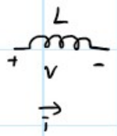
6.1 Inductance, Capacitance, and Mutual Inductance

Wednesday, February 27, 2019 2:52 PM

- Inductors & Capacitors
  - Passive circuit elements, cannot generate energy (like resistor)
  - can store/deliver energy that was previously stored.
  - Energy stored
    - Inductor: electromagnetic field, created by moving charge. (current)
    - Capacitor: electrostatic field, created by displaced charges. (voltage)

(need to be under AC)

- Inductor
  - Unit: Henry (H)



  - a coil of wire
  - $v = L \cdot \frac{di}{dt}$
  - Current is constant: $V=0$, just like a short circuit.
  - Current can only change gradually.

    otherwise $V \to \infty$ ⇒ An inductor "opposes" any change in current.

  Voltage can change instaneously.

  - $\boxed{i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)}$ • Remember L, $i(t_0)$!

    $v\,dt = L \cdot di \Rightarrow \int_{t_0}^{t} v\,d\tau = L\int_{i(t_0)}^{i(t)} dx$

    $= L(i(t) - i(t_0))$

    $\therefore\ i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$

    $t_0 = 0 \to i(t) = \frac{1}{L}\int_{0}^{t} v\,d\tau + i(0)$

  - $v(t)$ will change polarity if change the sign of slope of i.

e.g. $v = \begin{cases} 0 & t<0 \\ 20te^{-10t} & t \geq 0 \end{cases}$, $L = 100\,mH$, $i(t)$?

$$i(t) = \frac{1}{L}\int_0^t 20te^{-10t}\,dt + 0$$
$$= \left(-\frac{te^{-10t}}{10} - \frac{e^{-10t}}{100} + \frac{1}{100}\right) * 200$$
$$= -20te^{-10t} - 2e^{-10t} + 2$$

t	e^{-10t}
1	$-\frac{e^{-10t}}{10}$
0	$\frac{e^{-10t}}{100}$

- Power & Energy in the Inductor
  - $P = \boxed{L \cdot i \cdot \frac{di}{dt}} = vi = v \cdot \left(\frac{1}{L}\int_{t_0}^t v\,d\tau + i(t_0)\right)$
  - Energy: $\boxed{W = \frac{1}{2}Li^2}$ • Remember that $W_{eq} \neq \sum W_i$, because some w will be trapped in Ls.

$P = \frac{dW}{dt} = Li\frac{di}{dt}$ $\therefore dW = Li\,di$ $W = \int_0^i Ly\,dy = \frac{Li^2}{2}$

e.g. $v(t) = \begin{cases} 0, & t<0 \\ 5V, & t \geq 0 \end{cases}$, $i(0)=0$, $L = 100mH$

(a) Current $i(t)$?

(b) $p(t)$, $w(t)$?

(c) does i, p, w become 0 if shut down v at $t=10$?

(a) $i = \frac{1}{L}\int_0^t 5\,d\tau + 0 = 5t \times 10 = 50t$ (A), $t>0$

(b) $p(t) = V \cdot i = 5 \cdot 50t = 250t$ (W)

two soln
$w(t) = \frac{1}{2} \cdot Li^2 = \frac{1}{2} \cdot 2500t^2 \cdot 0.1 = 125t^2$ (J)
$w(t) = \int_0^t P\,dt + w(0) = 125t^2$ (J)

(c) $p(t) = vi = 0$

~~i~~, w doesn't go to 0 immediately.

$i = \frac{1}{L}\int_0^{10} v\,d\tau + \frac{1}{L}\int_{10}^t v\,d\tau = 500 + 0 = 500$ (A)

$W = \int_0^{10} P\,dt + \int_{10}^t P\,dt = 125 \times 10^2 = 1.25 \times 10^4$ (J)

- Inductors in series $L_{eq} = \sum_{i=1}^n L_i$

$V_j = L_j \cdot \frac{di}{dt} \Rightarrow V_{tot} = \sum_{j=1}^n L_j \frac{di}{dt}$

