

# IE6600 - Modeling\_Fung

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## Inputting Bejing Air Quality Dataset

### Data pre-processing

```
# Input data by using read_csv and make easy data formating----  
mydata<-read.csv("C:\\\\Users\\\\kings\\\\Downloads\\\\df.csv")  
#mydata  
#summarize the data  
summary(mydata)
```

```
##      year      month      day      hour  
##  Min.   :2013   Min.   : 1.000   Min.   : 1.00   Min.   : 0.0  
##  1st Qu.:2014   1st Qu.: 4.000   1st Qu.: 8.00   1st Qu.: 6.0  
##  Median :2015   Median : 7.000   Median :16.00   Median :12.0  
##  Mean   :2015   Mean   : 6.524   Mean   :15.72   Mean   :11.5  
##  3rd Qu.:2016   3rd Qu.:10.000   3rd Qu.:23.00   3rd Qu.:17.0  
##  Max.   :2017   Max.   :12.000   Max.   :31.00   Max.   :23.0  
##      PM2.5      PM10      SO2      NO2  
##  Min.   : 2.00   Min.   : 2.0   Min.   : 0.2856   Min.   : 1.026  
##  1st Qu.:21.00   1st Qu.: 36.0   1st Qu.: 3.0000   1st Qu.: 24.000  
##  Median : 55.00   Median : 82.0   Median : 7.0000   Median : 43.000  
##  Mean   : 79.19   Mean   :104.2   Mean   :15.6534   Mean   : 50.373  
##  3rd Qu.:109.00   3rd Qu.:144.0   3rd Qu.:19.0000   3rd Qu.: 70.000  
##  Max.   :999.00   Max.   :999.0   Max.   :500.0000   Max.   :290.000  
##      CO        O3      TEMP      PRES  
##  Min.   : 100   Min.   : 0.2142   Min.   :-19.90   Min.   : 982.4  
##  1st Qu.: 500   1st Qu.: 12.0000   1st Qu.: 3.20   1st Qu.:1002.2  
##  Median : 900   Median : 45.0000   Median : 14.50   Median :1010.4  
##  Mean   :1213   Mean   : 57.1151   Mean   : 13.56   Mean   :1010.7  
##  3rd Qu.:1500   3rd Qu.: 80.0000   3rd Qu.: 23.30   3rd Qu.:1019.0  
##  Max.   :10000   Max.   :1071.0000   Max.   : 41.60   Max.   :1042.8  
##      DEWP      RAIN      wd      WSPM  
##  Min.   :-36.000   Min.   : 0.0000   Length:418946   Min.   : 0.000  
##  1st Qu.: -8.900   1st Qu.: 0.0000   Class :character  1st Qu.: 0.900  
##  Median :  3.100   Median : 0.0000   Mode  :character  Median : 1.400  
##  Mean   :  2.493   Mean   : 0.0646  
##  3rd Qu.: 15.100   3rd Qu.: 0.0000  
##  Max.   : 29.100   Max.   :72.5000  
##      station  
##  Length:418946  
##  Class :character  
##  Mode  :character
```

```

##  

##  

##  

#convert orginial data to standard date format  

mydata$Date<-as.Date(with(mydata,paste(year,month,day,sep="-")),"%Y-%m-%d")  

#mydata

```

## Calculating Mean of PM 2.5 and PM10

```

#mean of PM2.5 by year  

df1 <- mydata[, c("PM2.5", "year")] [order(mydata$year),] %>%  

  group_by(year) %>%  

  summarise(mean = mean(PM2.5)) %>%  

  arrange(desc(year))  

df1  

## # A tibble: 5 x 2  

##   year   mean  

##   <int> <dbl>  

## 1 2017   91.6  

## 2 2016   71.4  

## 3 2015   79.0  

## 4 2014   84.7  

## 5 2013   79.7  

#mean of PM10 by year  

df2 <- mydata[, c("PM10", "year")] [order(mydata$year),] %>%  

  group_by(year) %>%  

  summarise(mean = mean(PM10)) %>%  

  arrange(desc(year))  

df2  

## # A tibble: 5 x 2  

##   year   mean  

##   <int> <dbl>  

## 1 2017  110.  

## 2 2016  94.2  

## 3 2015  103.  

## 4 2014  115.  

## 5 2013  103.

```

## Plot graph by distribution of PM 2.5 and PM10

```

#distribution of PM10  

ggplot(mydata, aes(PM10)) +  

  geom_histogram(bins = 15, aes(y = ..density..), fill = "purple") +  

  geom_density(alpha = 0.2, fill = "purple") +  

  ggtitle("Distribution of PM10")

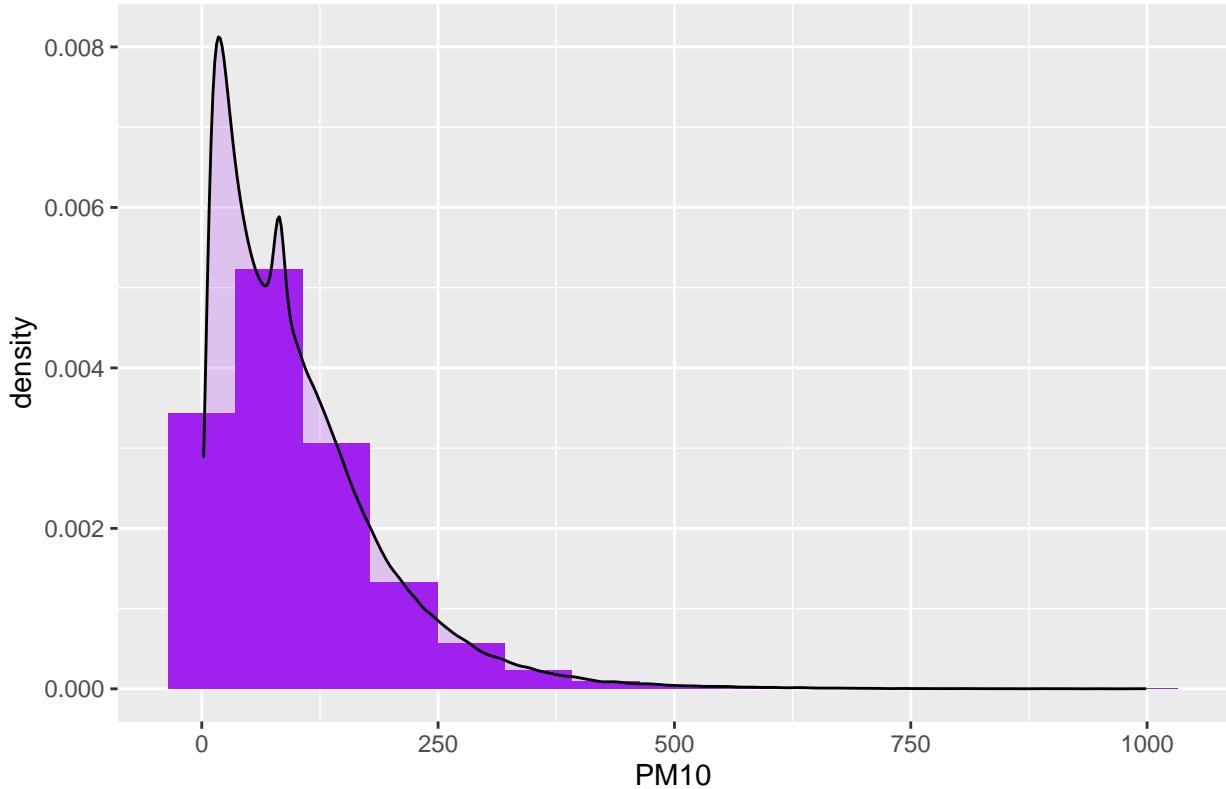
```

```

## Warning: The dot-dot notation ('..density..') was deprecated in ggplot2 3.4.0.
## i Please use 'after_stat(density)' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.

