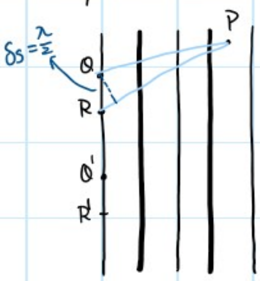
# 34.1 diffraction of light

Wednesday, February 6, 2019 8:22 PM

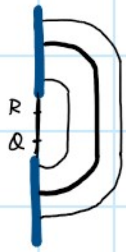
- Dual Nature of Light : Wave + Particle
- Diffraction is observed with light waves but only when the width of the opening through which the light passes is not much greater than the wavelength of the light.
    - Visible light, $\lambda$ on the order of $1\mu m$, diffracts through apertures up to hundreds of $\mu m$.

- Situation that produce diffraction.
    - A planar wave (Width of wavefront much greater than $\lambda$)




- Since $\delta s = \frac{\lambda}{2}$, the wavelets centered on P, Q cancel each other.
- On a planar wavefront, many pairs of points like P, Q can be found.
- the only place that will not be canceled is the point where P ⊥ wavefront.
- Under this condition, diffraction will not occur.
- The intensity is also uniform.

    - A barrier ( Width > wavefront)




Only a part of wavefront can go through.

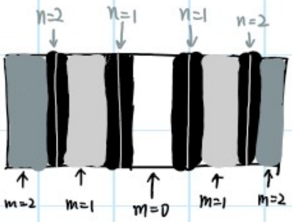
# 34.2 Diffraction Gratings

Thursday, February 7, 2019 10:45 AM

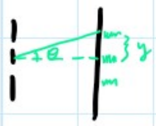
- When electromagnetic wave of a laser beam pass through a pair of narrow slits ( Width < $\lambda$)


  - Interference fringes the dark and bright bands appears on the screen place toward the slits.
  - The bright fringes are labeled by fringe order m.

  The central bright fringe is zeroth-order bright fringe (m=0)




  - dark fringes order denoted by n




  - $d\sin\theta_m = \pm m\lambda$ to locate the place of bright fringes. ($m \in \mathbb{Z}$)

    d: distance between slits. D: distance between fringes. $\theta_m \uparrow, D\uparrow$

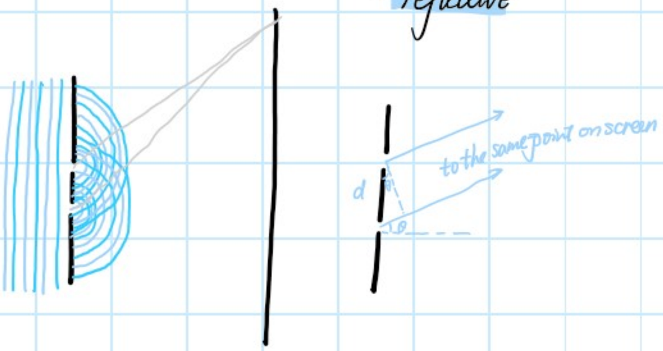
- There're two dark fringes between each pair of bright fringes.
  - Principal Maxima: The brightest fringes
  - Secondary Maxima: The fainter fringes. (as wide as the dark fringes)
  - If there're nth slits, the condition for complete destructive interference is that each wave must differ in phase by $\frac{1}{N}$ of a cycle from its immediate neighbors.
  - $d\sin\theta_k = \pm\frac{k}{N}\lambda$ ($\frac{k}{N} \notin \mathbb{Z}$, or it will be constructive)

    N: number of slits.
  - There're N-1 dark fringes between bright fringes & N-2 secondary maxima between each pair of principal maxima.

- Diffraction Grating: A barrier that contains a very large number of slits.
  - The interference of a planar electromagnetic wave as it passes through many closely spaced narrow slits is due to the diffraction occurs at the slits.

diffraction occurs at the slits.

