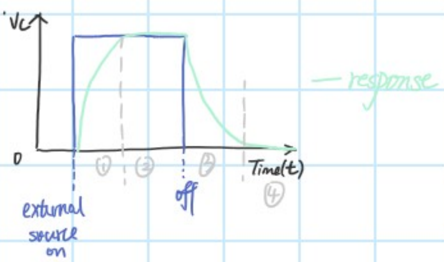
1st order circuit and Inductors

Monday, March 11, 2019 2:42 PM

- 1^{st}- order circuit : Circuit whose voltages and currents are described by 1^{st} order Differential equation.
  - RL circuits $V_L = L\frac{di}{dt}$
  - RC circuits $i_C = C\frac{dv}{dt}$

- Response : Currents and voltages that arise when energy is acquired or released by a circuit
  - Natural Response : No external sources
  - Step Response : Sudden applation of DC current or voltage
  - Under DC sources
    - Transient state
    - Steady-state

e.g.



Step response ①,②

Natural response ③,④

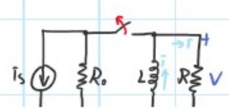
Transient state ①,③

Steady state ②,④

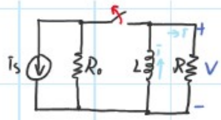
e.g. Natural Response under RL circuit

Switch is open at $t=0$.

$L\cdot\frac{di}{dt} + Ri = 0$



Switch is opened t=0.



$L \cdot \frac{di}{dt} + Ri = 0$

$\frac{di}{i} = -\frac{R}{L} dt$

$\ln i = -\frac{R}{L} t + C$

$i = Ce^{-\frac{R}{L}t} = i(0)e^{-\frac{R}{L}t}$

$i_L(0^-) = i_s = I_0$ (Inductors do not allow instaneous changes in i)

$\therefore$ natural response $= i(t) = I_0 e^{-\frac{R}{L}t}$

$v(t) = i(t) \cdot R = I_0 R e^{-\frac{R}{L}t}$

$V(0^-) = 0$, $V(0^+) = I_s \cdot R = I_0 R$

0: the instant when the switch is open.

when $i_s = 1A$, $R = 10\Omega$, $L = 1mH$

$i(t) = I_0 e^{-\frac{R}{L}t} = e^{-10000t}$ A, $t > 0$

$v(t) = RI_0 e^{-10000t} = 10e^{-10000t}$ V

- $i(t) = I_0 e^{-\frac{R}{L}t}$ for RL circuit.

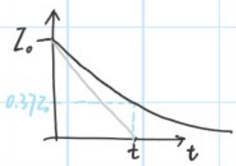
  time constant $\tau = \frac{L}{R}$ $i(t) = I_0 e^{-\frac{t}{\tau}}$ (natural response)

  $\frac{di}{dt} = -\frac{I_0}{\tau} e^{-\frac{t}{\tau}} \Rightarrow \frac{di}{dt}(0^+) = -\frac{I_0}{\tau}$ (slope at t=0)

  tangent: $i_0(t) = i_0 - \frac{i_0}{\tau} t$

  $\downarrow$

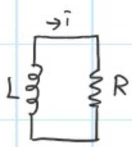
  $i_0(t) = 0$, $t = \tau$



  $t = \tau$, $i(t) = 0.37 I_0$

  $\boxed{t = 5\tau}$, $i(t) < 0.01 I_0 \rightarrow$ steady state

- Power & Energy



Power dissipated in R

$P = Vi = i^2 R = I_0^2 R e^{-\frac{2t}{\tau}}$ $t > 0$

Energy dissipated in R

$W_R = \int^t I_0^2 R e^{-\frac{2t}{\tau}} dt = -\frac{\tau}{2} I_0^2 R e^{-\frac{2t}{\tau}} \Big|^t = \frac{1}{2} L I_0^2 (1 - e^{-\frac{2t}{\tau}})$

Energy dissipated in R

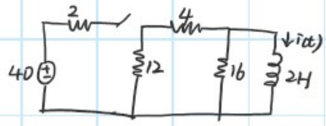
$$W_R = \int_0^t I_0^2 R e^{-\frac{2t}{\tau}} dt = -\frac{\tau}{2} I_0^2 R e^{-\frac{2t}{\tau}} \Big|_0^t = \frac{1}{2} L I_0^2 (1 - e^{-\frac{2t}{\tau}})$$

Energy delivered by L

$$\Delta W_L = W_L(t) - W_L(0) = \frac{1}{2} L i^2(t) - \frac{1}{2} L i^2(0)$$

$$= \frac{1}{2} L I_0^2 (e^{-\frac{2t}{\tau}} - 1)$$

e.g. Calculate $i(t)$. (Swith open from $t=0$)



