# 3.1

Wednesday, October 24, 2018 3:44 PM

- $2^{nd}$ orde DEs

$$\frac{d^2y}{dt^2} = f(t, y, \frac{dy}{dt})$$

e.g. $\frac{d^2y}{dt^2} = \cos t \cdot y - (\frac{dy}{dt})^2 \cdot t - \sin(y(t))^t$

- linear $2^{nd}$ order DEs

have the form: $\frac{d^2y}{dt^2} + p(t) \cdot \frac{dy}{dt} + q(t) y = g(t)$

- Homogeneous DE : $g(t) = 0$

e.g. $\frac{d^2y}{dt^2} - 5 \cdot \frac{dy}{dt} + 6y = 0$

Complementary solution

$y_1(t) = e^{2t}$

$y_2(t) = e^{3t}$ are solns $\Rightarrow \underline{y_c(t)} = C_1 e^{2t} + C_2 e^{3t}$

Also, $C_1 y_1 + C_2 y_2$ is the solution.

$\Rightarrow (\frac{d^2}{dt^2} - 5\frac{d}{dt} + 6) \cdot y = 0$

$L \cdot y = 0$

it is satisfied only when homogeneous DE.

$\because L \cdot y_1 = 0, \ L \cdot y_2 = 0$

$\therefore L(y_1 + y_2) = L y_1 + L y_2 = 0$

- Steps: $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 6t^2 - 10t + 8.$

I. solve homogeneous version

$y_c(t)$

II. Find any solution to the DE

$y_p(t)$ (particular)

III. Every solution will be

$y(t) = y_c(t) + y_p(t)$

- Constant coefficient

$$a\frac{d^2y}{dt^2}+b\cdot\frac{dy}{dt}+cy=0$$

e.g. $\frac{d^2y}{dt^2}-y=0$

guess $e^{rt}=y$

$re^{rt}=y'$

$r^2e^{rt}=y''$

$r^2e^{rt}-e^{rt}=0$

$r^2-1=0$

$r=\pm1$

$y(t)=C_1e^t+C_2e^{-t}$

e.g. $y''-5y'+6y=0$

$\Rightarrow e^{rt}(r^2-5r+6)=0$

$r^2-5r+6=0$

$(r-2)(r-3)=0$

$r=2,3.$

$y(t)=C_1e^{2t}+C_2e^{3t}$

e.g.2. $\frac{d^2y}{dt^2}+y=0$ characteristic equation

$r^2+1=0$

$$r=\frac{-0\pm i\sqrt{-(0^2-4)}}{2}$$

$=\pm i$

$y(t)=C_1e^{it}+C_2e^{-it}$

$\because e^{it}=\cos t+i\sin t$, make $C_1=\frac{1}{2}, C_2=\frac{1}{2}$

$\therefore y(t)=\frac{1}{2}(\cos t+i\sin t)-\frac{1}{2}(\cos(-t)+i\sin(t))$

$= i\sin t \qquad (\frac{1}{2}e^{it}+\frac{1}{2}e^{-it}=\cos t)$

$y = \sin t$ should be sln.

$y(t) = C_1 \cos t + C_2 \sin t$ .

# 3.2

Friday, October 26, 2018 3:28 PM

- Complex numbers symbol $i$, $i^2=-1$

  $\mathbb{C}$ extends $\mathbb{R}$ by making $a+bi$, $a\in\mathbb{R}$.

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  Add/Sub: $a+bi\pm(c+di) = a\pm c+(b\pm d)i$

  Mult: $(a+bi)(c+di) = ac-bd+(bc+ad)i$

  Div: $\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{(a+bi)(c-di)}{c^2+d^2}$

- $ay''+by'+cy=0$

  assume $y=e^{rt}$

  $e^{rt}(ar^2+br+c)=0$

  $r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

  $y=Ae^{\frac{-b+\sqrt{b^2-4ac}}{2a}t}+Be^{\frac{-b-\sqrt{b^2-4ac}}{2a}t}$

  $=Ae^{(\lambda+\mu)t}+Be^{(\lambda-\mu)t}$

  $\lambda=-\frac{b}{2a}$, $\mu=\frac{\sqrt{b^2-4ac}}{2a}$

  e.g. $y''+y=0$

  $r^2e^{rt}+e^{rt}=0$

  $r=\pm i$

  $y=Ae^{it}+Be^{-it}$

  $\therefore \begin{cases} \sin t = A_1e^{it}+B_1e^{-it} \\ \cos t = A_2e^{it}+B_2e^{-it} \end{cases}$

  $\because$ $\sin t$ odd $\therefore \sin(-t)=-\sin t$

  $A_1e^{-it}+B_1e^{it}=-A_1e^{it}-B_1e^{-it}$

  $A_1=-B_1$

  $\because$ $\cos t$ even $\therefore \cos t=\cos(-t)$

  $A_2e^{it}+B_2e^{-it}=A_2e^{-it}+B_2e^{it}$

$$A_2 e^{it} + B_2 e^{-it} = A_2 e^{-it} + B_2 e^{it}$$

$$A_2 = B_2$$

$\therefore$ $\sin t = A_1 e^{it} - A_1 e^{-it}$

$\cos t = A_2 e^{it} + A_2 e^{-it}$

$\because$ $\sin' t = \cos t$ && $\sin^2 t + \cos^2 t = 1$

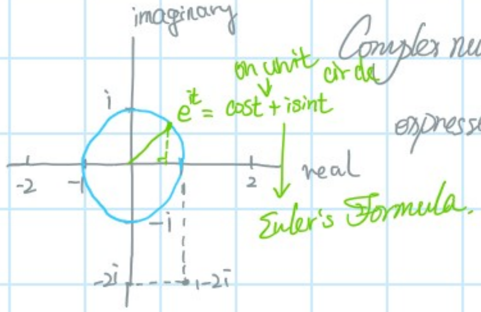
$A_1 = \frac{1}{2i}$ , $A_2 = \frac{1}{2}$

