MIE1622H: Assignment 1

Mean-Variance Portfolio Selection Strategies Report

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Introduction

The main purpose of the report is to compare the optimized portfolios built with four different trading strategies in portfolio values and interpret the performance of the four strategies over two years, with a total of 12 trading periods.

In the rest of the report, the following part would be covered. First, by completing the code of four strategies in Python, the portfolio values before and after the strategy applied in each trading period were produced. After that, a figure has been plotted to demonstrate the daily value of portfolio of each strategy for the entire trading period. Two more plots were constructed to illustrate the dynamic changes in portfolio allocation of the 20 assets for "Minimum variance" portfolio strategy and for "Maximum Sharpe ratio" portfolio strategy. Through the plots, one of the strategies would be selected to implement. At last, a discussion would be made to compare the four strategies with each other and with other variations. Also, suggestions of improvement of the selected strategies would be presented.

Question 1: Implement Investment Strategies in Python

The initial portfolio value is 1000012.93 USD, holding 902 shares of "HOG" and 17500 shares of "VZ". The initial cash account is 0 USD.

The four different portfolio re-balancing strategies are implemented in Python, *x_optimal* is the optimal portfolio and *cash_optimal* is the amount in cash account:
"Buy and hold" strategy, a strategy which hold the initial portfolio, 902 shares of "HOG" and 17500 shares of "VZ" for the entire investment periods.

```
def strat_buy_and_hold(x_init, cash_init, mu, Q, cur_prices):
    x_optimal = x_init
    cash_optimal = cash_init
    return x_optimal, cash_optimal
```

"Equally weighted" (also known as "1/n") portfolio strategy, a strategy which maintain the assets weight are equal through the entire investment periods, while the number of units of each asset could differ. In this case, we have a total number of 20 assets.

$$w_i^t = \frac{1}{20} = \frac{v_i * x_i}{V}$$

```
def strat_equally_weighted(x_init, cash_init, mu, Q, cur_prices):
    total_value = np.dot(cur_prices,x_init)+cash_init
    weight_optimal = np.array([1/len(x_init)]*len(x_init)) # Equal weight = 1/n
    allocated_value = weight_optimal*total_value

    x_optimal = np.floor(allocated_value/cur_prices) # Rounding procedure
    transaction_fee = 0.005 * np.dot(cur_prices,abs(x_optimal-x_init))
    cash_optimal = total_value - np.dot(cur_prices,x_optimal) - transaction_fee

    return x_optimal, cash_optimal
```

By allocating the total portfolio value at the beginning of the trading period with the weight, the optimal portfolio can be calculated by dividing allocated value with the current price of stocks and round the units of each stock down to the nearest integer. Then, the cash account

value can be calculated with the optimal portfolio. For the rest two strategies, the logic is also suitable.

"Minimum variance" portfolio strategy, a strategy that invest on the portfolio with the minimum variance. Variance $w^T Q w$ need to be minimized, while w is the portfolio weight and Q is the covariance matrix of the 20 assets.

$$\min w^T Q w$$
$$s.t\ \Sigma_i w_i = 1$$
$$w \geq 0$$

```
def strat_min_variance(x_init, cash_init, mu, Q, cur_prices):
    total_value = np.dot(cur_prices,x_init)+cash_init

    n = len(x_init)

    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize) # Minimize objective
    c  = [0.0] * n # No linear objective
    lb = [0.0] * n
    ub = [1.0] * n

    Atilde = []
    for k in range(n):
        Atilde.append([[0,1],[1,0]]) # One column of ones times corresponding weights

    var_names = ["w_%s" % i for i in range(1,n+1)]
    cpx.linear_constraints.add(rhs=[1.0,0], senses="EE")
    cpx.variables.add(obj=c, lb=lb, ub=ub, columns=Atilde, names=var_names)

    qmat = [[list(range(n)), list(2*Q[k,:])] for k in range(n)] # Sparse matrix of Q
    cpx.objective.set_quadratic(qmat) # Quadratic objective
    cpx.parameters.threads.set(6)
    cpx.set_results_stream(None)
    cpx.set_warning_stream(None)
    cpx.solve()
    weight_optimal = np.array(cpx.solution.get_values())

    allocated_value = weight_optimal*total_value

    x_optimal = np.floor(allocated_value/cur_prices) #Rounding procedure
    transaction_fee = 0.005 * np.dot(cur_prices,abs(x_optimal-x_init))
    cash_optimal = total_value - np.dot(cur_prices,x_optimal) - transaction_fee

    return x_optimal, cash_optimal
```

