CHAPTER 2

Kinematics: Description of Motion

I. Chapter Objectives

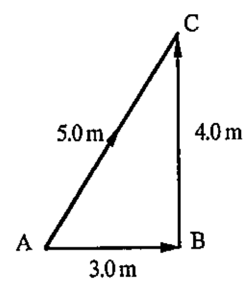
Upon completion of this chapter, you should be able to:

1. define distance and calculate speed, and explain what is meant by a scalar quantity.
2. define displacement and calculate velocity, and explain the difference between scalar and vector quantities.
3. explain the relationship between velocity and acceleration, and perform graphical analyses of acceleration.
4. explain the kinematic equations of constant acceleration, and apply them to physical situations.
5. use the kinetic equations to analyze free fall.

II. Chapter Summary and Discussion

1. Distance and Speed; Displacement and Velocity (Sections 2.1 – 2.2)

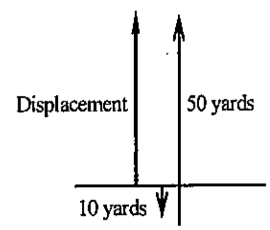
Motion is related to change of position. The length traveled in changing position may be expressed in terms of **distance**, the actual path length between two points. Distance is a scalar quantity, which has only a magnitude with no direction. The direct straight line pointing from the initial point to the final point is called **displacement** (change in position). Displacement measures only the change in position, not the details involved in the change in position. Displacement is a vector quantity, which has both magnitude and direction. In the figure shown, an object goes from point A to point C by following paths AB and BC. The distance traced is 3.0 m + 4.0 m = 7.0 m, and the displacement is 5.0 m in the direction of the arrow.



For motion in one dimension along the x-axis, the displacement between two points, x_1 and x_2, is simply the vector subtraction between x_1 and x_2: $\Delta x = x_2 - x_1$, where the Greek letter Δ (delta) is used to represent a change or difference in a quantity. For example, if an object moves from a point at $x_1 = 2.0$ m to another point at $x_2 = 4.0$ m, its displacement is $\Delta x = 4.0\text{ m} - 2.0\text{ m} = +2.0$ m. The positive sign here indicates the direction of the displacement as along the positive x-axis; however, if the motion is reversed, then $\Delta x = 2.0\text{ m} - 4.0\text{ m} = -2.0$ m. (What is the meaning of the negative sign here?)

Example 2.1: In a soccer game, a midfielder kicks the ball back 10 yards to a goalkeeper. The goalkeeper then kicks the ball straight up the field 50 yards to a forward. What is the distance traveled by the soccer ball? What is the displacement of the soccer ball?

Solution:



Sketch a diagram of the situation. For clarity, the arrows are laterally (horizontally) displaced. It is obvious from the diagram that the soccer ball first traveled 10 yards backward and then 50 yards forward. Thus, the *distance traveled* is 10 yards + 50 yards = 60 yards.

Displacement is the straight line from the initial position to the final position. The ball was displaced only 50 yards − 10 yards = 40 yards **straight up the field.**

Try to solve this problem without the diagram. You will find it is very difficult to do. That is why you are encouraged to try to draw a diagram to help solve a problem.

In a description of motion, the rate of change of position may be expressed in terms of speed and velocity.

Average speed is defined as the distance traveled divided by the time interval to travel that distance. $\bar{s} = \frac{d}{\Delta t}$, where $\bar{s}$ is average speed, d is distance traveled, and Δt is time interval (change in time).

Instantaneous speed is the speed at a particular time instant (Δt is infinitesimally small or close to zero). Because distance is a scalar quantity with no direction, so are average speed and instantaneous speed. Either average speed or instantaneous speed tells us only how fast objects are moving.

Average velocity is defined as *displacement* divided by the time interval, $\bar{v} = \frac{\Delta x}{\Delta t}$, where $\bar{v}$ is average velocity, Δx is displacement (change in position), and Δt is time interval. The sign of the displacement, positive or negative for one-dimensional motion, indicates the direction of displacement. **Instantaneous velocity,** v, is the velocity (both magnitude and direction) at a particular instant of time (Δt is close to zero). Because displacement is a vector quantity, so are average velocity and instantaneous velocity. Either average velocity or instantaneous velocity tells us not only how fast but in which directions objects are moving. The sign of velocity, positive or negative for one-dimensional motion, indicates the direction of velocity.

The SI units of speed and velocity are m/s.

Example 2.2: If the play described in Example 2.1 took 5.0 s, what is the average speed of the soccer ball? What is the average velocity of the soccer ball?

Solution:

Average speed: $\bar{s} = \dfrac{d}{\Delta t} = \dfrac{60\ \text{yd}}{5.0\ \text{s}} = 12\ \text{yd/s}.$

Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{40\ \text{yd straight up the field}}{5.0\ \text{s}} = 8.0\ \text{yd/s straight up the field.}$

2. Acceleration (Section 2.3)

Acceleration is the rate of change of velocity with time. *Note*: It is the change in *velocity* (a vector), not speed (a scalar); hence, acceleration is also a vector.

Average acceleration is defined as the change in velocity divided by the time interval to make the change, $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v - v_o}{t - t_o}$, where $\bar{a}$ is average acceleration, Δv is change in velocity, and Δt is time interval.

Instantaneous acceleration is the acceleration at a particular instant of time (Δt is close to zero). Because velocity is a vector quantity, so are average acceleration and instantaneous acceleration. The SI units of acceleration are (m/s)/s or m/s^2. In many of the topics you are going to study, the motion will have a constant acceleration. If the acceleration is constant, then the average acceleration is equal to the instantaneous acceleration.

A common misconception about velocity and acceleration has to do with their directions. Because velocity has both magnitude and direction, a change in either magnitude (speed) or direction or both will result in a change in velocity, and therefore acceleration. We can accelerate objects either by speeding them up or slowing them down (change the magnitude of velocity) and/or by changing their direction of travel (even though the speed remains constant). We often call the gas pedal of a car an *accelerator*. Can we call the brake pedal an accelerator? Can we call the steering wheel an accelerator? The answers are yes to both questions. (Why?)

For motion in one dimension, when the velocity and acceleration of an object are in the same direction (they have the same directional signs), the magnitude of the velocity (speed) increases and the object speeds up (acceleration). When the velocity and acceleration are in opposite directions, the magnitude of the velocity (speed) decreases and the object slows down (deceleration). The Learn by Drawing on page 42 in the textbook graphically illustrates this point.

Integrated Example 2.3:

An object moving in the positive x-axis has a decrease in the magnitude of velocity from 5.0 m/s to 1.0 m/s in 2.0 s. (a) The acceleration is (1) positive, (2) zero, or (3) negative. Explain. (b) What is the average acceleration? What does your result mean?

(a) Conceptual Reasoning:

(a) Since the velocity is positive and the magnitude of the velocity is decreasing, the acceleration must be opposite to the direction of velocity. Therefore, the acceleration is (3) negative.

(b) Quantitative Reasoning and Solution: Listing the given quantities and note that the velocities are positive.

Given: $v_0 = +5.0$ m/s, $v = +1.0$ m/s, $t = 2.0$ s.

Find: $\bar{a}$.

According to the definition of average acceleration,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = \frac{+1.0 \text{ m/s} - (+5.0 \text{ m/s})}{2.0 \text{ s}} = \frac{-4.0 \text{ m/s}}{2.0 \text{ s}} = -2.0 \text{ m/s}^2.$$

The negative sign means that the acceleration is to the left, opposite the velocity vector (deceleration). Physically, the result means that the object *decreases* its velocity by 2.0 m/s every second or 2.0 m/s^2.

