Solution to Problem Set 2

Solution P2.1

AM (i): Vertically oriented.
TV (ii): The antenna is parallel to the ground and perpendicular to the direction from which the signal comes.
FM (i)(ii): The antenna may be oriented in any direction as long as the antenna is on the plane perpendicular to the direction from which the signal is coming. This is because the antenna will always intercept the electric field that is rotating around a circular path in that plane.

Solution P2.2

(a) $6e^{i\frac{\pi}{4}}$
(b) $8e^{i\frac{\pi}{2}}$
(c) $-3i-2$
(d) $-6i$
(e) None
(f) None

Solution P2.3

(a)

$$\overline{E} = \hat{x}2\cos(kz-\omega t+\frac{\pi}{2}) + \hat{y}\cos(kz-\omega t+\frac{\pi}{4})$$

At the point $z = z_0$, $\overline{E}$ will trace a trajectory

$$x = 2\cos(kz_0-\omega t+\frac{\pi}{2}) = -2\sin(kz_0-\omega t)$$
$$y = \cos(kz-\omega t+\frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(kz_0-\omega t) - \frac{1}{\sqrt{2}}\sin(kz_0-\omega t)$$

The equation describing this trajectory is

$$\frac{x^2}{2} - \sqrt{2}xy + 2y^2 = 1$$

which is the equation of an ellipse. Therefore, the wave is elliptically polarized.

(b) Consider the following linearly polarized plane wave:

$$\begin{aligned}\overline{E} &= \hat{x}\cos(kz-\omega t)\\ &= \left[\frac{1}{2}\hat{x}\cos(kx-\omega t)+\frac{1}{2}\hat{y}\sin(kz-\omega t)\right] + \left[\frac{1}{2}\hat{x}\cos(kx-\omega t)-\frac{1}{2}\hat{y}\sin(kz-\omega t)\right]\end{aligned}$$

This is the superposition of two circularly polarized waves.

(c) Consider the following circularly polarized plane wave:

$$\overline{E} = \hat{x}\cos(kz-\omega t+\pi/4) + \hat{y}\cos(kz-\omega t-\pi/4)$$

This is the superposition of two linearly polarized waves.

Solution P2.4

$\lambda = \frac{1}{A} = \frac{1}{100} = 0.01\,(\mathrm{m})$

The wave propagates in the $+\hat{z}$ direction.
At $z = 0$,

$$\overline{E}(z=0,t) = E_0\left[\hat{x}\cos(\omega t) - \hat{y}\sin(\omega t)\right]$$

This is a left-hand circularly polarized plane wave.

