Project 1 Design Report

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1 Overview

The project of Module 1 in Quantitative Engineering Analysis involved the creation of a monohull displacement boat. This boat was to be fabricated from ABS foam. Other requirements included an aluminum mast (50 cm long, 0.5 cm radius), up to 300 g worth of ballast, an angle of vanishing stability between $120^o - 140^o$, and the ability to take a payload of two soda cans (approximately 760 g total).

This report will begin with the background and terminology needed to understand the contents, such as definitions of monohull displacement, AVS, and payload. As it assumes no prior knowledge of boats, buoyancy, or forces such as righting moments, it can be skipped or skimmed by readers with experience in this area. However, it will also explain the coordinate system that will be used throughout the report.

Directly following are the design considerations—broken into subsections—which takes into account the main requirements of the project and analyzes how to achieve them. This begins by examining whether or not the vessel will float through brief computations surrounding mass and volume, moves to the levelness of the boat's deck while considering center of mass, transitions to analysis of the righting moment and angle of vanishing stability of the boat, and finally concludes by briefly examining speed considerations.

The next section, Proposed Design, takes the considerations from above and applies them in creating a boat that meets the requirements. Here, the center of buoyancy and righting moment curve are analyzed and a design is proposed and visualized.

After testing, a final section shall be added, comparing the calculated results with those measured with the physical model.

2 Background and Terminology

Basic terminology includes:

- Hull: the main body of the boat.
- Bow: front of the boat.
- Stern: back of the boat.
- Mast: a tall pole of metal or wood, usually to hold the sail.
- Deck: the horizontal top of the boat.
- Port and Starboard: when facing the bow, port is the left and starboard is the right.

- Beam: the largest width of the boat.
- Keel: a beam running along the central bottom of the boat.
- Ballast: heavy material added to a boat to increase stability.
- Capsize: to flip over.
- Floating Level: when the deck is horizontal and parallel to the surface of the water.
- Waterline: the plane or line where the water goes up the hull.
- Payload: the boat's cargo.
- Boat Waterlines: the cross-sections of the boat in the horizontal direction, showing the shape of the length and width. If the boat is vertical and level, this is the plane of the water's surface. In this report, this is the xy-plane.
- Boat Stations: cross-sections made by vertical slices along the length of the boat. These are in the xz-plane.
- Boat Buttocks: the cross-sections made by vertical slices across the width of the boat, or the yz-plane.
- Heel Angle: degree of rotation around the x-axis, or tipping in the port or starboard direction.

The center of mass is a point representing the average position of mass in a system. The center of buoyancy is the center of mass of displaced water. The buoyancy force a distributed force equivalent to the weight of displaced water. Displaced water is the volume of water an object pushes out of the way, which can be up to the volume of the object.

Floating occurs when the average density (mass per unit volume) of the object is less than or equal to the density of water ($1.0\frac{g}{cm^3}$). Alternately, an object floats when the buoyancy force is equal to the gravitational force.

Wetted surface area is the surface area of the boat beneath the waterline. The volume of the boat beneath the waterline is the displaced water, and is used to calculate center of buoyancy.

The righting moment is the moment created by the buoyancy force correcting the tipping of the boat. A boat is floating levelly (at a 0^o angle) if the center of mass and center of buoyancy are in a vertical line, and the righting moment is zero at that point. The angle of vanishing stability (AVS) is the point at which the righting moment becomes zero and the boat rolls over.

There are several different styles of hull. This project is concerned only with a monohull displacement boat, or a single hull. These can be styled as flat-bottomed, round, or "V"-shaped. Flat-bottomed boats tend to be less stable when tipping.

3 Design Considerations

We focused on three questions when designing our boat. 1. Does it float? 2. Does it float flat? and 3. Does it have an angle of vanishing stability between 120^o and 140^o?

We based our calculations off of a boat with the generalized equation:

$$z = \frac{d}{w^n}|y|^n - d\sqrt{1 - (\frac{x}{l})^2}$$

where d is the distance between the deck and the lowest point on the hull, w is half the width of the boat at its widest point, l is half the length from stern to prow, and n is a number we varied to find the optimal boat shape.

Our research showed that flat-bottomed boats were suboptimal, and we knew a soda can would not fit into a narrow triangle, so we constrained n to be between 1 and 2.5. We also used a width of $20cm$ in order to create a sturdy boat.