3. Graphical Interpretation (Sections 2.2 – 2.3)

Graphical analysis is often helpful in understanding motion and its related quantities. In algebra, we learned that if $y = mx + b$, then m is the slope of the graph of y versus x. If we take $t_0 = 0$, then $\bar{a} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t}$, or $v = v_0 + \bar{a}\, t$. That is, the slope of a velocity-versus-time graph gives the average acceleration.

In general, on a position-versus-time graph, we can extract the average velocity by finding the slope of a line connecting two points. Instantaneous velocity is equal to the slope of a straight line tangent to the curve at a specific time. For a velocity-versus-time graph, the average acceleration is the slope of a straight line connecting two points, and instantaneous acceleration is the slope of a straight line tangent to the curve at a specific time. The area under the curve in a velocity-versus-time graph gives the change in position (displacement), and the area under the curve in an acceleration-versus-time graph yields the change in velocity.

Example 2.4: The graph represents the position of a particle as a function of time.

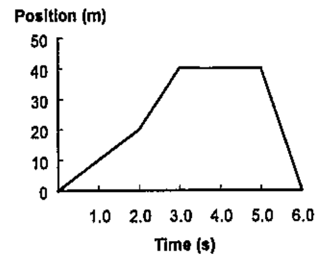
(a) What is the velocity at 1.0 s?

(b) What is the velocity at 2.5 s?

(c) What is the velocity at 4.0 s?

(d) What is the average velocity from 0 to 4.0 s?

(e) What is the average velocity for the 6.0-s interval?



Solution:

(a) The velocity is a constant between 0 s and 2.0 s so the velocity at 1.0 s is the same as the average velocity between 0 s and 2.0 s. $v = \dfrac{\Delta x}{\Delta t} = \dfrac{20\text{ m} - 0\text{ m}}{2.0\text{ s} - 0\text{ s}} = 10\text{ m/s}.$

(b) The velocity is a constant between 2.0 s and 3.0 s so the velocity at 2.5 s is the same as the average velocity between 2.0 s and 3.0 s. $v = \dfrac{\Delta x}{\Delta t} = \dfrac{40\text{ m} - 20\text{ m}}{3.0\text{ s} - 2.0\text{ s}} = 20\text{ m/s}.$

(c) The line is horizontal between 3.0 s and 5.0 s. So slope of the line is zero, and thus $v = 0$.

(d) Average velocity is the slope of a line connecting the points at $t = 0$ and 4.0 s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{40\text{ m} - 0\text{ m}}{4.0\text{ s} - 0\text{ s}} = 10\text{ m/s}.$$

(e) Average velocity is the slope of a line connecting the points at $t = 0$ and 6.0 s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0\text{ m} - 0\text{ m}}{6.0\text{ s} - 0\text{ s}} = 0\text{ m/s}.$$

Example 2.5: The graph represents the velocity of a particle as a function of time.

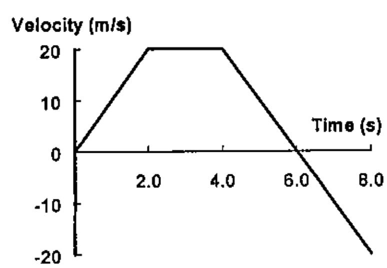
(a) What is the acceleration at 1.0 s?

(b) What is the acceleration at 3.0 s?

(c) What is the average acceleration between 0 and 5.0 s?

(d) What is the average acceleration for the 8.0-s interval?

(e) What is the displacement for the 8.0-s interval?



Solution:

(a) The acceleration is constant between 0 s and 2.0 s so the acceleration at 1.0 s is the same as the average acceleration between 0 s and 2.0 s.

$$a = \frac{\Delta v}{\Delta t} = \frac{20\ \text{m/s} - 0\ \text{m/s}}{2.0\ \text{s} - 0\ \text{s}} = 10\ \text{m/s}^2.$$

(b) The line is horizontal between 2.0 s and 4.0 s so the slope of the line is zero, and thus $a = 0$.

(c) Average acceleration is the slope of a line connecting the points at $t = 0$ and 5.0 s.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{10\ \text{m/s} - 0\ \text{m/s}}{5.0\ \text{s} - 0\ \text{s}} = 2.0\ \text{m/s}^2.$$

(d) Average acceleration is the slope of a line connecting the points at $t = 0$ and 8.0 s.

$$a = \frac{\Delta v}{\Delta t} = \frac{-20\ \text{m/s} - 0\ \text{m/s}}{8.0\ \text{s} - 0\ \text{s}} = -2.5\ \text{m/s}^2.$$

(e) The net area equals the displacement.

The area of a rectangle is length × width, and the area of a triangle is $\frac{1}{2}$× base × height.

$\Delta x_{0\text{-}2} = \frac{1}{2}(2.0\ \text{s} - 0\ \text{s})(20\ \text{m/s}) = 20\ \text{m}$; $\Delta x_{2\text{-}4} = (4.0\ \text{s} - 2.0\ \text{s})(20\ \text{m/s}) = 40\ \text{m}$;

$\Delta x_{4\text{-}6} = \frac{1}{2}(6.0\ \text{s} - 4.0\ \text{s})(20\ \text{m/s}) = 20\ \text{m}$; $\Delta x_{6\text{-}8} = \frac{1}{2}(8.0\ \text{s} - 6.0\ \text{s})(-20\ \text{m/s}) = -20\ \text{m}$.

Thus, $\Delta x = 20\ \text{m} + 40\ \text{m} + 20\ \text{m} + (-20\ \text{m}) = 60\ \text{m}$.

4. Kinematic Equations (Constant Acceleration) (Section 2.4)

Our discussion is restricted to **motions with constant accelerations.** In a motion with constant acceleration, the acceleration does not change with time; however, the constant acceleration can be zero or a negative or a positive non-zero constant. Zero acceleration simply means that the velocity is a constant (no acceleration). For positive velocity, a negative acceleration means deceleration (speed decrease) and a positive acceleration means acceleration (speed increase). For negative velocity, a negative acceleration means acceleration (speed increase) and a positive acceleration means deceleration (speed decrease).

The symbols used in the kinematic equations are: v_o, initial velocity; v, final velocity; a, acceleration; $x - x_o$, displacement; t, time interval. Be aware that the terms *initial* and *final* are relative. The end of one event is always the beginning of another. There are three general equations and two algebraic combinations of these equations that provide calculation convenience.

$x = x_o + \bar{v}t$	Final position = initial position + average velocity × times interval.
$\bar{v} = \frac{v + v_o}{2}$	Average velocity $= \frac{\text{final velocity + initial velocity}}{2}$.
$v = v_o + at$	Final velocity = initial velocity + acceleration × time interval.

$x = x_o + v_o t + \frac{1}{2}at^2$ Final position = initial position + initial velocity × time interval

$+\frac{1}{2}$ × acceleration × time interval squared.

$v^2 = v_o^2 + 2a(x - x_o)$ Final velocity squared = initial velocity squared

$+ 2$ × acceleration × displacement.

Among the five equations listed, the last three can be used to solve the majority of kinematic problems. Which equation should you select in solving a particular problem? The equation you select must have the unknown quantity in it and everything else must be given, because you can solve only for one unknown in one equation.

Note that $(x - x_o)$, final position minus initial position, is the displacement of the object. This is explicitly expressed in $v^2 = v_o^2 + 2a(x - x_o)$. The equation $x = x_o + v_o t + \frac{1}{2}at^2$ can also be written as $(x - x_o) = v_o t + \frac{1}{2}at^2$.

In many kinematic problems, we can set $x_o = 0$ at the initial time. In other words, we can select a coordinate system so the object is at the origin when $t = 0$. The preceding equations can then be simplified, and the final position, x, will be the same as the displacement because $x - x_o = x - 0 = x$.