$t<0$, L behave like a wire

$$i_1(t) = \frac{40}{2+(12\|4)} = \frac{40}{5} = 8A$$

$$i(t) = 8 \cdot \frac{12}{12+4} = 6A, \quad i(0) = i(0^-) = 6A$$

$t \geqslant 0$

$R_{eq} = 16 \| 16 = 8\Omega$

$\therefore i = 6e^{-\frac{t}{4}}$

Response of RC circuits

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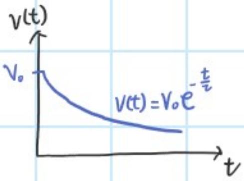
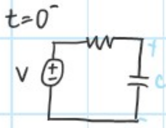
- $V(t) = V_0 e^{-\frac{t}{RC}}$ $t \geqslant 0$, $V_0 = V$ (unit: V) (unitless)

  $\tau = RC$ unit: s^{-1}





  - $i(t) = \frac{V(t)}{R} = \frac{V_0}{R} e^{-\frac{t}{\tau}}$ for $t \geqslant 0$

- *Power & Energy in the resistor*
  - *Power dissipated by R.*

    $$P = Vi = \frac{V_0^2}{R} e^{\frac{-2t}{\tau}}, \quad t \geqslant 0^+$$

  - *Energy dissipated by R:*

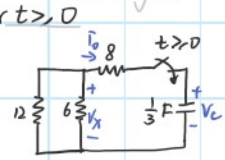
    $$W_R = \int_0^t P dt = \int_0^t \frac{V_0^2}{R} e^{-\frac{2t}{\tau}} dt = \frac{1}{2} C V_0^2 (1 - e^{-\frac{2t}{\tau}})$$

    Energy change by C:

    $$\Delta W_C = W_C(t) - W_0(t) = \frac{1}{2} C V_0^2 (e^{-\frac{2t}{\tau}} - 1)$$

  - $t \to \infty$, $V_C \to 0$, *when fully charged, C acts like short circuit.*

e.g. $V_C(0) = 60V$. find $V_C(t)$, $V_x(t)$, $i_o(t)$ for $t \geqslant 0$ (switch is open)



$R_{eq} = (12 // 6) + 8 = 4 + 8 = 12\Omega$

$\tau = RC = 12 \times \frac{1}{3} = 4$

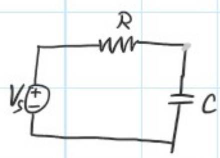
$V_C(t) = 60 e^{-0.25t}$ V

$V_x(t) = V_C(t) \cdot \frac{12//6}{12} = 20 e^{-0.25t}$

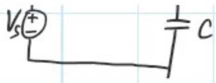
$i_o(t) = C \cdot \frac{dV_C}{dt} = -\frac{V_C}{12}$

- *Step response of R-C*
  - $V_C(t) = V_s + (V_o - V_s) e^{-\frac{t}{\tau}}$, $\tau = RC$

    V_o: *initial voltage*



$V_C(t) = V_S + (V_0 - V_S)e$, $\tau = RC$



V_0 · initial voltage

$$\frac{V_c(t) - V_s}{R} + C\frac{dV}{dt} = 0$$

current direction

$$\frac{dV_c}{V_c - V_s} = -\frac{1}{RC}dt \rightarrow \int_{V_0}^{V_c(t)} \frac{dx}{x - V_s} = -\frac{1}{RC}\int_0^t dy$$

$$V_c(t) = V_s + (V_0 - V_s)e^{-\frac{t}{RC}} = V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, \ \tau = RC$$

$$i_0 = \frac{V_s - V_0}{R}, \ i(\infty) = 0$$

$$i(t) = \frac{V_s - V_0}{R}e^{-\frac{t}{\tau}}$$

Step Response of RL

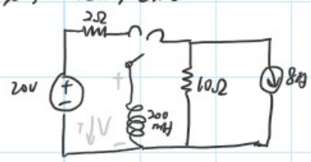
Friday, March 15, 2019 2:45 PM

- $I(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-\frac{R}{L}t} = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-\frac{t}{\tau}}$

  $V_L = (V_s - I_0 R)e^{-\frac{t}{\tau}}$

e.g. Assume that the switch in the circuit has been in position b for a long time and at $t=0$ it moves to position a.

Find (a) $i(0^+)$; (b) $v(0^+)$; (c) τ; (d) $i(t), t \geq 0$; (e) $v(t)$, $t \geq 0^+$



(a) inductor act as short.

