Problem Set No. 4

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Reading assignment: Section 3.1, 3.3, 2,1, 2.2 ; J. A. Kong, "*Electromagnetic Wave Theory,*" EMW Publishing, 2008.

Problem P4.1

Consider a conductive uniaxial medium with

$$\overline{\overline{\epsilon}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

and

$$\overline{\overline{\sigma}} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}$$

Find dispersion relations for this medium when a plane wave is propagating in $\hat{x}$ direction. Explain the operation of a polaroid with this model by assuming $\sigma_z/\sigma \gg 1$. Show that a piece of polaroid turns any wave into a linearly polarized wave.

Problem P4.2

Consider a circularly polarized electromagnetic wave normally incident upon a slab as shown in the figure. The incident electric field is expressed by

$$\overline{E}_i = E_o \hat{x} e^{ik_y y} + \alpha E_o \hat{z} e^{i(k_y y + \beta)}.$$

In this problem, neglect the reflection of the slab.

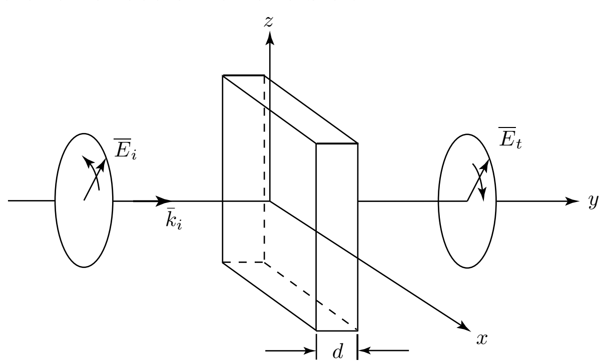

Fig. 1

(a) Let the incident wave be left-hand circularly polarized and assume that both α and β are positive, what is α and what is β?

(b) Let the slab be a uniaxial medium with the permittivity tensor

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

where $\epsilon_x = \epsilon_y = 4\epsilon_o$, $\epsilon_z = 9\epsilon_o$ and the permeability $\mu = \mu_o$. Inside the uniaxial slab, what is the wave number k_y for the $\hat{x}$-polarized electric wave in terms of the wave number in free space k_o, where $k_o = \omega\sqrt{\mu_o\epsilon_o}$?

(c) For the uniaxial slab as in Part (b), let the incident wave be left-hand circularly polarized, what is the minimum thickness d in terms of the wavelength in free space λ_o, where $\lambda_o = 2\pi/k_o = 2\pi/\omega\sqrt{\mu_o\epsilon_o}$, such that the output electric field is right-hand circularly polarized?

Problem P4.3

A break in a high-voltage DC power line occurs at $z = 0$ at time $t = 0$, as shown in Fig.2. The line was carrying a DC voltage V_o and DC current I_o before the break occurred. Assume that the tree is non-conducting.

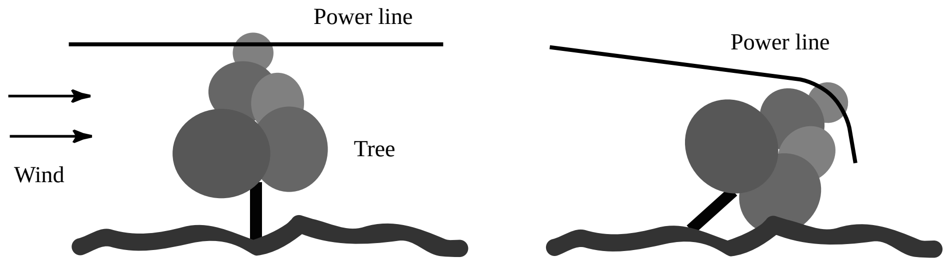

Fig. 2

(a) Sketch I and V on the line at some time t after the break has occurred, but before any reflections from the source and load ends. The characteristic impedance of the line is Z_0.

(b) Consider a 600 kV line, carrying a power of 10^3 megawatts, with a characteristic impedance of $500\,\Omega$ (two-wire line). What is the peak voltage on the line after the break occurs?

Problem P4.4

A very long transmission line with characteristic impedance Z_0 and wave velocity $v = c$ has a shunt resistor of unknown vlue R_L at an unknown location $z = \ell$. A measurement of the voltage at the input, $V_0(t)$, with a unit step generator applied to the line, yields the result shown in Fig. 3.

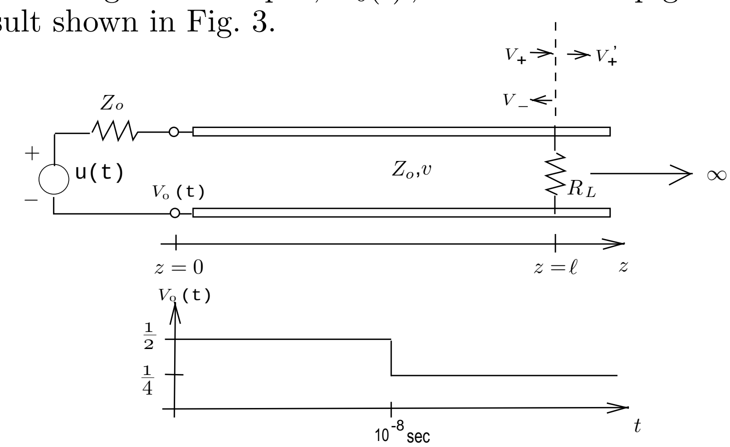

Fig. 3

(a) What is ℓ?

(b) What is R_L?

(c) Sketch the voltage and current distribution on the line at the time $t = 1.5\ell/v$.