Linear Algebra MATH 325: Assignment 4

(Due in class, February 9)

Problem 1: Determine the eigenvalues and the eigenspaces of the following two matrices

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix},$$

and if one of them is not diagonalizable give a Jordan basis for it. (For the definition of a Jordan basis see my notes Part 3, page 3.)

Problem 2: Let $A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$.

(i) Show that 1 and 2 are eigenvalues of A and that A is not diagonalizable.

(ii) Decompose the vector $v = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ into a sum of generalized eigenvectors.

Problem 3: Show that the 4×4-matrix $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ is not diagonalizable.

Problem 4: Recall that the trace $tr(A)$ of a $n \times n$-matrix $A = (a_{ij})$ is defined as

$$tr(A) := \sum_{i=1}^{n} a_{ii}.$$

Show that $tr(A \cdot B) = tr(B \cdot A)$ for all $n \times n$-matrices A, B.