"Maximum Sharpe ratio" portfolio strategy, a strategy that invest on the portfolio that maximize the Sharpe ratio. μ_i is the mean of each asset, r_f is the daily risk-free rate, and Q is the covariance matrix of the 20 assets. We can solve y through CPLEX optimization solver and calculated the optimal portfolio weight w^* by using the equation $w^* = \frac{y}{\kappa}$.

$$\min y^T Q y$$
$$s.t. \Sigma(\mu_i - r_f) y_i = 1$$
$$\Sigma y_i = \kappa$$
$$\kappa \geq 0$$

```
def strat_max_Sharpe(x_init, cash_init, mu, Q, cur_prices):
    total_value = np.dot(cur_prices,x_init)+cash_init

    n = len(x_init)+1
    r_rf = 0.025
    daily_rf = r_rf/252
    diff = mu - daily_rf

    coe_k = np.zeros((20,1))
    Q = np.hstack((Q,coe_k))
    coe_k = np.zeros((1,21))
    Q = np.vstack((Q,coe_k)) # New row and column for risk free asset

    Atilde = []
    for k in range(20):
        Atilde.append([[0,1],[diff[k],1]])
    Atilde.append([[0,1],[0,-1]])

    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize)
    c  = [0.0] * n
    lb = [0.0] * n
    ub = [np.inf] * n

    var_names = ['y_%s'% i for i in range(1,n+1)]
    cpx.linear_constraints.add(rhs=[1.0,0],senses='EE')
    cpx.variables.add(obj=c,lb=lb,ub=ub,columns=Atilde,names=var_names)

    qmat = [[list(range(n)), list(2*Q[k,:])] for k in range(n)]
    cpx.objective.set_quadratic(qmat) # Quadratic objective
    cpx.parameters.threads.set(6)
    cpx.set_results_stream(None)
    cpx.set_warning_stream(None)
    cpx.solve()

    w_maxSharpe = np.array(cpx.solution.get_values()) # Optimal weight
    weight = w_maxSharpe[0:20]/w_maxSharpe[20]

    allocated_value = weight*total_value

    x_optimal = np.floor(allocated_value/cur_prices) # Rounding procedure
    transaction_fee = 0.005 * np.dot(cur_prices,abs(x_optimal-x_init))
    cash_optimal = total_value - np.dot(cur_prices,x_optimal) - transaction_fee

    return x_optimal, cash_optimal
```

A rounding procedure is designed in all strategies except the "Buy and Hold" strategy as it holds the initial portfolio unchanged. *np.floor()* function is used to round the unit of each asset down to the nearest integer.

A validation procedure is contained in loops when performing these strategies:

```
# Validation Procedure
if cash[strategy, period-1] < 0:
    # Adjust the portfolio with reallocation of the cash by weight
    cur_portfolio_value = np.dot(cur_prices,curr_positions) + curr_cash
    weight = (cur_prices * x[strategy, period-1])/cur_portfolio_value
    excess_cash = abs(cash[strategy, period-1])*weight
    excess_stock = np.ceil(excess_cash/cur_prices) # The units of stocks need to sell
    x[strategy, period-1] = x[strategy, period-1] - excess_stock # New optimal protfolio
    new_tran_fee = np.dot(cur_prices , abs(x[strategy, period-1]-curr_positions)) * 0.005
    # New cash account value
    cash[strategy, period-1] = cur_portfolio_value - np.dot(cur_prices,x[strategy, period-1]) - new_tran_fee
```

If the amount in cash account is negative, the portfolio needs to be re-evaluated by selling excess stocks. The excess stocks are calculated through the current weight multiply the negative amount in cash account, then divided by current prices. And using a *np.ceil()* function here is to avoid selling to less, so that the cash will be negative again. After that, the amount in cash account need to be rebalanced.