Example 2.6: An object starts from rest and accelerates with a constant acceleration of 5.0 m/s^2. Find its velocity and displacement after 3.0 s.

Solution: Given: $v_o = 0$ (starts from rest), $a = 5.0$ m/s^2, $t = 4.0$ s.

Find: v and $(x - x_o)$.

From $v = v_o + at$, we have $v = 0 + (5.0\text{ m/s}^2)(4.0\text{ s}) = 20$ m/s.

Also $x - x_o = v_o t + \frac{1}{2}at^2 = (0)(3.0\text{ s}) + \frac{1}{2}(5.0\text{ m/s}^2)(4.0\text{ s})^2 = 40$ m.

Both velocity and displacement are in the direction of the motion.

Example 2.7: An automobile decelerates uniformly from 25 m/s to a stop while traveling 100 m. What is the acceleration of the automobile?

Solution: Given: $v_o = 25$ m/s, $v = 0$ (to a stop), $(x - x_o) = 100$ m.

Find: a.

Since $v^2 = v_o^2 + 2a(x - x_o)$, $a = \dfrac{v^2 - v_o^2}{2(x - x_o)} = \dfrac{(0)^2 - (25\text{ m/s})^2}{2(100\text{ m})} = -3.1\text{ m/s}^2$.

Because a is negative, it is in a direction opposite the velocity or motion.

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5. Free Fall (Section 2.5)

Objects in motion solely under the influence of gravity are said to be in **free fall**. A free fall does not necessarily mean a falling object. A vertically rising object is also said to be in free fall. The magnitude of **acceleration due to gravity** is often expressed by the symbol g. Near the surface of the Earth, the acceleration due to gravity is $g = 9.80 \text{ m/s}^2$ (downward) and near the surface of the Moon, it is $g = 1.7 \text{ m/s}^2$.

Note that g itself is a positive quantity, 9.80 m/s^2. If we use the upward direction as the positive reference direction, then we say the acceleration due to gravity is $a = -g = -9./80 \text{ m/s}^2$ (downward 9.80 m/s^2); however, if we use the downward direction as the positive reference direction, then the acceleration due to gravity is $a = +g = +9./80 \text{ m/s}^2$ (still downward 9.80 m/s^2).

Because free fall is in the vertical direction, and we often choose the upward direction as the positive y-axis, we replace the x's with y's and a's with $-g$'s in the kinematic equation. The results are

$$y = y_o + \bar{v}t, \quad \bar{v} = \frac{v + v_o}{2}, \quad v = v_o - gt, \quad y = y_o + v_o t - \tfrac{1}{2}gt^2, \quad v^2 = v_o^2 - 2g(y - y_o), \text{ where } g = 9.80 \text{ m/s}^2.$$

In many free-fall situations, we can take $y_o = 0$ to simplify the application of the preceding equations. Also note that $(y - y_o)$ is the displacement of the object.

Example 2.8: A ball is thrown upward with an initial velocity near the surface of the Earth. When it reaches the highest point

(a) its velocity is zero, and its acceleration is nonzero.

(b) its velocity is zero, and its acceleration is zero.

(c) its velocity is nonzero, and its acceleration is zero.

(d) its velocity is nonzero, and its acceleration is nonzero.

Solution:

The answer is (a), not (b), as you might think. The velocity has to change its direction at the highest point (goes from positive to negative) and so it is zero; however, the acceleration is not zero there. The acceleration is a constant 9.80 m/s^2 downward, independent of velocity. Stop and think, what if both the velocity and acceleration were zero at the highest point? Would the ball fall down from the highest point?

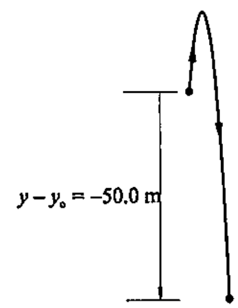
Example 2.9: A ball is thrown upward with an initial velocity of 10.0 m/s from the top of a 50.0-m-tall building.

(a) With what velocity will the ball strike the ground?

(b) How long does it take the ball to strike the ground?

Solution: Given: $(y - y_o) = -50.0$ m (displacement), $v_o = +10.0$ m/s.

Find: (a) v (b) t.



$(y - y_o)$, the difference between the final position and the initial position, is, by definition, displacement, not distance. When the ball strikes the ground, it will have been displaced –50.0 m, or fallen 50 m below the launch point.

(a) $v^2 = v_o^2 - 2g(y - y_o) = (+10.0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(-50.0 \text{ m}) = 1.08 \times 10^3 \text{ m}^2/\text{s}^2$.

So $v = \sqrt{1.08 \times 10^3 \text{ m}^2/\text{s}^2} = \pm 32.9 \text{ m/s}$.

The positive answer is discarded, because the ball is falling right before it lands (moving downward).

Therefore, $v = -32.9$ m/s.

(b) From $v = v_o - gt$, we have

$$t = \frac{v_o - v}{g} = \frac{(+10.0 \text{ m/s} - (-32.9 \text{ m/s})}{9.80 \text{ m/s}^2} = \frac{42.9 \text{ m/s}}{9.80 \text{ m/s}^2} = 4.38 \text{ s.}$$

Try to solve this problem without using the overall displacement concept. You could break it into two phases. First, you would have to find out how high the ball goes, then second, determine the velocity when it strikes the ground, and the total time it is in the air.

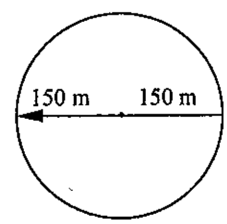
III. Mathematical Summary

Average speed	$\bar{s} = \dfrac{d}{\Delta t}$ (2.1)	Defines average speed.
Average velocity	$\bar{v} = \dfrac{\Delta x}{\Delta t}$ or $x = x_o + \bar{v}\, t$ (2.3)	Defines average velocity or expresses position in terms of average velocity and time interval.
Average acceleration	$\bar{a} = \dfrac{\Delta v}{\Delta t}$ (2.5)	Defines average acceleration.
Kinematic equation	$\bar{v} = \dfrac{v + v_o}{2}$ (2.9)	Defines average velocity for motion with constant acceleration.
Kinematic equation	$v = v_o + at$ (2.8)	Relates final velocity with initial velocity, acceleration, and time (constant acceleration only).

Kinematic equation	$x = x_o + \frac{1}{2}(v + v_o)t$ (2.10)	Relates displacement with initial velocity, final velocity, and time (constant acceleration only).
Kinematic equation	$x = x_o + v_o t + \frac{1}{2}at^2$ (2.11)	Relates displacement with initial velocity, acceleration, and time (constant acceleration only).
Kinematic equation	$v^2 = v_o^2 + 2a(x - x_o)$ (2.12)	Relates final velocity with initial velocity, acceleration, and displacement (constant acceleration only).
Equation (free fall)	$v = v_o - gt$ (2.8')	Relates final velocity with initial velocity, acceleration, and time.
Kinematic equation	$y = y_o + \frac{1}{2}(v + v_o)t$ (2.10')	Relates displacement with initial velocity, final velocity, and time.
Equation (free fall)	$y = y_o + v_o t - \frac{1}{2}gt^2$ (2.11')	Relates displacement with initial velocity, acceleration, and time.
Equation (free fall)	$v^2 = v_o^2 - 2g(y - y_o)$ (2.12')	Relates final velocity with initial velocity, acceleration, and displacement.

IV. Solutions of Selected Exercises and Paired Exercises

10. Displacement is the change in position.



So the magnitude of the displacement for half a lap is $\boxed{300 \text{ m}}$.

For a full lap (the car returns to its starting position), the displacement is $\boxed{\text{zero}}$.

18. (a) The average velocity is $\boxed{\text{(1) zero}}$ because the displacement is zero for a complete lap.