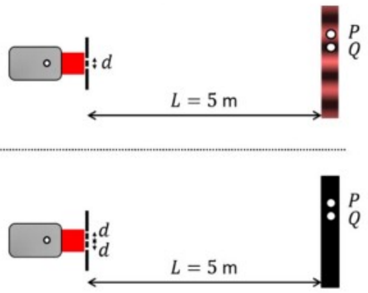
- Diffraction Grating { transmissive / reflective




★★★★

**many choice question**

As shown in the top image, light with wavelength λ incident on two slits separated by a very small distance d produces an interference pattern on a screen 5 m away. Point P is the 1st maxima and point Q is the 1st minima. Now a third slit, also separated by a distance d, is added. Which of the following statements are correct for the new setup? Select all that apply.




A.

Point P is still the 1st maxima

B.

Point P is now the 2nd maxima

C.

Point P is now the 1st minima

D.

Point Q is now the 1st maxima

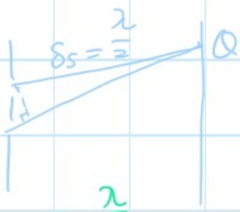
E.

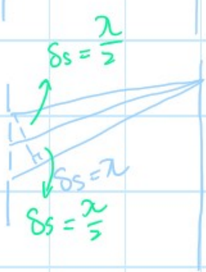
Point Q is still the 1st minima

F.

Point Q is now a secondary maxima

AF θ doesn't change.

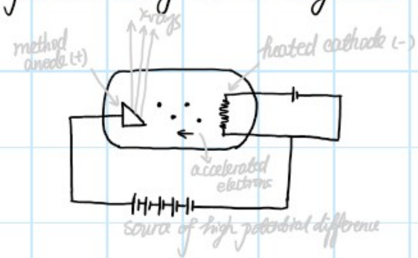



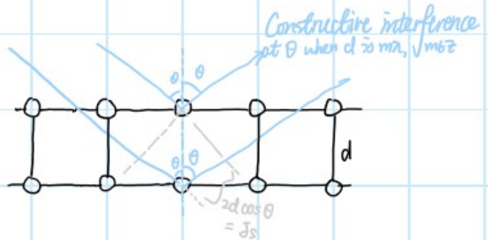
# 34.3 x-ray diffraction

Tuesday, February 12, 2019 10:35 AM

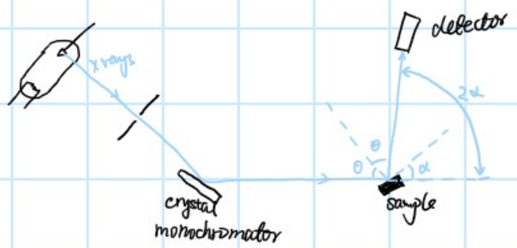
- X-rays : electromagnetic waves that have wavelengths ranging from 0.01nm to 10nm. (more than 100 times less than λ of visible light)
    - produced by cathode ray tube




- x-rays can be used to study atomic arrangements in solid.
    - Many solids 're crystalline - atoms arranged in 3-dim
    - crystal lattice regularly spaced grid.
    - Interference of x-rays diffracted by adjacent planes of a crystal




- Bragg condition $2d\cos\theta = m\lambda,\ m \in \mathbb{Z}$ ⇒ constructive interference
    - only at θ, strong diffraction will occur. otherwise weak diffraction.
- Measure α to see atomic arrangements




slightly rotate the sample, measure the intensity on detector

Bragg angle α : angle between x-rays and the sample surface.

**Bragg angle α**: angle between x-rays and the sample surface.

$2d\cos\theta = m\lambda$ , $2\theta = 180 - 2\alpha \Rightarrow \theta = 90 - \alpha$

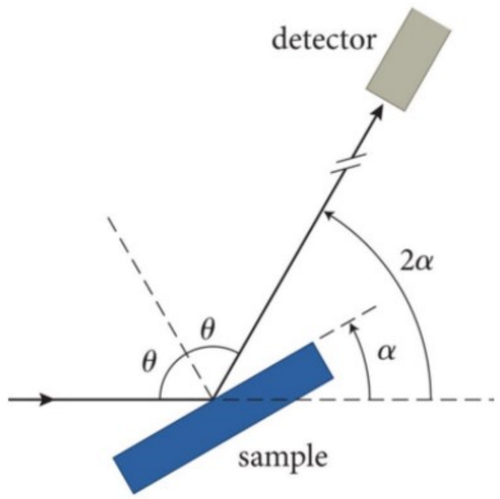
$\because \cos\theta = \sin\alpha$

$\therefore$ $2d\sin\alpha = m\lambda$

e.g.

