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# 15-State Extended Kalman Filter Design for INS/GPS Navigation System

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**Abstract**—Nowadays, navigation system has been receiving high demand for various kind of applications. Among these systems, Global positioning system (GPS) and Inertial navigation system (INS) are the most popular. In this paper, a 15-state Extended Kalman Filter is designed to integrate INS and GPS in a flexible way compared with many conventional integration. Based on the loosely coupled GPS/INS integration, the proposed scheme can switch back and forth between feed forward and feedback aiding methods. Thus, the system can reduce the position and velocity errors compared to conventional integration method. To verify the technique, a simulation model is created using Simulink/MATLAB. The data is obtained from Micro PSU BP3010 IMU sensor and HI-204 GPS receiver. The simulation result confirms the benefit of integrated system in both open and urban areas, and suitable for real-time implementation.

**Index Terms**—kalman filter, navigation, INS/GPS

## I. INTRODUCTION

In recent years, navigation and control for vehicle are important and widely used in civil and military applications. One of the common used navigation systems is the Global Positioning System (GPS). GPS is a satellite-based navigation system, which provides accurate position and velocity information worldwide [1]. However, the operational capability of GPS degrades in harsh environments such as urban and forest areas, where GPS signals may be partially or completely blocked by buildings and dense foliage. Besides, a GPS receiver does not provide attitude data and the slow update frequency.

Inertial Navigation System (INS) is a system of sensors designed to measure specific force and angular rates with respect to an inertial frame to provide velocity, position and attitude [2]. INS is a self-contained system, so it is autonomous regardless of the operational environment. However, the accuracy of an INS is diminished over time

by the accumulation of systemic errors such as inertial alignment errors and inertial sensor [3].

Integration of GPS with an inertial Navigation System improves the quality and integrity of each navigation system: use of GPS permits calibration of inertial instrument biases, and the INS can be used to improve the tracking and reacquisition performance of the GPS receiver [4].

In the integration of GPS and INS, the Kalman filter plays a significant role. Being a recursive estimator, a Kalman filter can process the linear model and estimate the state vector which has a minimum variance based on the information at the moment and its prior value in the past. In INS/GPS integration system the Kalman filter combine the navigation signal from both GPS and INS, estimate the errors then compensate back to the original input. The distinction of Kalman filter is that: it uses only navigation state at the present and previous, hence it costs less memory than traditional filters. Besides that, a Kalman filter could use one of little measurement information (e.g. position) to estimate to provide additional information (heading, pitch, roll, etc...) which is useful for semi-automated navigation.

In reality, the navigation process is always non-linear. However this problem can be solved by the Extended Kalman Filter.

## II. WORKING PRINCIPLES

### A. Global Positioning System (GPS)

GPS is a Global Positioning System based on satellite technology. The fundamental technique of GPS is to measure the ranges between the receiver and a few simultaneously observed satellites. The positions of the satellites are forecasted and broadcasted along with the GPS signal to the user. Through several known positions (of the satellites) and the measured distances between the receiver and the satellites, the position of the receiver can be determined. The position change, which can be also determined, is then the velocity of the receiver. The most

important applications of the GPS are positioning and navigating [5]. GPS consists of 3 segments: the Space Segment consists of 24 satellites distributed in six orbital planes, the Control Segment monitors the operation of satellite and maintains system functionalities, and the User Segment consists of GPS receivers and user communities. Although GPS is a high-tech system, it still exists errors by six major causes (not including selective availability error): satellite ephemeris, satellite clock, ionospheric group delay, tropospheric group delay, multipath and receiver measurement errors [6].

### B. Inertial Navigation System (INS)

The operation of an inertial navigation system depends upon the laws of classical mechanics as formulated by Newton. With ability to measure specific force using an accelerometer it is possible to calculate a change in velocity and position by performing successive integration of the acceleration with respect to time [3].

