T5/MATH4402/2024-25/2nd

THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH4402 Analysis II

Tutorial 5 (Mar 21)

Chapter 4

1. Let $A \in \mathbb{R}^{n\times n}$ be a symmetric matrix and $\det A \neq 0$. Let $b \in \mathbb{R}^n$ and $c \in \mathbb{R}$. Define

$$M = \{x \in \mathbb{R}^n : x^T Ax + 2x^T b = c\}.$$

   (a) For what values of c (in terms of A and b) will M be an $(n-1)$-dimensional submanifold of $\mathbb{R}^n$? Prove your answer.

   (b) Find the tangent $(n-1)$-plane of M at x_0 when M is an $(n-1)$-dimensional submanifold of $\mathbb{R}^n$.

2. Let $M \subseteq \mathbb{R}^4$ be the set of solutions of the system of equations

$$\begin{cases} xu + yu^2v - 2 = 0 \\ xu^3 + y^2v^4 - 2 = 0. \end{cases}$$

   Define $g : \mathbb{R}^4 \longrightarrow \mathbb{R}$ by

$$g(x, y, u, v) = xu + vy.$$

   Show that g does not have a local extremum at $(1, 1, 1, 1) \in M$.

3. Extremize

$$g(x, y, z) = 3x^2 + 3y^2 + 4z^2 + 4xy + 2yz + 2xz$$

   on the unit sphere $x^2 + y^2 + z^2 = 1$.

Chapter 5 (Section 5.1)

4. (*Theorem 5.7*) Let A be a closed rectangle in $\mathbb{R}^n$ and let $f : A \longrightarrow \mathbb{R}$ be a bounded function. Prove that f is integrable over A if and only if for any $\epsilon > 0$, there exists a partition P of A such that

$$U(f, P) - L(f, P) < \epsilon.$$

5. (*Corollary 5.8*) Let A be a closed rectangle in $\mathbb{R}^n$ and let $f : A \longrightarrow \mathbb{R}$ be a continuous function. Prove that f is integrable over A.