**multiple choice question**

X-rays with a wavelength of $2.0\times10^{-10}$ m are incident on a crystal lattice sample as shown. The smallest Bragg angle at which $20°$
Which of the following statements is correct?




A.

The atoms in the crystal have a diameter of $1.1\times10^{-10}$ m

B.

The atoms in the crystal have a diameter of $3.0\times10^{-10}$ m

C.

The planes of the crystal are separated by $1.1\times10^{-10}$ m

D.

The planes of the crystal are separated by $3.0\times10^{-10}$ m

$2d\sin\alpha = \lambda$

$d = 2.9\times10^{-10}$ m

D

# 34.4 matter waves

Tuesday, February 12, 2019 11:29 AM

- Electrons behave both like waves & particles.
  - Electron $\lambda$ depends on speed.

    $v\uparrow, \lambda\downarrow$

    spacing between spots $\propto \lambda$

- Wave-particle duality: the possession of both wave properties and particle properties
  - be observed for all subatomic particles
  - de Broglie wavelength $\lambda = \frac{h}{p}$, $h = 6.626 \times 10^{-34}\ J \cdot s$

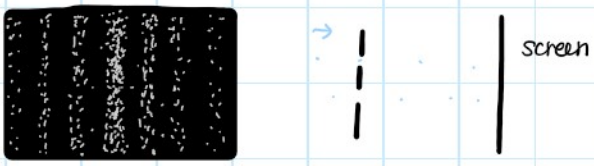
    $h$ → Planck's constant

# 34.5 Photons

Thursday, February 14, 2019 11:33 AM

- Photons: Particles of light; basic unit of a light wave & carries a certain amount of energy.
  - Energy of a photon: hf, f: the frequency of the photon.
  - Photons represent the quantum of electromagnetic energy.

    (Cannot be subdivided)




  - if it's detector on the slit, each photon is detected by either of the slit;

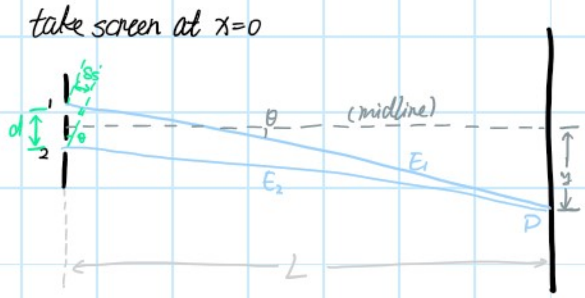
    but the photon passes through both slits simultaneously.

# 34.6 Multiple-slit interference

Thursday, February 14, 2019 11:48 AM

- Path difference $\delta s$

take screen at $x=0$




$\therefore$ The electric field of the two waves:

$E_1 = E_0 \sin \omega t$

$E_2 = E_0 \sin(\omega t + \phi)$

$\because \frac{\phi}{2\pi} = \frac{\delta s}{\lambda}, \quad \delta s = d\sin\theta$

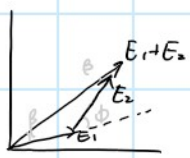
$\therefore \phi = \frac{2\pi}{\lambda} \cdot d\sin\theta$

for bright fringe $2d\sin\theta_m = m\lambda, \ m \in \mathbb{Z}$

$\therefore \phi = \pm m \cdot 2\pi, \quad \sin\theta_m = \frac{m\lambda}{2d}$

for dark fringe,

$\phi_n = \pm(2n-1)\pi$, plug in, $\sin\theta_n = \pm\frac{(n-\frac{1}{2})}{d}\lambda$




$\phi = 2\beta$

$\vec{E}_1 + \vec{E}_2 = \vec{E}_{12} = 2E_0 \cdot \cos\beta = 2E_0 \cos\frac{\phi}{2}$

$\because S = \frac{1}{\mu_0} EB = \frac{E(E/c_0)}{\mu_0}$

$\therefore S = \frac{4E_0^2}{\mu_0 c_0} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \cdot \sin^2 \omega t.$

The maximum intensity of interference pattern:

$S_{av} = 4S_{0,av} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

twice the sum of two interfering waves.

