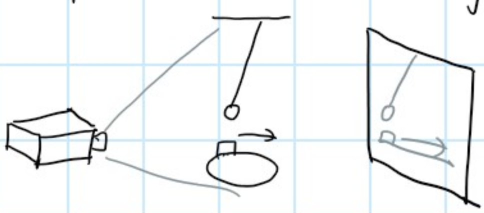
15.1,2,3

Tuesday, January 8, 2019 2:06 PM

- Periodic Motion : Any motion that repeats itself at a regular time interval.
  - period T, frequency $f = \frac{1}{T}$, amplitude A
    - Periodic motion is characterized by a continuous conversion between potential and kinetic energy in a closed system.

- A system that exhibits equal periods for all amplitute is isochronons.
  - The $x(t)$ curve for an isochronous oscillation are all sine functions.
    - therefore they're sinusoidal curves.
  - Simple Harmonic Motion (SHM) : Any periodic motion that yields a sinusoidal $x(t)$ curve.
  - isochronous $\Leftrightarrow$ SHM
  - SH Oscillator system that executes such motion.
  - Experiment : The shadow of the ball projected will be the same as that of circular motion at constant speed. $\Rightarrow$ move synchronously



- Fourier's Theorem any periodic function can be written as a sum of sinusoidal functions of $f_n = \frac{n}{T}$, $n > 1$, $n \in \mathbb{Z}$ $f_n = nf_1$, since $n \in \mathbb{Z}$ $\therefore T = T_1 = \frac{1}{f_1}$
  - Any periodic motion can be treated as a superposition of simple harmonic motion.

    Any motion that is non repetitive can be thought of as periodic motion that has an infinitely long period.
  - Time dependent motion of any sys can be expressed in Sinusoidal funs.

- Time dependent motion of any sys can be expressed in Sinusoidal fns.
- Fundamental frequency / first harmonic $f_1 = \frac{1}{T}$, lowest in sum; first component has

  others: higher harmonics

  $f_n = nf_1 = n/T$

- Fourier analysis the breaking down of a function into harmonic components

  Fourier series the resulting sum of sinusoidal function

  Fourier synthesis the creation of periodic functions by adding sinusoidal functions together

  - spectrum: plot of A_n^2 with respect to f.

15.4

Tuesday, January 8, 2019 11:27 PM

- Unstable equilibrium
  Neutral equilibrium Moving the ball in any direction has no effect on its subsequent acceleration
  Stable equilibrium
- In the absence of friction, a small displacement of a system
- Oscillations arise from an interplay between inertia and a restoring force.

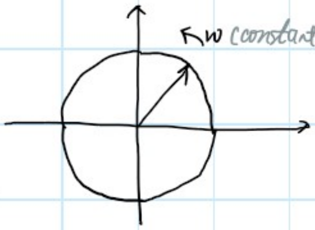
  when the object gets back equilibrium position, it has a nonzero velocity, and its inertia causes it to overshoot the eq. position.
- $T \uparrow$ $m\uparrow$, $T\downarrow$ when $F_{restore}\uparrow$

  $\underset{\text{moment of inertia}}{\underline{I}} = mr^2$, $m\uparrow$, $I\uparrow$ $\quad$ $a\uparrow$

15.5-15.7

Thursday, January 10, 2019 12:01 AM

- Reference circle
  Phasor: the arrow
  simple harmonic motion



can be express by a rotating arrow of length A whose tip traces out a circle → angular frequency rad/s → s^{-1}

- $\omega = \frac{2\pi}{T}$ length of phasor: A
- angular frequency
- Phase (unitless) angle of phasor relative to the positive horizontal axis.

$$\Phi(t) = \omega t + \boxed{\Phi_i}$$ → initial phase

- $x(t) = A\sin(\omega t + \Phi_i)$

- $x(t) = A\sin(\omega t + \Phi_i)$ [General SHM]
  - $v(t) = A\omega \cdot \cos(\omega t + \Phi_i)$