$L_{eq} = \sum L_j$

- Inductors in parallel $\frac{1}{L_{eq}} = \sum_{i=1}^n \frac{1}{L_i}$

$i_j = \frac{1}{L_j}\int_{t_0}^t v\,dt + i_j(0)$

$i_{tot} = \sum_{j=1}^n \frac{1}{L_j}\int_{t_0}^t v\,dt + i_j(0)$

$$i_{tot} = \sum_{j=1}^{n} \frac{1}{L_j} \int_{t_0}^{t} v\,dt + i_j(0)$$

$$= i_{tot} = \left(\int_{t_0}^{t} v\,dt + \qquad \cdot \sum_{j=1}^{n} \frac{1}{L_j}\right.$$

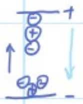
$$i(t_0) = \sum_{j=1}^{n} i_j(t_0)$$

Capacitance

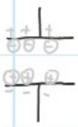
Tuesday, March 5, 2019 9:01 AM

- Capacitor: Consists of two conductor separated by an insulator or dielectric material. Unit: Farad (F)
  - $C = \frac{\varepsilon A}{d}$
    - $\varepsilon = \varepsilon_r \varepsilon_0$ permitivity of dielectric (ε_r: relative permitivity)

      $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m Electrical permittivity of free space
  - $q = CV$ charge stored in the capacitor (V: voltage)
  - practical range: pF ~ μF



the atoms are polarized

- 

  Displacement current

  $$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = C \cdot \frac{dV}{dt}$$

  - If v is a constant, $i = 0$

    (If DC, $i = 0$)
  - v cannot change instaneously, or $i \to \infty$

    but i can.
  - $V(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + V(t_0)$

    $i = C\frac{dV}{dt}$

    $i\,dt = C\,dV$

    $V(t) - V(t_0) = \frac{1}{C}\int_{t_0}^{t} i\,dt$

    $V(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + V(t_0)$
  - $V(t)$ and $i(t)$ doesn't come to max at the same time
  - when $\frac{dV}{dt}$ change sign, $i(t)$ change its sign.
- Power $P = vi = Cv\frac{dV}{dt}$ or $P = \frac{i}{C}\left(\int_{t_0}^{t} i\,dt + V(t_0)\right)$

  Energy $W = \frac{1}{2}CV^2$ $P = \frac{dW}{dt}$

- Power $P = vi = Cv\frac{dv}{dt}$

Energy $W = \frac{1}{2}CV^2$ $\quad P = \frac{dW}{dt}$

$Pdt = dW$

$W = \frac{1}{2}CV^2$

e.g. $C = 1mF$, $i(t) = \begin{cases} 0, & t<0 \\ 1mA & 0 \le t \le 10s \\ 0 & t > 10s \end{cases}$, $V(0)=0$

(1) find $v(t)$
(2) $p(t)$, $w(t)$

(1) $V(t) = \int_0^t i(t)\,dt + 0$

$= t, \quad t \in [0, 10]$

$V(t) = \int_0^{10} i(t)\,dt + 0$

$= 10, \quad t \in [10, +\infty)$

(2) $P = iV = 10t, \quad t \in [0, 10]$

$P = iV = 0, \quad t \in (10, +\infty)$

$W = \frac{1}{2}CV^2 = \frac{1}{2}Ct^2 \quad t \in [0, 10]$

$W = \frac{100}{2}C = 50 \times 10^{-3} = 5 \times 10^{-2}\,J, \quad t \in (10, +\infty)$

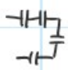
- **Capacitors in Parallel** $\boxed{C_{eq} = \Sigma C_i}$

$i_i = C_i \frac{dv}{dt}$

$i_{tot} = \Sigma i_i = \frac{dv}{dt}\Sigma C_i$

$\therefore C_{eq} = \Sigma C_i$

- **Capacitors in Series** $\boxed{\frac{1}{C_{eq}} = \Sigma \frac{1}{C_i}}$



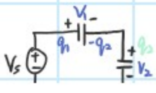
$i = i_i = C_i \frac{dV_i}{dt}$

$V_i = \frac{1}{C_i}\left(\int_{t_0}^t i\,d\tau + V_i(0)\right)$

$V = \Sigma V_i = \Sigma \frac{1}{C_i} \cdot \left(\int_{t_0}^t i\,d\tau + V(0)\right)$

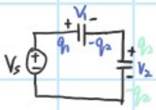
$\frac{1}{C_{eq}} = \Sigma \frac{1}{C_i}$