Question 2: Analyze your results

Output for 12 periods

Here are the results of all strategies that used to rebalance the portfolio bimonthly from 2020 to 2021:

Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = $ 1000012.93, value end = $ 893956.75

Strategy "Equally Weighted Portfolio", value begin = $ 990894.80, value end = $ 892945.31

Strategy "Minimum Variance Portfolio", value begin = $ 992762.96, value end = $ 916121.67

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990063.94, value end = $ 922166.34

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39

Strategy "Equally Weighted Portfolio", value begin = $ 931128.27, value end = $ 862105.12

Strategy "Minimum Variance Portfolio", value begin = $ 955863.06, value end = $ 850847.29

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 962128.89, value end = $ 1017291.45

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = $ 937916.81, value end = $ 913415.30

Strategy "Equally Weighted Portfolio", value begin = $ 830867.21, value end = $ 933903.54

Strategy "Minimum Variance Portfolio", value begin = $ 826537.62, value end = $ 853624.64

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 974438.74, value end = $ 1175843.03

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = $ 905419.63, value end = $ 994693.42

Strategy "Equally Weighted Portfolio", value begin = $ 927502.48, value end = $ 1060455.46

Strategy "Minimum Variance Portfolio", value begin = $ 855948.35, value end = $ 981094.11

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1219722.56, value end = $ 1607156.24

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18

Strategy "Equally Weighted Portfolio", value begin = $ 1068063.99, value end = $ 998951.36

Strategy "Minimum Variance Portfolio", value begin = $ 982848.02, value end = $ 942330.62

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1641142.05, value end = $ 1554381.07

Period 6: start date 11/02/2020, end date 12/31/2020

Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.67

Strategy "Equally Weighted Portfolio", value begin = $ 1007789.79, value end = $ 1193997.53

Strategy "Minimum Variance Portfolio", value begin = $ 950764.28, value end = $ 1005495.76

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1552982.30, value end = $ 1790408.17

Period 7: start date 01/04/2021, end date 02/26/2021

Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.15

Strategy "Equally Weighted Portfolio", value begin = $ 1180456.03, value end = $ 1266854.46

Strategy "Minimum Variance Portfolio", value begin = $ 1003520.02, value end = $ 974692.95

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1738669.31, value end = $ 1853474.22

Period 8: start date 03/01/2021, end date 04/30/2021

Strategy "Buy and Hold", value begin = $ 957791.42, value end = $ 1019731.31

Strategy "Equally Weighted Portfolio", value begin = $ 1297218.45, value end = $ 1398494.53

Strategy "Minimum Variance Portfolio", value begin = $ 975012.32, value end = $ 1087613.70

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1901869.64, value end = $ 2061649.68

Period 9: start date 05/03/2021, end date 06/30/2021

Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85

Strategy "Equally Weighted Portfolio", value begin = $ 1397370.18, value end = $ 1458915.38

Strategy "Minimum Variance Portfolio", value begin = $ 1087429.09, value end = $ 1076273.58

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2053119.43, value end = $ 2016330.98

Period 10: start date 07/01/2021, end date 08/31/2021

Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.12

Strategy "Equally Weighted Portfolio", value begin = $ 1466315.59, value end = $ 1517377.31

Strategy "Minimum Variance Portfolio", value begin = $ 1076305.39, value end = $ 1086086.21

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2015220.37, value end = $ 2121352.09

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41

Strategy "Equally Weighted Portfolio", value begin = $ 1513145.16, value end = $ 1563053.46

Strategy "Minimum Variance Portfolio", value begin = $ 1080556.14, value end = $ 1056718.23

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2102080.92, value end = $ 2144068.69

Period 12: start date 11/01/2021, end date 12/31/2021

Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35

Strategy "Equally Weighted Portfolio", value begin = $ 1584422.82, value end = $ 1646208.63

