1.1 lines&linear equations

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- e.g. $2x+y=7$ (*) is a <u>linear equation</u>

  (1,5) is a soln to *.

  e.g. If s_1 is any real number, $x=s_1$, $y=7-2s_1$,
  then $(s_1, 7-2s_1)$ is a soln to (*)

  $\{(s_1, 7-2s_1) \mid s_1 \in \mathbb{R}\}$ is solution set for (*)

- System of linear equations

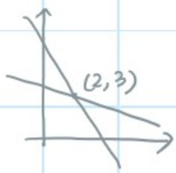
  e.g. $\begin{cases} 2x+y=7 \\ x+2y=8 \end{cases}$ (**)

  is a system of linear equations

  (2,3) is a solution to (**)

  {(2,3)} is a solution set of (**)

  can be solved by relating the lines
  (geometry)



- Soln to a LE is an n-tuple $(s_1, s_2, \ldots s_n)$ that satisfies every equation in the system.
  - Soln set : the collection of all solns.

- Consistent If a linear system has at least one solution.
  Inconsistent
  - free parameter para that can be any real num
    ⇨ gives general solution

- Leading variable a variable that appear in the first term in

- Leading variable: a variable that appear in the first term in at least one equation (with nonzero coefficient)
- Triangular system/form
  - Properties
    - Every var is the leading variable of 1 equation — 1 to 1.
    - number of equations = number of variables
    - Have one solution

e.g. $a_1x_1 + x_2 + a_3x_3 = 0$ is not triangular.

$b_1x_3 + b_2x_2 = b_3$ (consider it as a matrix)

$c_1x_2 = c_2$

  - method to solve: back substitution

- Echelon Systems: a var is a leading variable for at most one equation.
  - method to solve: back substitution
  - The indices of leading variables are strictly increasing from top to bottom.
  - There're no solns, exactly one soln, or infinitely many solns.
    parallel / n planes (or hyperplanes) have one intersection point / intersect at a line or same plane
  - Free variable in echelon form: Any var that is not a leading variable

- Two LS are equivalent if they have same set of solns.
- Elementary operations (transformation between equivalent sys of equations)
  - Interchange any of equation
  - Multiply any of equation by c ($c \neq 0, c \in \mathbb{R}$)
  - Add a multiple of one equation to another equation, & replace it.

e.g. $\begin{cases} x_1 - 3x_2 + 2x_3 = -1 \\ 2x_1 - 5x_2 - x_3 = 2 \\ -4x_1 + 13x_2 - 12x_3 = 11 \end{cases}$

$$\begin{bmatrix} 1 & -3 & 2 & -1 \\ 2 & -5 & -1 & 2 \\ -4 & 13 & -12 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & -1 \\ 0 & 1 & -5 & 4 \\ 0 & 1 & -4 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & -1 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{cases} x_1 = 50 \\ x_2 = 19 \\ x_3 = 3 \end{cases}$$

1.2 Linear Systems&matrices

Wednesday, January 9, 2019 5:05 PM

- $$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \quad \ddots \quad \vdots \quad \vdots \\ a_{m1}x_n + \cdots + a_{mn}x_n = b_m \end{cases} \qquad (*)$$

  $$\begin{pmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{pmatrix}$$ The augmented matrix for $*$

- Elementary row operation
- e.g. $$\begin{cases} 2x_1 - 3x_2 + 10x_3 = -2 \\ x_1 - 2x_2 + 3x_3 = -2 \\ -x_1 + 3x_2 + x_3 = 4 \end{cases}$$

$$\begin{pmatrix} 2 & -3 & 10 & -2 \\ 1 & -2 & 3 & -2 \\ -1 & 3 & 1 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 3 & -2 \\ 0 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 3 & -2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

take $x_3 = s_1$

$\therefore x_2 = 2 - 4s_1$

$x_1 - 4 + 8s_1 + 3s_1 = -2$

$x_1 = -11s_1 + 2$

- Two matrices are equivalent if one can be transformed into another by elementary row operations.
- def. A leading term in a matrix is the left most non-zero term in a row
- def. A row with all 0 is a zero-row

  (A row with any non-zero term(s) is a non-zero row.
- def. Echelon form of a matrix
  - every leading term is in the column to the left of the leading term in the row below it.
  - Any zero rows are at the buttom.

- Any zero rows are at the button.

- Echelon form matrix ≠ Echelon form equations
- Pivot
  - Pivot position: position containing a leading term.
  - Pivot column: a column containing a pivot position.

$$\begin{pmatrix} 5 & 4 & 3 & 1 \\ 0 & 2 & 4 & 8 \\ 0 & 7 & 5 & 2 \end{pmatrix}$$

○ pivot.

  - Pivot: a nonzero number in a pivot position
- Process to take a matrix & put it into echelon form

  Gaussian Elimination

- More general: Gaussian-Jordan Elimination

$$\begin{pmatrix} 1 & -2 & 3 & -2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 & -2 \\ 0 & 2 & 8 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

① Change all leading variable to 1.

(divide by leading var)

② Use row operations to make pivot position the only non-zero term in the pivot column.

- def. A matrix is in reduced echelon form if
  - in echelon form
  - All pivot positions contain a 1.
  - The only non-zero term in any pivot column is in the pivot position.
- Theorem. a given matrix is equivalent to a unique matrix in reduced echelon form.

- e.g. $\begin{pmatrix} -4 & 2 & -2 & 10 \\ 1 & 0 & 1 & -3 \\ 3 & -1 & 1 & -8 \end{pmatrix}$ to reduced echelon form

$$= \begin{pmatrix} 1 & 0 & 1 & -3 \\ 0 & -1 & -2 & 1 \\ 0 & 2 & 2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 1 & -3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

HW1

Wednesday, January 9, 2019 1:21 PM

Determine if the statement is true or false.

A linear system with three equations and five variables must be consistent.

- (•) True
- ( ) False

e.g. $x_1+x_2+x_3+x_4+x_5=10$

$3x_1+3x_2+3x_3+3x_4+3x_5=10$

...

Determine if the statement is true or false.

Different sequences of row operations can lead to different reduced echelon forms for the same matrix.

- (•) True
- ( ) False

✗

Reduced echelon forms are unique, but echelon forms are not.