An INS system often consists of three accelerometers and three gyroscopes in order to measure the accelerations in three dimensions and the rotation rates around three axes. The development of MEMS technology has been a stimulus to widen the application area of INS. Today, an Inertial Measurement Unit (IMU) even includes a three-degree of freedom gyroscope and a three-degree of freedom accelerometer [1, 6].

There are two kinds of INS: gimbals INS and strapdown INS. The gimbals one uses gimbals with pivots to keep INS in stable with ground. This type is complicate and rare in use. Another type of INS is the strapdown system that eliminates the use of gimbals which is simple for motion analysis. In this case, the gyros and accelerometers are mounted directly to the structure of the vehicle or strapped on the body segment [7].

Strapdown mechanization (or INS mechanization) is the process of determining the navigation states (position, velocity and attitude) from the raw inertial measurements through solving the differential equations describing the system motion. Mechanization differential equations in the local level frame [8]:

$$\begin{pmatrix} \dot{r}^n \\ \dot{v}^n \\ \dot{C}_b^n \end{pmatrix} = \begin{pmatrix} D^{-1}v^n \\ C_b^n f^b - (2\Omega_{ie}^n + \Omega_{en}^n)v^n + g^n \\ C_b^n (\Omega_{ib}^b - \Omega_{in}^b) \end{pmatrix} \quad (1)$$

where:  $r^n = [\varphi \ \lambda \ h]^T$ ;  $v^n = [V_N \ V_E \ V_U]^T$ ;  $D^{-1}$  is a  $3 \times 3$  matrix whose non zero elements are functions of the user's latitude and height;  $C_b^n$  is transformation matrix from b-frame to n-frame;  $\Omega_{ie}^n, \Omega_{en}^n, \Omega_{ib}^b, \Omega_{in}^b$  are skew-symmetric matrix of corresponding respective angular velocity vector;  $f^b$  is special force vector in b-frame,  $g^n$  is gravity vector expressed in the n-frame.

The structure of the algorithm is given by the scheme in Fig. 1.

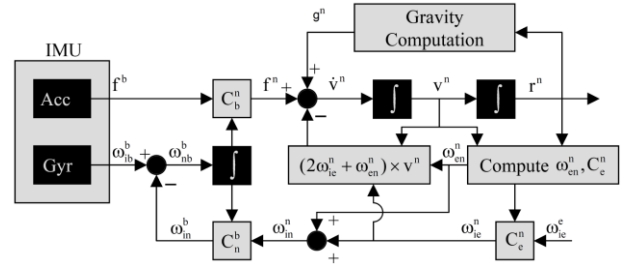


Figure 1. Scheme of INS mechanization defined for local navigation frame [8]

### C. INS/GPS Integration System

There are three main types of INS/GPS integration: loosely-coupled, tightly-coupled and ultra-tightly coupled.

In loosely-coupled integration, the position and velocity output of the GPS receiver and inertial sensors data are integrated in a Kalman filter.

Tightly-coupled integration uses an estimation technique to integrate inertial sensors readings with raw GPS data (i.e. pseudoranges and pseudorange rates) to get the vehicle position, velocity, and orientation.

For the ultra-tightly-coupled integration, there is a basic difference in the architecture of the GPS receiver compared to those used in loose and tight integration. Here, the receiver comprises a bank of single vector delay lock loop instead of a bank of independent code and carrier tracking loops. The information from INS is used as an integral part of the GPS receiver, thus, INS and GPS are no longer independent navigators, and the GPS receiver itself accepts feedback [9].

## III. PROPOSED INS/GPS INTERGRATED SOLUTION

### A. Proposed Integrated System

The integration of GPS and INS in this paper based on loosely-coupled approach. The input of Kalman Filter is the mixed of GPS and INS errors. After the filtering process, the random noises mostly come from GPS are removed, the remained INS errors are added to INS output to get the correct navigation value. Another concern in commonly INS/GPS system is the difference in each system's update rate. An INS system always has higher update rate than a GPS system that means from time to time, the system has to operate without the presence of GPS information. Moreover, GPS signal could suffer from external environment and may lost, causing an absence of GPS in relative long time. To deal with these situations, we use the configuration with the ability to switch back and forth between feed forward mode and feedback mode (Fig. 2).