Strategy "Minimum Variance Portfolio", value begin = $ 1054122.96, value end = $ 1048183.18

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2113215.60, value end = $ 2217523.12

Below is the plot which demonstrate the daily value change of the portfolio over the year 2020 and 2021:

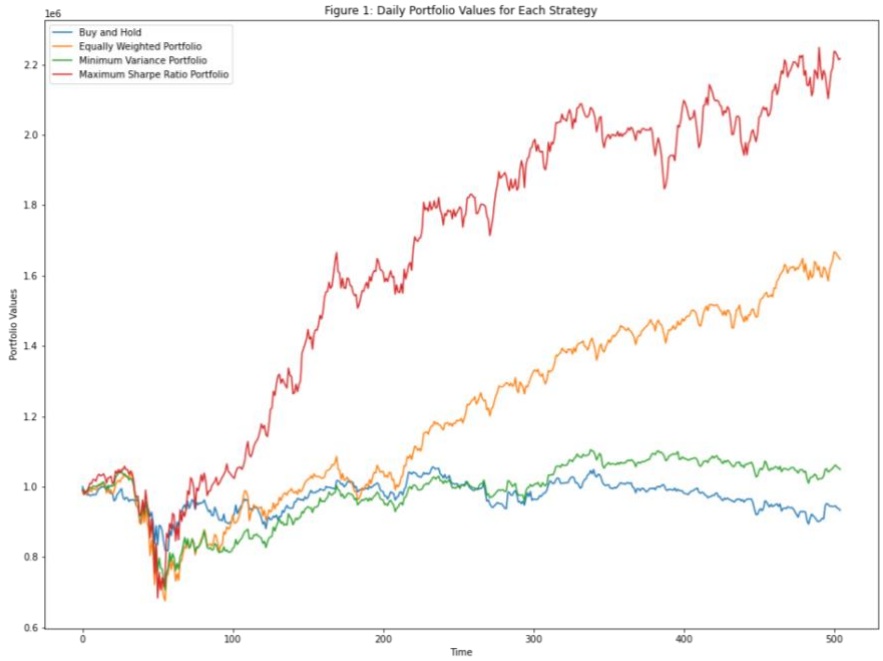

Figure 1: Daily Portfolio Values for Each Strategy

Next is two graphs that show the dynamic change in portfolio allocations for “Minimum Variance” portfolio strategy and “Maximum Sharpe ratio” portfolio strategy

