Linear Algebra MATH 325: Assignment 8

(Due in class, April 1)

Problem 1:

(i) Show that the map

$$\langle -,- \rangle \, : \, \mathbb{C}^2 \times \mathbb{C}^2 \longrightarrow \mathbb{C}\,,$$

$$\langle \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \rangle \;:=\; 3a_1\bar{b}_1 + a_1\bar{b}_2 + a_2\bar{b}_1 + 2a_2\bar{b}_2$$

is an inner product on $\mathbb{C}^2$.

(ii) Give an orthogonal basis of $\mathbb{C}^2$ with respect to this inner product.

Problem 2: Let $(V, \langle -,- \rangle)$ be the inner product space $\mathbb{R}^4$ with respect to the usual Euclidean inner product. Let

$$w_1 = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 6 \end{pmatrix}, \; w_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \; \text{and } w_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix},$$

and $W \subseteq V = \mathbb{R}^4$ be the subspace generated by these vectors. Give a orthogonal basis for W.

Problem 3: Let V be a $\mathbb{C}$-vector space with inner product $\langle -,- \rangle$. Determine all complex numbers λ such that the map

$$(v, w) \longmapsto \lambda \cdot \langle v, w \rangle$$

is also an inner product on V?

Problem 4: Let $A = \begin{pmatrix} 5 & 1 \\ 1 & 1 \end{pmatrix}$. Find a 2×2-matrix S with real coefficients such that

$$S^T \cdot A \cdot S \;=\; \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$