$i(0^-) = \frac{24A}{2\Omega} = 12A$

(b)

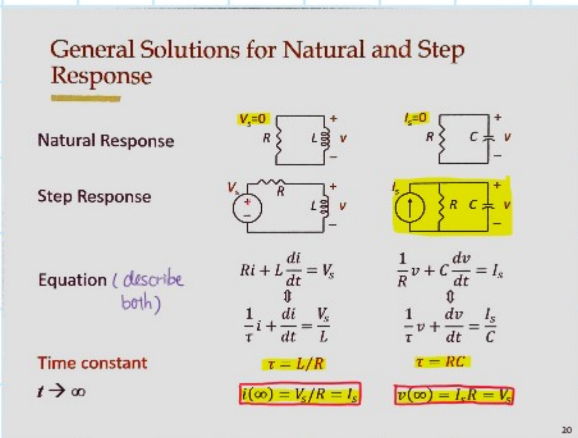
$V(0^+) = -10(8+12) = -200V$

(c) $\tau = 20mS = \frac{200m}{10\Omega} = 20mS$

(d) $i(t) = i_f + [i(0^+) - i_f]e^{-\frac{t}{\tau}}$, $i_f = -8A$

$i(t) = -8 + 20e^{-50t}$ A

(e) $v(t) = L \cdot \frac{di}{dt} = -200e^{-50t}$



- All equations are of the form $\frac{x}{\tau} + \frac{dx}{dt} = K$
  - τ: time constant K: constant
- final steady state is reached with $\frac{dx_f}{dt} = 0 \rightarrow x_f = K\tau$

- final steady state is reached with $\frac{dx_f}{dt} = 0 \rightarrow x_f = K\tau$

$x(t)$ changes exponentially from $x(t_0)$ to x_f at time constant τ

$\rightarrow x(t)$ contains $(x(t_0) - x_f)e^{-\frac{t-t_0}{\tau}}$

$$x(t) = x(\infty) + (x(t_0) - x(\infty))e^{-\frac{t-t_0}{\tau}}$$

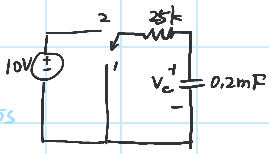
Sequential Switching

Friday, March 15, 2019 3:12 PM

- When there're more than one switching operation.

e.g. Switch move to 2 at $t=0$ (previous 1). Then return to 1 at $t=4$s.

find $v_c(t)$, $t \geq 0$



$t \in [0, 4]$, $V_c(\infty) = 10$

$V_c(0) = 0$, $\tau = 25k \times 0.2m = 5s$

$V_c(t) = 10(1 - e^{-\frac{t}{5}})$

$(V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty))e^{-0.2t})$

$t \geq 4$

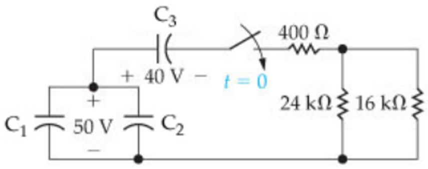
$V_c(4) = 10(1 - e^{-0.8t}) = 5.5$, $V_c(\infty) = 0$

$V_c(t) = 5.5e^{-\frac{t}{5}}$

Prob

Monday, March 18, 2019 12:57 AM

At the time the switch is closed in the circuit in (Figure 1), the voltage across the paralleled capacitors is 50 V and the voltage on the C_3 capacitor is 40 V. Take that C_1 = 470 nF, C_2 = 970 nF and C_3 = 510 nF.



What percentage of the initial energy stored in the three capacitors is dissipated in the 24 $\mathrm{k\Omega}$ resistor?

Express your answer using two decimal places.

▸ **View Available Hint(s)**

% diss = 0.33 %

What percentage of the initial energy stored in the three capacitors is dissipated in the 400 Ω resistor?

Express your answer using two decimal places.