    $a(t) = -A\omega^2 \cos(\omega t + \Phi_i)$
  - $a = -\omega^2 x$

    $\Sigma F = -m\omega^2 x$
  - $W = -\int_{x_0}^{x} m\omega^2 x\,dx = -m\omega^2 \cdot \left(\frac{x^2}{2}\right)\Big|_{x_0}^{x} = -m\omega^2\left(\frac{x^2}{2} - \frac{x_0^2}{2}\right) = \Delta K$

    $= \left(\frac{x_0^2}{2} - \frac{x^2}{2}\right)m\omega^2$

    $\Delta K = -\Delta U \Rightarrow \Delta U = \left(\frac{x^2}{2} - \frac{x_0^2}{2}\right)m\omega^2$

    $E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 x^2$

    $= \frac{1}{2}m\omega^2\cos^2(\omega t + \phi_i) + \frac{1}{2}m\omega^2 \sin^2(\omega t + \phi_i)$

    $= \frac{1}{2}m\omega^2$

- Spring

$F = -kx \Rightarrow a = -\frac{kx}{m} = -\frac{k}{m}x$

$F=-kx \Rightarrow a=-\frac{kx}{m}=-\frac{k}{m}x$

$\therefore \frac{k}{m}=\omega^2$

$\therefore \omega=\sqrt{\frac{k}{m}}$

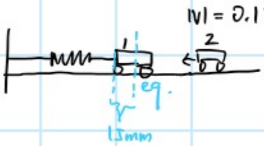
e.g.1. $m=0.5kg$ $k=14N/m$, calculate $x(t)$, $V_x(t)$ at $t=2.0$.

$x(t)=A\cdot\sin(\sqrt{\frac{k}{m}}t+\frac{\pi}{2})$ $\quad V_i=0$

$x(2)=0.03\sin(\sqrt{28}\times2+\frac{\pi}{2})$ $\quad \cos(\phi_i)=0$

$V(2)=\sqrt{28}\cdot0.03\cos(2\sqrt{28}+\frac{\pi}{2})$ $\quad \phi_i=\frac{\pi}{2}$

e.g.2. $m_1=0.5$, $k=14N/m$, after collision (elastic), remove 2. $m_1=m_2$



(a) Max compression of spring

(b) t that reach max compression

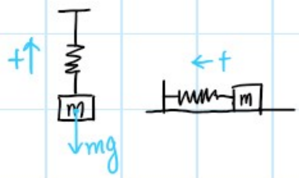
(a) $\frac{1}{2}kx_0^2+\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $\quad$ (b) $v_0=0$ $\quad$ A has changed!

$\frac{1}{4}\times0.001=7x^2$ $\quad \therefore \phi_i=\frac{\pi}{2}$ $\quad v=0.1m/s$, $\phi=\arcsin 0.1$

$x^2=\frac{0.001}{28}$ $\quad \therefore x=0.024\sin(\sqrt{28}t+\frac{\pi}{2})=\cancel{\sqrt{\frac{0.001}{28}}}\ 24$

$x=\cancel{\sqrt{\frac{0.001}{28}}}$ 24mm

e.g.3. $m=0.5kg$, $k=100N/m$, compare f of two situation



$x_{eq}=\frac{5}{100}=\frac{1}{20}$ m

x: the change of length from eq. position

Vertically: $\Sigma F=ma=k(x+x_0)-mg$

$a=\frac{k}{m}x$

Horizontally: $\Sigma F=ma=kx$

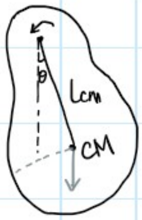
$\therefore f_{vertical}=f_{horizontal}$

- Restoring torques

$\Sigma\tau_\theta=I\alpha_\theta$

$\tau_\theta=-K(\theta-\theta_0)$ $\quad$ K torsional constant

- Pendulum



$\tau_\theta = -l_{CM} \cdot mg \cdot \sin\theta \quad \sin\theta \approx \theta$