3.1 Designing a Boat that Floats

In order to float, the average density of the boat needs to be less than that of water ($1000\frac{kg}{m^3}$, or $1\frac{g}{cm^3}$. The foam used in fabrication has a density of approximately $31.7\frac{kg}{m^3}$, ($.0317\frac{g}{cm^3}$), and would easily float on its own. However, the aluminum mast has a density of $2.70\frac{g}{cm^3}$ and a mass of $96.2g$, and the ballast and soda cans add approximately 300 g and 760 g respectively, while adding either no or negligible volume.

We thus can find that, added all together, the added weight comes out to:

$$96.2g + 300g + 760g = 1156.2g$$

This indicates that the density will be equal to that of water if the boat has a volume of approximately $1156cm^3$, assuming the mass of the foam to be negligble. Each can is $12.2cm$ long and has a radius of $3.3cm$. If both cans stood upright, the minimum length of the boat would still have to have a length of at least $15.2cm$ (adding an extra centimeter for the mast and the width of the walls and a width of $7.6cm$. Assuming a rectangular prism:

$$1156.2/(15.2 * 7.6) = 10cm$$

which shows that the cans would have to stick out of the boat in order to make it dense enough to sink. As we intended to have the cans laying horizontally in order to keep the center of mass as low as possible, the length had to be greater than $25cm$, and we knew that as long as our volume stayed larger than $1156.2cm^3$, it would float.

3.2 A Boat that Floats Flat

The depth of our boat varied significantly as we tried to adjust our center of mass. Eventually, we found that a depth of $15cm$ was best for being able to fit the soda cans. As the boat is symmetrical in the x direction (along the length), we disregarded the third dimension for our initial calculations for level floating.

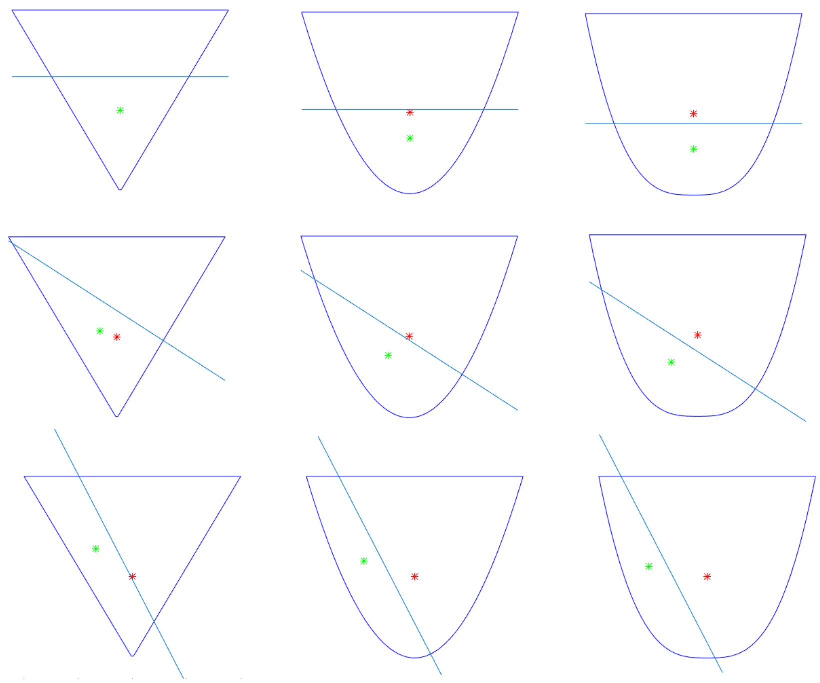


Figure 1: The waterline, center of mass, and center of buoyancy for three different values of n (1, 2, and 3 from left to right) and three different angles (0, 30, and 60).

Figure 1 demonstrates that as n increases, the waterline lowers due to an increase of volume low on the boat. The center of mass being above the waterline causes less stability, so we knew we wanted to keep n as low as possible.

By varying values of n and the angle, we were able to output this plot:

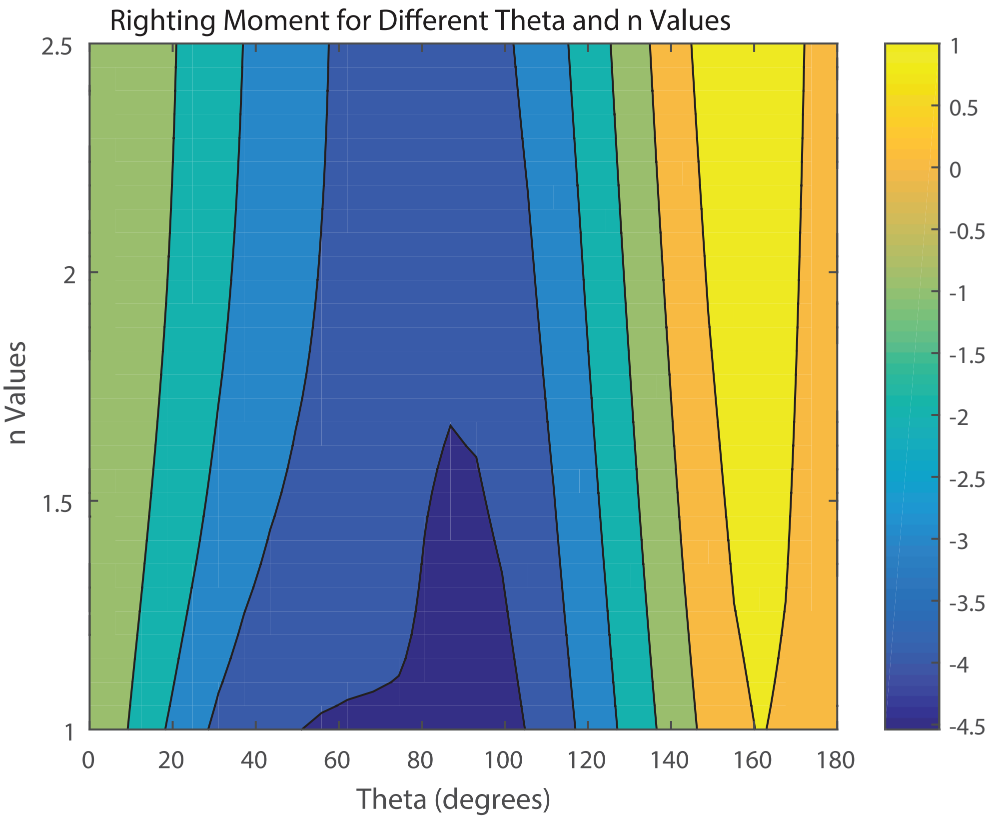


Figure 2: A negative value of the righting moment (displayed by the colorbar) indicates a righting moment returning the boat to vertical.

Figure 2 shows that, with the added masses at the depths we wanted them, the righting moment would return our boat to vertical for all the values of n we were considering. From this, we knew our boat would float vertically. We also tried to ensure that our center of mass would be as low as possible by putting our mast as far down as it could be, our ballast along the bottom of the boat, and the cans low in the hull.

3.3 Angle of Vanishing Stability

Figure 2 displays an estimate for the angle of vanishing stability. Looking at this graph, we decided that an n of 1.75 was optimal for both fitting the cans (assuming the length to be $50cm$) and we moved to a three dimensional analysis. By finding the waterline and the center of buoyancy for different heeling angles, as well as experimenting with moving the payload vertically, we were able to calculate the righting moment. We used the following equations:

$$ra = (CoB_x - CoM_x)i + (CoB_y - CoM_y)j \tag{1}$$

$$F_{buoyancy} = 9.8M_{boat}cos(\theta)i + 9.8M_{boat}sin(\theta)j \tag{2}$$

$$rm = ra \times F_{buoyancy} \tag{3}$$

where ra is the righting arm, CoB is the center of buoyancy, CoM is the center of mass, M_{boat} is the mass of the boat, and rm is the righting moment.