- Voltage division with Capacitors



$q_1 = q_2$

$C_{eq} = \frac{C_1C_2}{C_1+C_2}$



$q_1 = q_2$

$C_{eq} = \frac{C_1C_2}{C_1+C_2}$

$\because$ Energy conservation

$\frac{1}{2}C_{eq}V_s^2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

$V_s = V_1 + V_2$

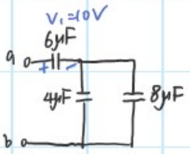
$C_1V_1 = C_2V_2 \Leftrightarrow q_1 = q_2$

∘ $V_1 = \frac{C_2}{C_1+C_2}V_s$, $V_2 = \frac{C_1}{C_1+C_2}V_s$

$V_2 = V_s - V_1$ $V_2C_2 = C_1(V_s - V_1)$

(AP. 6.5 doesn't obey $C_1V_1 = C_2V_2$, the wire is not neutral)
So can't use $\frac{1}{2}C_{eq}V_s^2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

- Determine Cab and vab at ab



$C_{eq} = 8+4 = 12\mu F$

$C_{ab} = \frac{C_{eq}\cdot 6}{C_{eq}+6} = \frac{12\times 6}{18} = 4\mu F$

$C_1V_1 = C_{eq}V_{eq}$

$V_{eq} = \frac{C_1}{C_{eq}}\cdot V_1$

$V_{ab} = V_{eq} + V_1 = \frac{C_1 + C_{eq}}{C_{eq}}V_1 = \frac{6+12}{12}\cdot 10 = 15V$

☆
e.g. DRAM (Dynamic Random Access Memory)

- 1-bit memory storage:

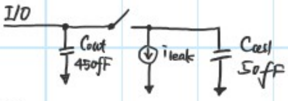
  Vcell > 1.5 V true (1)

  Vcell < 1.5V false (0)

- The switch is closed to (a) Write data (b) Read data

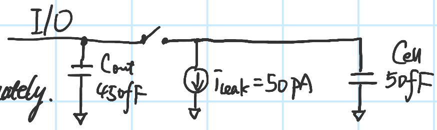
  Switch is realized with a transistor that has a certain leakage current i_{leak}, the leads have a certain C_{out}

1° Write data: store "1", apply 3V to I/O, then open it. What's Qcell?



$Q = CV = 50\times 10^{-15}F \times 3V = 150fC$

2° Storing data: due to i_{leak}, charges and voltage will decrease gradually.



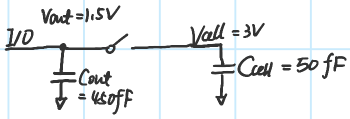
How long is it until $V_{cell}(t) = 0.5\,V_{cell}(0)$?

$$V(t) = \frac{1}{C_{cell}}\int_0^t i(t)\,dt + V(0) \qquad 0.5V(0) = 1.5V$$

$$= \frac{1}{50\times10^{-15}}\int_0^t -50\times10^{-12}\,dt + 3 \qquad \therefore\ t = \frac{1.5}{10^3} = 1.5ms$$

$$= -10^3 t + 3$$

3° Reading data: Assume this is done before any significant leakage happens. I/O is charged to 1.5V before the switch is closed again. What's the voltage change at I/O after the switch is closed?



$Q_i = Q_f$ (charge conservation)

$Q_i = 1.5V \times 450fF + 3 \times 50fF$

$Q_f = V(450fF + 50fF) \qquad \therefore V = 1.65V$

- Charge conservation can only be used when there's no dissipation of charge.
- however, energy not conserved in 3°

$E_i = \frac{1}{2}C_{cell}V_{cell}^2 + \frac{1}{2}C_{out}V_{out}^2 = 731.25fJ$

$E_f = \frac{1}{2}(C_{cell} + C_{out})\cdot V^2 = 680.625fJ$

$\therefore$ a resistor need to be inserted between the switch and Cell.

Probs

Saturday, March 9, 2019 5:33 PM

The voltage across the terminals of a 250 nF capacitor is

$$v = \begin{cases} 50\ \mathrm{V}, & t \le 0; \\ A_1 e^{-4000t} + A_2 t e^{-4000t}\ \mathrm{V}, & t \ge 0, \end{cases}$$

where t is in seconds. The initial current in the capacitor is 400 mA. Assume the passive sign convention.

(1)
What is the initial energy stored in the capacitor?

Express your answer to three significant figures and include the appropriate units.

