5.2 (a) Clearly posterior expected loss is Risk where $P_1 = P(y=1|x)$

$$R(\hat{y}=0|x) = \lambda_{01} P(y=1|x) = \lambda_{01} P_1$$

and $$R(\hat{y}=0|x) = \lambda_{10} P(y=0|x) = \lambda_{10} P_0 = \lambda_{10}(1-P_1)$$

So we will predict $\hat{y}=0$

if $R(\hat{y}=0|x) < R(\hat{y}=1|x)$

$$\lambda_{01} P_1 < \lambda_{10}(1-P_1)$$

$$\underline{\underline{P_1 < \frac{\lambda_{10}}{\lambda_{01}+\lambda_{10}} = \theta}}$$

(b) if $\dfrac{\lambda_{10}}{\lambda_{01}+\lambda_{10}} = 0.1 = \dfrac{1}{10} = \dfrac{1}{1+9}$

then $\lambda_{10} = 1$ and $\lambda_{01} = 9$ (Note: Not unique) clearly loss matrix will be

predicted $\hat{y}$	True y: 0	1
0	0	9
1	1	0

(Note any multiple of 1 and 9 will also give same threshold 0.1

5.3 posterior expected loss/Risk

(a) cost of rejecting is λ_r

cost of picking most probable class is $j = \arg\max_{l} P(y=l|X)$ is

$$\sum_{i \neq j} \lambda_s P(y=i|x)$$ [cost of picking right class is 0]

so pick j if

$$\lambda_r \geq \sum_{i \neq j} \lambda_s P(y=i|X)$$

$$\frac{\lambda_r}{\lambda_s} \geq 1 - P(y=j|X)$$ [probability sum to one]

or $P(y=j|x) \geq 1 - \frac{\lambda_r}{\lambda_s}$

otherwise choose reject.

Note if we decide to choose a class we has to choose

$$j = \arg\max_i P(y=i \mid x)$$

if we choose other class $K \neq j$ we will incur more cost.

ie cost of choosing K will be

$$\sum_{i \neq K} \lambda_s P(y=i \mid x) = \lambda_s \left(1 - P(y=K \mid x)\right)$$

$$\geq \lambda_s \left(1 - P(y=j \mid x)\right)$$

because $j = \arg\max_i P(y=i \mid x)$

(b) if $\frac{\lambda_r}{\lambda_s} = 0$ there is no cost of rejecting.

as $\frac{\lambda_r}{\lambda_s} \to 1$

$$P(y=j \mid x) \geq 1-1 \geq 0$$

cost of rejecting increases. Above inequality for most probable class is satisfied more and more. we always accept the most probable class. [redacted]