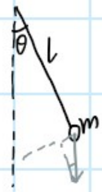
$\approx -l_{CM} \cdot mg \cdot \theta$

$\because \tau = I\alpha$

$\therefore \frac{d^2\theta}{dt^2} = \frac{-m l_{CM} g}{I}\theta$

$\therefore \omega = \sqrt{\frac{mg l_{CM}}{I}}$

- Simple pendulum



$\tau = mg \cdot \sin\theta l \sim mg\theta l$

$\tau = I\alpha = mr^2\alpha = ml^2\alpha$

$\therefore l^2\alpha = g\theta l$

$\alpha = \frac{g}{l}\theta$

$\omega = \sqrt{\frac{g}{l}}$ (or plug in $\sqrt{\frac{mg l_{CM}}{I}}$

- Damped oscillations
  - Mechanical energy $E = E_0 e^{-\frac{t}{\tau}}$, $\tau = \frac{m}{b}$, $\omega_d = \sqrt{\omega^2 - (\frac{1}{2\tau})^2}$

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

$\Rightarrow$ Soln: $E_0 e^{-\frac{t}{\tau}}$

15.8 damped oscillations

Thursday, January 24, 2019 11:08 AM

- damped oscillation an oscillation with slow conversion of E to thermal energy due to a damping force.
  - (horizontal direction) { restoring force
    damping force a force due to friction or drag that dissipates energy of oscillation
- assume damping force (drag forces exerted by air or liquids at low speed tend to be proportional to the v_{obj})

is $\vec{F}_d = -b\vec{v}$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\Rightarrow \quad x(t) = Ae^{-\frac{b}{2m}t}\sin(w_d t + \phi_i)$$

$$w_d = \sqrt{\frac{k}{m} - \frac{b^2}{m^2}} = \sqrt{w^2 - \left(\frac{b}{2m}\right)^2}$$

- time constant $\tau = \frac{m}{b}$

$$x_{MAX} = Ae^{-\frac{t}{2\tau}}$$

$$E(t) = E_0 \cdot e^{-\frac{t}{\tau}} = \frac{1}{2}mw^2x_{max}^2 = \frac{1}{2}mw^2A^2 \cdot e^{-\frac{t}{\tau}}$$

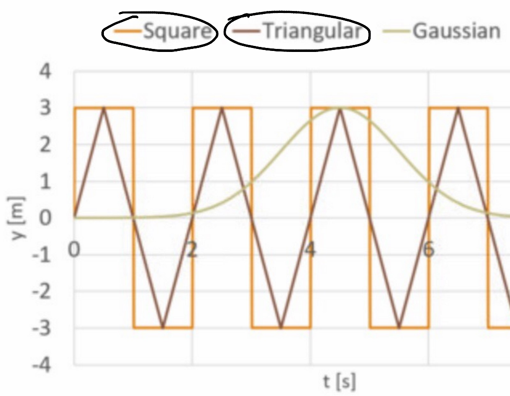
Prob

Tuesday, January 8, 2019 11:24 PM

Which one oscillates faster?

- (•) Springs oscillate with the same frequency.
- ( ) The spring with the heavier ball.
- ( ) The spring with the lighter ball.
- ( ) It is impossible do determine without additional data.

Select all of the following functions that can be written as the sum of sinusoidal functions.



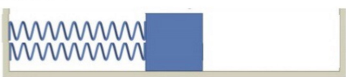
since Gaussian is not periodic

A student makes a simple harmonic oscillator using two identical springs and a mass. She connects the springs and mass in two different configurations, as shown. Which configuration has the higher oscillation frequency?

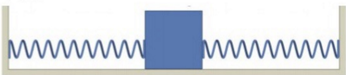
A. Case 1
B. Case 2
C. They are the same.
D. Not enough information is given.

restoring force same

Case 1



Case 2



Quiz: Determining amplitude

An oscillator with angular frequency of $\omega = 1.00$ s^{-1} has initial displacement of $x_0 = 1.00$ m and initial velocity of $v_0 = 1.72$ m/s. What is the amplitude of oscillation?

