《量子信息基础》期末考试

考试范围：

请重点复习**如下习题内容及其相关的课堂 PPT**：

(Text book* Problem 5.6)
Imagine two non-interacting particles, each of mass *m*, in the infinite square well. If one is in the state ψ_n ($\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$), and the other in state $\psi_l (l \neq n)$, calculate $\langle(x_1 - x_2)^2\rangle$, assuming (a) they are distinguishable particles, (b) they are identical bosons, and (c) they are identical fermions.

(Text book* Problem 5.23, 注意是教材的第二版)
Support you had three (non-interacting) particles, in thermal equilibrium, in a one-dimensional harmonic potential, with a total energy $E = (9/2)\hbar\omega$

(a) If they are distinguishable particles (but all with the same mass), what are the possible occupation-number configurations, and how many distinct (three particle) states are there for each one? What is the most probable configuration? If you picked a particle at random and measured its energy, what values might you get, and what is the probability of each one? What is the most probable energy?
(b) Do the same for the case of identical fermions (ignoring spin).
(c) Do the same for the case of identical bosons (ignoring spin).

Put 10 distinguishable particles into 4 different quantum states to let the final configuration to be $(4, 3, 2, 1)$ as the macrostate. Calculate the number of microstates in this configuration.

Consider a system of two particles, they have six possible states $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6$. Calculate the number of the microstates of the system, in the following conditions:

(1) Two particles are bosons;
(2) Two particles are fermions;
(3) Two particles are distinguishable.

The binary entropy is a concave function. Prove
$$S_{bin}(px_1 + (1-p)x_2) \geq pS_{bin}(x_1) + (1-p)S_{bin}(x_2)$$
where $0 \leq p, x_1, x_2 \leq 1$.

(Text book* Problem 3.4)

(a) Show that the sum of two Hermitian operators is Hermitian.

(b) Suppose $\hat{Q}$ is Hermitian, and α is a complex number. Under what condition (on α) is $\alpha\hat{Q}$ Hermitian?

(c) When is product of two Hermitian operators hermitian?

(d) Show that the position operator ($\hat{x}$) and the Hamiltonian operator $\left(\hat{H} = -\left(\frac{\hbar^2}{2m}\right)\frac{d^2}{dx^2} + V(x)\right)$ are hermitian.

A Hermitian operator $\hat{A}$ has a complete orthonormal set of eigenfunctions $|\psi_n\rangle$ with associated eigenvalues α_n. Show that we can always write

$$\hat{A} = \sum_i \alpha_i |\psi_i\rangle\langle\psi_i|$$

(Text book* Problem 3.27)

Let $\hat{Q}$ be an operator with a complete set of orthonormal eigenvectors:

$$\hat{Q}|e_n\rangle = q_n|e_n\rangle$$

(a) Show that $\hat{Q}$ can be written in terms of its spectral decomposition:

$$\hat{Q} = \sum_n q_n|e_n\rangle\langle e_n|$$

Hint: An operator is characterized by its action on all possible vectors, so what you must show is that

$$\hat{Q}|\alpha\rangle = \left\{\sum_n q_n|e_n\rangle\langle e_n|\right\}|\alpha\rangle$$

for any $|\alpha\rangle$.

(b) Another way to define a function of $\hat{Q}$ is via the spectral decomposition:

$$f(\hat{Q}) = \sum_n f(q_n)|e_n\rangle\langle e_n|$$

Show that this is equivalent to

$$e^{\hat{Q}} \equiv 1 + \hat{Q} + \frac{1}{2}\hat{Q}^2 + \frac{1}{3!}\hat{Q}^3 + \cdots$$

when $f(\hat{Q}) = e^{\hat{Q}}$.

An operator $\hat{Q}$ has the complete sets of Eigen wave functions $\{|a_n\rangle\}$ and $\{|b_n\rangle\}$ in the A and B representations respectively. Assuming $\{|a_n\rangle\}$ and $\{|b_n\rangle\}$ are connected by unitary transformation

$$|b_n\rangle = \hat{U}|a_n\rangle$$

prove that

$$\hat{Q}_{(B)} = \hat{U}\hat{Q}_{(A)}\hat{U}^{\dagger}$$

(Text book* Problem 3.16)

Show that two noncommuting operators cannot have a complete set of common eigenfunctions. Hint: Show that if $\hat{P}$ and $\hat{Q}$ have a complete set of common eigenfunctions, then $[\hat{P}, \hat{Q}]f = 0$ for any function in Hilbert space.