$\approx 4S_{0,av} \cdot \cos^2\left(\pi d \frac{y}{L\lambda}\right)$

$\sin\theta \sim \tan\theta = \frac{y}{L}$

$\sin\theta \approx \tan\theta = \frac{y}{L}$

- Distance D between adjacent intensity maxima of this pattern

  $D = \frac{L}{d}\lambda$

  $m\lambda = d\frac{y_m}{L}$, $(m+1)\lambda = d\frac{y_{m+1}}{L}$

  $D = y_{m+1} - y = (m+1-m)\cdot\lambda\cdot L/d = \frac{L}{d}\lambda$

**multiple choice question**

As shown, light with wavelength 532 nm incident on a diffraction grating (many slits with the same separation) produces an interference pattern on a screen. If we used a white light instead, which of the following statements is correct?

A. All the maxima would be white.

B. The central maxima would be white, but the others would form a rainbow.

C. All the maxima would form a rainbow.

θ changed since λ changed. B.

- Resolved the wavelengths that can be distinguished from one another in the spectrum.
  - Resolution: the smallest wavelength difference that can be distinguished.

# 34.7 Thin-film Interference

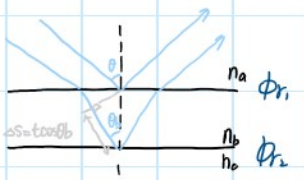
Thursday, February 14, 2019 4:03 PM

- **film**: a thin, transparent material whose thickness is comparable to the wavelengths of visible light.

  - film
    - suspend in air
    - supported on a much thicker material with a different index of refraction.

  - film thickness $t$

    index of refraction $n_b$

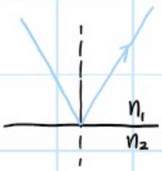



$\phi = \dfrac{4\pi n_b t \cdot \cos\theta_b}{\lambda} + \phi_{r_2} - \phi_{r_1}$ $\Rightarrow$ $\phi_{r_1}, \phi_{r_2}$ are either $0$ or $\pi$

$\lambda$ in vaccum.

because of phase shift.

- Normal incidence $\phi = \dfrac{4\pi n_b t}{\lambda} + \phi_{r_2} - \phi_{r_1}$




- $n_1 < n_2$ : A phase shift of $\pi$ occurs

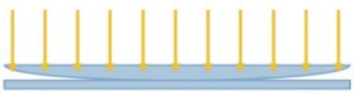
  $\because n_1 < n_2 \therefore C_2 < C_1, \rho_2 > \rho_1$

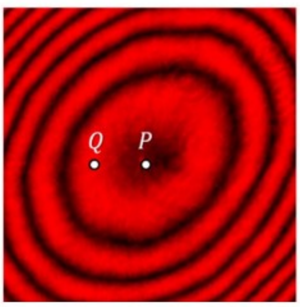
  $\therefore$ wave is inverted due to fixed end

  $n_1 > n_2$, nothing changes.

☆☆☆

Light shines down on a curved piece of glass that rests on a flat glass as shown. The bottom image shows the reflected light. Point P has minimum intensity and point Q has maximum intensity. Which of the following statements is/are correct? Select all that apply.