▸ **View Available Hint(s)**

% diss = 0.03 %

Part A.

$$E_{tot} = \frac{1}{2}(C_1+C_2)50^2 + \frac{1}{2}C_3 40^2 = 2.208\times10^{-3} W$$

$$C_{eq} = \frac{(470+970)\times510}{470+970+510} = 376.6\times10^{-9} F$$

$$R_{eq} = 9.6+0.4 = 10 k\Omega \qquad \therefore \tau = C_{eq}\cdot R_{eq} = 3.766\times10^{-3} s$$

1° remember to time C. $i = C\frac{dv}{dt}$

$$V_{eq} = 50-40 = 10V$$

2° remember to divide current.

$$v(t) = 10e^{-266t}$$

$$i_{24k}(t) = 376.6\times10^{-9}\times(-266)\times10e^{-266t}\times\frac{16}{16+24} = 4\times10^{-4}e^{-266t}$$

$$P = i_{24k}^2 R = \overset{16\times10^{-8}}{\cancel{10^{-6}}}\times24\times10^{3} e^{-532t} = \overset{3.84}{\cancel{24}}\times10^{-3}e^{-532t}$$

$$W_{24k} = \int_0^\infty P dt = \frac{24\times10^{-3}}{-532}e^{0} = \overset{7.2\times10^{-6}}{\cancel{4.5\times10^{-5}}}$$

$$\frac{W_{24k}}{E_{tot}}\% = \frac{7.2\times10^{-6}}{2.208\times10^{-3}}\times100 = 0.33\%$$

B. $i_{400} = 10^{-3} e^{-266t}$

$i^2R = 10^{-6} \times 400 e^{-532t} = 4\times10^{-4} e^{-532t}$

$W_{400} = \int_0^\infty 4\times10^{-4} e^{-532t} = \frac{4\times10^{-4}}{-532} = 7.52\times10^{-7} J$

$\frac{W_{400}}{E_{tot}} = \frac{7.52\times10^{-7}}{2.208\times10^{-6}} \times 100 = 0.03\%$

C. $i_{16k} = 10^{-3} e^{-266t} \times \frac{24}{24+16} = 6\times10^{-4} e^{-266t}$

$i^2R = 3.6\times10^{-7} e^{-532t} \times 16\times10^{3} = 5.76\times10^{-3} e^{-532t}$ W

$W_{16k} = \int_0^\infty 5.76\times10^{-3} e^{-532t} = \frac{5.76\times10^{-3}}{-532} = 1.08\times10^{-5} J$

$\frac{W_{16k}}{E_{tot}} = \frac{1.08\times10^{-5}}{2.208\times10^{-3}} \times 100 = 0.49\%$

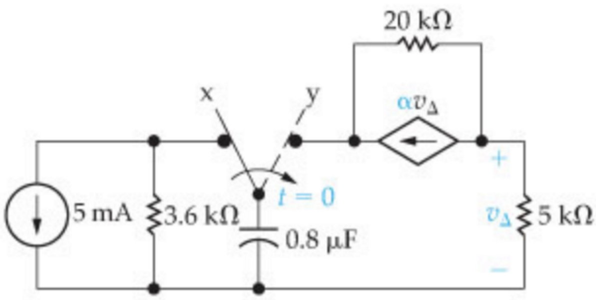
D. $1 - 0.49\% - 0.33\% - 0.03\% = 0.9915 = 99.15\%$

Review | Constants

The switch in the circuit seen in the figure has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y. (Figure 1)

Figure

1 of 1



Part A

Find α so that the time constant for $t > 0$ is 40 ms.

Express your answer to three significant figures and include the appropriate units.

$\alpha = 2.50\times10^{-4}\ \frac{\text{A}}{\text{V}}$

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✔ Correct

Part B

For the found α, find v_Δ.

Express your answer in volts in terms of t, where t is in seconds.

$v_\Delta = -\frac{9}{5}e^{-\frac{t}{4\cdot10^{-2}}}\ \text{V}$

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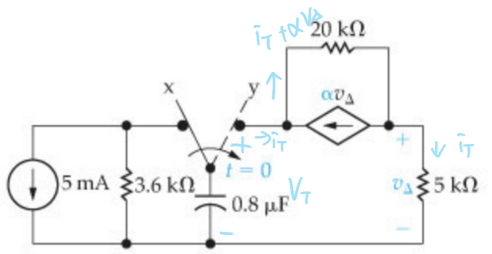

A.

$$\begin{cases} V_T = 20k(i_T + \alpha V_\Delta) + 5k\, i_T \\ V_\Delta = 5k\, i_T \end{cases}$$

$\therefore\ V_T = 25k\, i_T + \alpha 10^2 k\, i_T$

$\therefore\ R_{Th} = \frac{V_T}{i_T} = 25k + \alpha 100k^2$

$R_{Th} \cdot C = 4\times10^{-2}$

$R_{Th} = 25k + \alpha 100k^2 = 50k$

$\alpha = \frac{25k}{100k^2} = 2.5\times10^{-4}\ A/V$

B.

$V_0 = 18V$

$V_c(t) = 18e^{-\frac{t}{40m}}$

$V_\Delta = -18\times\frac{5}{50}e^{-\frac{t}{40m}}$