(b) The average speed is $\bar{s} = \frac{d}{\Delta t} = \frac{2\pi r}{\Delta t} = \frac{2\pi(500 \text{ m})}{50 \text{ s}} = \boxed{63 \text{ m/s}}$.

20. (a) The average speed is $\bar{s} = \frac{d}{\Delta t} = \frac{2(7.1 \text{ m})}{2.4 \text{ s}} = \boxed{5.9 \text{ m/s}}$.

(b) The average velocity is $\boxed{\text{zero}}$, as the ball is caught at the initial height so the $\boxed{\text{displacement is zero}}$.

24. (a) $\bar{v} = \frac{\Delta x}{\Delta t}$, So

$\bar{v}_{AB} = \frac{1.0 \text{ m} - 1.0 \text{ m}}{1.0 \text{ s} - 0} = \boxed{0}$; $\bar{v}_{BC} = \frac{7.0 \text{ m} - 1.0 \text{ m}}{3.0 \text{ s} - 1.0 \text{ s}} = \boxed{3.0 \text{ m/s}}$;

$\bar{v}_{CD} = \dfrac{9.0\text{ m} - 7.0\text{ m}}{4.5\text{ s} - 3.0\text{ s}} = \boxed{1.3\text{ m/s}}$; $\bar{v}_{DE} = \dfrac{7.0\text{ m} - 9.0\text{ m}}{6.0\text{ s} - 4.5\text{ s}} = \boxed{-1.3\text{ m/s}}$;

$\bar{v}_{EF} = \dfrac{2.0\text{ m} - 7.0\text{ m}}{9.0\text{ s} - 6.0\text{ s}} = \boxed{-1.7\text{ m/s}}$; $\bar{v}_{FG} = \dfrac{2.0\text{ m} - 2.0\text{ m}}{11.0\text{ s} - 9.0\text{ s}} = \boxed{0}$;

$\bar{v}_{BG} = \dfrac{2.0\text{ m} - 1.0\text{ m}}{11.0\text{ s} - 1.0\text{ s}} = \boxed{0.10\text{ m/s}}$.

(b) $\boxed{\text{The motion of BC, CD, and DE are not uniform}}$ since they are not straight lines.

(c) The object changes its direction of motion at point D. So it has to stop momentarily and $v = \boxed{0}$.

30. To the runner on the right, the runner on the left is running at a velocity of

$+4.50\text{ m/s} - (-3.50\text{ m/s}) = +8.00\text{ m/s}$. So it takes $\Delta t = \dfrac{\Delta x}{\bar{v}} = \dfrac{100\text{ m}}{8.00\text{ m/s}} = \boxed{12.5\text{ s}}$.

They meet at $(4.50\text{ m/s})(12.5\text{ s}) = \boxed{56.3\text{ m (relative to runner on left)}}$.

40. $15.0\text{ km/h} = (15.0\text{ km/h}) \times \dfrac{1000\text{ m}}{1\text{ km}} \times \dfrac{1\text{ h}}{3600\text{ s}} = 4.167\text{ m/s}$, $65.0\text{ km/h} = 18.06\text{ m/s}$.

So $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{18.06\text{ m/s} - 4.167\text{ m/s}}{6.00\text{ s}} = \boxed{2.32\text{ m/s}^2}$.

44. $75\text{ km/h} = (75\text{ km/h}) \times \dfrac{1000\text{ m}}{1\text{ km}} \times \dfrac{1\text{ h}}{3600\text{ s}} = 20.8\text{ m/s}$, $30\text{ km/h} = 8.33\text{ m/s}$.

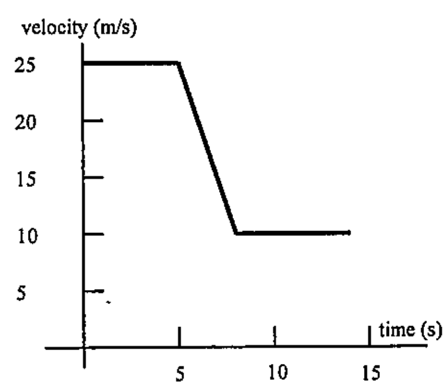
So $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{8.33\text{ m/s} - 20.8\text{ m/s}}{6.0\text{ s}} = \boxed{-2.1\text{ m/s}^2}$.

The negative sign indicates that the acceleration vector is in opposite direction of velocity.

52. (a) See the sketch on the right.

(b) The acceleration is negative as the object slows down (assume velocity is positive).

$v = v_o + at = 25\text{ m/s} + (-5.0\text{ m/s}^2)(3.0\text{ s}) = \boxed{10\text{ m/s}}$.



(c) $x = x_1 + x_2 + x_3$

$= (25\text{ m/s})(5.0\text{ s})$

$+ (25\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(-5.0\text{ m/s}^2)(3.0\text{ s})^2$

$+ (10\text{ m/s})(6.0\text{ s})$

$= 237.5\text{ m} = \boxed{2.4 \times 10^2\text{ m}}$.

(d) $\bar{s} = \dfrac{d}{\Delta t} = \dfrac{237.5\text{ m}}{14.0\text{ s}} = \boxed{17\text{ m/s}}$.

62. Given: $v_o = 0$, $a = 2.0 \text{ m/s}^2$, $t = 5.00$ s. Find: v and x (take $x_o = 0$).

(a) $v = v_o + at = 0 + (2.0 \text{ m/s}^2)(5.0 \text{ s}) = \boxed{10 \text{ m/s}}$.

(b) $x = x_o + v_o t + \frac{1}{2}at^2 = 0 + 0(5.00 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s}^2)(5.0 \text{ s})^2 = \boxed{25 \text{ m}}$.

66. Given: $v_o = 0$, $v = 560 \text{ km/h} = 155.6 \text{ m/s}$, $x = 400$ m (take $x_o = 0$). Find: t and a.

(a) From $x = x_o + \bar{v}\,t = \dfrac{v_o + v}{2}\,t$, we have $t = \dfrac{2x}{v_o + v} = \dfrac{2(400 \text{ m})}{0 + 155.6 \text{ m/s}} = \boxed{5.14 \text{ s}}$.

(b) Also from $v = v_o + at$, the acceleration is $a = \dfrac{v - v_o}{t} = \dfrac{155.6 \text{ m/s} - 0}{5.14 \text{ s}} = \boxed{30.3 \text{ s}}$.

72. $40 \text{ km/h} = (40 \text{ km/h}) \times \dfrac{1000 \text{ m}}{1 \text{ km}} \times \dfrac{1 \text{ h}}{3600 \text{ s}} = 11.11 \text{ m/s}$.

During reaction, the car travels a distance of $d = (11.11 \text{ m/s})(0.25 \text{ s}) = 2.78$ m.

So the car really has only 13 m – 2.78 m = 10.2 m to come to rest.

Let's calculate the stopping distance of the car (take $x_o = 0$).

Given: $v_o = 11.1$ m/s, $v = 0$, $a = -8.0 \text{ m/s}^2$. Find: x. (Take $x_o = 0$.)

From $v^2 = v_o^2 + 2a(x - x_o)$, we have $x = \dfrac{v^2 - v_o^2}{2a} = \dfrac{0 - (11.1 \text{ m/s})^2}{2(-8.0 \text{ m/s}^2)} = 7.70$ m.

So it takes the car only $2.78 \text{ m} + 7.70 \text{ m} = \boxed{10.5 \text{ m}}$ (< 13 m) to stop.

$\boxed{\text{Yes}}$, the car will stop before hitting the child.

76. (a) The answer is $\boxed{(3)\ t_1 > t_2}$. Because the object is accelerating its velocity increases. Therefore it will spend less time in traveling the second 3.00 m.