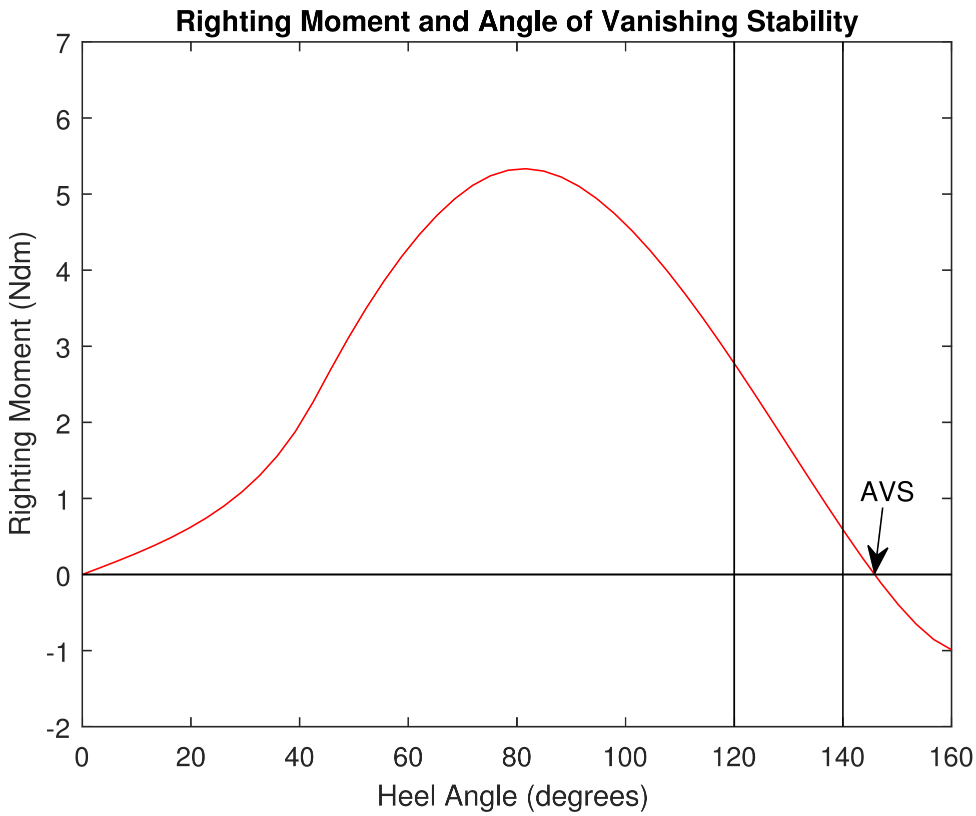


Figure 3: This shows that the angle of vanishing stability is slightly past 140^o. Here, a positive righting moment indicates the boat being returned to vertical and level.

3.4 Speed

While we briefly considered speed, we decided that it was secondary to the other calculations. Prior knowledge suggested a non-flat prow would improve speed, so by creating a curve along the xz-plane, we improved the racing potential.

4 Proposed Design

Our final hull design follows the equation:

$$z = \frac{15}{10^{1.75}}|y|^{1.75} - 15\sqrt{1-(\frac{x}{25})^2}$$

where all the measurements are in cm.

The length was determined by trying to find the balance between size and the position of the cans inside the boat. They had to be as low as possible to benefit stability while still fitting inside the hull. This also determined depth, in order to compensate for the mast. We were finally able to position the cans so that their center of mass was approximately 7 cm above the lowest point of the boat.

Figure 3 is the righting moment curve for this design. However, there is some slight idealism, as the calculations did not include the foam and were optimistic about how the payload would sit within the boat.

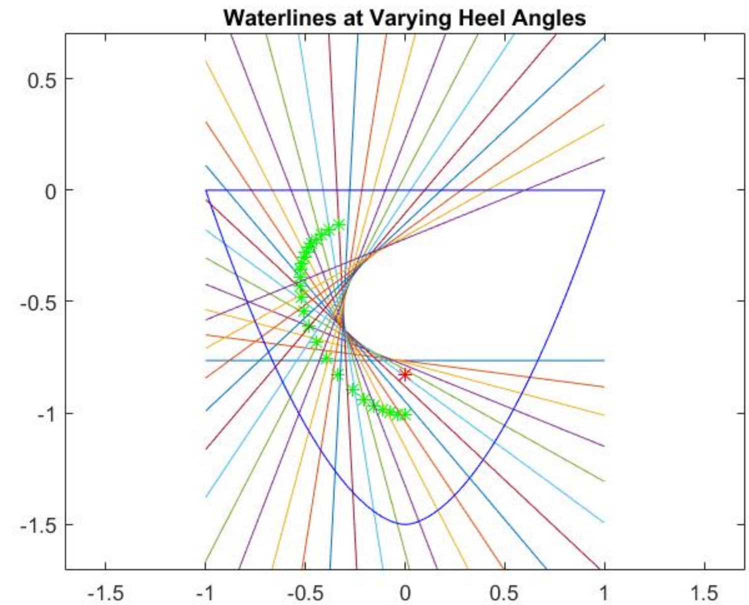
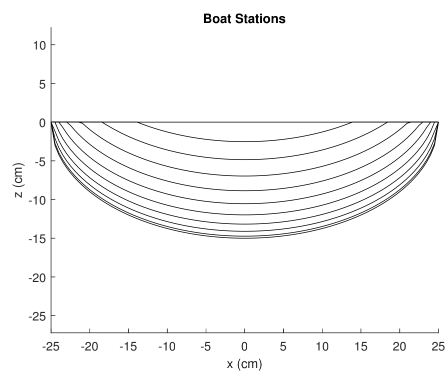


Figure 4: Each line represents a different heel angle. The red shows where the center of mass is, whereas the green shows the movement of the center of buoyancy as the boat heels farther.