```

## Distribution of PM10



```
theme(axis.title = element_text(), axis.title.x = element_text())
```

```

## List of 2
## $ axis.title :List of 11
##   ..$ family      : NULL
##   ..$ face        : NULL
##   ..$ colour      : NULL
##   ..$ size        : NULL
##   ..$ hjust       : NULL
##   ..$ vjust       : NULL
##   ..$ angle       : NULL
##   ..$ lineheight  : NULL
##   ..$ margin      : NULL
##   ..$ debug       : NULL
##   ..$ inherit.blank: logi FALSE
##   ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.x:List of 11
##   ..$ family      : NULL
##   ..$ face        : NULL

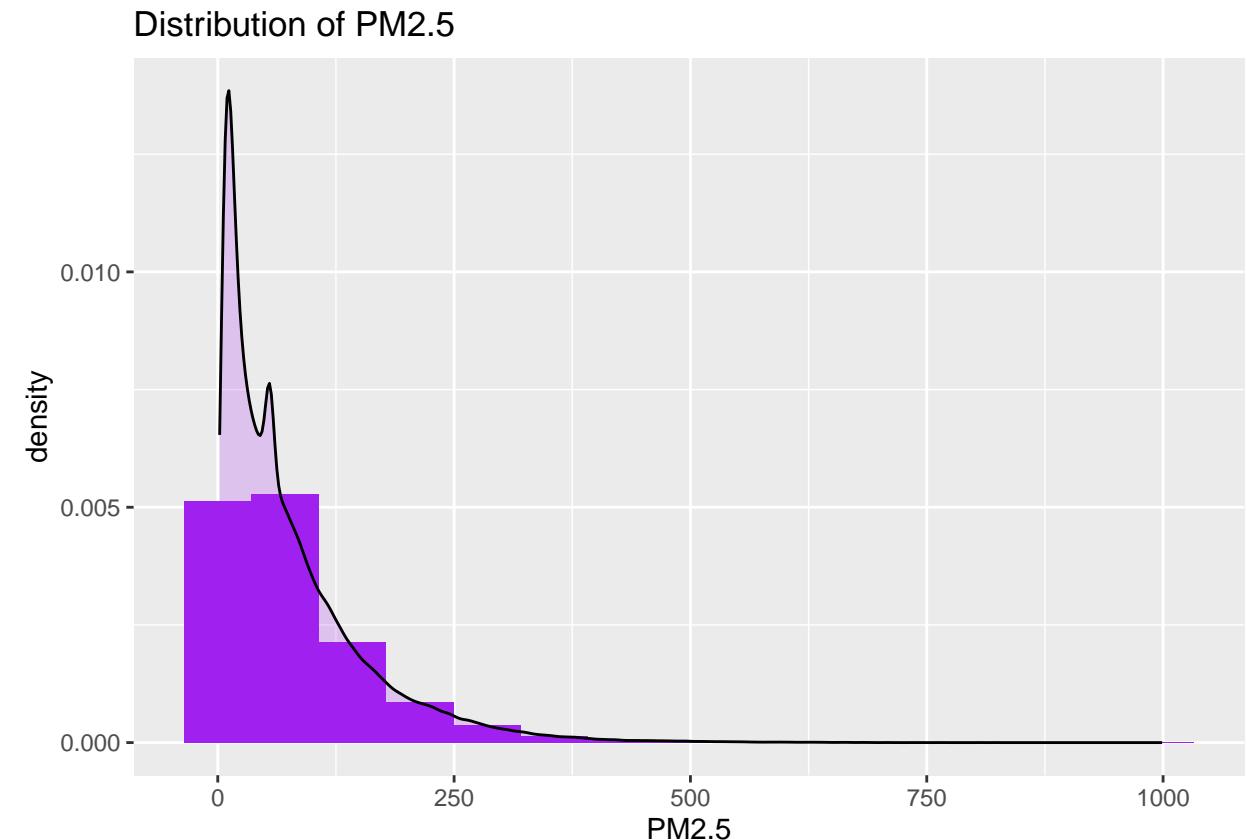
```

```

## ..$ colour      : NULL
## ..$ size        : NULL
## ..$ hjust       : NULL
## ..$ vjust       : NULL
## ..$ angle       : NULL
## ..$ lineheight  : NULL
## ..$ margin      : NULL
## ..$ debug       : NULL
## ..$ inherit.blank: logi FALSE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## - attr(*, "class")= chr [1:2] "theme" "gg"
## - attr(*, "complete")= logi FALSE
## - attr(*, "validate")= logi TRUE

#distribution of PM2.5
ggplot(mydata, aes(PM2.5)) +
  geom_histogram(bins = 15, aes(y = ..density..), fill = "purple") +
  geom_density(alpha = 0.2, fill = "purple") +
  ggtitle("Distribution of PM2.5")

```



```

theme(axis.title = element_text(), axis.title.x = element_text())

## List of 2
## $ axis.title  :List of 11
##   ..$ family      : NULL

