AI-1110 Assignment 1

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April 2023

Problem - 11.16.4.9

1. **Question:** If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?

   Solution: Let us define the digits set **T** and sample space **S** such that

   $$T \coloneqq \{0, 1, 3, 5, 7\}$$

   and

   $$S \coloneqq \{x \mid (x \geq 5001) \wedge (\text{digits}(x) \in T)\}$$

   Thus, $\mid S \mid = 2 * 5^3 - 1$ because first digit-place(i,e. leftmost digit) of $x \in S$ can have digits from $\{5, 7\}$. So, we have 2 choices. The next 3 places can be filled in $\mid T \mid = 5$ ways. Finally, we exclude the case of 5000. So, in total we have $2 * 5^3 - 1$ elements in S.

   NOTE: A number is divisible by 5 $\iff$ its last digit is 0 or 5.

   (i) Let E_1 denote the event where the numbers $\in S$ and repetition of digits from T is allowed. Then we can pick 2 digits $\{5, 7\}$ as the left-most digits, while we can have $\{0, 5\}$ as the right-most digits. Whereas, the middle two digit places can be filled in 5 ways. We also exclude the case where the number formed is 5000. So, we have a total of $2*2*5^2-1$ ways. So,

   $$\mathcal{P}(E_1) = \frac{2 * 2 * 5^2 - 1}{2 * 5^3 - 1} = \frac{99}{249} = \frac{33}{83}.$$

(ii) Let E_2 denote the event where the numbers $\in S$ and repetition of digits from T is **not** allowed. Then we have 2 possibilities for left-most position $\{5, 7\}$. Let us consider that the left-most position has 5. It implies that right-most position is bound to be 0 for E_2 to occur. The middle 2 positions can be filled with digits from $T - \{0, 5\}$. That is, we can do it in ${}^3P_2$ ways. And if the left-most position has 7, then right-most position has 2 possibilities $\{0, 5\}$. The middle positions can then have ${}^3P_2$ possibilities of filling up. Hence, total number of ways is $(1 *^3 P_2 * 1) + (1 *^3 P_2 * 2)$. So,

$$\mathcal{P}(E_2) = \frac{(1 *^3 P_2 * 1) + (1 *^3 P_2 * 2)}{2 * 5^3 - 1} = \frac{6 + 12}{249} = \frac{6}{83}.$$

THE END