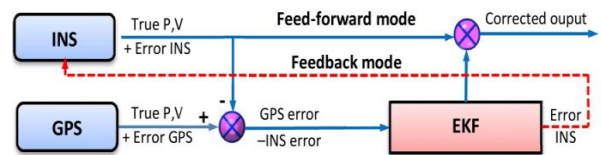


Figure 2. Feedback and feed-forward mode.

Feedback mode: Assume when GPS signal is lost, since there is no presence of GPS information, the Kalman Filter block enable prediction mode which use the last corrected value to estimate the current state using a dynamic model. Since the measurement signal is interrupted, all the measurement equation and Kalman gain computation are obsoleted.

Feed-forward mode when GPS has its signal back, the feedback is removed, the Kalman Filter block enable feed-forward mode which use INS and GPS information to process as usual.

### B. State Space Model and Kalman Filter

From navigation error equations, a state model is settled which is required for the operation of Kalman filter. In this paper, the error state vector  $\delta x(t)$  and the measurement noise vector  $u(t)$  are defined as

$$\delta x(t) = [\delta r^{eT} \quad \delta v^{eT} \quad \varepsilon^{eT} \quad \delta f^{bT} \quad \delta \omega_{ib}^{bT}]^T \quad (2)$$

$$u(t) = [u_{acc}^T(t) \quad u_{gyro}^T(t)]^T \quad (3)$$

where  $\delta r^{eT}, \delta v^{eT}, \varepsilon^{eT}$  are the position, velocity and attitude errors, respectively;  $\delta f^{bT}, \delta \omega_{ib}^{bT}$  are the acceleration and angular rate errors;  $u_{acc}(t), u_{gyro}(t)$  are accelerometer noise and gyro noise.

Continuous state space model is written as

$$\delta \dot{x}(t) = F(t)\delta x(t) + G(t)u(t) \quad (4)$$

where  $F(t)$  is the  $15 \times 15$  matrix and  $G(t)$  is the  $15 \times 6$  matrix

$$F(t) = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & -2\Omega_{ie}^e & -S_f^e & C_b^e & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & -\Omega_{ie}^e & 0_{3 \times 3} & C_b^e \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \quad (5)$$

$$G(t) = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ C_b^e & 0_{3 \times 3} \\ 0_{3 \times 3} & C_b^e \\ 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}$$

$S_f^e$  is the skew symmetric matrix of specific force  $(f_x^e, f_y^e, f_z^e)$ ,  $\Omega_{ie}^e$  is the skew symmetric matrix of the Earth rotation rate with respect to i-frame.

The measurement vector is defined as

$$\delta z(t) = [\delta v^e] \quad (6)$$

The measurement equation

$$\delta z(t) = H(t)\delta x(t) + V(t) \quad (7)$$

where  $V(t)$  is the velocity different between the GPS and INS systems. The form of  $H$  is the  $15 \times 3$  matrix

$$H(t) = \begin{cases} [0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3}], & \text{when GPS is available} \\ [0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3}], & \text{else} \end{cases} \quad (8)$$

The state space model for the error dynamic is nonlinear since 2 matrices  $F(t)$  and  $G(t)$  contain time-variable component. Consequently we need a linearized process so that we could apply the principle of Kalman filter without truly linear dynamics or sensors - and usually with remarkably great success [1].

The discrete state space model reads

$$\delta x_k = \Phi_k \delta x_{k-1} + u_k \quad (9)$$

with:  $\Phi_k \approx I + F(kT_s)T_s$ ,  $T_s$  is sample period,  $u_k$  is process noise, assumed to be white and zero mean.

The measurement equation can be written in discrete form

$$\delta y_k = H_k \delta x_k + w_k \quad (10)$$

Let  $y_k$  be the difference between the GPS and INS velocity estimate and  $w_k$  the error in the GPS velocity estimates.

