Problem Set No. 1

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Reading assignment: Section 1.1, J. A. Kong,"*Electromagnetic Wave Theory*", EMW Publishing, 2008

Problem P1.1

Electromagnetic waves satisfy all of the Maxwell equations. Consider, in free space, the following electric field vectors:

$$
\begin{aligned}
\overline{E}_1 &= \hat{z}\cos(\omega t - kz) \\
\overline{E}_2 &= (\hat{x}+\hat{z})\cos\left(\omega t + k|x-z|/\sqrt{2}\right) \\
\overline{E}_3 &= (\hat{x}+\hat{z})\cos(\omega t + ky)
\end{aligned}
$$

(a) Do these electric field vectors satisfy the wave equation

$$\left(\nabla^2 - \mu_o\epsilon_o\frac{\partial^2}{\partial t^2}\right)\overline{E} = 0$$

What is the relationship between ω and k?

(b) Find the corresponding magnetic field vectors for each of the three electric field vectors.

(c) Which of the three fields qualify as electromagnetic waves? For those not qualified as electromagnetic waves, you should state which of the Maxwell equations are violated. Show that for those qualified as electromagnetic waves, the electric and magnetic field vectors are perpendicular to the direction of propagation.

Problem P1.2

The known spectrum of electromagnetic waves covers a wide range of frequencies. Electromagnetic phenomena are all described by Maxwell's equations and, by convention, are generally classified according to wavelengths or frequencies. Radio waves, television signals, radar beams, visible light, X rays, and gamma rays are examples of electromagnetic waves.

(a) Give in meters the wavelengths corresponding to the following frequencies:
  (i) 60 Hz
  (ii) AM radio (535–1605 kHz)
  (iii) FM radio (88–108 MHz)
  (iv) C-band (4–6 GHz)
  (v) Visible light ($\sim 10^{14}$ Hz)
  (vi) X-rays ($\sim 10^{18}$ Hz)

(b) Give in Hertz the temporal frequencies corresponding to the following wavelengths:
  (i) 1 km, (ii) 1 m, (iii) 1 mm, (iv) 1 μm, (v) 1 Å.

(c) Give in unit K_o the spatial frequencies corresponding to the wavelengths in (b).

Problem P1.3

Prove the following vector identities, all of which are frequently used in this course.

$\nabla \times \left(\nabla \times \overline{E}\right) = \nabla \left(\nabla \cdot \overline{E}\right) - \nabla^2 \overline{E}$

$\nabla \cdot \left(\overline{E} \times \overline{H}\right) = \overline{H} \cdot \left(\nabla \times \overline{E}\right) - \overline{E} \cdot \left(\nabla \times \overline{H}\right)$

$\nabla \cdot \left(\nabla \times \overline{A}\right) = 0$

$\nabla \times (\nabla \phi) = 0$

Problem P1.4

Consider a wave which has the form of $\cos(kz - \omega t)$, where the phase term is $\phi = kz - \omega t$. Take derivative of the phase ϕ with respect to time t and let $\partial \phi / \partial t = 0$ to calculate the wave velocity. Can we calculate the wave velocity by taking derivative of the phase with respect to z? Why?

Run the following code in Matlab and you will get a figure showing the propagation of this wave. Trace the line where each point has the same phase as the origin. Then the slope of the line represents the wave velocity. Here we let $k = 1$ first. What will happen if k is increased?

```
k =1;
omega = 2;
[z,t] = meshgrid(0:0.01:10, 0:0.01:10);
wave = cos(k*z - omega*t);
pcolor(t,z,wave);shading flat;colorbar;
xlabel('t');ylabel('z');axis square
```