MIE1622H: Assignment 2

Risk-Based and Robust Portfolio Selection Strategies Report

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Introduction

The main purpose of the report is to compare the optimized portfolios built with seven different trading strategies in portfolio values and interpret the performance of the strategies from 2020 to 2021, with a total of 12 trading periods. Compared to Assignment 1, Assignment 2 has three additional strategies: "Equal risk contributions" portfolio strategy, "Leveraged equal risk contributions" portfolio strategy and "Robust mean-variance optimization" portfolio strategy.

In the rest of the report, the following part would be covered. First, by completing the code of seven strategies in Python, the portfolio values before and after the strategy applied in each trading period were produced. After that, a figure has been plotted to demonstrate the daily value of portfolio of each strategy for the entire trading period. And a graph demonstrated the maximum drawdown of the portfolio for each trading strategies were plotted. Three more plots were constructed to illustrate the dynamic changes in portfolio allocation of the 20 assets for "Robust mean-variance optimization" portfolio strategy, "Minimum variance" portfolio strategy and for "Maximum Sharpe ratio" portfolio strategy. Through the plots, "Equal risk contributions" portfolio strategy, "Leveraged equal risk contributions" portfolio strategy and "Robust mean-variance optimization" portfolio strategy were compared with each other. And these three strategies were compared with the four strategies implemented in Assignment 1 to make a selection for managing own portfolio. At last, daily closing prices for the 20 assets in 2008 and 2009 were applied to the seven strategies. Same graphs were plotted, and relative performance were discussed. One of the strategies were chose for the management of portfolio during 2008 and 2009.

1 Implement Investment Strategies in Python

The initial portfolio value is 1000012.93 USD, holding 902 shares of "HOG" and 17500 shares of "VZ". The initial cash account is 0 USD.

The three portfolio re-balancing strategies, "Equal risk contributions" portfolio strategy, "Leveraged equal risk contributions" portfolio strategy and "Robust mean-variance optimization" portfolio strategy, are implemented in Python, *x_optimal* is the optimal portfolio and *cash_optimal* is the amount in cash account:

Equal risk contributions" portfolio strategy

This strategy computes a portfolio with an equal risk to standard deviation for each period and re-balance accordingly. For this strategy, IPOPT optimizer were used, the objective function, constraints, gradient of objective function, and jacobian of constraints were defined at first.

$$\begin{aligned} \min_{w} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_i(Qw)_i - w_j(Qw)_j\right)^2 \\ \text{s.t.} \quad & \textstyle\sum_{i=1}^{n} w_i = 1 \\ & w \geq 0 \end{aligned}$$

```
def strat_equal_risk_contr(x_init, cash_init, mu, Q, cur_prices):

    portfolio_value = np.dot(cur_prices,x_init) + cash_init

    class erc(object):
        def __init__(self):
            pass

        def objective(self, x):
            # The callback for calculating the objective
            y = x * np.dot(Q, x)
            fval = 0
            for i in range(n):
                for j in range(i,n):
                    xij = y[i] - y[j]
                    fval = fval + xij*xij
            fval = 2*fval
            return fval

        def gradient(self, x):
            # The callback for calculating the gradient
            grad = np.zeros(n)
            y = x * np.dot(Q, x)
            #  use finite differences to check the gradient
            for i in range(n):
                for j in range(n):
                    diff1 = np.dot(Q[i],x) + np.dot(Q[i][i],x[i])
                    diff2 = np.dot(Q[i][j], x[i])
                    delta_g = (y[i]-y[j]) * (diff1 - diff2)
                    grad[i] = grad[i] + delta_g
                grad[i] = 2 * 2 * grad[i]
            return grad
        def constraints(self, x):
        # The callback for calculating the constraints
            return [1.0] * n

        def jacobian(self, x):
        # The callback for calculating the Jacobian
            return np.array([[1.0] * n])

    n = len(x_init)
    w0 = (x_init*cur_prices)/portfolio_value # initial weight distribution
    lb = [0.0] * n  # lower bounds on variables
    ub = [1.0] * n  # upper bounds on variables
    cl = [1]        # lower bounds on constraints
    cu = [1]        # upper bounds on constraints


    # Define IPOPT problem
    nlp = ipopt.Problem(n=len(w0), m=len(cl), problem_obj=erc(), lb=lb, ub=ub, cl=cl, cu=cu)

    # Set the IPOPT options
    nlp.add_option('jac_c_constant'.encode('utf-8'), 'yes'.encode('utf-8'))
    nlp.add_option('hessian_approximation'.encode('utf-8'), 'limited-memory'.encode('utf-8'))
    nlp.add_option('mu_strategy'.encode('utf-8'), 'adaptive'.encode('utf-8'))
    nlp.add_option('tol'.encode('utf-8'), 1e-10)
    w_optimal, info = nlp.solve(w0)

    allocated_money = w_optimal * portfolio_value
    x_optimal = np.floor(allocated_money/cur_prices)                        #Convert the
    transaction_fee = np.dot(cur_prices, abs(x_optimal-x_init))*0.005      #0.5% of tran
    cash_optimal = portfolio_value - np.dot(cur_prices,x_optimal) - transaction_fee #cash = asset

    return x_optimal, cash_optimal
```