In this paper Extended Kalman Filter technique is used. It works as following:

Prediction step

$$\delta \hat{x}_k^- = \Phi_k \cdot \delta \hat{x}_{k-1} \quad (11)$$

$$P_k^- = \Phi_k P_{k-1} \Phi_k^T + Q_{d,k} \quad (12)$$

The Kalman gain is computed according as following:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (13)$$

Correction step

$$\delta \hat{x}_k = \delta \hat{x}_k^- + K_k (y_k - H_k \delta \hat{x}_k^-) \quad (14)$$

$$P_k = P_k^- - K_k H_k P_k^- \quad (15)$$

When the GPS signal is lost, the extended Kalman filter scheme proves to be inefficient in tracking the travel route. Therefore, a linear Kalman filter is configured to operate in the situation.

The state vector of the scheme is defined as:

$$\delta x_0^T(t) = [\delta v^{eT}]^T \quad (16)$$

Then the transition matrix is  $A = I_{3 \times 3}$

The operation of the proposed system is represented in the diagram Fig. 3.

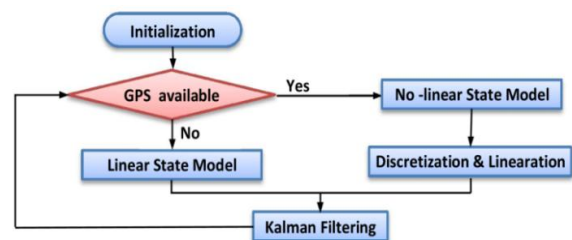


Figure 3. The operation of proposed scheme.

#### IV. EXPERIMENTS AND RESULTS

In this section, some simulation and performance results are shown.

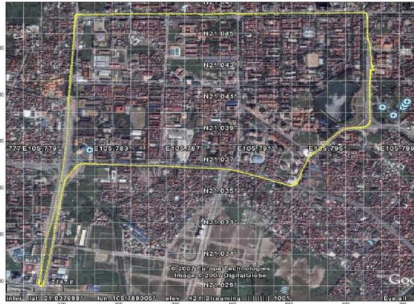


Figure 4. Trajectory of the experimental vehicle.

The input data of the simulation model comes from Micro PSU BP3010 INS system and Haicom HI-204 GPS receiver. After setting up the navigation devices, the several tests were carried out to create the experiment trajectory described in Fig. 4. The experiment is taken place at Cau Giay district, Hanoi, with a distance about 5000m. The simulation model is developed using SIMULINK in Matlab environment.

##### A. Open Area Experiment

Trajectory Comparison between two systems show the trajectory results obtained with GPS alone and with INS/GPS integrated system. It could be seen that the position accuracy of the integrated system is matched with the GPS alone system (see Fig. 5).

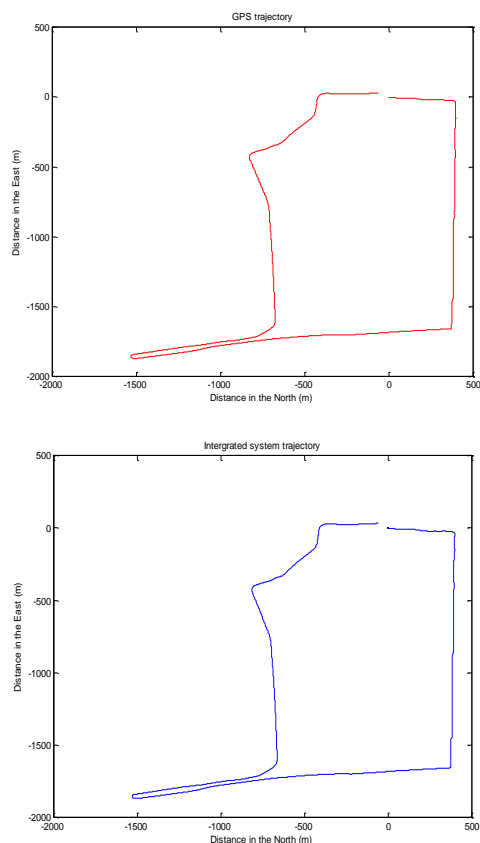


Figure 5. Trajectory comparison between two systems.