```

```

## ..$ face      : NULL
## ..$ colour    : NULL
## ..$ size      : NULL
## ..$ hjust     : NULL
## ..$ vjust     : NULL
## ..$ angle     : NULL
## ..$ lineheight: NULL
## ..$ margin    : NULL
## ..$ debug     : NULL
## ..$ inherit.blank: logi FALSE
## ...- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.x:List of 11
##   ..$ family    : NULL
##   ..$ face      : NULL
##   ..$ colour    : NULL
##   ..$ size      : NULL
##   ..$ hjust     : NULL
##   ..$ vjust     : NULL
##   ..$ angle     : NULL
##   ..$ lineheight: NULL
##   ..$ margin    : NULL
##   ..$ debug     : NULL
##   ..$ inherit.blank: logi FALSE
## ...- attr(*, "class")= chr [1:2] "element_text" "element"
## - attr(*, "class")= chr [1:2] "theme" "gg"
## - attr(*, "complete")= logi FALSE
## - attr(*, "validate")= logi TRUE

```

## Plot graph by relationship of PM 2.5 and other variables

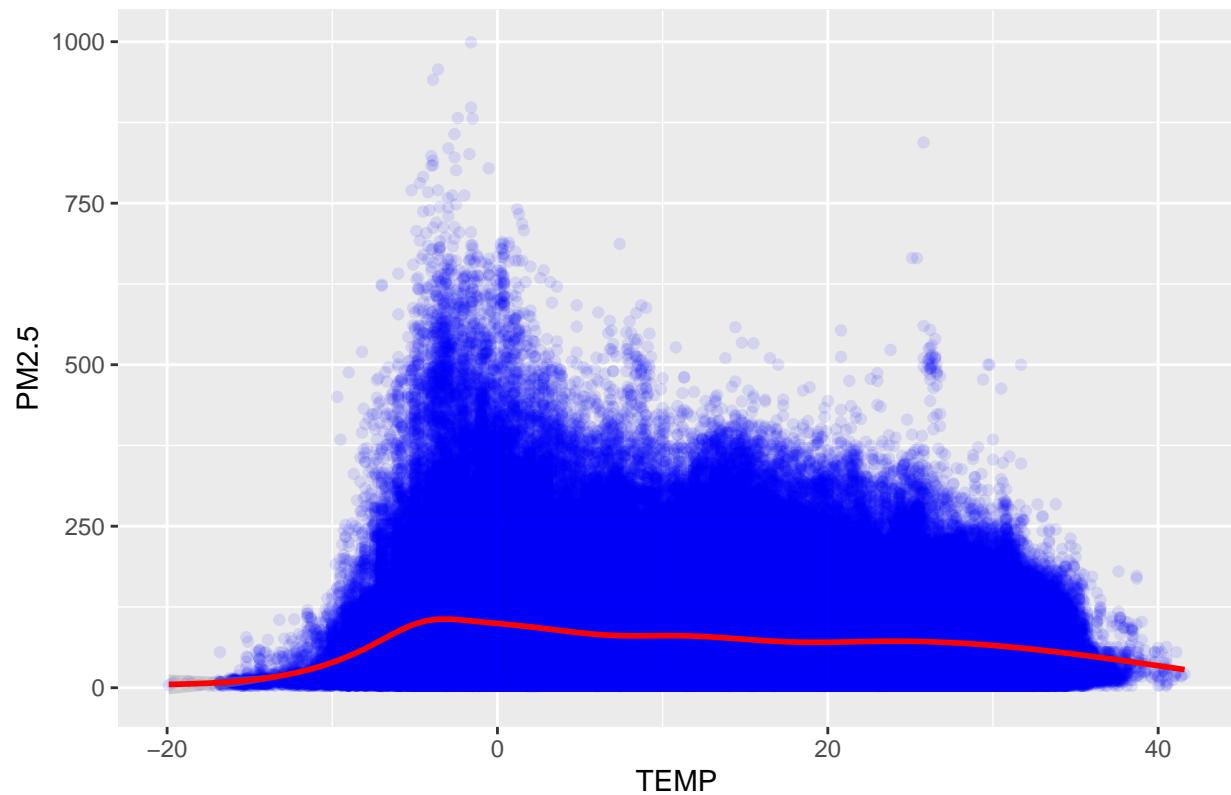
```

#plot the relationship between Temperature and PM2.5
ggplot(mydata, aes(x = TEMP, y = PM2.5)) +
  geom_point(alpha = 0.1, color = "blue") +
  geom_smooth(color = "red") +
  ggtitle("Relationship Between Temperature and PM2.5")

## 'geom_smooth()' using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'

