Linear Algebra MATH 325: Assignment 2

(Due in class, January 23)

Problem 1: Consider a matrix
$$A = \begin{pmatrix} 1 & 4 \\ 3 & 1 \end{pmatrix}$$
and let $\alpha_A : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $v \mapsto A \cdot v$ be the corresponding $\mathbb{R}$-linear map. Find the matrix of the linear map α_A with respect to the basis
$$v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$
of $\mathbb{R}^2$.

Problem 2: Let $A = \begin{pmatrix} 2 & 2 \\ 0 & 5 \end{pmatrix}$. What are the A-invariant subspaces in $\mathbb{R}^2$?

Problem 3: Give an example of two 2×2-matrices with real coefficients which have the same characteristic polynomial but which are not conjugate. (Recall that two $n \times n$-matrices A and B are called *conjugate* if there is an invertible $n \times n$-matrix S, such that $B = S^{-1} \cdot A \cdot S$.)

Problem 4: Let A be a 2×2-matrix with only one eigenvalue $\lambda = 3$. (E) conjecture Show that $(3 \cdot \mathrm{I}_2 - A)^2 = 0$. Recall that the symbol I_2 denotes the 2×2-identity matrix:
$$\mathrm{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$