Once we decided upon these parameters, we were able to render the following contour lines:



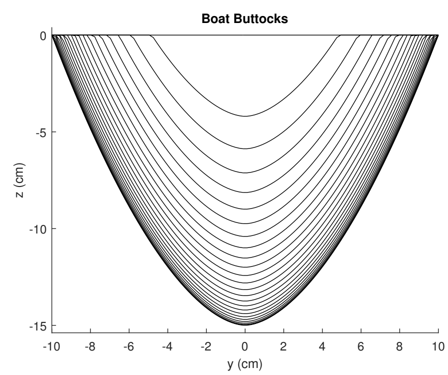


Figure 5: Station contours of the boat. Figure 6: Contour lines of the buttocks.

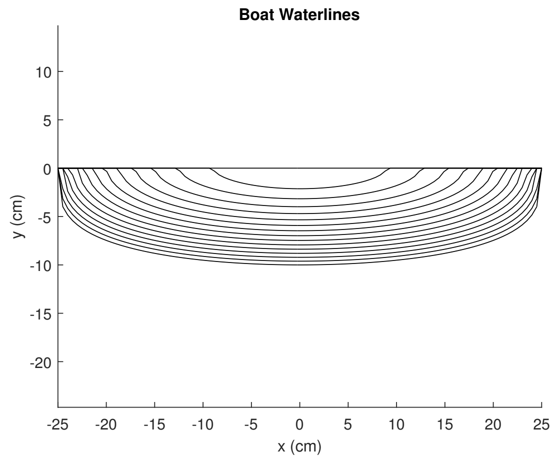


Figure 7: Contour waterlines over the port side of the boat.

From this, our final boat will appear as shown in Figure 8.

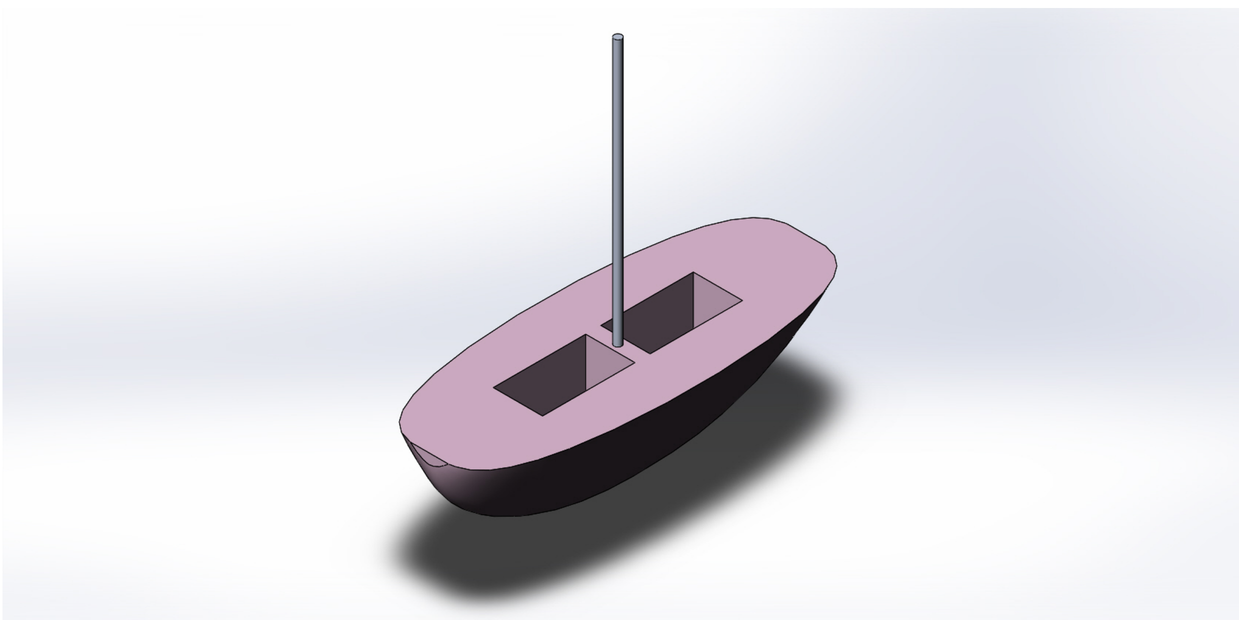


Figure 8: Assembled boat model.