(b) For the first 3.00 m:

Given: $v_o = 0$, $a = 2.00 \text{ m/s}^2$, $x = 3.00$ m (take $x_o = 0$). Find: t.

From $x = x_o + v_o t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}at^2$, we have $t_1 = \sqrt{\dfrac{2x}{a}} = \sqrt{\dfrac{2(3.00 \text{ m})}{2.00 \text{ m/s}^2}} = \boxed{1.73 \text{ s}}$.

At the end of the first 3.00 m, the velocity of the object is

$v = v_o + at = 0 + (2.00 \text{ m/s}^2)(1.73 \text{ s}) = 3.46$ m/s, which is then the initial velocity for the second 3.00 m.

For the second 3.00 m:

Given: $v_o = 3.46$ m/s, $a = 2.00 \text{ m/s}^2$, $x = 3.00$ m. Find: t.

From $x = x_o + v_o t + \frac{1}{2}at^2$, we have $3.00 \text{ m} = 0 + (3.46 \text{ m/s})t_2 + \frac{1}{2}(2.00 \text{ m/s}^2)t_2^2$.

Reducing to quadratic equation, $t^2 + 3.46t - 3.00 = 0$. Solving, $t_2 = \boxed{0.718 \text{ s}}$ or –4.18 s.

80. (a) From $v^2 = v_o^2 + 2a(x - x_o)$, we have $x - x_o = \dfrac{v^2 - v_o^2}{2a} = \dfrac{0^2 - v_o^2}{2a} = -\dfrac{v_o^2}{2a}$.

Taking $x_o = 0$, so $(x - x_o) = x$ is proportional to v_o^2. If v_o doubles, then x becomes 4 times as large.

The answer is then $\boxed{(3)\ 4x}$.

(b) $\dfrac{x_2}{x_1} = \dfrac{v_{2o}^2}{v_{1o}^2} = \dfrac{60^2}{40^2} = 2.25$. So $x_2 = 2.25\,x_1 = 2.25\,(3.00\text{ m}) = \boxed{6.75\text{ m}}$.

90. Taking $y_o = 0$, then $y = y_o + v_o t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2$. So $y_1 = -\frac{1}{2}gt^2$ and $y_2 = -\frac{1}{2}g(t-1)^2$.

The distance between the two positions is

$\Delta y = y_1 - y_2 = -\frac{1}{2}g\,[t^2 - (t-1)^2] = -\frac{1}{2}g\,[t^2 - t^2 + 2t - 1] = -\frac{1}{2}g\,[2t - 1]$.

Therefore, Δy $\boxed{\text{increases}}$ as t increases.

96. Given: $v_o = 15$ m/s, $v = 0$ (maximum height). Find: y. (Take $y_o = 0$.)

From $v^2 = v_o^2 - 2g(y - y_o)$, we have $y = \dfrac{v_o^2 - v^2}{2g} = \dfrac{(15\text{ m/s})^2 - (0)^2}{2(9.80\text{ m/s}^2)} = \boxed{11\text{ m}}$.

102. (a) Given: $v_o = 80.00$ mi/h $= 35.76$ m/s, $v = 0$ (max height), Find: $(y - y_o)$.

From $v^2 = v_o^2 - 2g(y - y_o)$, we have $y - y_o = \dfrac{v_o^2 - v^2}{2g} = \dfrac{0^2 - (35.76\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 65.2$ m.

Therefore the minimum ceiling height is $65.2\text{ m} + 1.00\text{ m} = \boxed{66.2\text{ m}}$.

(b) $y - y_o = 65.2\text{ m} - 10.0\text{ m} = 55.2$ m. With $v = 0$,

$v_o = \sqrt{2g(y - y_o)} = \sqrt{2(9.80\text{ m/s}^2)(55.2\text{ m})} = \boxed{32.9\text{ m/s}}$.

108. (a) Given: $v_o = 0$, $(y - y_o) = -10.0$ m (downward). Find: v.

$v^2 = v_o^2 - 2g(y - y_o) = -2(9.80\text{ m/s}^2)(-10.0\text{ m}) = 196\text{ m}^2/\text{s}^2$. So $v = -14.0\text{ m/s} = \boxed{14.0\text{ m/s}}$ downward.

(b) Given: $v = 0$ (max height), $(y - y_o) = 4.00$ m. Find: v_o.

From $v^2 = v_o^2 - 2g(y - y_o)$, we have $v_o = \sqrt{v^2 + 2g(y - y_o)} = \sqrt{0 + 2(9.80\text{ m/s}^2)(4.00\text{ m})} = \boxed{8.85\text{ m/s}}$.

(c) Falling: Also $v = v_o - gt$, so $t_1 = \dfrac{v_o - v}{g} = \dfrac{0 - (-14.0\text{ m/s})}{9.80\text{ m/s}^2} = 1.43$ s.

Rising: $t_2 = \dfrac{8.85\text{ m/s} - 0}{9.80\text{ m/s}^2} = 0.90$ s.

Therefore the total time is $1.43\text{ s} + 0.903\text{ s} = \boxed{2.33\text{ s}}$.

114. The height of each floor is $\frac{509 \text{ m}}{101} = 5.040$ m.

The height for 89 floors is then $89(5.040 \text{ m}) = 448.6$ m. At midpoint, the height is 224.3 m.

1008 m/min = 16.8 m/s and 610 m/min = 10.2 m/s.

(a) Up. $x = 224.3$ m (taking $x_o = 0$), $v_o = 0$, $v = 16.8$ m/s. Find: a.

From $v^2 = v_o^2 + 2a(x - x_o)$, we have $a_{up} = \frac{v^2 - v_o^2}{2x} = \frac{(16.8 \text{ m/s})^2 - 0}{2(224.3 \text{ m})} = \boxed{0.629 \text{ m/s}^2}$.

Down. $x = 224.3$ m (taking $x_o = 0$), $v_o = 0$, $v = 10.2$ m/s. Find: a.

$a_{down} = \frac{v^2 - v_o^2}{2x} = \frac{(10.2 \text{ m/s})^2 - 0}{2(224.3 \text{ m})} = \boxed{0.232 \text{ m/s}^2}$.

(b) Also $v = v_o + at$, so $t_{up} = \frac{v - v_o}{a} = \frac{16.8 \text{ m/s} - 0}{0.629 \text{ m/s}^2} = 26.70$ s.

However this is the time to accelerate to the peak speed. After that, the elevator needs to slow down to zero. So the total upward time is twice or 53.4 s.

$t_{down} = \frac{10.2 \text{ m/s} - 0}{0.232 \text{ m/s}^2} = 43.97$ s. Similarly, the total downward time is 87.94 s.

The time difference is $87.94 \text{ s} - 53.4 \text{ s} = \boxed{34.5 \text{ s}}$.

V. Practice Quiz

1. If you run around a complete circle of radius 25 m in 100 s, the magnitude of your average velocity is
(a) zero. (b) 0.20 m/s. (c) 0.50 m/s. (d) 1.0 m/s. (e) 3.14 m/s.

2. An object moving in the positive x-axis experiences an acceleration of +5.0 m/s^2. This means the object is
(a) traveling 5.0 m in every second.
(b) traveling at 5.0 m/s in every second.
(c) changing its velocity by 5.0 m/s.
(d) increasing its velocity by 5.0 m/s in every second.

3. A car starts from rest and travels 100 m in 5.0 s. What is the magnitude of the constant acceleration?
(a) zero (b) 5.0 m/s^2 (c) 8.0 m/s^2 (d) 10 m/s^2 (e) 40 m/s^2

4. An object is thrown straight up. When it is at the highest point
(a) both its velocity and acceleration are zero.
(b) neither its velocity nor its acceleration is zero.
(c) its velocity is zero, and its acceleration is not zero.
(d) its velocity is not zero, and its acceleration is zero.

