THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH4402 Analysis II

Tutorial 4 (Mar 7)

Chapter 4 (Section 4.1)

1. Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ by
$$f(x, y) = x^3 - 6xy + y^2.$$
Find all values $c \in \mathbb{R}$ for which the entire level set $f^{-1}(\{c\})$ is a 1-dimensional submanifold of $\mathbb{R}^2$.

2. Decide whether the solution set of the system of equations
$$x^3 + y^3 + z^3 = 1 \quad \text{and} \quad x + y + z = 0$$
is a 1-dimensional submanifold of $\mathbb{R}^3$.

3. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be of class C^1 and let
$$M = \{(x, y, f(x, y)) : (x, y) \in \mathbb{R}^2\}.$$
Show that M is a 2-dimensional submanifold of $\mathbb{R}^3$.

4. Prove that any k-dimensional vector subspace of $\mathbb{R}^n$ is a k-dimensional submanifold of $\mathbb{R}^n$.

5. Let $M = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 - 2xz - 4 = 0\}$.
   (a) Show that M is a 2-dimensional submanifold of $\mathbb{R}^3$.
   (b) Let $x_0 = (2, \sqrt{3}, 1) \in M$. Find $T_{x_0}M$ and the equation of the tangent 2-plane at x_0.