A. At point P, $\phi=0$

B. At point P, $\phi=\pi$

C. At point P, $\phi=2\pi$

D. At point Q, $\phi=0$

E. At point Q, $\phi=\pi$

F. At point Q, $\phi=2\pi$

C. At point P, ϕ=2π

D. At point Q, ϕ=0

E. At point Q, ϕ=π

F. At point Q, ϕ=2π

P: $\phi = \pi$ for destructive interference. BF

Q: $\phi = 0$ or $2\pi$ ⇒ P is the least of phase shift, ∴ only $2\pi$

if plug in: $\phi = \frac{4\pi n t}{\lambda} + \phi_{r_1} - \phi_{r_2}$, $t=0$

$\phi = \phi_{r_1} - \phi_{r_2} = \pi$, $\frac{4\pi t n}{\lambda} > 0$ ∴ only can increase.

# 34.8 diffraction at a single-slit barrier

Tuesday, February 19, 2019 8:33 AM

- General condition for dark fringes

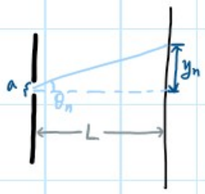
  $\sin\theta_n = \pm n\frac{\lambda}{a}$ $(n \in \mathbb{Z})$ $a$: width of aperture

  - $\sin\theta_n > 0$ : dark fringes above middle line

    $\sin\theta_n < 0$ : dark fringes below middle line

- $y_n = \pm n\frac{\lambda L}{a}$

  $y_n = L \cdot \tan\theta_n \approx L \cdot \sin\theta = \pm L \cdot n \cdot \frac{\lambda}{a}$

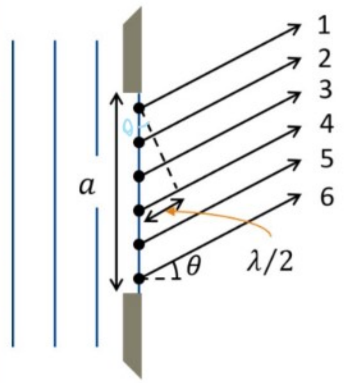



  - distance between fringes increases with decreasing $a$.

    $a \downarrow$, more wave spreads out after passing through the slit.

  - If $a < \lambda$, treat as pt source, no diffraction will occur.

Parallel wavefronts are incident on a slit of width a, as shown. Consider the six wavelets going at an angle θ. The path length difference of the wavelet pair (1, 4) is λ/2. Do the wavelets cause a bright fringe, dark fringe, or neither on a screen a distance L away, where L is much much larger than d?




A.
Bright fringe

B.
Dark fringe

C.
Neither

$\frac{\lambda}{6}$

$\Downarrow$

$\frac{5}{6}\lambda$

$\frac{4}{6}\lambda \Rightarrow \frac{2}{3}\lambda$

$\frac{3}{6}\lambda \Rightarrow \frac{\lambda}{2}$

B.

$\therefore n \in \mathbb{Z}$ , $\therefore$ dark fringe.

# 34.9 Circular apertures and limits of resolution

Tuesday, February 19, 2019 8:49 AM

- When light passes through circular aperture, diffraction is also circular.
  - Airy disk: central bright fringe
    - $\sin\theta_1 = 1.22\frac{\lambda}{d}$ the first dark fringe

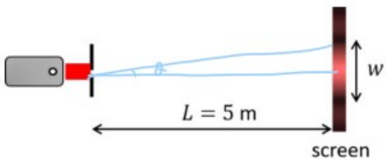
- Rayleigh's criterion: condition for distinguishability (angular separation between the two objs is at least $\theta_1$
  - minimum angular separation $\theta_r \approx \sin\theta_r = 1.22\frac{\lambda}{d}$ (in radian)

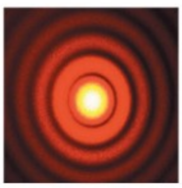


  - when $\theta \geq \theta_r$, then two obj satisfy Rayleigh's criterion
  - The smallest diameter to which light can be focused is about $2.5\lambda$
  - Diffraction limit: the diffraction-determined minimum size of features in an image.

**numerical question**

A red laser with wavelength λ=650 nm illuminates a hole with diameter d=0.50×10$^{-3}$ m. What is the width of the central maximum, w, on a screen L=5.0 m away? Answer in meters, but only include the number.
Enter 3.14E-15 for 3.14×10$^{-15}$.