$\therefore$ $\sin t = \frac{e^{it} - e^{-it}}{2i}$ , $\cos t = \frac{e^{it} + e^{-it}}{2}$

$\therefore$ $i \cdot \sin t + \cos t = \frac{2e^{it}}{2} = e^{it}$

when $t = \frac{\pi}{2}$

$e^{\frac{\pi}{2} i} = i \cdot 1 + 0 = i \Rightarrow i = e^{\frac{\pi}{2} i}$




Complex numbers can be expressed on this coordinate

- Hyperbolic Trig.

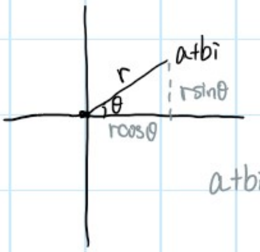
$\sinh(t) = \frac{e^t - e^{-t}}{2}$ , $\cosh(t) = \frac{e^t + e^{-t}}{2}$

Properties: $\cos(it) = \cosh(t)$

$\sin(it) = i \sinh(t)$

$e^{i\pi} + 1 = 0$, plug in $t = \pi$.

- Polar coordinate




$a = r \cdot \cos\theta$. $b = r \sin\theta$

$a + bi = r \cdot \cos\theta + r \cdot \sin\theta i$

$= r(\cos\theta + \sin\theta i)$

$= re^{i\theta}$

$= r(\cos\theta + i\sin\theta)$

$= re^{i\theta}$

e.g. $2y'' + 3y' + 2y = 0$

assume $y = e^{rt}$

$e^{rt}(2r^2 + 3r + 2) = 0$

$\therefore \; r = \frac{-3 \pm \sqrt{9-16}}{4} = \frac{-3 \pm \sqrt{7}i}{4}$

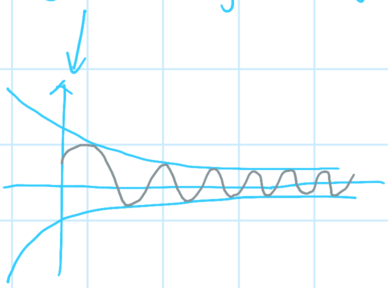
$\therefore \; y = Ae^{\frac{-3+\sqrt{7}i}{4}t} + Be^{\frac{-3-\sqrt{7}i}{4}t}$

$= e^{-\frac{3}{4}t}\left(Ae^{\frac{\sqrt{7}}{4}it} + Be^{-\frac{\sqrt{7}}{4}it}\right)$

$= e^{-\frac{3}{4}t}\left(\check{A}\cos(\frac{\sqrt{7}}{4}t) + \hat{B}i\sin(\frac{\sqrt{7}}{4}t)\right)$

$y = e^{\lambda t}(\tilde{A}\cos(\mu t) + \tilde{B}i\sin(\mu t))$ $\quad \lambda = -\frac{3}{4}, \mu = \frac{\sqrt{7}}{4}$.

real component of r.



# HW 3.1, 3.2

Sunday, October 28, 2018 12:51 PM

$a+bi = re^{i\theta}$

Write each complex number in complex polar notation: $r=\sqrt{a^2+b^2} = a\cdot\cos\theta = b\cdot\sin\theta$

$(4.445 + 0.365\, i) =$ 4.460 $e^{i\, 0.082}$

$(2.152 + 1.795\, i) =$ 2.802 $e^{i\, 0.695}$

$\boxed{e^{(ct+di)} = \cos(ct+d) + i\sin(ct+d)}$

Write each complex exponential function as a sum of its real and imaginary parts:

$(4.445 + 0.365\, i)\, e^{(2.236+4.041 i)t} =$ 4.460 $e^{2.236\, t}\cos($ 4.041 $t+$ 0.082 $) + i$ 4.460 $e^{2}$

$(2.152 + 1.795\, i)\, e^{(0.341+1.201 i)t} =$ 2.802 $e^{0.341\, t}\cos($ 1.201 $t+$ 0.695 $) + i$ 2.802 $e^{0}$

Find the general (real) solution of the differential equation ( $y' = \frac{dy}{dx}$ ):

$y'' + 8\, y' + 113/4\, y=0$

i is already contained

y(x) = $C_2e^{-4x}\sin\left(\frac{7}{2}x\right) + C_1e^{-4x}\cos\left(\frac{7}{2}x\right)$ | $C_2e^{-4x}\sin\left(\frac{7x}{2}\right) + C_1e^{-4x}\cos\left(\frac{7}{\ }\right.$

$y(x) = C_1e^{\lambda}\cdot\sin(\mu x) + C_2e^{\lambda}\cdot\cos(\mu x)$

$ar^2+br+c=0$

$\lambda = \frac{-b}{2a}$ $\mu = \frac{\sqrt{\Delta}}{2a}$ (without i)

Find the unique solution that satisfies the initial conditions:
$y(0) = -4$ and $y'(0)=9$

y(x) = $-2e^{-4x}\sin\left(\frac{7}{2}x\right) - 4e^{-4x}\cos\left(\frac{7}{2}x\right)$

.236 $^{t}\sin($ 4.041 $t+$ 0.082 $)$

.341 $^{t}\sin($ 1.201 $t+$ 0.695 $)$

d in $C_2$.

$\boxed{\dfrac{7x}{2}\Big)}$