5. Human reaction time is usually greater than 0.10 s. If your lab partner holds a ruler between your fingers and releases it without warning, how far can you expect the ruler to fall before you catch it?
(a) at least 3.0 cm (b) at least 4.9 cm (c) at least 6.8 cm (d) at least 9.8 cm (e) at least 11.0 cm

6. Which one of the following quantities is an example of a scalar?
(a) displacement (b) speed (c) velocity (d) acceleration

7. A ball is thrown vertically upward with a speed v_o. An identical second ball is thrown upward with a speed $2v_o$ (twice as fast). What is the ratio of the maximum height of the second ball to that of the first ball? (How many times higher does the second ball go than the first ball?)
(a) 4:1 (b) 2:1 (c) 1.7:1 (d) 1.4:1 (e) 1:1

8. A car starts from rest and accelerates for 4.0 m/s^2 for 5.0 s, then maintains that velocity for 10 s and then decelerates at the rate of 2.0 m/s^2 for 4.0 s. What is the final speed of the car?
(a) 20 m/s (b) 16 m/s (c) 12 m/s (d) 10 m/s (e) 8.0 m/s

9. An object moves 5.0 m north and then 3.0 m east. Find both the distance traveled and the magnitude of the displacement.
(a) 8.0 m; 5.8 m (b) 5.8 m; 8.0 m (c) 8.0 m; 4.0 m (d) 4.0 m; 8.0 m (e) 5.8 m, 34 m

10. A car with a speed of 25.0 m/s brakes to a stop. If the maximum deceleration of the car is 10.0 m/s^2, what is the minimum stopping distance?
(a) 0.032 m (b) 0.80 m (c) 1.3 m (d) 31 m (e) 6.3×10^2 m

11. An object moves in the positive x-axis. At $x = 10$ m, its speed is 5.0 m/s. At 4.0 s later, the object is at $x =$ 70 m. What is its acceleration?
(a) 1.0 m/s^2 (b) 5.0 m/s^2 (c) 10 m/s^2 (d) 15 m/s^2 (e) 20 m/s^2

12. A stone is thrown vertically upward at an initial speed of 10 m/s from a height of 20 m above the ground. How long is the stone in the air before it hits the ground?
(a) 1.2 s (b) 2.0 s (c) 3.3 s (d) 4.5 s (e) 10 s

Answers to Practice Quiz:

1. a 2. d 3. c 4. c 5. b 6. b 7. a 8. c 9. a 10. d 11. b 12. c

CHAPTER 3

Motion in Two Dimensions

I. Chapter Objectives

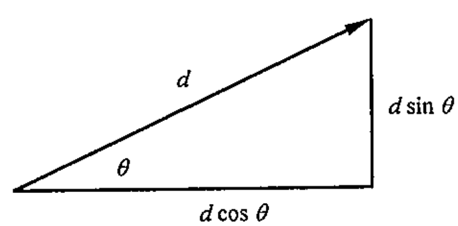
Upon completion of this chapter, you should be able to:

1. analyze motion in terms of its components, and apply the kinematic equations to components of motion.
2. learn vector notation, be able to add and subtract vectors graphically and analytically, and use vectors to describe motion in two dimensions.
3. analyze projectile motion to find position, time of flight, and range.
4. understand and determine relative velocities through vector addition and subtraction.

II. Chapter Summary and Discussion

1. Components of Motion (Section 3.1)

Motion in two dimensions, or curvilinear motion, is motion in which an object moves in a plane that can be described by a rectangular coordinate system. When such motion is analyzed, quantities are usually resolved into rectangular **components**. The diagram on the right shows the displacement vector $\vec{\mathbf{d}}$ being resolved into $x = d\cos\theta$ and $y = d\sin\theta$, the rectangular components. Similarly, velocity and acceleration vectors, $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$, are resolved into $v_x = v\cos\theta$, $v_y = v\sin\theta$, and $a_x = a\cos\theta$, $a_y = a\sin\theta$, their horizontal and vertical components, respectively.



Note: The angle θ used in the preceding calculations is the angle relative to the x-axis.

Once the displacement, velocity, and acceleration vectors are resolved into their respective components, we can apply the kinematic equations from Chapter 2 to the motion in the x- and y-directions. For example:

$$v_x = v_{xo} + a_x t, \quad x = x_o + v_{xo} t + \tfrac{1}{2} a_x t^2,$$
$$v_y = v_{yo} + a_y t, \quad y = y_o + v_{yo} t + \tfrac{1}{2} a_y t^2, \quad \text{etc.}$$

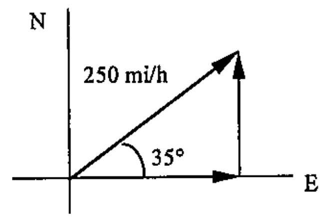
The key to success in solving two-dimensional motion is to resolve the motion into components. Remember to treat the components as independent, i.e., a_x having nothing to do with a_y, and so forth; however, the time in all the equations is the same, providing a common link. Always think about resolving vectors into components when working problems in two dimensions.

Example 3.1: An airplane is moving at a velocity of 250 mi/h in a direction 35° north of east. Find the components of the plane's velocity in the eastward and northward directions.

Solution: Given: $\vec{v}$ = 250 mi/h in a direction 35° north of east,

or v = 250 mi/h and θ = 35°.

Find: v_E and v_N.



$v_E = v \cos\theta = (250 \text{ mi/h}) \cos 35° = 205 \text{ mi/h};$

$v_N = v \sin\theta = (250 \text{ mi/h}) \sin 35° = 143 \text{ mi/h}.$

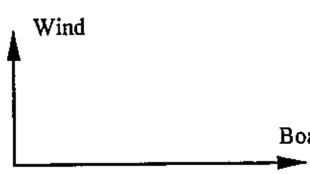
Example 3.2: A boat travels with a speed of 5.0 m/s in a straight path on a still lake. Suddenly, a steady wind pushes the boat perpendicular to its straight-line path with a speed of 3.0 m/s for 5.0 s. Relative to its position just when the wind started to blow, where is the boat at the end of this time?

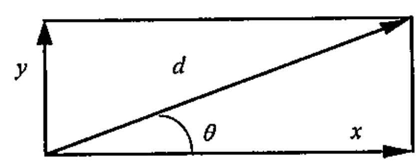
Solution: Chose the x-axis in the original direction of the bpat and the y-axis in the direction of the wind.

Given: $v_{xo} = 5.0$ m/s, $a_x = 0$, $v_{yo} = 3.0$ m/s, $a_y = 0$, $t = 5.0$ s.

Find: x and y.

Both motions are motions with constant velocity (zero acceleration). Choose the straight path of the boat as the x-axis and the direction of wind as the y-axis. Take both $x_o = 0$ and $y_o = 0$.





$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2 = 0 + (5.0 \text{ m/s})(5.0 \text{ s}) + 0 = 25 \text{ m},$

$y = y_o + v_{yo}t + \frac{1}{2}a_y t^2 = 0 + (3.0 \text{ m/s})(5.0 \text{ s}) + 0 = 15 \text{ m}.$

Or $\quad d = \sqrt{x^2 + y^2} = \sqrt{(25 \text{ m})^2 + (15 \text{ m})^2} = 29 \text{ m},$

and $\quad \theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{15 \text{ m}}{25 \text{ m}} = \tan^{-1} 0.60 = 31°.$

---

2. Vector Addition and Subtraction (Section 3.2)

Vector addition can be done geometrically with the triangle method or the parallelogram method for two vectors and with the polygon method for more than two vectors. **Vector subtraction** is a special case of vector addition because $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$, and a negative vector is defined as a vector having the same magnitude but opposite direction of the positive vector. For example, the negative vector of a velocity vector at 45 m/s north is simply a velocity vector at 45 m/s south. A convenient and consistent scale must be used for adding or subtracting vectors geometrically.

