## Linear Algebra MATH 325: Assignment 8

( It is for practice, not for marking )

**Problem 1:** Let $i = \sqrt{-1} \in \mathbb{C}$. Recall that every complex number can be written

$$z = r \cdot e^{i \cdot \alpha} = r \cdot (\cos(\alpha) + i \cdot \sin(\alpha))$$

for some real number $r \geq 0$ and some $\alpha \in \mathbb{R}$. For which $r, \alpha$ is the $\mathbb{C}$-linear map

$$\ell_z \,:\, \mathbb{C} \longrightarrow \mathbb{C}\,,\; w \longmapsto z \cdot w$$

hermitian, for which unitary?

**Problem 2:** Find an unitary $3 \times 3$-matrix $U$ with first column $\begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$.

**Problem 3:** Let

$$\langle -, - \rangle \,:\, \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

be an inner product on $\mathbb{R}^n$. Show that there exists an invertible $n \times n$-matrix $A$, such that

(a) $A^T = A$, and

(b) $\langle v, w \rangle \,=\, v^t \cdot A \cdot w$ for all $v, w \in \mathbb{R}^n$.

**Problem 4:** Let $\langle -, - \rangle$ be the usual inner product on $\mathbb{R}^2$. Does there exists a real $2 \times 2$-matrix $A \neq \mathrm{I}_2$, such that

$$\langle A \cdot v, A \cdot w \rangle \,=\, \langle v, w \rangle$$

for all $v, w \in \mathbb{R}^2$? If such a matrix $A$ exists, what is $\det A$?