

The entropy of all system at zero absolute temperature is a universal constant that can be taken to be zero. $\lim_{T \rightarrow 0} S(X, T) = 0$

Consequence:

1. $S(T=0, X) = 0$ for all coordinates X

$$\lim_{T \rightarrow 0} \left. \frac{\partial S}{\partial X} \right|_T = 0$$

2. Heat capacity must vanish as $T \rightarrow 0$

Since $S(T, X) - S(0, X) = \int_0^T dT' \frac{C_X(T')}{T'}$ and the integral diverges

as $T \rightarrow 0$ unless $\lim_{T \rightarrow 0} C_X(T) = 0$

3. Thermal expansivities also vanish as $T \rightarrow 0$ since

$$\alpha_J = \frac{1}{X} \left. \frac{\partial X}{\partial T} \right|_J \stackrel{\text{Maxwell relation}}{=} \frac{1}{X} \left. \frac{\partial S}{\partial J} \right|_T \stackrel{!}{=} 0$$

weaker statement for 3rd law 4. impossible to cool any system to absolute zero in a finite number of steps.

