Problem Set No. 5

Reading assignment: Section 2.2, 2.3 ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2008.

Problem P5.1

Digital systems on printed circuit boards often incorporate source terminations instead of load terminations to minimize the dissipated power and to reduce the magnitude of propagating voltage waveforms. Consider the digital system in the figure. The driver is modeled by a voltage source $V_s = 1$ V and a source resistance $R_s = 25\ \Omega$. The transmission line has a characteristic impedance $Z_0 = 100\ \Omega$ and a propagation velocity $v = c/2 = 1.5 \cdot 10^8$ m/s due to the geometry and dielectric constant of the the printed circuit board. Digital receivers often have very low input capacitance so that the load can be modeled as an open circuit in the frequency range of interest. The source termination is implemented by inserting a series resistance $R_1 = 75\ \Omega$ so that the total input resistance matches the characteristic impedance of the transmission line.

(a) The length of the transmission line is measured to be 30 cm. Calculate the round trip propagation time for a signal transient.

(b) A low to high transition is modeled by a switch that closes at $t = 0$. If you measure the voltage at V_1, you might conclude that the quality of the signal is poor. Plot the voltage at V_1 versus time assuming zero volts initially on the line.

(c) Now assume that the signal quality measurement is made at the digital receiver. Plot the voltage V_2 versus time assuming zero volts initially on the line.

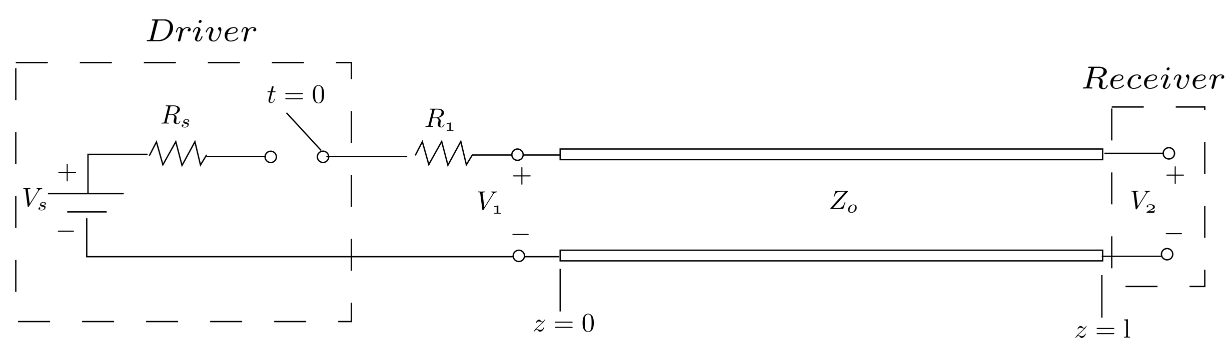

Fig. 1

Problem P5.2

Consider a TEM transmission line as shown in the following figure. The characteristic impedance of the transmission line is Z_0 and its length is $l = 2\lambda$, where λ is the wavelength of the wave in the line. The load impedance is Z_L. The current on the line is given by

$$I(z) = I_o \cos kz.$$

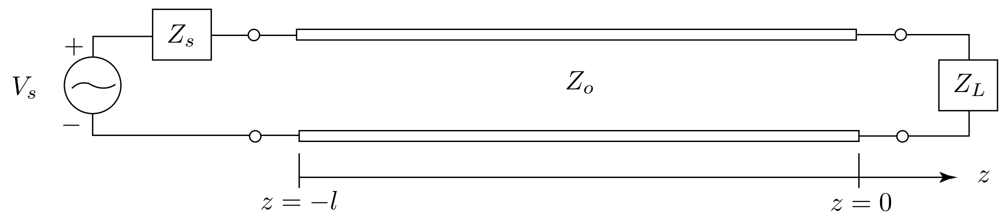

Fig. 2

(a) What is V_+ and what is V_- in terms of Z_o and I_o?
(b) What is the reflection coefficient at the load Γ_L?
(c) Show that the load impedance $Z_L = 0$.
(d) The real voltage in space and time is defined as $V(z,t) = \mathrm{Re}\{V(z)e^{j\omega t}\}$. Let I_o and Z_o be real, write down the expression and sketch the voltage on the line at $\omega t = \pi/2$.
(e) Let the voltage of the source be $V_s = I_o Z_o$, what is the source impedance Z_s in terms of Z_o?

Problem P5.3

Consider the transmission line system which is composed of three parts $T1$, $T2$ and $T3$ as shown in Fig. 3. All of them have the same length ℓ. $T1$ and $T2$ have the characteristic impedance Z_o while $T3$ has the characteristic impedance $2Z_o$. $T2$ is open at the right end while $T3$ is short at the right end.

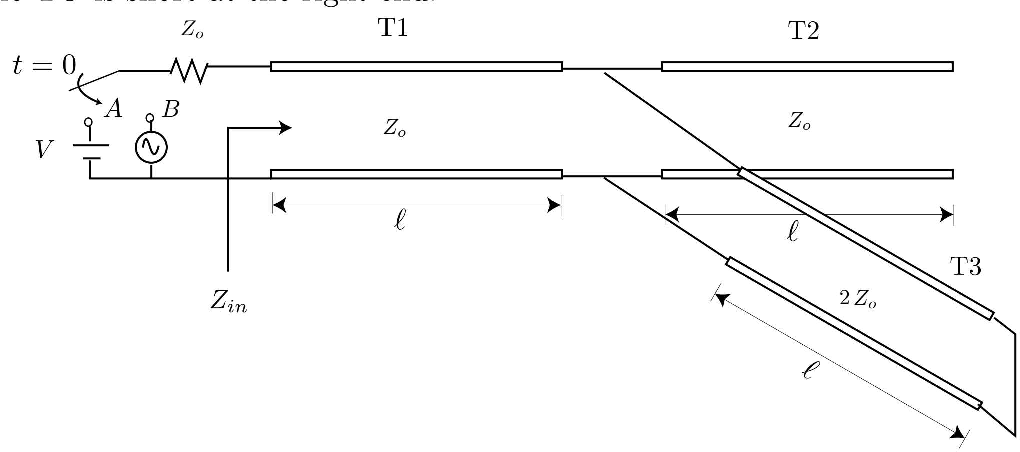

Fig. 3

(a) If at time $t = 0$ the switch is connected to A, sketch the voltage form $V(z)$ on $T1$ at the time $t = 1.5\ell/v$. Here v is the speed of the wave.
(b) Now the switch is connected to B and after a long time the sinusoidal steady state has been built up. Calculate the input impedance Z_{in} at the input end. ($k\ell = \pi$.)