To further evaluate the accuracy of system, the Euclidian distance is used to find the deviation between GPS and proposed system. The result is shown in Fig. 6.

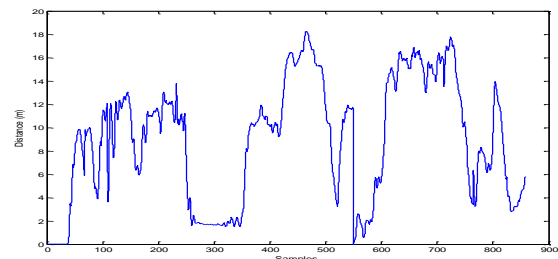


Figure 6. The deviation between GPS and integrated system.

From the graph, the deviation of two systems is fluctuated from zero to about 18m of the peak. Note that this is the relative of the integrated system with a low grade GPS receiver. It would be better if compared with the real trajectory.

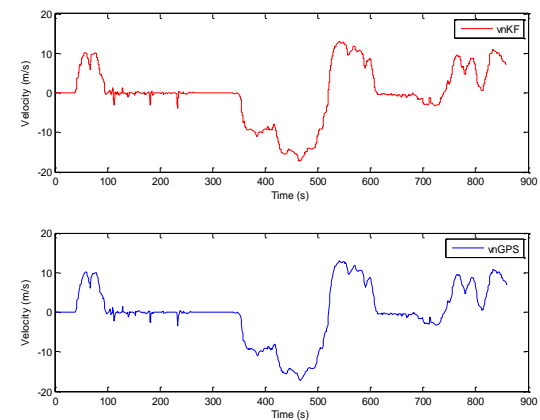


Figure 7. Comparison of velocity to the North.

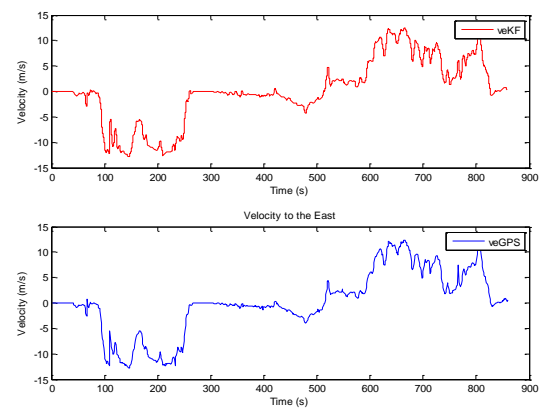


Figure 8. Comparison of velocity to the East.

Fig. 7 and Fig. 8 described the velocity of vehicle measured by the integrated system and the GPS only, respectively. The similar result is obtained which shows that the accuracy of Kalman is comparable with the output of GPS receiver.

The integrated system could provide the information about roll, pitch and heading angles of vehicle, as shown in Fig. 9 and Fig. 10.

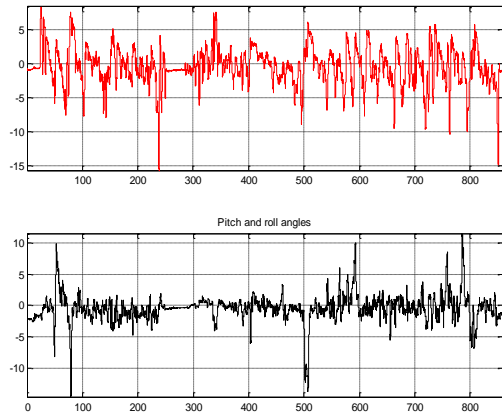


Figure 9. Pitch and roll angles of proposed system.

