《量子信息基础》第4章第一部分：

1. In a quantum system, the Eigen energy and wavefunctions are . When , the system is in the ground state of . A perturbation occurs for , which is Calculate the probabilities for the system evolves into the state of when .
2. (Text book\* Problem 11.6)

Solve Equation 11.17 to second order in perturbation theory, for the general case .

1. (Text book\* Problem 11.3)

Solve Equation 11.17 for the case of a *time-independent* perturbation, assuming that and . Check that . Comment: Ostensibly, this system oscillates between “pure ” and “some ”. Doesn’t this contradict my general assertion that no transitions occur for time-independent perturbations? No, but the reason is rather subtle: In this case and are not, and never were, eigenstates of the Hamiltonian—a measurement of the energy *never* yields or . In time-dependent perturbation theory we typically contemplate turning *on* the perturbation for a while, and then turning it *off* again, in order to examine the system. At the beginning, and at the end, and are eigenstates of the exact Hamiltonian, and only in this context does it make sense to say that the system underwent a transition from one to the other. For the present problem, then, assume that the perturbation was turned on at time , and off again at time *t*—this doesn’t affect the *calculations*, but it allows for a more sensible interpretation of the result.

1. (Text book\* Problem 11.10)

You could derive the spontaneous emission rate (Equation 11.63) without the detour through Einstein’s *A* and *B* coefficients if you knew the ground state energy density of the electromagnetic field, , for then it would simply be a case of stimulated emission (Equation 11.54). To do this honestly would require quantum electrodynamics, but if you are prepared to believe that the ground state consists of one photon in each classical mode, then the derivation is fairly simple:

1. To obtain the classical modes, consider an empty cubical box, of side , with one corner at the origin. Electromagnetic fields (in vacuum) satisfy the classical wave equation

where *f* stands for any component of ***E*** or of ***B***. Show that separation of variables, and the imposition of the boundary condition *f* = 0 on all six surfaces yields the standing wave patterns

with

There are two modes for each triplet of positive integers , corresponding to the two polarization states.

1. The energy of a photon is (Equation 4.92), so the energy in the mode is

What, then, is the total energy per unit volume in the frequency range , if each mode gets one photon? Express your answer in the form.

and read off . Hint: refer to Figure 5.3.

1. Use your result, together with Equation 11.54, to obtain the spontaneous emission rate. Compare Equation 11.63.

\* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).