Linear Algebra MATH 325: Assignment 7

(Due in class, March 22)

Problem 1: Find all real numbers a, b such that the map

$$\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right) \longmapsto \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}^T \cdot \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

is an inner product on $\mathbb{R}^2$? Here T is the transpose.

Problem 2: Which of the following two maps are inner products on $\mathbb{R}^3$?

(a)

$$\langle \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \rangle := x_1y_1 + 2x_2y_2 + 3x_3y_3 .$$

(b) $\langle \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \rangle :=$

$$x_1y_1 + 2x_1y_2 + x_1y_3 + 2x_2y_1 + x_2y_2 + x_3y_1 - x_3y_3 .$$

Problem 3: Give an orthonormal basis of the orthogonal complement of $v = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ in the inner product space $\mathbb{R}^3$ with the usual Euclidean inner product.

Problem 4: Let $\langle -, - \rangle$ be the usual Euclidean scalar product on $\mathbb{R}^n$ and $\|v\| := \sqrt{\langle v, v \rangle}$ the associated norm. Let $v, w \in \mathbb{R}^n \setminus \{0\}$. Show that $\|v\| = \|w\|$ if and only if $v + w$ is orthogonal to $v - w$.