Problem Set No. 3

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Reading assignment: Section 1.3, 1.5, 3.1 ; J. A. Kong, "*Electromagnetic Wave Theory,*" EMW Publishing, 2008.

Problem P3.1

According to the Bohr model of an atom, electrons revolve around the nucleus in quantized orbits with radii $R = n\hbar/mv$ where n is an integer, m is the electron mass and v is the electron velocity. Letting the nucleus be a positive charge of Ze, calculate R by equating the centrifugal force with the Lorentz force. Estimate the radius for a hydrogen atom with $Z = 1$.

Problem P3.2

(a) At the operating frequency (2.5 GHz) of a microwave oven, the permittivity for bottom round steak is about $\epsilon = 40\epsilon_0$ and the conductivity $\sigma = 2$ mho/meter. What is the penetration depth? Compare this penetration depth to that of polystyrene foam which has the permittivity $\epsilon = 1.03\epsilon_0$ and conductivity $\sigma = 4 \times 10^{-6}$ mho/meter.

(b) Earth is considered to be a good conductor when $\omega\epsilon/\sigma \ll 1$. Determine the highest frequency for which earth can be considered a good conductor if $\ll 1$ means less than 0.1. Assume $\sigma = 5 \times 10^{-3}$ mho/meter and $\epsilon = 10\epsilon_0$.

(c) Aluminum has $\epsilon = \epsilon_0$, $\mu = \mu_0$ and $\sigma = 3.54 \times 10^7$ mho/m. If an antenna for VHF reception is made of wood coated with a layer of aluminum and if its thickness ought to be five times greater than the skin depth of the aluminum at that frequency, determine the thickness of the aluminum layer. Is ordinary aluminum foil thick enough for that purpose? Use $f = 100$ MHz. Ordinary aluminum is approximately 1/1000 inch thick.

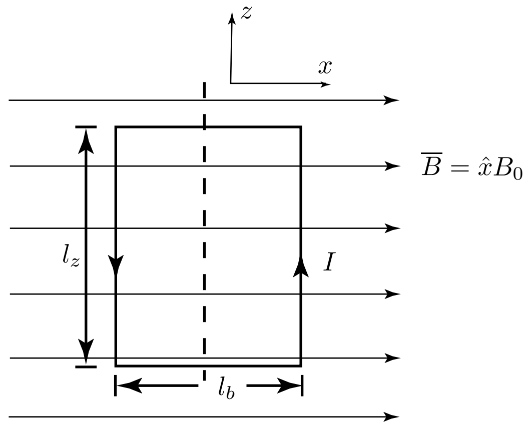
(d) Calculate skin depths for sea water at frequencies 100 Hz and 5 MHz. Sea water can be characterized by conductivity $\sigma = 4$ mho/m, permittivity $\epsilon = 80\epsilon_0$, and permeability $\mu = \mu_0$ at those frequencies.

(e) A ship at the ocean surface wishes to communicate electromagnetically with a deeply submerged vehicle 100 meters below the surface. Consider a ULF signal at 1 KHz propagating down into the sea water. What fraction of the incident power density reaches the submerged vehicle?

Problem P3.3

The magnetic moment $\overline{M}$ is analogous to the expression for the mechnical angular momentum. We will show that the magnetic torque can be expressed by $\overline{T} = \overline{M} \times \overline{B}$.

(a) As shown in the figure below, a current loop is placed in a uniform magnetic field. Given $\overline{T} = \overline{r} \times \overline{F}$, use Lorentz force law to show that $\overline{T} = \hat{z}IAB_0$. By writing $\overline{M} = -\hat{y}IA$ and $\overline{B} = \hat{x}B_0$, show that $\overline{T} = \overline{M} \times \overline{B}$.



(b) As shown in the figure below, a loop carrying a current of I_l with normal $\hat{n} = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y})$ is placed a distance d above a straight wire, which is carrying a current of I_0. Calculate the magnetic moment of the current loop and the magnetic field generated by straight wire at the loop's position. Using these two values, calculate the torque vector, $\overline{T}$, of loop. In what direction does the loop move due to the torque?