```

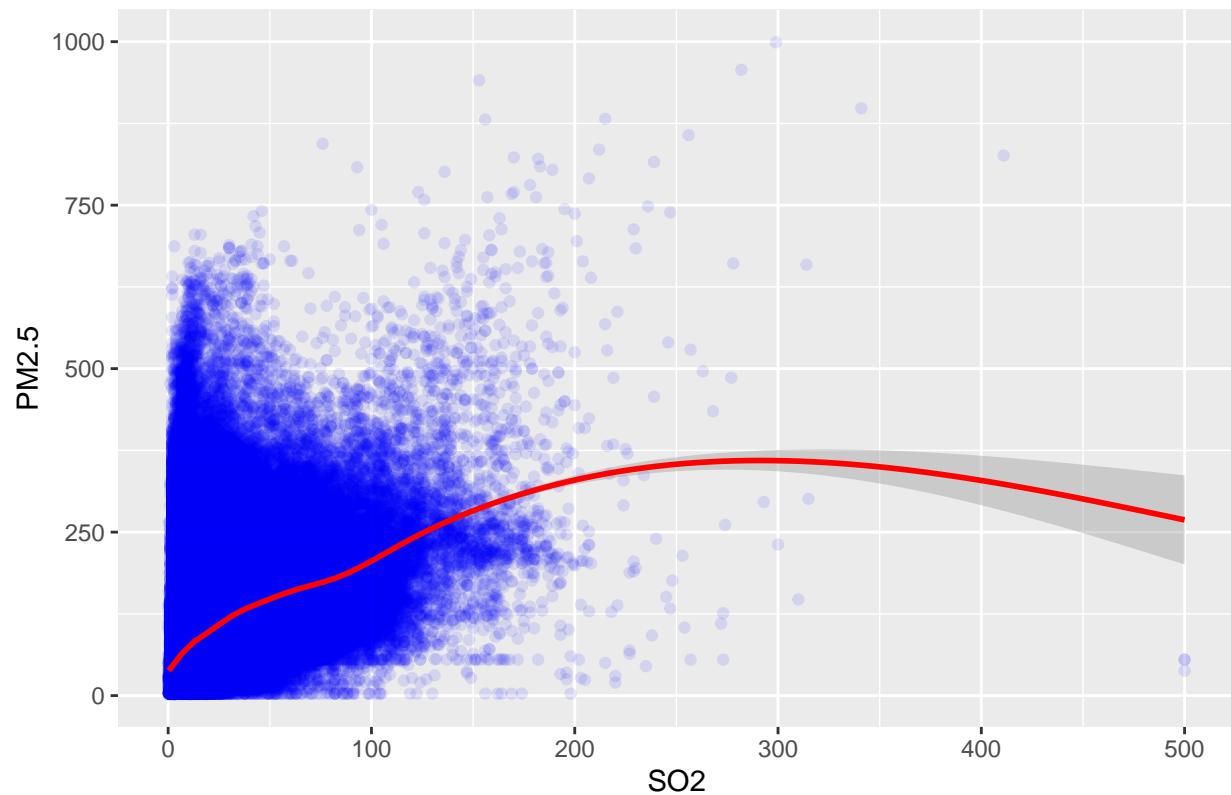
## Relationship Between Temperature and PM2.5



```
#plot the relationship between SO2 and PM2.5
ggplot(mydata, aes(x = SO2, y = PM2.5)) +
  geom_point(alpha = 0.1, color = "blue") +
  geom_smooth(color = "red") +
  ggtitle("Relationship Between SO2 and PM2.5")

## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

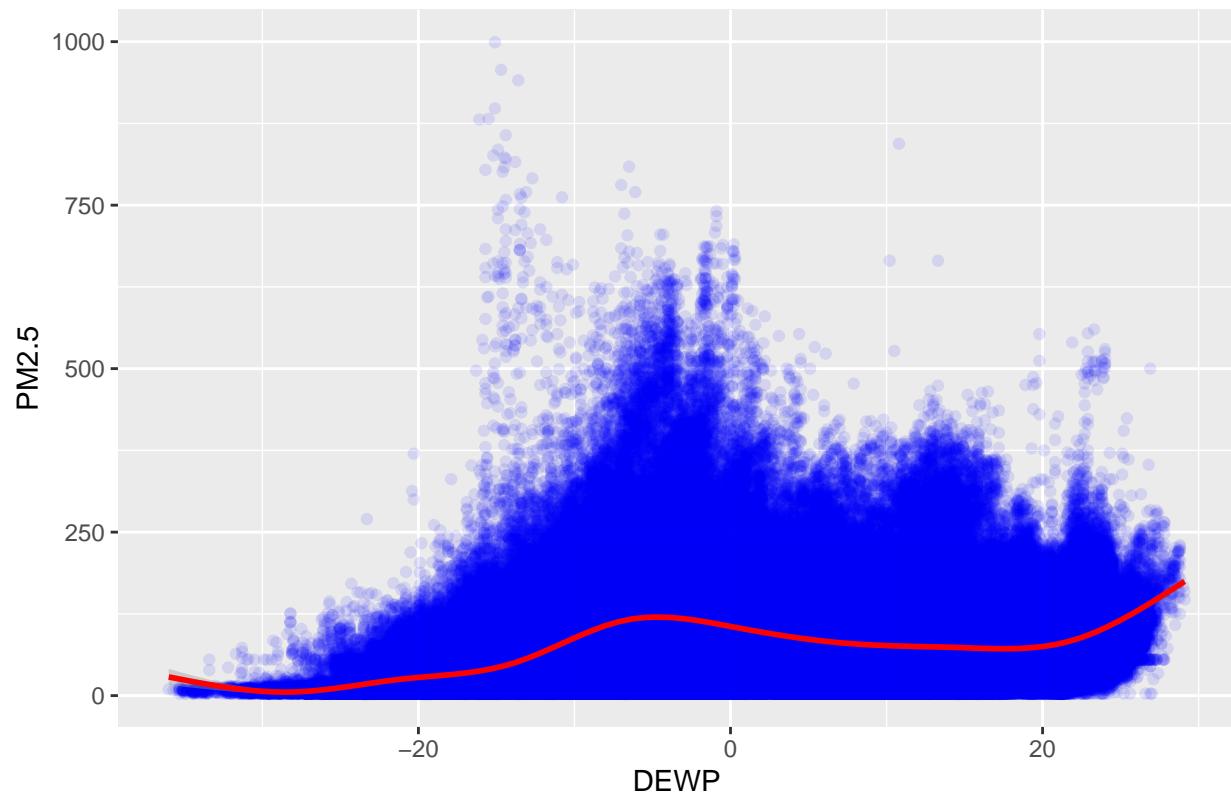
## Relationship Between SO<sub>2</sub> and PM2.5



```
#plot the relationship between DewPoint and PM2.5
ggplot(mydata, aes(x = DEWP, y = PM2.5)) +
  geom_point(alpha = 0.1, color = "blue") +
  geom_smooth(color = "red") +
  ggtitle("Relationship Between DewPoint and PM2.5")
```

```
## 'geom_smooth()' using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

