A ADDITIONAL PROOFS

Lemma 4.1. Given an almost-satisfying graph $(L \cup \{v\}, R)$, each vertex $u \in R$ that connects v is involved in all local solutions within $(L \cup \{v\}, R)$.

Proof. Suppose $(L' \cup \{v\}, R')$ is a local solution and does not involve u, i.e., $u \notin R'$. Then, we consider another subgraph $(L' \cup \{v\}, R' \cup \{u\})$, which is larger than $(L' \cup \{v\}, R')$. We derive a contradiction by showing that $(L' \cup \{v\}, R' \cup \{u\})$ is a k-biplex since (1) each vertex $v' \in L'$ disconnects at most k vertices from $R' \cup \{u\}$ (note that $R' \cup \{u\}$ is subset of R and v' disconnects at most k vertices from R), (2) v disconnects at most k vertices from $R' \cup \{u\}$ (note that v disconnects at most k vertices from R' and v connects u), (3) each vertex $u' \in R'$ disconnects at most k vertices from $L' \cup \{v\}$ (note that $(L' \cup \{v\}, R')$ is a k-biplex) and (4) vertex u disconnects at most k vertices from $L' \cup \{v\}$ (note that u disconnects at most k vertices from L' and u connects v). □

Lemma 4.2. Let $R' = R'' \cup R_{keep}$ and $R'' = R''_1 \cup R''_2$, where $R''_1 \subseteq R^1_{enum}$, $R''_2 \subseteq R^2_{enum}$, and $|R''_1 \cup R''_2| \leq k$. There does not exist a subset L' of L such that $(L' \cup \{v\}, R')$ is a local solution if (1) $|R''_1 \cup R''_2| < k$ and (2) $R^1_{enum} \backslash R''_1 \neq \emptyset$.

Proof. This can be verified by contradiction. Suppose there exists a subset $L' \subseteq L$ such that $(L' \cup \{v\}, R')$ is a local solution. Then, another subgraph $(L' \cup \{v\}, R' \cup \{u\})$, where $u \in R^1_{enum} \backslash R''_1$, would be a k-biplex since (1) each vertex $v' \in L'$ disconnects at most k vertices from $R' \cup \{u\}$ (note that $R' \cup \{u\}$ is subset of R and v' disconnects at most k vertices from R), (2) v disconnects at most k vertices from $R' \cup \{u\}$ (note that v disconnects u and only those vertices in R'' and $|R''| < k$), (3) each vertex $u' \in R'$ disconnects at most k vertices from $L' \cup \{v\}$ (since $(L' \cup \{v\}, R')$ is a local solution) and (4) u disconnects at most k vertices from $L' \cup \{v\}$ (note that u disconnects at most $k-1$ vertices from L since $u \in R^1_{enum}$ and u disconnects v since $u \in R_{enum}$). □

Lemma 4.3. In the subgraph $(L' \cup \{v\}, R')$, where $L' = L$ and $R' = R_{keep} \cup R''_1 \cup R''_2$, all vertices except for those in R''_2 disconnect at most k vertices and vertices in R''_2 disconnect exactly $(k+1)$ vertices.

Proof. This can be verified as follows. (1) Each vertex $v' \in L$ disconnects at most k vertices in R' (this is because $\overline{\delta}(v', R) \leq k$ since (L, R) is a k-biplex, and $R' \subseteq R$). (2) Vertex v disconnects at most k vertices in R' (this is because $\overline{\delta}(v, R') = \overline{\delta}(v, R'' \cup R_{keep}) = \overline{\delta}(v, R'') \leq |R''| \leq k$. Note that v connects every vertex in R_{keep} by the definition of R_{keep}). (3) Each vertex $u \in R_{keep}$ disconnects at most k vertices in $L \cup \{v\}$ (this because u disconnects at most k vertices in L given that (L, R) is a k-biplex and u connects v by the definition of R_{keep}). (4) Each vertex u_1 in R''_1 disconnects at most k vertices in $L \cup \{v\}$ (this is because $\overline{\delta}(u_1, L) \leq k-1$ by the definition of R^1_{enum} and u_1 disconnects v). (5) Each vertex u_2 in R''_2 disconnects *exactly* $(k+1)$ vertices in $L \cup \{v\}$ (this is because $\overline{\delta}(u_2, L) = k$ by the definition of R^2_{enum} and u_2 disconnects v). □