▸ **View Available Hint(s)**

w = 313 $\mu\mathrm{J}$

(2)
Evaluate the coefficient A_1.

Express your answer to three significant figures and include the appropriate units.

▸ **View Available Hint(s)**

A_1 = 50.0 V

(3)
Evaluate the coefficient A_2.

Express your answer in volts per second using three significant figures.

▸ **View Available Hint(s)**

A_2 = 1.80×10^6 $\mathrm{V/s}$

(4)
Select the correct expression for the capacitor current for $t \ge 0$.

▸ **View Available Hint(s)**

- ○ $i = (0.4 + 900t)e^{-4000t}\ \mathrm{A}$
- ○ $i = (0.4 + 1800t)e^{-4000t}\ \mathrm{A}$
- ○ $i = (0.2 - 1800t)e^{-4000t}\ \mathrm{A}$
- ○ $i = (0.4 - 900t)e^{-4000t}\ \mathrm{A}$
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- ◉ $i = (0.4 - 1800t)e^{-4000t}\ \mathrm{A}$
- ○ $i = (0.2 + 900t)e^{-4000t}\ \mathrm{A}$
- ○ $i = (0.2 - 900t)e^{-4000t}\ \mathrm{A}$

(1)

$W_i = \frac{1}{2} C V_i^2 = \frac{1}{2} \times 250 \times 10^{-9} \times 50^2 = 3.125 \times 10^{-4}\ \mathrm{J}$

(1)

$W_i = \frac{1}{2} C V_i^2 = \frac{1}{2} \times 250\times10^{-9} \times 50^2 = 3.125 \times 10^{-4}$ J

(2)

$V_0 = 50$ V

$\therefore \; A_1 + 0 = 50$

$A_1 = 50$ V

(3)

$i = C \cdot \frac{dv}{dt}$

$i_0 = 0.4 = 250\times10^{-9} \times (A_1 \times (-4000) + A_2)$

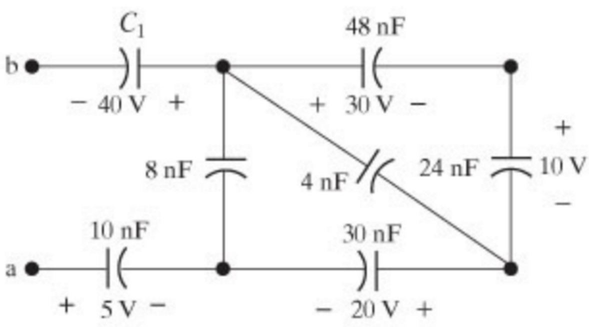
$A_2 = 1.8\times10^6$ V/s

(4)

$i = C \cdot \frac{dv}{dt}$

$= 250\times10^{-9}\left[(50\times(-4000) + 1.8\times10^6) + (-4000)\times1.8\times10^6 t e^{-4000t}\right]$

$= (0.4 - 1800t)e^{-4000t}$



(1)

Find the equivalent capacitance with respect to the terminals a, b for the circuit shown in (Figure 1). Suppose that C_1 = 19 nF .

Express your answer to three significant figures and include the appropriate units.

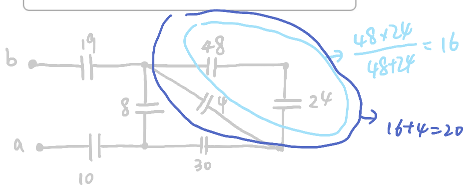
C_{eq} = 4.94 nF

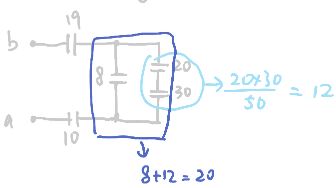
(2)

Determine the the initial voltage drop for the equivalent capacitance with respect to the terminals a, b for the circuit shown in (Figure 1).

Express your answer to three significant figures and include the appropriate units.

$V_{0\,ab}$ = -15.0 V

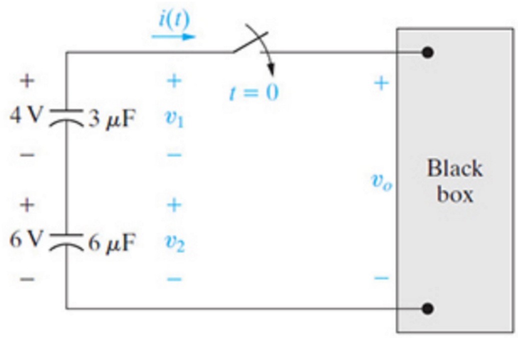




$C_{eq} = 1/\left(\frac{1}{20} + \frac{1}{19} + \frac{1}{10}\right) = 4.94$ nF

(2) Voltage drop : Negative.

