Linear Algebra MATH 325: Assignment 3

(Due in class, February 1)

Problem 1: Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$. Compute A^{-1} using the Cayley-Hamilton theorem. (Hint: compute the characteristic polynomial $P_A(T)$ of A and analyze the equality $P_A(A) = 0$.)

Problem 2: Let $A = \begin{pmatrix} 2 & 9 \\ -1 & 8 \end{pmatrix}$ and $\alpha_A : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$, $v \mapsto A \cdot v$, the corresponding linear map.

(i) Show that A has only one eigenvalue and that A is not diagonalizable.

(ii) Find a basis of $\mathbb{C}^2$, such that the matrix of α_A with respect to this basis is

$$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix},$$

where λ is the only eigenvalue of A.

Problem 3: Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Show that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is a generalized eigenvector for the eigenvalue 1 of A.

Problem 4: Let A be a complex non zero $n \times n$-matrix, and $f(T)$ a complex polynomial of minimal degree ≥ 1, such that $f(A) = 0$. Show that $f(T)$ divides the characteristic polynomial of A. (Hint: apply the division algorithm.)