity of adaptive strategies in the face of climate change, offering a blueprint for stakeholders across various sectors to mitigate risks associated with extreme weather. By harnessing advanced modeling techniques, our study equips stakeholders with data-driven strategies to confront the challenges of extreme weather, paving the way for a resilient future through informed insurance practices, prudent development planning, and focused preservation efforts.

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