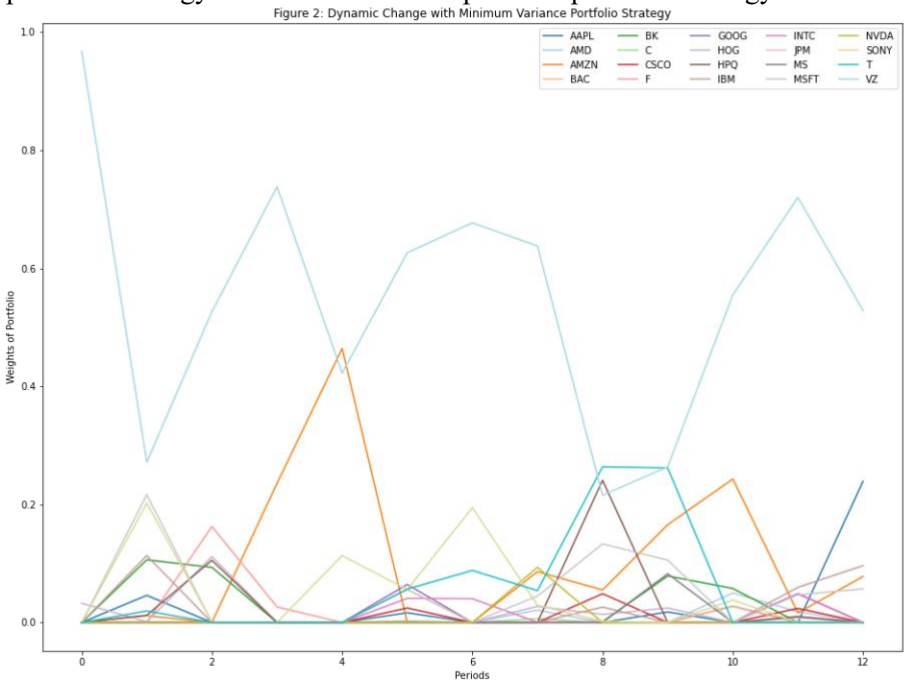

Figure 2: Dynamic Change with Minimum Variance Portfolio Strategy

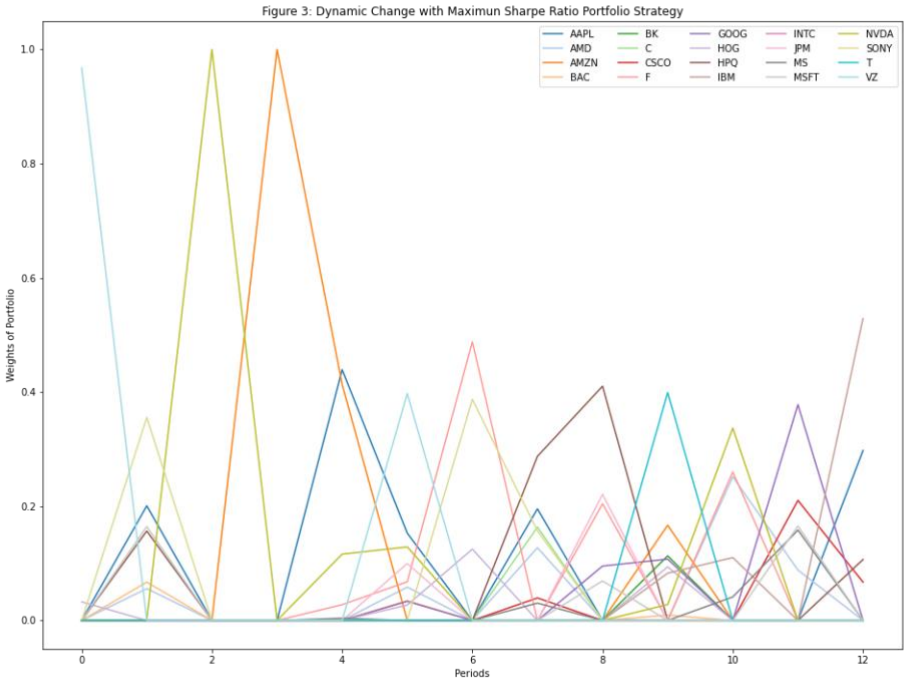

Figure 3: Dynamic Change with Maximum Sharpe Ration Portfolio Strategy

Compare Trading Strategies and discuss performance

From the figure 1, "Maximum Sharpe ratio" portfolio strategy produces the best results in portfolio re-balancing, "Equally weighted" portfolio strategy follows next. "Minimum variance" portfolio strategy and "Buy and hold" strategy show similar performance. However, around day 50, the portfolio value of all four strategies has a dramatic decrease and rise again soon.

In dynamic change figures, for "Minimum variance" portfolio strategy, asset "VZ" accounts for a great portion (>20%) in all periods, this may be due to that asset "VZ" has the lowest variance. The weight of the rest stocks is all below 20% with a few exceptions, such as asset "AMZN" in period 3, which exceed the portion of "VZ".

For "Maximum Sharpe ratio" portfolio strategy, the portfolio allocation is hard to find a general pattern. But in period 2 and 3, asset "NVDA" and asset "AMZN" have a percentage which are very close to 1, respectively. In other periods, the strategy suggests buying an allocation with multiple stocks, at least 4 stocks.

By comparing the daily value and dynamic change figures, it is found that "Maximum Sharpe ratio" portfolio strategy considers both risk and return, which is important in further investment, and the portfolio value of "Maximum Sharpe ratio" portfolio strategy is much higher than other three strategies, thus, I will select "Maximum Sharpe ratio" portfolio strategy to manage my own portfolio.