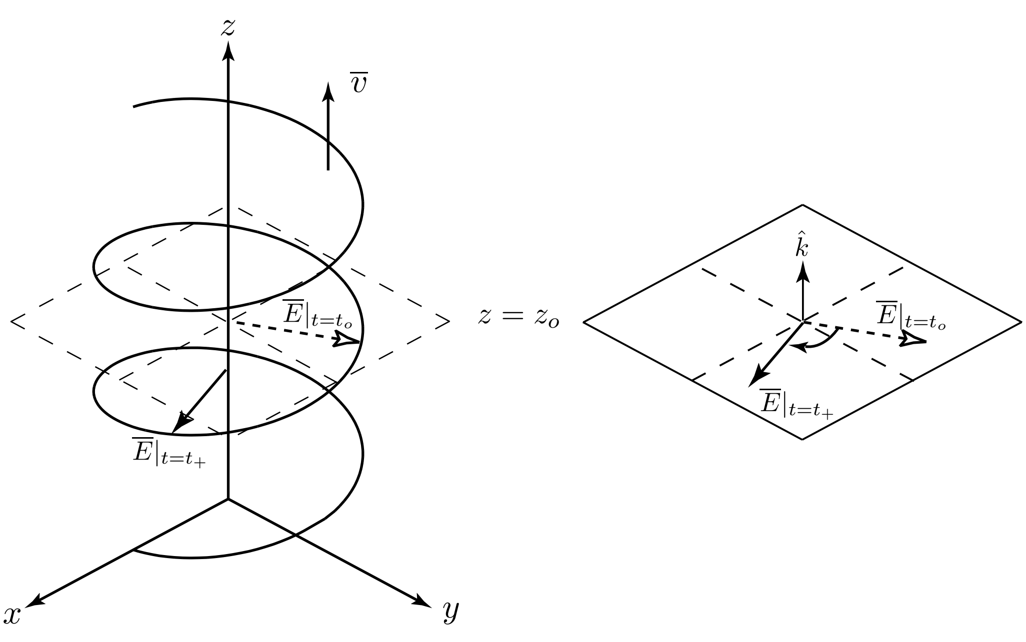
At $t = 0$,

$$E_x = E_0 \cos\left(\frac{2\pi}{\lambda}z\right)$$
$$E_y = E_0 \sin\left(\frac{2\pi}{\lambda}z\right)$$

The parametric equation of a helix is

$$x = R\cos\left(\frac{2\pi}{p}z\right)$$
$$y = R\sin\left(\frac{2\pi}{p}z\right)$$

where p is the pitch of the helix. Thus the locus of the tip point of the electric field vector measured along the z axis is a right-handed helix with the pitch $p = \lambda$. From the figure below, we see that $\overline{E}|t_0 \times \overline{E}|t_+$ is in the direction of $-\hat{k}$, thus the wave is temporally l.h.c.p.



At $\omega t = \pi/4$,

$$E_x = E_0 \cos\left[\frac{2\pi}{\lambda}(z - \frac{\lambda}{8})\right]$$
$$E_y = E_0 \sin\left[\frac{2\pi}{\lambda}(z - \frac{\lambda}{8})\right]$$

The helical locus advances along $+\hat{z}$ without rotating!

Solution P2.5

(a) Speed of light $c = 3 \times 10^8$m/sec
Distance between Sun and Earth $D = 150 \times 10^9$m
Travelling time of light from Sun to Earth $T = D/C = 500\text{sec} = 8.33\text{min}$

(b) Power received by the Earth $P_r = S(\text{power density}) \times A(\text{cross section area})$

$$\begin{aligned} A &= \pi r^2 = \pi \times (6.4 \times 10^6)^2 \text{m}^2 \\ S &= 1.5\text{kW/m}^2 \\ P_r &= 1.93 \times 10^{14}\text{kW} \end{aligned}$$

(c) The total power radiated by the Sun is

$$\begin{aligned} P_{total} &= S(\text{power density at distance } D) \\ &\quad \times A_T(\text{Total surface area of sphere with radius } D) \\ A_T &= 4\pi D^2 = 4\pi \times (150 \times 10^9)^2 \text{m}^2 \\ S &= 1.5\text{kW/m}^2 \\ \text{Mass of Sun } m &= 2 \times 10^{30}\text{kg} \\ \text{Efficiency } E_{eff} &= 1\% \end{aligned}$$

Total time that Sun can radiate

$$\begin{aligned} T &= \frac{mc^2 E_{eff}}{P_{Total}} = \frac{mc^2 E_{eff}}{S 4\pi D^2} \\ &= \frac{(2 \times 10^{30}) \times (3 \times 10^8)^2 \times 0.01}{(1.5 \times 10^3) \times 4\pi \times (150 \times 10^9)^2} = 4.2 \times 10^{18}\text{sec} \approx 1.34 \times 10^{11}\text{years} \end{aligned}$$

(d)

$$\begin{aligned} \text{Poynting power } S &= P(\text{power density per Hz}) \times W(\text{bandwith}) \\ &= 10^{-20}\text{Wm}^{-2}\text{Hz}^{-1} \times 10^9\text{Hz} = 10^{-11}\text{Wm}^{-2} \end{aligned}$$

$$\text{Since } S = E^2/2\eta, \text{ we have}$$

$$E = \sqrt{2\eta S} = \sqrt{2 \times 120\pi \times 10^{-11}} = 8.68 \times 10^{-5} \frac{\text{volt}}{\text{m}}$$