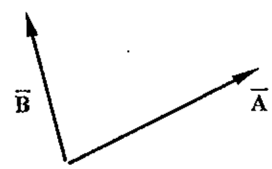
Example 3.3: Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given. Show

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ with the triangle method.

(b) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ with the parallelogram method.

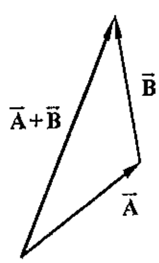
(c) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ with the triangle method.

(d) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ with the parallelogram method.

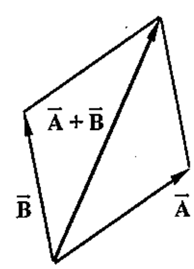


Solution:

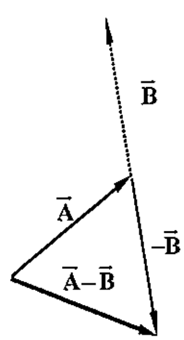
(a)



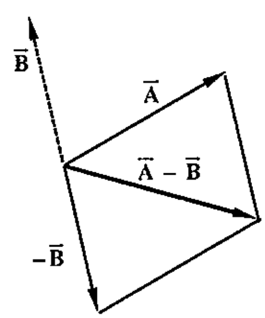
(b)



(c)



(d)



Vector addition is conveniently done by the **analytical component method**. The recommended procedure is as follows:

(1) Resolve the vectors to be added into their x- and y-components. Include directional signs (positive or negative) in the components.

(2) Add, algebraically, all the x-components together and all the y-components together to get the x- and y-components of the resultant vector, respectively.

(3) Express the resultant vector using (a) the component form, e.g., $\vec{\mathbf{C}} = C_x\,\hat{\mathbf{x}} + C_y\,\hat{\mathbf{y}}$, or

(b) the magnitude-angle form, e.g., $C = \sqrt{C_x^2 + C_y^2}$,

$\theta = \tan^{-1}\dfrac{C_y}{C_x}$ (relative to the positive x-axis).

A detailed treatment of this procedure can be found on page 78 in the textbook.

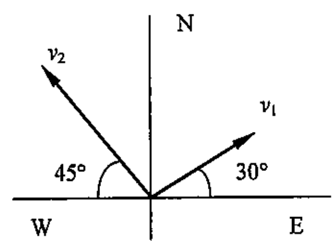
Example 3.4: Use the analytical component method to find the resultant velocity of the following two velocities:

(i) $\vec{\mathbf{v}}_1 = 35$ m/s 30° north of east;

(ii) $\vec{\mathbf{v}}_2 = 55$ m/s 45° north of west.

Solution:

(1) Resolve the vectors to be added into their x- and y-components.



$v_{1x} = v_1 \cos 30° = (35 \text{ m/s}) \cos 30° = 30.3$ m/s;

$v_{1y} = v_1 \sin 30° = (35 \text{ m/s}) \sin 30° = 17.5$ m/s.

$v_{2x} = -\,v_2 \cos 45° = -\,(55 \text{ m/s}) \cos 45° = -38.9$ m/s, ($-x$ direction)

(Why is the x-component of $\vec{\mathbf{v}}_2$ negative?)

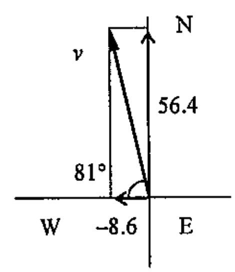
$v_{2y} = v_2 \sin 45° = (55 \text{ m/s}) \sin 45° = 38.9$ m/s.

(2) Add components.

$v_x = v_{1x} + v_{2x} = 30.3 \text{ m/s} + (-38.9 \text{ m/s}) = -8.6$ m/s,

$v_y = v_{1y} + v_{2y} = 17.5 \text{ m/s} + 38.9 \text{ m/s} = 56.4$ m/s.

(3) Express the resultant vector.



First, we draw the resultant velocity vector based on the components obtained in the previous procedure. We know that the x-component is -8.6 m/s, and the y-component is 56.4 m/s.

In component form: $\vec{\mathbf{v}} = \vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 = -8.6 \text{ m/s}\,\hat{\mathbf{x}} + 56 \text{ m/s}\,\hat{\mathbf{y}}$;

In magnitude-angle form: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-8.6 \text{ m/s})^2 + (56.4 \text{ m/s})^2} = 57$ m/s,

and $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{56.4 \text{ m/s}}{-8.6 \text{ m/s}} = 81°$ north of west.

Note: Once you have the components for the resultant [at the end of step (2)], you need to draw a diagram showing the x- and y-components of the resultant vector to determine the angle and the quadrant in which the vector is located. It is very difficult to determine the location of the vector without this diagram.

3. Projectile Motion (Section 3.3)

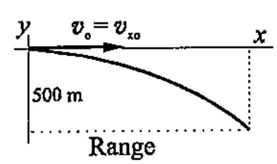
Projectile motion is a motion in two dimensions (horizontal and vertical) with the vertical motion under the action of gravity only (downward). Because the action of gravity is in the vertical direction, the horizontal motion has zero acceleration if air resistance is ignored. The vertical motion is a free fall and so the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\ \text{m/s}^2$, if the upward direction is chosen positive. This two-dimensional motion is analyzed using components; that is, the horizontal quantities are independent of the vertical quantities and vice versa; however, the time of flight—the time the projectile spends in the air—is the common quantity for both the horizontal and vertical motions.

Applying the general kinematic equations in component form to projectile motion ($a_x = 0$ and $a_y = -g$ with the upward direction chosen positive), we have $v_x = v_{xo}$, $v_y = v_{yo} - gt$, $x = x_o + vt$, $y = y_o + v_{yo}t - \frac{1}{2}gt^2$, and so on. Again, the key to success in solving projectile motion is to resolve the motion into components, treat the components as independent, and use the same time of flight for both motions. Always think about resolving vectors into components. Usually, the time of flight is something you have to find first, since it is a common quantity for both motions.

Example 3.5: A package is dropped from an airplane traveling with a constant horizontal speed of 120 m/s at an altitude of 500 m. What horizontal distance does the package travel before hitting the ground?

Solution: Given: horizontal motion ($x_o = 0$) (taking it in the $+x$-direction) $v_{xo} = 120$ m/s,

vertical motion ($y_o = 0$) (up as positive) $v_{yo} = 0$, $y = -500$ m.



Find: x (the range).

Because the horizontal distance (range) is $x = x_o + v_{xo}t = v_{xo}t$, we have to find the time of flight t first.

From the vertical motion, we use $y = y_o + v_{yo}t - \frac{1}{2}gt^2 = v_{yo}t - \frac{1}{2}gt^2$.

Thus, $-500\ \text{m} = 0 - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$, and $t = 10.1$ s.

Therefore, $x = (120\ \text{m/s})(10.1\ \text{s}) = 1.21 \times 10^3\ \text{m} = 1.21$ km.

Note: The quantities such as initial velocities and displacements have to be treated independently. For example, the initial horizontal velocity is 120 m/s, and the initial vertical velocity is 0 m/s. The 120 m/s can be used *only* in the horizontal motion, and the 0 m/s can be used *only* in the vertical motion. A common mistake is to use the 120 m/s for the vertical motion. It is imperative to list the horizontal and vertical quantities separately.

Example 3.6: A golfer hits a golf ball with a velocity of 35 m/s at an angle of 25° above the horizontal. If the point where the ball is hit and the point where the ball lands are at the same level,

(a) how much time does the ball spend in the air?