Question 3: Discuss possible improvements to trading strategies

Different variations of strategies

In this part, two variations of strategies have been made. One is "Buy Equally and Hold, which is to select "1/n" portfolio at beginning of period 1 and hold till the end of period 12. And as the significant decrease has been discovered in daily value figure, the other variation is to reduce it. The other variation is "Hold and Maximum Sharpe ratio", holding the initial

portfolio at period 1 and apply the "Maximum Sharpe ratio" strategy for the rest the periods. The design of these two variations and the output are in Appendix.

A plot of daily value of the original four strategies and two new variations is shown below:

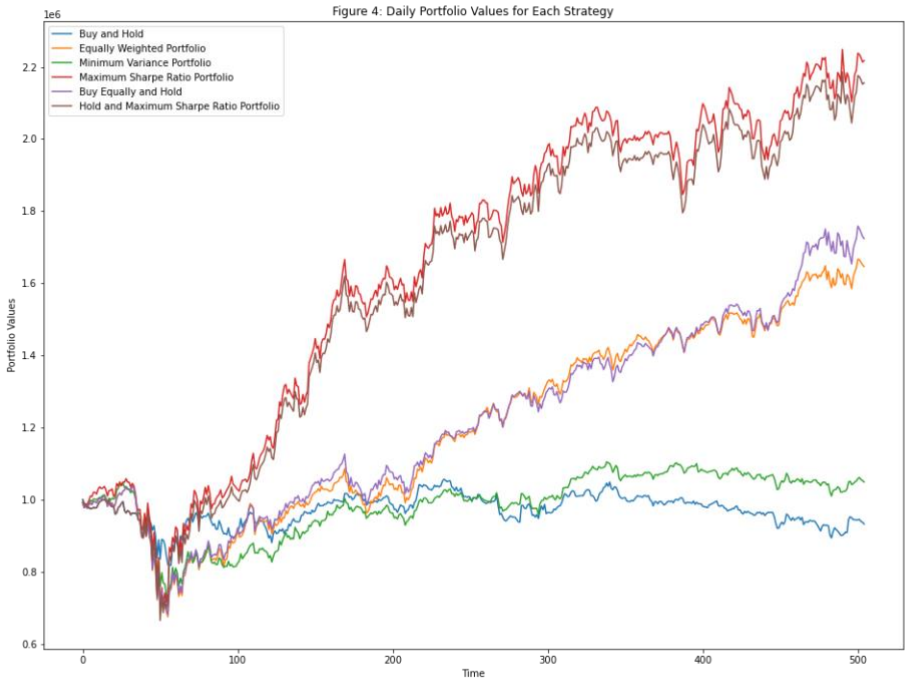

Figure 4: Daily Portfolio Values for Each Strategy

So, as can see from the above graph, the "Buy Equally and Hold" shows a similar trend as the "Equally Weighted Portfolio". These two lines are overlapped with each time for some time, so it is hard to tell which of the variation and the original strategy "Equally Weighted Portfolio" is better. Also, "Hold and Maximum Sharpe Ratio" is close and share a similar pattern with the "Maximum Sharpe ratio". But, obviously, "Maximum Sharpe ratio" portfolio strategy still has a higher return. Therefore, these two variations did not achieve better result. I will still select "Maximum Sharpe ratio" portfolio strategy to manage my own portfolio.

Suggestion on Improvements of "Maximum Sharpe ratio" Strategy

- First, the rounding procedure designed now is to round the units of each stock down to the nearest integer, which may not be very precise. An advanced rounding procedure can be designed to either round up or round down based on the combination of return and variance.
- In the assignment, we simply assumed that the daily risk-free rate is the same. So, in real life, the daily risk-free rate could be fluctuated, some might be higher than the one used in function. Thus, using real daily risk-free rate may achieve a more accurate portfolio value.
- And last, although the general trend of the daily value of "Maximum Sharpe ratio" is increasing, still a few decreases in values happened. I suggest that rather than re-balancing bimonthly, the trading period could be extended, such as seasonally or half a year.