$\frac{W}{2} = L\cdot\tan\theta$

$\sin\theta = \frac{\lambda}{d}\cdot 1.22$

$\theta = \sin^{-1}\left(\frac{\lambda}{d}\cdot 1.22\right)$

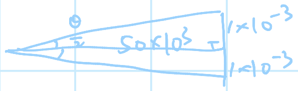
$\therefore W = 1.6\times10^{-2}\,m$

Session 46332343

**numerical question**

A spy satellite is 50.0 km above Earth's surface. What diameter must the lens be so that it can resolve objects 2.00 mm apart? Assume the light has a wavelength of 400 nm.
Enter your answer in meters, but only include the number.

$\theta$ is small, $\therefore \theta \approx \tan\theta \approx \sin\theta$




$$\theta = 2 \times \frac{10^{-3}}{50 \times 10^{3}}$$

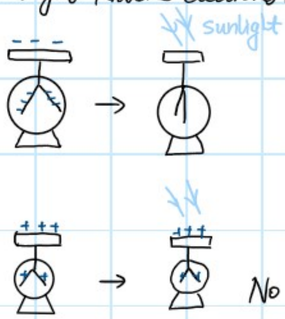
$$\frac{1.22\lambda}{d} = \theta$$

$$d = \frac{1.22\lambda}{\theta} = 12.2\,m.$$

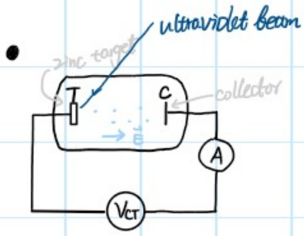
# 34.10 Photon Energy & Momentum

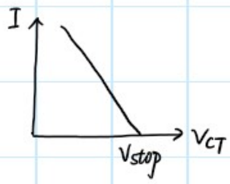
Tuesday, February 19, 2019 10:40 AM

- Photoelectric effect
  - Light knocks electrons out of the zinc plate




No effect, because opposite-charge attraction causes the ejected electrons to be attracted back to plate.

- 



The reason that UV light blocker blokes the effect.

Low-frequency light doesn't eject electrons

  - $V_{CT} \geq V_{stop}$, there'll be no current.

    ∴ Maximum kinetic energy $K_{max}$ doesn't depend on light intensity.

  - K is different for each electron.

    (initial location different)

  - $K_{max} = eV_{stop}$

    $\Delta K = -eV_{CT} = K_f - K_i \quad \because K_f = 0$

    $\therefore \Delta K = -K_i$

    $K_{MAX} = K_i = eV_{CT}$

- $E_{photon} = hf = K_{max} + E_0$
  - $E_0$: work-function minimal energy to free the electron
    - Is a property of metal (doesn't change if metal doesn't change)

- $E_0$: work function minimum energy to free the electron
    - Is a property of metal (doesn't change if metal doesn't change)
- To determine the Planck's constant, use

$$V_{stop} = \frac{h}{e} \cdot f - \frac{E_0}{e}$$

- $p_{photon} = \frac{h}{\lambda_{photon}} = \frac{h f_{photon}}{c_0} = \frac{E_{photon}}{c_0}$

plug in $E^2 - (c_0 p)^2 = (m c_0^2)^2 = 0$

$\therefore$ ○ $m_{photon} = 0$

$\therefore$ ○ Photons have no internal energy, no internal structure

e.g. A photon enters a piece of glass, $n_{glass} = 1.5$. What happens to its

(1) speed — (1) speed $\downarrow$, $\because n \uparrow$

(2) frequency — (2) $f$ doesn't change $\Rightarrow$ source doesn't change.

(3) wavelength — (3) $c = \lambda f$, $c \downarrow$, $\lambda \downarrow$

(4) energy — (4) $E = hf$, doesn't change.