(b) what horizontal distance does the ball travel before landing (range)?

Solution: Given: horizontal motion ($x_o = 0$)

(taking it in the $+x$-direction)

$v_{xo} = v_o \cos\theta = (35 \text{ m/s}) \cos 25°$

$= 31.7 \text{ m/s}$;

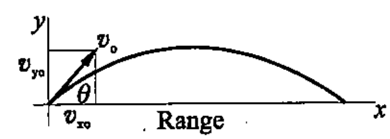
vertical motion ($y_o = 0$)

(up as positive)

$v_{yo} = v_o \sin\theta = (35 \text{ m/s}) \sin 25°$

$= 14.8 \text{ m/s}$

$y = 0$ (on landing).



Find: (a) t (b) x.

(a) When the golf ball lands, $y = 0$; and from $y = y_o + v_{yo}t - \frac{1}{2}gt^2$, we have

$0 = 0 + (14.8 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$. Solving the quadratic equation for $t = 0$ or 3.02 s.

The $t = 0$ answer corresponds to the position at the start ($x_o = 0$ and $y_o = 0$) and the $t = 3.02$ s corresponds to the landing position (x = the range and $y = 0$), so the time of flight is 3.0 s, (to two significant figures).

(b) $x = x_o + v_{xo}t = 0 + (31.7 \text{ m/s})(3.02 \text{ s}) = 96$ m.

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4. Relative Velocity (Section 3.4)

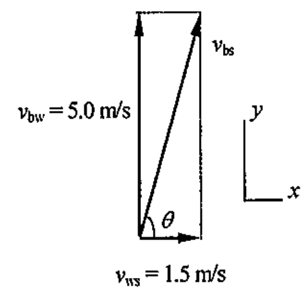
Physical phenomena can be observed from different **frames of reference.** The velocity of a ball tossed by a passenger in a moving car will be measured differently by a passenger on the car than by an observer on the sidewalk. In fact, any velocity we measure is **relative.** The velocity of a moving car is measured relative to the ground, and the revolving motion of the Earth around the Sun is relative to the Sun, etc. Relative velocity can be determined with vector addition or subtraction. The symbols used in relative velocity such as $\vec{v}_{cg}$ (where c stands for car and g stands for ground) means the velocity of *a car relative to the ground.*

Integrated Example 3.7:

A river has a current with a velocity of 1.5 m/s due east. A boat whose speed in still water is 5.0 m/s is directed north across a 100-m-wide river. (a) The velocity of the boat relative to the shore is directed (1) north of east, (2) due north, (3) due east, or (4) north of west. Explain. (b) How long does it take the boat to reach the opposite shore? (c) What is the velocity of the boat relative to the shore?

(a) Conceptual Reasoning:

We use the following subscripts: w = water, b = boat, s = shore.



The velocity of the boat relative to the shore, $\vec{v}_{bs}$, is the vector addition of the velocity of the boat relative to the water, $\vec{v}_{bw}$, and the velocity of the water relative to the shore, $\vec{v}_{ws}$. That is, $\vec{v}_{bs} = \vec{v}_{bw} + \vec{v}_{ws}$. From the drawing, it can be seen that the direction of the velocity of the boat relative to the shore is (1) north of east.

(b) Quantitative Reasoning and Solution:

Taking the x-axis in the direction of the current and the y-axis directly across the river.

Given: $v_{ws} = 1.5$ m/s, $v_{bw} = 5.0$ m/s, $y = 100$ m.

Find: (b) t (c) $\vec{v}_{bs}$.

(b) From the concept of components of motion, the time it takes the boat to reach the opposite shore is

$$t = \frac{y}{v_{bw}} = \frac{100\text{ m}}{5.0\text{ m/s}} = 20\text{ s}.$$

(c) Again, The velocity of the boat relative to the shore, $\vec{v}_{bs}$, is the vector addition of the velocity of the boat relative to the water, $\vec{v}_{bw}$, and the velocity of the water relative to the shore, $\vec{v}_{ws}$. That is, $\vec{v}_{bs} = \vec{v}_{bw} + \vec{v}_{ws}$.

Note: The pattern of the subscripts is helpful in problem solving. On the right side of the equation, the two inner subscripts are the same (w). The outer subscripts (b and s) are sequentially the same as those for the relative velocity on the left side of the equation. This pattern is a good check to see if you have written the relative velocity equation correctly!

Because the direction of $\vec{v}_{bw}$ and $\vec{v}_{ws}$ are perpendicular to each other, we can use Pythagorean theorem to calculate the magnitude of $\vec{v}_{bs}$.

$$v_{bs} = \sqrt{v_{bw}^2 + v_{ws}^2} = \sqrt{(5.0\text{ m/s})^2 + (1.5\text{ m/s})^2} = 5.2\text{ m/s},$$

The angle θ shown in the diagram is

$$\theta = \tan^{-1}\left(\frac{5.0\text{ m/s}}{1.5\text{ m/s}}\right) = 73°\text{ north of east.}$$

This is the direction of the velocity of the *boat relative to the shore.*

Example 3.8: If the person on the boat in Example 3.5 wants to travel directly across the river,

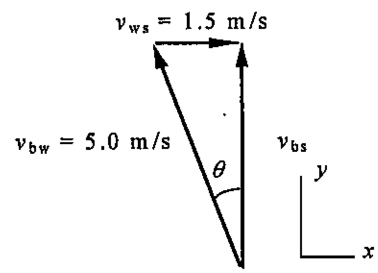
(a) at what angle upstream must the boat be directed?

(b) with what speed will the boat cross the river?

(c) how long will it take the boat to reach the opposite shore?

Solution:

To travel directly across the river, the velocity of the boat *relative to the shore* must be directly across the river. The vector form of the relative-velocity equation in Example 3.5, $\vec{v}_{bs} = \vec{v}_{bw} + \vec{v}_{ws}$ is still valid.



(a) $\theta = \sin^{-1}\frac{v_{rs}}{v_{br}} = \sin^{-1}\frac{1.5\text{ m/s}}{5.0\text{ m/s}} = 17°$ upstream from a line straight across the river.

(b) From the triangle in the diagram, we have $v_{bs}^2 + v_{ws}^2 = v_{bw}^2$,

so $v_{bs} = \sqrt{v_{bw}^2 - v_{ws}^2} = \sqrt{(5.0\text{ m/s})^2 - (1.5\text{ m/s})^2} = 4.8$ m/s.

(c) The time is then $t = \frac{y}{v_{bs}} = \frac{100\text{ m}}{4.8\text{ m/s}} = 21$ s.

III. Mathematical Summary

Components of Initial velocity	$v_x = v\cos\theta$ $v_y = v\sin\theta$	(3.1a) (3.1b)	Relates the x- and y-components to the magnitude and the angle of the initial velocity. (θ is from x-axis.)
Components of Displacement	$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2$ $y = y_o + v_{yo}t + \frac{1}{2}a_y t^2$	(3.3a) (3.3b)	Relates the displacement components to initial velocity components, acceleration components, and time (constant acceleration only).
Components of Velocity	$v_x = v + a_x t$ $v_y = v_{yo} + a_y t$	(3.3c) (3.3d)	Relates the velocity components to initial velocity components and acceleration components (constant acceleration only).
Vector Representation	$C = \sqrt{C_x^2 + C_y^2}$ $\theta = \tan^{-1}\left\lvert\frac{C_y}{C_x}\right\rvert$	(3.4a) (3.6)	Magnitude-angle form.
Vector Representation	$\mathbf{C} = C_x\hat{\mathbf{x}} + C_y\hat{\mathbf{y}}$	(3.7)	Component form.