Laplace

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- Warm-up: $\mathcal{L}\{e^{7t}\}$

$= \int_0^\infty e^{-st} \cdot e^{7t} \cdot dt$

$= \lim_{A\to\infty} \int_0^A e^{-st} e^{7t} dt$

$= \lim_{A\to\infty} \frac{e^{7t} \cdot e^{-st}}{7} + \frac{s}{7}\int e^{-st} e^{7t} \cdot dt$

$= \lim_{A\to\infty} \frac{e^{7t} \cdot e^{-st}}{7} \cdot \frac{1}{1-\frac{s}{7}} \Big|_0^A dt$

$s > 7$

$= \frac{1}{7} \times \frac{1}{\frac{s}{7}-1}$

$= \frac{1}{7} \times \frac{7}{s-7}$

$= \frac{1}{s-7}$

Turns out $\mathcal{L}$ is a linear operator.

$\mathcal{L}\{c \cdot f(t) + d g(t)\} = c \cdot \mathcal{L}\{f(t)\} + d\,\mathcal{L}\{g(t)\}$

e.g. $\mathcal{L}\{8 + 6t - \sin(3t) + e^{2t}\}$

$= 8\mathcal{L}\{1\} + 6\mathcal{L}\{t\} - \mathcal{L}\{\sin 3t\} + \mathcal{L}\{e^{2t}\}$

$= \frac{8}{s} + \frac{6}{s^2} - \frac{3}{9+s^2} + \frac{1}{s-2}$

- c is constant.

$\mathcal{L}\{c\} = \frac{c}{s}$

$\mathcal{L}\{ct\} = \frac{c}{s^2}$

$\mathcal{L}\{\sin ct\} = \frac{c}{s^2+c^2}$

$\mathcal{L}\{e^{ct}\} = \frac{1}{s-c}$ $(c>0)$

$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ $s>0$

$\mathcal{L}\{\cos ct\} = \frac{s}{s^2+c^2}$

- e.g. $\mathcal{L}\{t^n e^{at}\}$

$= \int_0^\infty t^n e^{at} e^{-st}$

e.g. $\mathcal{L}\{t^n e^{at}\}$

$= \int_0^\infty t^n e^{at} e^{-st}$

$= \lim_{A\to\infty} \int_0^A t^n e^{at-st}\,dt$

$$\begin{array}{ll}
t^n & e^{(a-s)t} \\
n t^{n-1} & \frac{e^{(a-s)t}}{a-s} \\
n(n-1)t^{n-2} & \frac{e^{(a-s)t}}{(a-s)^2} \\
n!\,t \quad \cdots & \frac{e^{(a-s)t}}{(a-s)^n} \\
n! & \frac{e^{(a-s)t}}{(a-s)^{n+1}}
\end{array}$$

$\sum_{i=0}^{n} (-1)\,(n-i)!\,t^i \frac{e^{(a-s)t}}{(a-s)^{n+1-i}}$

$s > a$

$t=A$, $t=0$

$= 0 - n!\frac{1}{(a-s)^{n+1}}$

$= \frac{n!}{(s-a)^{n+1}}$

Similarly,

$\mathcal{L}\{e^{at}\cdot \sin(bt)\}$

$= \frac{b}{(s-a)^2+b^2} \quad s > a.$

Whenever multiply a e^{at}, substitute s with $s-a$

- Inverse Laplace

bijection between function & their Laplace.

{injective, surjective}

$\mathcal{L}\{f(t)\} = F(s)$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$

e.g. $\mathcal{L}^{-1}\{\frac{s+3}{s^2+9}\}$

$\mathcal{L}^{-1}\{\frac{s}{s^2+9} + \frac{3}{s^2+9}\}$

$= \cos 3t + \sin 3t$

e.g. $\mathcal{L}^{-1}\{\frac{7s^2-20s+125}{s(s^2+25)}\}$ Partial Fraction.

$= \mathcal{L}^{-1}\{\frac{A}{s} + \frac{B}{s^2+25}\}$

$As^2+25A+Bs = 7s^2-20s+125$

$25A = 125 \Rightarrow A = 5$

$Bs = 2s^2-20s$

$B\cdot s = s(2s-20)$

$B = 2s - 20$

$$= \mathcal{L}^{-1}\left\{ \frac{5}{s} + \frac{2s}{s^2+25} + \frac{-20}{s^2+25} \right\}$$

$$= 5 + 2\cos 5t - 4\sin 5t$$

6.2 HW

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8. **2/2 points** | Previous Answers

Compute $F(s)$, the Laplace transform of $f(t) = e^{1t}(t^2 + 3t + 5)$

$$F(s) = \boxed{\frac{5}{s-1} + \frac{3}{(s-1)^2} + \frac{2}{(s-1)^3}}$$ ✓

7. **2/2 points** | Previous Answers

Find the inverse Laplace transform, $f(t)$, of the function:

$$F(s) = \frac{18}{(s-3)^3}$$

$$f(t) = \boxed{9e^{3t}t^2}$$ ✓