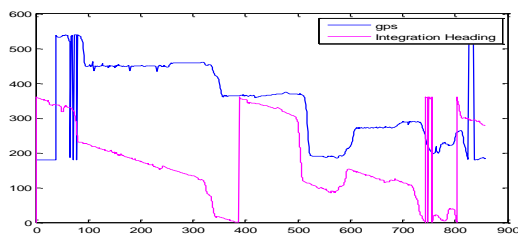


Figure 10. Heading angle of proposed system.

The figures below represent the attitude of a land vehicle, thus the average pitch and roll of the integrated system is around zeros, except where the object take the turn, e.g. at 500s-550s. In that time the heading angle degrade rapidly along with heading information, and the roll angle is fluctuated. This result is matched with the dynamic model of proposed system.

#### B. Urban Area Experiment

To simulate the outage time gap of GPS system, we removed the logged data from GPS, in the different time and different duration.

Straight test when GPS outage is occurred when the vehicle travel on a straight road. Using the same INS and GPS devices, GPS outage from 270s to 300s. The trajectory result is represented in Fig. 11. It could be seen from the figure that the integration system could performance well in this test. The trajectory of the integrated system is similar to the information from GPS.

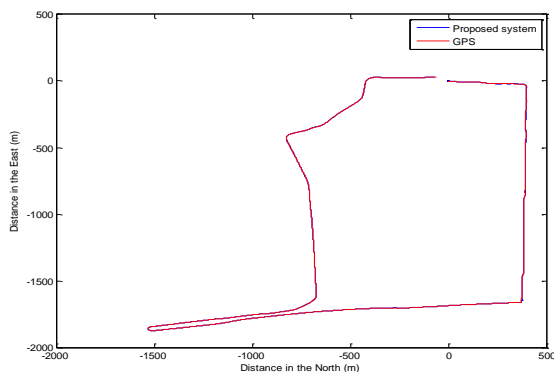


Figure 11. Straight trajectory result.

Fast turn test when GPS outage or occurred at the turning point of the two-way road. The condition of this test is similar to the previous test, except for the GPS outage period is 50s, which is 20sec longer than the straight test (GPS outage from 500s to 550s). The trajectory result is represented in Fig. 12. In this experiment, without the GPS aiding, the system error could rise to about 700m, but it still tracks the motion of vehicle.

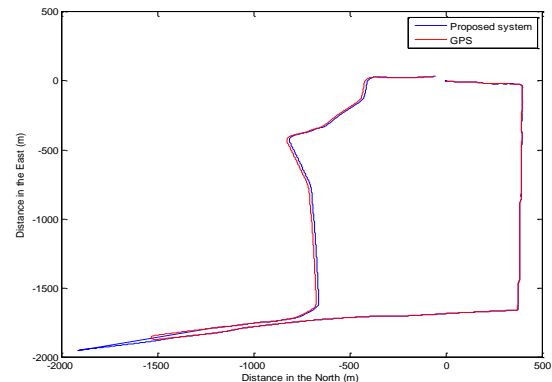


Figure 12. Fast-turn trajectory result.

#### V. CONCLUSION

In this paper, a configuration of INS/GPS integrated system is given for compensating the limitation of using each navigation system separately. With the system integrated using this configuration, it's possible to navigate with relative high accuracy when GPS signal is presented and continue to keep a good track even when the GPS signal is lost. In order to achieve this performance, a 15-state Extended Kalman Filter is implemented to deal with the non-linear issue of the system dynamic. Besides that, a linear Kalman filter is also presented and operated when the GPS outage occur. Along with two filtering scheme, a switching mechanism is given to brings the feature of automatically switch from the extended Kalman filtering scheme to the linear Kalman filtering scheme whenever GPS signal is lost, and switch back to the extended Kalman filtering scheme on condition that the system receive GPS information. A simulated model is developed to evaluate the performance of the proposed system. With the observed data from real IMU sensors and GPS receiver, the result shows the overall accuracy of the integrated system is quite good. In the situation of GPS outage, the integrated system is also achieved better performance when changing to linear Kalman filter scheme. In the future work, this configuration could be used to implement to a DSP system in order to create a real-time navigation system.

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