Enter the answer in meters, but only include the number.

$A \sin\phi_i = 1$

$A \cos\phi_i = 1.72$

$A^2 = 1^2 + 1.72^2$

$A \approx 1.99$

Fourier analysis of a particular spectrum includes the harmonic frequencies 1113 Hz , 1431 Hz , and 1590 Hz

Part A

What is the fundamental frequency f_1, assuming that $f_1 > 100$ Hz?

Express your answer with the appropriate units.

find the smallest

f_1 = 159 Hz

$1590-1431=159$

$1431-1113=318=159\times2$

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✔ **Correct**

Part A

Which of these forces could result in simple harmonic motion? The positive values of $F(x)$ correspond to the forces acting in the positive x direction.

Check all that apply.

- [x] $F(x) = -5x + 9$
- [ ] $F(x) = 5x$
- [x] $F(x) = -5x$
- [ ] $F(x) = -5x^2$
- [ ] $F(x) = 5x^2$
- [ ] $F(x) = -5(x-9)^2$

restoring force is always proportional to x inverse.

obtain from $x = A\sin(wt+\phi)$

$F=ma=-Aw^2\sin(wt+\phi)$

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✔ **Correct**

Constants | Periodic Table

A gong makes a loud noise when struck. The noise gradually gets less and less loud until it fades below the sensitivity of the human ear. The simplest model of how the gong produces the sound we hear treats the gong as a damped harmonic oscillator. The tone we hear is related to the frequency f of the oscillation, and its loudness is proportional to the energy of the oscillation.

Part A ✔

If the loudness drops to 90 % of its original value in 5.0 s , what is the time constant of the damped oscillation?

Express your answer with the appropriate units.

τ = 47 s

Loudness $= A_0 e^{-\frac{t}{\tau}}$

$e^{-\frac{5}{\tau}} = 0.9$

$\tau = 47s$

Submit Previous Answers

✔ **Correct**

Significant Figures Feedback: Your answer 47.46 s was either rounded differently or used a different number of significant figures than required for this part.

Part B ✔

How long does it take for the sound to be 35 % as loud as it was at the start?

Express your answer with the appropriate units.

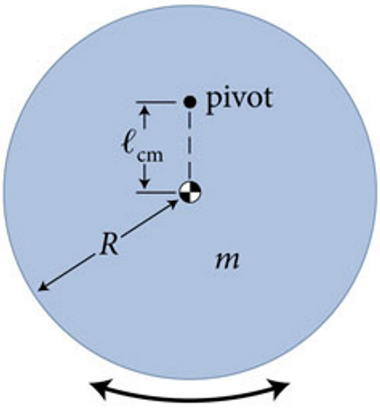
t_{35{\rm %} } = 50 s

$e^{-\frac{t}{47}} = 35\%$

Constants | Periodic Table

A uniform disk of mass m and radius R lies in a vertical plane and is pivoted about a point a distance ℓ_{cm} from its center of mass in (Figure 1). When given a small rotational displacement about the pivot

A uniform disk of mass m and radius R lies in a vertical plane and is pivoted about a point a distance ℓ_{cm} from its center of mass in (Figure 1). When given a small rotational displacement about the pivot, the disk undergoes simple harmonic motion.



$$= \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{mg l_{cm}}{I_{cm}}} \quad T = 2\pi\sqrt{\frac{I_{cm}}{mg l_{cm}}} = 2\pi\sqrt{\frac{\frac{MR^2}{2} + l_{cm}^2 \cdot M}{mg l_{cm}}}$$

$$= 2\pi\sqrt{\frac{R^2 + 2l_{cm}^2}{2mg l_{cm}}}$$

Part A

Determine the period of this motion. Use the notation l_{cm} for the distance ℓ_{cm}.

Express your answer in terms of π, acceleration due to gravity g, some or all of the variables m, R, and l_{cm}.

$$T = 2\pi\sqrt{\frac{R^2 + 2(l_{cm})^2}{2g l_{cm}}}$$

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✔ Correct