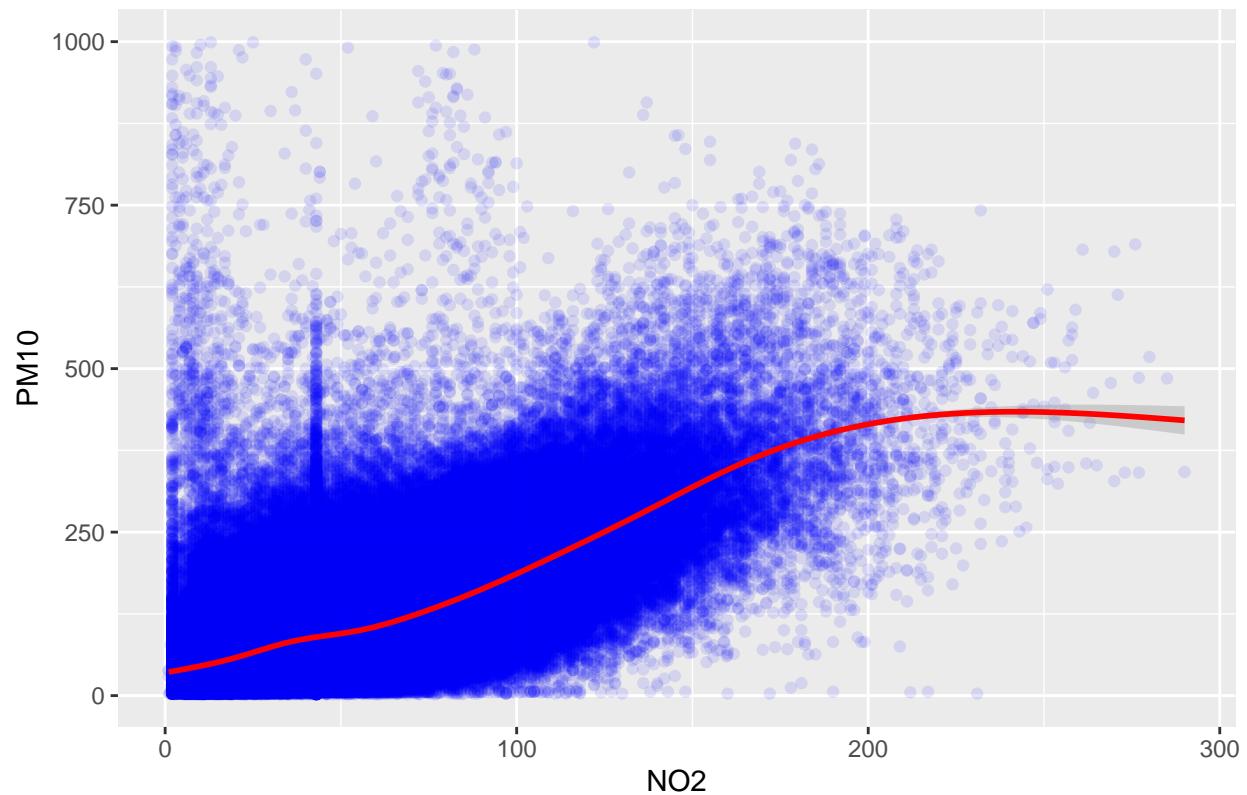
## Relationship Between DewPoint and PM2.5



```
#plot the relationship between NO2 and PM10
ggplot(mydata, aes(x = NO2, y = PM10)) +
  geom_point(alpha = 0.1, color = "blue") +
  geom_smooth(color = "red") +
  ggtitle("Relationship Between NO2 and PM10")

## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

## Relationship Between NO2 and PM10

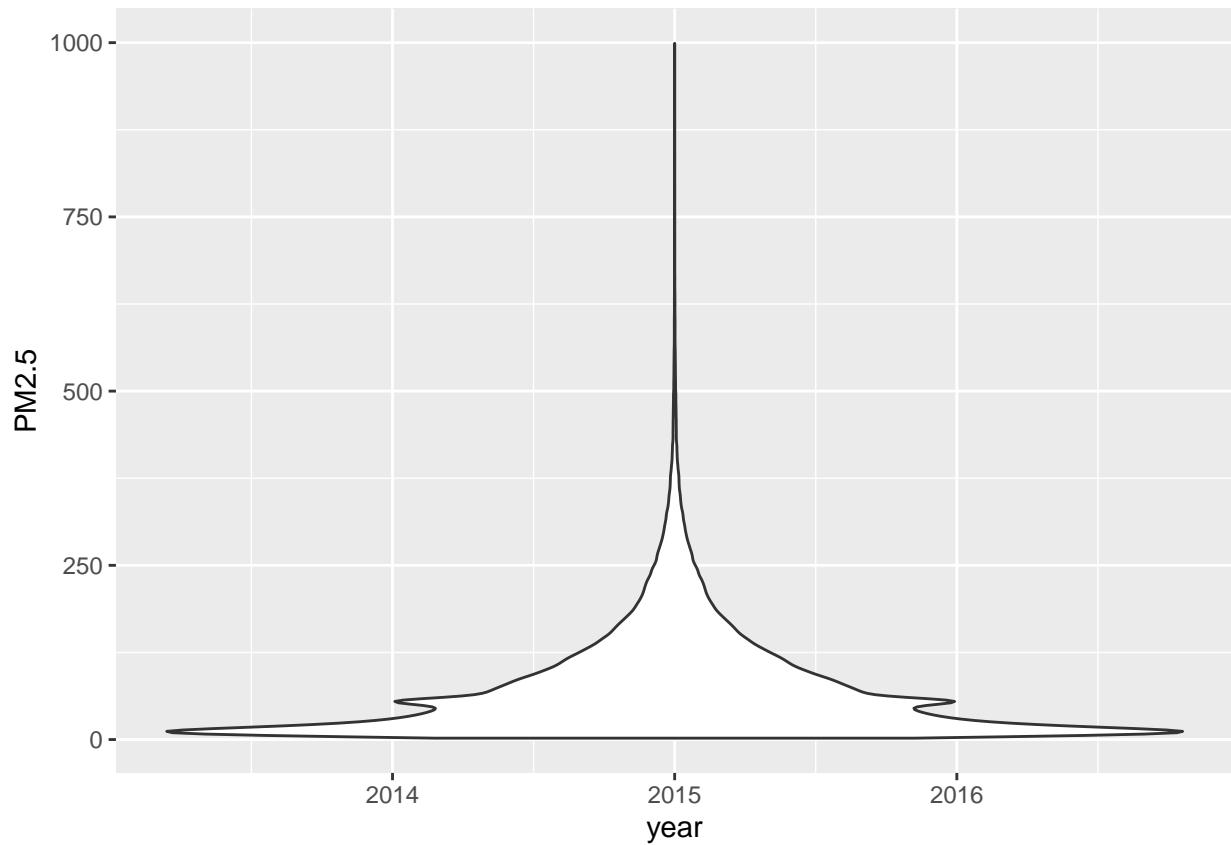


```
#plot the relationship between CO and PM2.5 according to station
ggplot(mydata, aes(x = CO, y = PM2.5, color = station)) +
  geom_point(size = 3, shape = 21) +
  scale_color_manual(values = c("#ff9900", "#660099", "#BD6263", "#8EA325", "#A9D179", "#84CAC0", "#F5AE6B",
    ggttitle("CO and PM2.5 According to Station")
```

## CO and PM2.5 According to Station



```
#plot violin graph to visualize PM 2.5
ggplot(mydata, aes(x = year, y = PM2.5)) +
  geom_violin()
```



### Linear Regression:set wind direction as a factor

```
#Regression of wind direction
mydata$wd <- as.factor(mydata$wd)

m1 <- lm(mydata$PM2.5 ~ mydata$year + mydata$PM10 + mydata$SO2 + mydata$CO + mydata$O3 + mydata$NO2+mydata$wd)
summary(m1)