Appendix:

Design of Variations

```
def strat_buy_equal_and_hold(x_init, cash_init, mu, Q, cur_prices,period):
    total_value = np.dot(cur_prices,x_init)+cash_init

    x_init = np.array(x_init)
    x_optimal = [672,1011,25,1454,1034,656,1089,5237,35,1376,2518,424,859,377,1006,316,830,720,1492,898]

    transaction_fee = 0.005 * np.dot(cur_prices,abs(x_optimal-x_init))
    cash_optimal = total_value - np.dot(cur_prices,x_optimal) - transaction_fee

    return x_optimal, cash_optimal
def strat_hold_and_max_Sharpe(x_init, cash_init, mu, Q, cur_prices, period):
    if period == 1:
        x_optimal = x_init
        cash_optimal = cash_init
    else:
        total_value = np.dot(cur_prices,x_init)+cash_init

        n = len(x_init)+1
        r_rf = 0.025
        daily_rf = r_rf/252
        diff = mu - daily_rf

        coe_k = np.zeros((20,1))
        Q = np.hstack((Q,coe_k))
        coe_k = np.zeros((1,21))
        Q = np.vstack((Q,coe_k))

        Atilde = []
        for k in range(20):
            Atilde.append([[0,1],[diff[k],1]])
        Atilde.append([[0,1],[0,-1]])

        cpx = cplex.Cplex()
        cpx.objective.set_sense(cpx.objective.sense.minimize)
        c  = [0.0] * n
        lb = [0.0] * n
        ub = [np.inf] * n

        var_names = ['y_%s'% i for i in range(1,n+1)]
        cpx.linear_constraints.add(rhs=[1.0,0],senses='EE')
        cpx.variables.add(obj=c,lb=lb,ub=ub,columns=Atilde,names=var_names)
        qmat = [[list(range(n)), list(2*Q[k,:])] for k in range(n)]
        cpx.objective.set_quadratic(qmat)
        cpx.parameters.threads.set(6)
        cpx.set_results_stream(None)
        cpx.set_warning_stream(None)
        cpx.solve()

        w_maxSharpe = np.array(cpx.solution.get_values())
        weight = w_maxSharpe[0:20]/w_maxSharpe[20]

        allocated_value = weight*total_value

        x_optimal = np.floor(allocated_value/cur_prices)
        transaction_fee = 0.005 * np.dot(cur_prices,abs(x_optimal-x_init))
        cash_optimal = total_value - np.dot(cur_prices,x_optimal) - transaction_fee

    return x_optimal, cash_optimal
```

Output

Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = $ 1000012.93, value end = $ 893956.75

Strategy "Equally Weighted Portfolio", value begin = $ 990894.80, value end = $ 892945.31

Strategy "Minimum Variance Portfolio", value begin = $ 992762.96, value end = $ 916121.67

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990063.94, value end = $ 922166.34

Strategy "Buy Equally and Hold", value begin = $ 990898.03, value end = $ 893204.97

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1000012.93, value end = $ 893956.75

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39

Strategy "Equally Weighted Portfolio", value begin = $ 931128.27, value end = $ 862105.12

Strategy "Minimum Variance Portfolio", value begin = $ 955863.06, value end = $ 850847.29

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 962128.89, value end = $ 1017291.45

Strategy "Buy Equally and Hold", value begin = $ 931799.85, value end = $ 870651.97

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 935672.83, value end = $ 989316.72

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = $ 937916.81, value end = $ 913415.30

Strategy "Equally Weighted Portfolio", value begin = $ 830867.21, value end = $ 933903.54

Strategy "Minimum Variance Portfolio", value begin = $ 826537.62, value end = $ 853624.64

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 974438.74, value end = $ 1175843.03

Strategy "Buy Equally and Hold", value begin = $ 839846.68, value end = $ 946377.09

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 947646.89, value end = $ 1143377.82

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = $ 905419.63, value end = $ 994693.42

Strategy "Equally Weighted Portfolio", value begin = $ 927502.48, value end = $ 1060455.46

Strategy "Minimum Variance Portfolio", value begin = $ 855948.35, value end = $ 981094.11

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1219722.56, value end = $ 1607156.24

