Problem Set No. 2

Reading assignment: Section 3.3 ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2008.

Problem P2.1

In a negative uniaxial medium ($\epsilon_z < \epsilon$), a wave vector $\overline{k}$ makes an angle $\theta_1 < \pi/2$ with the optic axis. The Poynting's vector $< \overline{S} >$ makes an angle θ_2 with the optic axis. Determine whether θ_2 is larger or smaller than θ_1. Sketch the curve of θ_2 vs θ_1 for $0 < \theta_1 < \pi/2$.

Problem P2.2

Consider a slab of thickness d, as shown in Fig. 1, with the following permittivity and conductivity tensors:

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \qquad \overline{\overline{\sigma}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

where $\epsilon_x = 12\epsilon_\circ$, $\epsilon_y = \epsilon_\circ$, $\epsilon_z = 4\epsilon_\circ$, and $\mu = \mu_\circ$. The conductivity for polarized wave in z-direction is $\sigma_z = 0.2\epsilon_\circ\omega$ mho/meter.

Let the incident electric field propagate in the $\hat{w}$-direction. Neglect reflections.

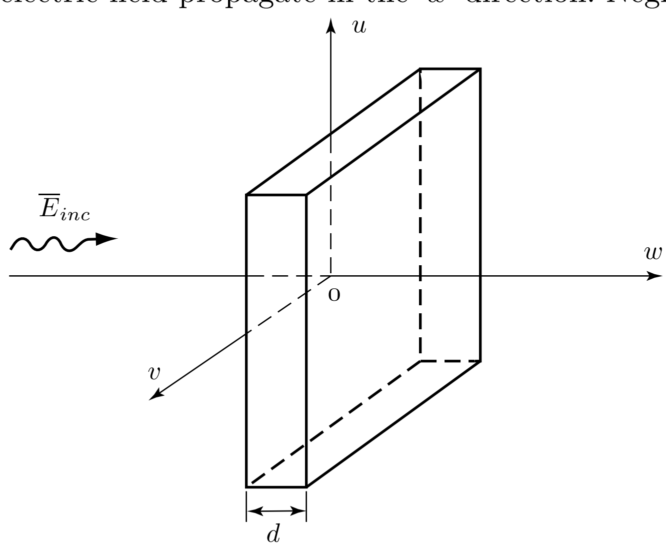

Fig. 1

Polarizer:

(a) Assign x, y, z to u, v, w (not necessarily in that order) so that, for any arbitrarily-polarized incident electric field and sufficiently thick slab, the transmitted field is linearly polarized.

(b) Determine the minimum thickness d in terms of free space wavelength such that the undesirable component of the incident field is attenuated by $1/e$.

Wave plate:

(c) Assign x, y, z to u, v, w (not necessarily in that order) so that, for a given linearly polarized incident electric field, the transmitted field is circularly polarized. Specify the axes so that there is no power absorption. Give an expression for an incident electric field such that, given the correct thickness d, the transmitted electric field is circularly polarized. What is the minimum correct d? Sketch the rotation of E field on the right surface (i. e. $w = d$) of the slab with this minimum d.

Problem P2.3

In a ferrite, the magnetic moment $\overline{M}$ roughly obeys the relationship $d\overline{M}/dt = g\mu_0\overline{M} \times \overline{H}$, where g is the gyromagnetic ratio. When a $\hat{z}$-directed dc magnetic field $\overline{H}_0$ (zeroth order) is present, the total fields take the form $\overline{H} = \hat{z}H_0 + \overline{H}_1$, $\overline{M} = \hat{z}M_0 + \overline{M}_1$, and $\overline{B} = \mu_0(\overline{H} + \overline{M})$.

(a) For the cross product $\hat{z} \times \overline{H}_1$, convert it to be $\overline{\overline{z}} \cdot \overline{H}_1$. What is $\overline{\overline{z}}$?

(b) To the first order approximation, $\overline{M}_1 \times \overline{H}_1$ is negeligible. Show that the permeability $\overline{\overline{\mu}}$ which satisfies $\overline{B}_1 = \overline{\overline{\mu}} \cdot \overline{H}_1$ has the form of

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu & i\mu_g & 0 \\ -i\mu_g & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

(c) Find dispersion relations for the first-order fields in kDB system.

(d) Show that Faraday rotation exists in the ferrite.