# Prob

Monday, February 11, 2019 4:45 PM

Constants | Periodic Table

Planar waves from a monochromatic light source are normally incident on a circular obstacle, which casts a shadow on a screen positioned behind the obstacle.

**Part A**

What do the wave properties of light predict about how dark the center of the shadow is?

- The center of the pattern should be brighter than its surrounding.
- The center of the pattern should be darker than its surrounding.

Submit Previous Answers

**Correct**

Since all rays diffracting around the edge of the circular obstacle are equidistant from the center, all the waves should strike the center in phase, and therefore interfere constructively.

Constants | Periodic Table

You shine a red laser beam on a diffraction grating and then shine a green laser beam on the grating.

**Part A**

Is the spacing of the bright fringes for the red beam greater than, smaller than, or equal to the spacing of the bright fringes for the green beam?

- equal
- greater
- smaller

Submit Previous Answers

**Correct**

Red light has a longer wavelength, so it will be diffracted to a larger angle than green light, for a given order of bright fringe. Since this holds for any value of $m$, the spacing for the bright fringes of red light is larger than that of green light, throughout the pattern.

$d\sin\theta = m\lambda$

$\sin\theta = \frac{m\lambda}{d}$

$\lambda\uparrow, \sin\theta\uparrow, D\uparrow$

$\lambda\uparrow, \theta\uparrow$

spacing between bright fringes $= \lambda \cdot \frac{\text{screen distance}}{\text{slit width}}$

$D = \frac{\lambda L}{a}$

Constants | Periodic Table

You are designing a thin transparent reflective coating for the front surface of a sheet of glass. The index of refraction of the glass is 1.52, and when it is in use the coated glass has air on both sides. Because the coating is expensive, you want to use a layer that has the minimum thickness possible, which you determine to be 104 nm.

**Part A**

What should the index of refraction of the coating be if it must cancel 540-nm light that hits the coated surface at normal incidence?

$n =$ 1.30

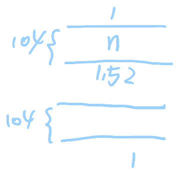
Submit Previous Answers

**Correct**

Provide Feedback

$n = \frac{L}{4t} = \frac{540}{4\times104} = 1.30$

if there's a phase change, n will be too large.




Constants | Periodic Table

A scientist notices that an oil slick floating on water when viewed from above has many different rainbow colors reflecting off the surface. She aims a spectrometer at a particular spot and measures the wavelength to be 750 nm (in air). The index of refraction of water is 1.33.

**Part A**

The index of refraction of the oil is 1.20. What is the minimum thickness $t$ of the oil slick at that spot?

**Express your answer in nanometers to three significant digits.**

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**Hint 1.** Thin-film interference

In thin films, there are interference effects because light reflects off the two different surfaces of the film. In this problem, the scientist observes the light that reflects off the air-oil interface and off the oil-water interface. Think about the phase difference created between these two rays. The phase difference will arise from differences in path length, as well as differences that are introduced by certain types of reflection. Recall that if the phase difference between two waves is $2\pi$ (a full wavelength) then the waves interfere constructively, whereas if the phase difference is $\pi$ (half of a wavelength) the waves interfere destructively.

**Hint 2.** Path-length phase difference

The light that reflects off the oil-water interface has to pass through the oil slick, where it will have a different wavelength. The total "extra" distance it travels is twice the thickness of the slick (since the light first moves toward the oil-water interface, and then reflects back out into the air).

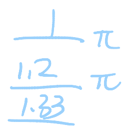
**Hint 3.** Phase shift due to reflections

Recall that when light reflects off a surface with a higher index of refraction, it gains an extra phase shift of $\pi$ radians, which corresponds to a shift of half of a wavelength. What used to be a maximum is now a minimum! Be careful, though; if two beams each reflect off a surface with a higher index of refraction, they will both get a half-wavelength shift, canceling out that effect.