##
## Call:
## lm(formula = mydata$PM2.5 ~ mydata$year + mydata$PM10 + mydata$SO2 +
##     mydata$CO + mydata$O3 + mydata$NO2 + mydata$wd)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -602.41  -13.11    0.49   12.51   894.94 
##
## Coefficients:
##             Estimate Std. Error t value            Pr(>|t|)    
## (Intercept) 323.0466663  90.8839074   3.554 0.000379 ***  
## mydata$year  -0.1658310   0.0451090  -3.676 0.000237 ***  
## mydata$PM10   0.5627373   0.0008683 648.091 < 0.0000000000000002 *** 
## mydata$SO2     0.0277592   0.0029614   9.374 < 0.0000000000000002 ***
```

```

## mydata$CO      0.0209087  0.0000730 286.421 < 0.0000000000000002 ***
## mydata$O3      0.0532714  0.0011711 45.489 < 0.0000000000000002 ***
## mydata$N02     0.1147336  0.0025187 45.553 < 0.0000000000000002 ***
## mydata$wdENE   -1.5494715 0.2646332 -5.855      0.0000000477 ***
## mydata$wdESE   1.5318550  0.2887373 5.305      0.00000011251 ***
## mydata$wdN     -3.0191627 0.2721007 -11.096 < 0.0000000000000002 ***
## mydata$wdNE    -1.4130597 0.2516705 -5.615      0.00000001970 ***
## mydata$wdNNE   -2.2885976 0.2778975 -8.235 < 0.0000000000000002 ***
## mydata$wdNNW   -4.7529146 0.2876131 -16.525 < 0.0000000000000002 ***
## mydata$wdNW    -6.6724568 0.2705992 -24.658 < 0.0000000000000002 ***
## mydata$wdS     -1.4592792 0.3108265 -4.695      0.00000266906 ***
## mydata$wdSE    -0.2885909 0.3056148 -0.944      0.345019
## mydata$wdSSE   -0.7959560 0.3191559 -2.494      0.012634 *
## mydata$wdSSW   -3.0193421 0.2983384 -10.121 < 0.0000000000000002 ***
## mydata$wdSW    -5.2934062 0.2790201 -18.971 < 0.0000000000000002 ***
## mydata$wdW     -6.7710134 0.3177255 -21.311 < 0.0000000000000002 ***
## mydata$wdWNW   -7.8804945 0.2900055 -27.174 < 0.0000000000000002 ***
## mydata$wdWSW   -5.3495092 0.3058519 -17.491 < 0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.32 on 418924 degrees of freedom
## Multiple R-squared:  0.8266, Adjusted R-squared:  0.8266
## F-statistic: 9.508e+04 on 21 and 418924 DF,  p-value: < 0.00000000000000022

#m1:#PM2.5 = 323.04 + -0.16(year) + 0.56(PM10) + 0.02(SO2)+0.02(co)+0.05(O3)+0.11(N02)-1.54(wdENE)-3.01
#-0.28(WdSE)-0.79(WdSSE)-3.01(wdSSW)-5.29(wdSW)-6.77(wdW)-7.88(wdWNW)-5.34(wdWSW)

m2 <- lm(mydata$PM10 ~ mydata$year+ mydata$PM2.5 + mydata$SO2 + mydata$CO + mydata$O3 +mydata$N02+mydata$wd
summary(m2)

```

```

##
## Call:
## lm(formula = mydata$PM10 ~ mydata$year + mydata$PM2.5 + mydata$SO2 +
##     mydata$CO + mydata$O3 + mydata$N02 + mydata$wd)
##
## Residuals:
##       Min     1Q   Median     3Q    Max 
## -875.47 -19.72   -8.75    9.22  938.23 
##
## Coefficients:
##             Estimate Std. Error t value     Pr(>|t|)    
## (Intercept) 375.2548557 114.2750409  3.284 0.001024 ** 
## mydata$year  -0.1860509  0.0567190 -3.280 0.001037 ** 
## mydata$PM2.5  0.8896774  0.0013728 648.091 < 0.0000000000000002 ***
## mydata$SO2    0.0963912  0.0037210 25.905 < 0.0000000000000002 ***
## mydata$CO     -0.0012710 0.0001004 -12.666 < 0.0000000000000002 ***
## mydata$O3     0.1320017  0.0014619  90.292 < 0.0000000000000002 ***
## mydata$N02    0.4634968  0.0030929 149.856 < 0.0000000000000002 ***
## mydata$wdENE  -2.1047449 0.3327398 -6.325 0.0000000025267023 *** 
## mydata$wdESE  1.2469200  0.3630570  3.435 0.000594 *** 
## mydata$wdN    1.8930801  0.3421693  5.533 0.00000003157373756 *** 
## mydata$wdNE   -2.4926198 0.3164317 -7.877 0.0000000000000335 *** 
## mydata$wdNNE -1.3701541 0.3494422 -3.921 0.00008820528114206 *** 
```

```

## mydata$wdNNW 5.0790467 0.3616691 14.043 < 0.0000000000000002 ***
## mydata$wdNW 8.5654268 0.3402332 25.175 < 0.0000000000000002 ***
## mydata$wdS 2.2972934 0.3908184 5.878 0.00000000415162155 ***
## mydata$wdSE 3.6315169 0.3842307 9.451 < 0.0000000000000002 ***
## mydata$wdSSE 3.7571700 0.4012583 9.363 < 0.0000000000000002 ***
## mydata$wdSSW 3.4116649 0.3751308 9.095 < 0.0000000000000002 ***
## mydata$wdSW 4.3152711 0.3509191 12.297 < 0.0000000000000002 ***
## mydata$wdW 4.4390231 0.3996564 11.107 < 0.0000000000000002 ***
## mydata$wdWNW 9.6738959 0.3646595 26.529 < 0.0000000000000002 ***
## mydata$wdWSW 2.5842081 0.3846890 6.718 0.00000000001849070 ***

## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ',' 1
##
## Residual standard error: 41.9 on 418924 degrees of freedom
## Multiple R-squared: 0.7884, Adjusted R-squared: 0.7884
## F-statistic: 7.433e+04 on 21 and 418924 DF, p-value: < 0.0000000000000002

```

```
#m2:#PM10 = 375.25 + -0.186(year) + 0.889(PM2.5) + 0.096(SO2)-0.001(co)+0.132(O3)+0.463(NO2)-2.104(wdEN)
#-0.28(WdSE)-0.79(WdSSE)-3.01(wdSSW)-5.29(wdSW)-6.77(wdW)-7.88(wdWNW)-5.34(wdWSW)
```

## Linear Regression:set station as a factor

```

#Linear regression of station
#In order to fit this regression model and tell R that the variable "program" is a categorical variable

mydata$station <- as.factor(mydata$station)