★★★ $i(t) = 20e^{-t}$ μA $(t>0)$



(1)

Replace the original capacitors with an equivalent capacitor and select the correct expression for $v_o(t)$ for $t \geq 0$.

▸ View Available Hint(s)

- $v_o(t) = -10e^{-t} - 10$ V
- $v_o(t) = -10e^{-t} - 6$ V
- $v_o(t) = -10e^{-t} - 4$ V
- $v_o(t) = 10e^{-t}$ V
- $v_o(t) = 10e^{-t} - 6$ V
- $v_o(t) = 10e^{-t} - 4$ V
- $v_o(t) = 10e^{-t} - 10$ V
- $v_o(t) = -10e^{-t}$ V

(2)

Select the correct expression for $v_1(t)$ for $t \geq 0$.

▸ View Available Hint(s)

- $v_1(t) = 3.33e^{-t} - 2.67$ V
- $v_1(t) = 3.33e^{-t} + 2.67$ V
- $v_1(t) = 6.67e^{-t} - 10.67$ V
- $v_1(t) = 6.67e^{-t} - 2.67$ V
- $v_1(t) = 6.67e^{-t} + 2.67$ V
- $v_1(t) = 3.33e^{-t} - 10.67$ V
- $v_1(t) = 3.33e^{-t} + 10.67$ V
- $v_1(t) = 6.67e^{-t} + 10.67$ V

(3)

Select the correct expression for $v_2(t)$ for $t \geq 0$.

▸ View Available Hint(s)

(3)
Select the correct expression for $v_2(t)$ for $t \geq 0$.

▶ **View Available Hint(s)**

- ○ $v_2(t) = 6.67e^{-t} + 10.67\ \mathrm{V}$
- ◉ $v_2(t) = 3.33e^{-t} + 2.67\ \mathrm{V}$
- ○ $v_2(t) = 6.67e^{-t} - 10.67\ \mathrm{V}$
- ○ $v_2(t) = 3.33e^{-t} + 10.67\ \mathrm{V}$
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- ○ $v_2(t) = 3.33e^{-t} - 10.67\ \mathrm{V}$
- ○ $v_2(t) = 3.33e^{-t} - 2.67\ \mathrm{V}$

(4)
How much energy is delivered to the black box in the time interval $0 \leq t < \infty$?

Express your answer to three significant figures and include the appropriate units.

▶ **View Available Hint(s)**

$w_{\text{delivered}} =$ 100 μJ

(5)
How much energy was initially stored in the series capacitors?

Express your answer to three significant figures and include the appropriate units.

▶ **View Available Hint(s)**

$w_{\text{inital}} =$ 132 μJ

(6)
How much energy is trapped in the ideal capacitors?

Express your answer to three significant figures and include the appropriate units.

▶ **View Available Hint(s)**

$w_{\text{trapped}} =$ 32.0 μJ

(1)
$$C_{eq} = \frac{3\times 6}{9} = 2\mu F$$

$$V_0 = -\frac{1}{C_{eq}}\int_0^t 20e^{-t}\,dt + 10$$

"–" because current direction

$$= 10e^{-t}\Big|_0^t + 10$$

$$= 10e^{-t}$$

(2)
$$V_1 = -\frac{1}{C_1}\int_0^t 20e^{-t}\,dt + 3$$

$$= 6.67\,e^{-t} - 2.67\ \ V$$

$= 6.67\, e^{-t} - 2.67 \quad V$

(4) $W_{delivered} = \int_0^{\infty} iv\, dt = \int_0^{\infty} 10e^{-t} \cdot \overset{\mu A}{\downarrow}20e^{-t}\, dt$

$= \int_0^{\infty} 200e^{-2t}\, dt$

$= -100e^{-2t} \Big|_0^{\infty}$

$= 100\ \mu J$

(5) $W = \frac{1}{2}CV^2 = \frac{1}{2}(3 \times 4^2 + 6 \times 6^2)$

$= 132\, \mu J$

(6) $132 - 100 = 32\mu J$