Strategy "Buy Equally and Hold", value begin = $ 942976.79, value end = $ 1098622.46

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1186017.35, value end = $ 1562736.51

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18

Strategy "Equally Weighted Portfolio", value begin = $ 1068063.99, value end = $ 998951.36

Strategy "Minimum Variance Portfolio", value begin = $ 982848.02, value end = $ 942330.62

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1641142.05, value end = $ 1554381.07

Strategy "Buy Equally and Hold", value begin = $ 1109956.13, value end = $ 1020908.38

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1595788.57, value end = $ 1511390.42

Period 6: start date 11/02/2020, end date 12/31/2020

Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.67

Strategy "Equally Weighted Portfolio", value begin = $ 1007789.79, value end = $ 1193997.53

Strategy "Minimum Variance Portfolio", value begin = $ 950764.28, value end = $ 1005495.76

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1552982.30, value end = $ 1790408.17

Strategy "Buy Equally and Hold", value begin = $ 1027978.17, value end = $ 1199797.10

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1510030.92, value end = $ 1740895.08

Period 7: start date 01/04/2021, end date 02/26/2021

Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.15

Strategy "Equally Weighted Portfolio", value begin = $ 1180456.03, value end = $ 1266854.46

Strategy "Minimum Variance Portfolio", value begin = $ 1003520.02, value end = $ 974692.95

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1738669.31, value end = $ 1853474.22

Strategy "Buy Equally and Hold", value begin = $ 1187472.47, value end = $ 1258210.64

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1690586.67, value end = $ 1802211.93

Period 8: start date 03/01/2021, end date 04/30/2021

Strategy "Buy and Hold", value begin = $ 957791.42, value end = $ 1019731.31

Strategy "Equally Weighted Portfolio", value begin = $ 1297218.45, value end = $ 1398494.53

Strategy "Minimum Variance Portfolio", value begin = $ 975012.32, value end = $ 1087613.70

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1901869.64, value end = $ 2061649.68

Strategy "Buy Equally and Hold", value begin = $ 1288118.25, value end = $ 1377833.98

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1849267.71, value end = $ 2004704.94

Period 9: start date 05/03/2021, end date 06/30/2021

Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85

Strategy "Equally Weighted Portfolio", value begin = $ 1397370.18, value end = $ 1458915.38

Strategy "Minimum Variance Portfolio", value begin = $ 1087429.09, value end = $ 1076273.58

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2053119.43, value end = $ 2016330.98

Strategy "Buy Equally and Hold", value begin = $ 1374468.42, value end = $ 1457664.34

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1996407.10, value end = $ 1960692.71

Period 10: start date 07/01/2021, end date 08/31/2021

Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.12

Strategy "Equally Weighted Portfolio", value begin = $ 1466315.59, value end = $ 1517377.31

Strategy "Minimum Variance Portfolio", value begin = $ 1076305.39, value end = $ 1086086.21

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2015220.37, value end = $ 2121352.09

Strategy "Buy Equally and Hold", value begin = $ 1465622.81, value end = $ 1538293.32

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 1959617.79, value end = $ 2062821.27

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41

Strategy "Equally Weighted Portfolio", value begin = $ 1513145.16, value end = $ 1563053.46

Strategy "Minimum Variance Portfolio", value begin = $ 1080556.14, value end = $ 1056718.23

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2102080.92, value end = $ 2144068.69

Strategy "Buy Equally and Hold", value begin = $ 1536102.13, value end = $ 1608098.64

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 2044071.18, value end = $ 2084939.14

Period 12: start date 11/01/2021, end date 12/31/2021

Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35

Strategy "Equally Weighted Portfolio", value begin = $ 1584422.82, value end = $ 1646208.63

Strategy "Minimum Variance Portfolio", value begin = $ 1054122.96, value end = $ 1048183.18

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2113215.60, value end = $ 2217523.12

Strategy "Buy Equally and Hold", value begin = $ 1628472.41, value end = $ 1724106.96

Strategy "Hold and Maximum Sharpe Ratio Portfolio", value begin = $ 2054888.00, value end = $ 2156316.52