#fit linear regression model
fit <- lm(PM2.5 ~ PM10 + SO2+N02+C0+O3+station, data = mydata)
summary(fit)

```

```

##
## Call:
## lm(formula = PM2.5 ~ PM10 + SO2 + N02 + C0 + O3 + station, data = mydata)
##
## Residuals:
##     Min      1Q      Median      3Q      Max 
## -601.37   -13.23     0.26    12.54   895.36 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -19.36164253  0.22870213 -84.659 < 0.0000000000000002 ***
## PM10          0.55757854  0.00086983  641.023 < 0.0000000000000002 ***
## SO2           0.02263090  0.00289788   7.809  0.000000000000576 ***
## N02           0.17725825  0.00261932  67.673 < 0.0000000000000002 ***
## C0            0.02078761  0.00007222  287.852 < 0.0000000000000002 ***
## O3            0.06581994  0.00110067  59.800 < 0.0000000000000002 ***
## stationChangping 1.75232492  0.25311203   6.923  0.0000000000442435 ***
## stationDingling  9.89969138  0.25798217  38.374 < 0.0000000000000002 ***
## stationDongsi   3.19562259  0.25174020  12.694 < 0.0000000000000002 ***
## stationGuanyuan 0.91201733  0.25123799   3.630          0.000283 ***

```

```

## stationGucheng      -4.55285130   0.25177386 -18.083 < 0.0000000000000002 ***
## stationHuairou     6.43547601   0.25724136  25.017 < 0.0000000000000002 ***
## stationNongzhanguan 1.35567414   0.25131501   5.394  0.00000006881978917 ***
## stationShunyi       7.34119512   0.25424860  28.874 < 0.0000000000000002 ***
## stationTiantan      1.82663843   0.25177366   7.255  0.00000000000040211 ***
## stationWanliu        -1.04904552   0.25156166  -4.170  0.00003044834656828 ***
## stationWanshouxigong -0.54635720   0.25160079  -2.172          0.029892 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.22 on 418929 degrees of freedom
## Multiple R-squared:  0.8276, Adjusted R-squared:  0.8276
## F-statistic: 1.257e+05 on 16 and 418929 DF,  p-value: < 0.0000000000000022

```

*#From the values in the Estimate column, we can write the fitted regression model:*

```
#PM2.5 = -19.36 + .55(PM10) + 0.02(SO2) + 0.17(NO2)+0.02(co)+0.06(O3)+1.75(Changping)+9.89(Dingling)+3.
```

## Ridge Model Regression

*#Ridge regression is a method we can use to fit a regression model when multicollinearity is present in the data.*  
*#To perform ridge regression, we'll use functions from the glmnet package. This package requires the re*

```
library(glmnet)
```

```
## Warning: package 'glmnet' was built under R version 4.2.3
```

```
## Loading required package: Matrix
```

```
##
```

```
## Attaching package: 'Matrix'
```

```
## The following objects are masked from 'package:tidyverse':
```

```
##
```

```
##     expand, pack, unpack
```

```
## Loaded glmnet 4.1-7
```

```
y <- mydata$PM2.5
x <- data.matrix(mydata[, c('PM10', 'SO2', 'NO2', 'CO', 'O3')])
#fit ridge model
```

*#we'll use the glmnet() function to fit the ridge regression model and specify alpha=0.*

*# ridge regression requires the data to be standardized such that each predictor variable has a mean of 0 and a standard deviation of 1.*

```
model <- glmnet(x, y, alpha = 0)
```

*#view the summary of model*

```

summary(model)

##          Length Class    Mode
## a0         100   -none- numeric
## beta       500   dgCMatrix S4
## df         100   -none- numeric
## dim         2    -none- numeric
## lambda     100   -none- numeric
## dev.ratio  100   -none- numeric
## nulldev     1    -none- numeric
## npasses     1    -none- numeric
## jerr        1    -none- numeric
## offset      1    -none- logical
## call         4    -none- call
## nobs        1    -none- numeric

#Next, we'll identify the lambda value that produces the lowest test mean squared error (MSE) by using k-fold cross-validation to find optimal lambda value

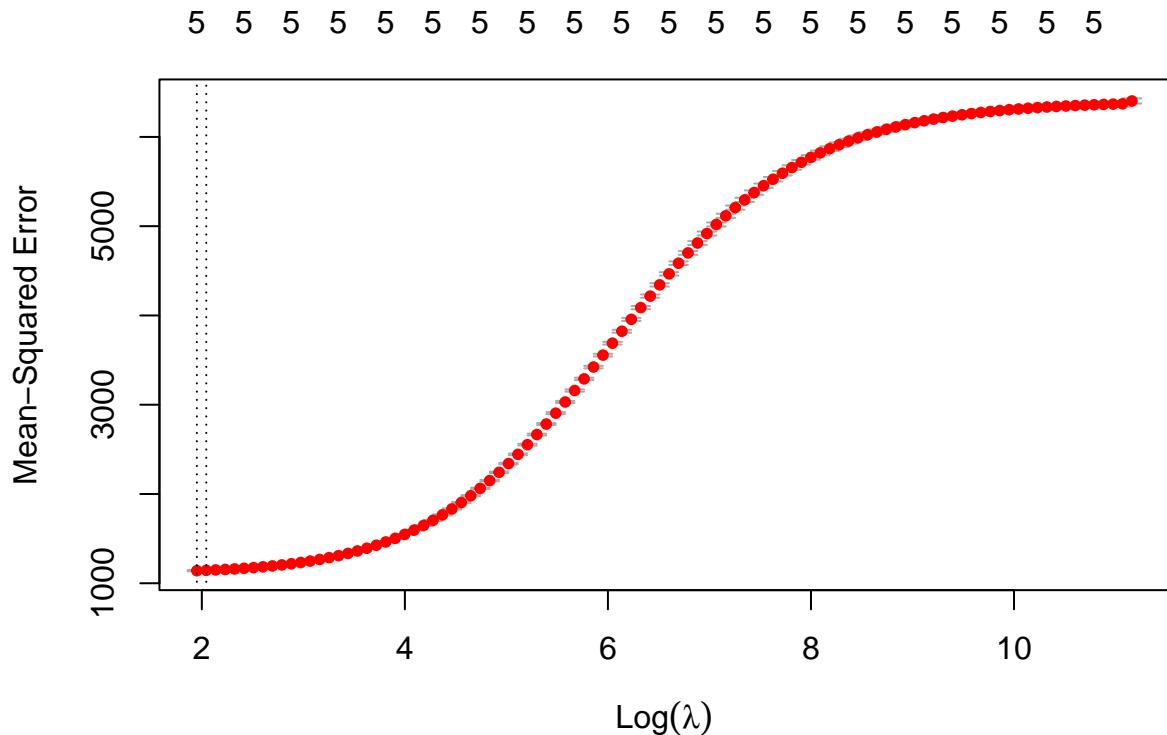
cv_model <- cv.glmnet(x, y, alpha = 0)