Constructive interference ✓

$2\pi = \frac{4\pi n t}{\lambda}$

$t = \frac{\lambda}{2n} = \frac{750\,\text{nm}}{2\times1.2} = 313\,\text{nm}$




$t =$ 313 nm

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**Correct**

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✓ Correct

$$t = \frac{\lambda}{2n} = \frac{750nm}{2\times1.2} = 313nm$$

$\frac{1.2}{1.33}$ $\pi$ $\pi$

Part B

Suppose the oil had an index of refraction of 1.50. What would the minimum thickness $t$ be now?

**Express your answer in nanometers to three significant digits.**

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$t =$ 125 nm

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✓ Correct

there's phase shift

$$\pi = \frac{4\pi n t}{\lambda}$$

$$t = \frac{\lambda}{4n} = \frac{750nm}{4\times1.5} = 125nm$$

$\frac{1}{1.5}$ / $1.33$

Part C

Now assume that the oil had a thickness of 200 nm and an index of refraction of 1.5. A diver swimming underneath the oil slick is looking at the same spot as the scientist with the spectromenter. What is the longest wavelength $\lambda_{water}$ of the light *in water* that is transmitted most easily to the diver?

**Express your answer in nanometers to three significant digits.**

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$\lambda_{water} =$ 451 nm

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$$\lambda_{air} = 2nt = 2\times1.5\times200 = 600nm$$

from oil to air ($n=1.5$ → $n=1$)

$$\lambda_1 \cdot n_1 = \lambda_2 n_2$$

$$\lambda_{water} = \frac{\lambda_{air} \cdot n_{air}}{n_{water}} = \frac{600nm}{1.33} = 451nm$$

✓ Correct

This problem can also be approached by finding the wavelength with the minimum reflection. Conservation of energy ensures that maximum transmission and minimum reflection occur at the same time (i.e., if the energy did not reflect, then it must have been transmitted to conserve energy), so finding the wavelength of minimum reflection must give the same answer as finding the wavelength of maximum transmission. In some cases, working the problem one way may be substantially easier, so you should keep both approaches in mind.

Constants | Periodic Table

The pupil of the human eye can vary in diameter from 2.00 mm in bright light to 8.00 mm in dim light. The eye has a focal length of about 25 mm, and the visible spectrum extends from 390 nm (violet) to 750 nm (red). Note that the light-sensitive cells on the retina have radii ranging from 0.75 $\mu m$ to 3.0 $\mu m$.

## Part A

What smallest Airy disk radius is possible for the eye?

**Express your answer with the appropriate units.**

$y_{r,min}$ = 1.5×10$^{-6}$ m

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✔ **Correct**

$y_r = 1.22 \cdot \frac{\lambda f}{d}$

$y_{r\,min} = 1.22 \frac{\lambda_{min} f}{d_{max}}$

$= 1.22 \times \frac{390nm \times 25mm}{8mm}$

$= 1.5 \times 10^{-6} m$

## Part B

What largest Airy disk radius is possible for the eye?

**Express your answer with the appropriate units.**

$y_{r\,max} = 1.22 \cdot \frac{\lambda_{max} f}{d_{min}}$

$= 1.22 \times \frac{750nm \times 25mm}{2mm}$

$= 1.14 \times 10^{-5} m$

$y_{r,max}$ = 1.14×10$^{-5}$ m

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✔ **Correct**

★★★

If the work function of a material is such that red light of wavelength 700 nm just barely initiates the photoelectric effect, what must the maximum kinetic energy of ejected electrons be when violet light of wavelength 400 nm illuminates the material?

**Express your answer with the appropriate units.**

$K_{max}$ = 2.13×10$^{-19}$ J

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✔ **Correct**

$\because hf = K_{max} + E_0$

$f = \frac{c}{\lambda}$

$E_0 = hf - K_{max}$

$hf_{red} - K_{max\,red} = hf_{vio} - K_{max\,vio}$, $K_{max\,red} = 0$

$K_{max\,vio} = h(f_{vio} - f_{red})$

$= 6.626 \times 10^{-34} J \cdot S \cdot 3 \times 10^8 m/s \cdot (\frac{1}{400nm} - \frac{1}{700nm})$

$= 2.13 \times 10^{-19} J$