$\hat{D}_x(a)$ is a translation operator in one dimension. When it applies to a wavefunction

$$\hat{D}_x(a)\psi(x) = \psi(x - a)$$

If $\hat{f}(x)$ is commutable with $\hat{D}_x(a)$, prove $\hat{f}(x) = \hat{f}(x - a)$.

In the EPRB experiment, state whether each of the following states is entangled (i.e. can it be factored into a product of states of the individual particles, in which case it is not entangled)

(a) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |0_1, 0_2\rangle)$

(b) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle)$

(c) $\frac{3}{5}|0_1, 1_2\rangle + \frac{4i}{5}|1_1, 1_2\rangle$

(d) $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle + |1_1, 1_2\rangle)$

Consider a Bell state $\frac{1}{\sqrt{2}}(|0_1, 0_2\rangle + |1_1, 1_2\rangle)$ of two photons.

(a) Express it now on a basis of $|\theta\rangle$ and $|\theta + \pi/2\rangle$ by rotating the original basis by an angle θ. Hint: on such a basis, $|1\rangle = \sin\theta|\theta\rangle + \cos\theta\,|\theta + \pi/2\rangle$ and also a similar expression for $|0\rangle$.

(b) In the new basis of such sate, show that the two photons will always come out of the same arm of each polarizer when two aligned polarizers are used to examine the pair of photons.

Consider the Bell state $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0_1, 0_2\rangle - |1_1, 1_2\rangle)$ of two photons, and suppose now that we wish to express it not on a basis of horizontal ($|0\rangle$) and vertical ($|1\rangle$) polarized states, but instead on a basis rotated by 45°, i.e., a new basis

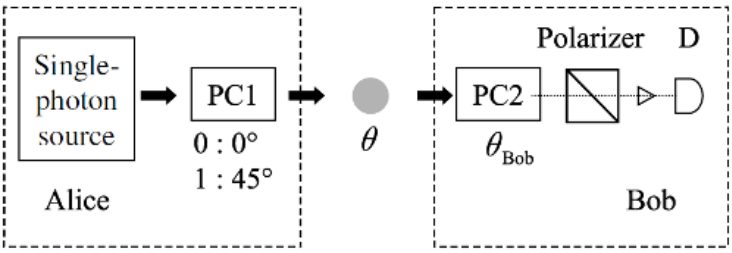
$$|+45\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|-45\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

(a) Show that, expressed on this particular basis, the resulting state is still a Bell state.

(b) Repeat part (a), but with the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0_1, 0_2\rangle + |1_1, 1_2\rangle)$. What difference do you note between the two results?

The following figure gives a schematic representation of a system designed to implement the B92 protocol using linearly polarized photons. Alice encodes her data according to the polarization angle θ of the photon, with $0^o \equiv 0$ and $45^o \equiv 1$. Bob makes measurements with a Pockels cell PC2 randomly set to rotate by an angle θ_{Bob} of either of 45^o or 90^o. A polarizer set to transmit perfectly for photons with $\theta = 0^o$ when $\theta_{Bob} = 0^o$ is placed after PC2, followed by a single-photon detector D.

(a) Describe the possible outcomes for both of Bob's measurement settings. Explain how this arrangement can be used for unambiguous transmission of bits.
(b) In the absence of losses, detector errors, and an eavesdropper, compare the fraction of Alice's bits that Bob receives in the B92 protocol to the fraction in the sifted data set of the BB84 protocol.
(c) How would an eavesdropper be detected in this scheme?



Quantum gate can be expressed by matrix as the following

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

In the meantime, it can be expressed by using Dirac notations

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|$$

Hence

$$X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$$

Write down the expression of the following gates using Dirac notations

(a) Z gate;

(b) H gate;

(c) P gate.

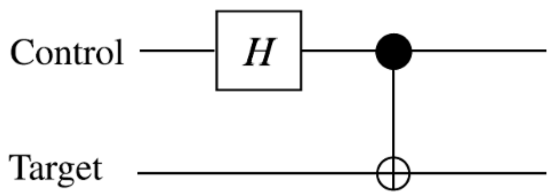
Calculate the output of the quantum circuit shown in the following figure when the input wave function is

(a) $|10\rangle$;

(b) $|01\rangle$;

(c) $|11\rangle$;

Assume that it is the first qubit that undergoes the Hadamard operation.



夏学期发布的全部选择题。