#find optimal lambda value that minimizes test MSE
best_lambda <- cv_model$lambda.min
best_lambda

## [1] 7.036282

#produce plot of test MSE by lambda value
plot(cv_model)

```



```
#The lambda value that minimizes the test MSE turns out to be 7.0362.
```

```
best_model <- glmnet(x, y, alpha = 0, lambda = best_lambda)
coef(best_model)
```

```
## 6 x 1 sparse Matrix of class "dgCMatrix"
##           s0
## (Intercept) -13.72545524
## PM10         0.49314785
## S02          0.09795717
## N02          0.22288125
## CO           0.02053946
## O3           0.06771158
```

```
#fitted model:PM2.5=-13.725+0.49PM10+0.09so2+0.22N02+0.02CO+0.067O3
```

#Lastly, we can analyze the final model produced by the optimal lambda value.

#We can use the following code to obtain the coefficient estimates for this model:

```

y_predicted <- predict(model, s = best_lambda, newx = x)

#find SST and SSE
sst <- sum((y - mean(y))^2)
sse <- sum((y_predicted - y)^2)

#find R-Squared
rsq <- 1 - sse/sst

rsq

```

## [1] 0.8218718

*#The R-squared turns out to be 0.8218. That is, the best model was able to explain 82.1% of the variation in the data.*

## Lasso regression

```

#Lasso regression

#To perform lasso regression, we'll use functions from the glmnet package. This package requires the regularized least squares algorithm.
#define response variable
y <- mydata$PM2.5

#define matrix of predictor variables
x <- data.matrix(mydata[, c('PM10', 'S02', 'N02', 'CO', 'O3')])

library(glmnet)

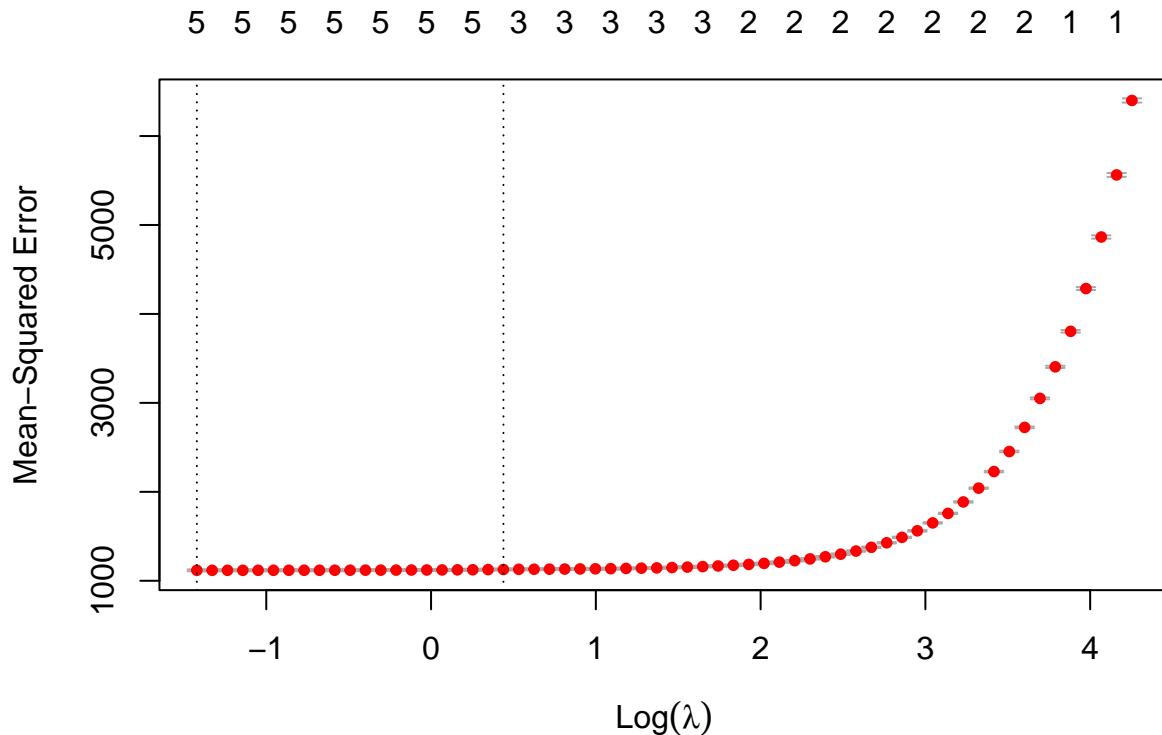
#perform k-fold cross-validation to find optimal lambda value
cv_model <- cv.glmnet(x, y, alpha = 1)

#find optimal lambda value that minimizes test MSE
best_lambda <- cv_model$lambda.min
best_lambda

## [1] 0.2413775

#produce plot of test MSE by lambda value
plot(cv_model)

```



```
#The lambda value that minimizes the test MSE turns out to be 0.21.
```

```
#Lastly, we can analyze the final model produced by the optimal lambda value.
```

```
#We can use the following code to obtain the coefficient estimates for this model:  
#find coefficients of best model  
best_model <- glmnet(x, y, alpha = 1, lambda = best_lambda)  
coef(best_model)
```

```
## 6 x 1 sparse Matrix of class "dgCMatrix"  
##                               s0  
## (Intercept) -13.89451463  
## PM10          0.56332051  
## S02           0.02401005  
## N02           0.11445905  
## CO            0.02099390  
## O3            0.04890430
```

```
#                               s0  
#(Intercept) -13.89451463  
#PM10          0.56332051  
#S02           0.02401005  
#N02           0.11445905  
#CO            0.02099390
```

```

#03          0.04890430

#model PM2.5=-13.894+0.56PM10+0.02SO2+0.11NO2+0.02CO+0.0403

#use fitted best model to make predictions
y_predicted <- predict(best_model, s = best_lambda, newx = x)
#y_predicted
#find SST and SSE
sst <- sum((y - mean(y))^2)
sse <- sum((y_predicted - y)^2)

#find R-Squared
rsq <- 1 - sse/sst
rsq

```

## [1] 0.8255041

*#The R-squared turns out to be 0.825. That is, the best model was able to explain 82.5% of the variation.*