“Leveraged equal risk contributions” portfolio strategy

The strategy borrows the money, which equals to the initial value of portfolio, in the first period and pay interest for each period throughout two years. The design of this strategy is similar with “Equal risk contributions” portfolio strategy.

```
def strat_lever_equal_risk_contr(x_init, cash_init, mu, Q, cur_prices):
    #Use IPOPT solver to calculate equal risk contribution
    class erc(object):
        def __init__(self):
            pass

        def objective(self, x):
            # The callback for calculating the objective
            y = x * np.dot(Q, x)
            fval = 0
            for i in range(n):
                for j in range(n):
                    xij = y[i] - y[j]
                    fval = fval + xij*xij
            fval = 2*fval
            return fval

        def gradient(self, x):
            # The callback for calculating the gradient
            grad = np.zeros(n)
            y = x * np.dot(Q, x)
            #use finite differences to check the gradient
            for i in range(n):
                for j in range(n):
                    diff1 = np.dot(Q[i],x) + np.dot(Q[i][i],x[i])
                    diff2 = np.dot(Q[i][j], x[i])
                    delta_g = (y[i]-y[j]) * (diff1 - diff2)
                    grad[i] = grad[i] + delta_g
                grad[i] = 2 * 2 * grad[i]
            return grad

        def constraints(self, x):
        # The callback for calculating the constraints
            return [1.0] * n

        def jacobian(self, x):
        # The callback for calculating the Jacobian
            return np.array([[1.0] * n])

    portfolio_value = np.dot(cur_prices,x_init) + cash_init
    borrow_value = init_value
    interest = borrow_value*0.025/6

    if period == 1:
        portfolio_value = portfolio_value + borrow_value        #200% long position in the beginning

    n = len(x_init)
    lb = [0.0] * n  # lower bounds on variables
    ub = [1.0] * n  # upper bounds on variables
    cl = [1]        # lower bounds on constraints
    cu = [1]        # upper bounds on constraints
    w0 = (x_init*cur_prices)/portfolio_value # Current weight
    # Define IPOPT problem
    nlp = ipopt.Problem(n=len(w0), m=len(cl), problem_obj=erc(), lb=lb, ub=ub, cl=cl, cu=cu)

    # Set the IPOPT options
    nlp.add_option('jac_c_constant'.encode('utf-8'), 'yes'.encode('utf-8'))
    nlp.add_option('hessian_approximation'.encode('utf-8'), 'limited-memory'.encode('utf-8'))
    nlp.add_option('mu_strategy'.encode('utf-8'), 'adaptive'.encode('utf-8'))
    nlp.add_option('tol'.encode('utf-8'), 1e-10)
    w_lerc, info = nlp.solve(w0)
    w_lerc = np.asarray(w_lerc)
    w_optimal = w_lerc*(1/w_lerc.sum())

    allocated_money = w_optimal*portfolio_value
    x_optimal = np.floor(allocated_money/cur_prices)                          #Convert th
    transaction_fee = np.dot(cur_prices, abs(x_optimal-x_init))*0.005        #0.5% of tra
    cash_optimal = portfolio_value - np.dot(x_optimal,cur_prices) - transaction_fee - interest #

    return x_optimal, cash_optimal
```

“Robust mean-variance optimization” portfolio strategy

The robust mean-variance optimization uses the equation below, and in designing, using a CPLEX optimizer to compute the portfolio:

$$
\begin{array}{lrcl}
\min & w^T Q w & & \\
\text{s.t.} & \sum_{i=1}^{n} w_i & = & 1 \\
 & \mu^T w & \geq & \varepsilon_{\text{ret}} \\
 & w^T \Theta w & \leq & \tilde{\varepsilon}_{\text{rob}} \\
 & w & \geq & 0
\end{array}
$$

```python
def strat_robust_optim(x_init, cash_init, mu, Q, cur_prices):
    portfolio_value = np.dot(cur_prices,x_init) + cash_init
    n = len(x_init)
    w0 = [1/n]*n                                        # Weight distribution for 1/n portfolio
    var_matr = np.diag(np.diag(Q))                          # Required portfolio robustness
    rob_init = np.dot(w0, np.dot(var_matr, w0))             # return estimation error of 1/n portfolio
    rob_bnd  = rob_init                                     # Target portfolio return estimation error is ret
    daily_rf = r_rf/252
    Portf_Retn = daily_rf

    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize)


    c  = [0.0] * n
    lb = [0.0] * n
    ub = [1.0] * n
    Atilde = []
    for k in range(n):
        Atilde.append([[0,1],[1.0,mu[k]]])
    var_names = ["w_%s" % i for i in range(1,n+1)]

    cpx.linear_constraints.add(rhs=[1.0,Portf_Retn], senses="EG")
    cpx.variables.add(obj=c, lb=lb, ub=ub, columns=Atilde, names=var_names)
    Qmat = [[list(range(n)), list(2*Q[k,:])] for k in range(n)]
    cpx.objective.set_quadratic(Qmat)
    Qcon = cplex.SparseTriple(ind1=var_names, ind2=range(n), val=np.diag(var_matr))
    cpx.quadratic_constraints.add(rhs=rob_bnd, quad_expr=Qcon, name="Qc")
    cpx.parameters.threads.set(4)
    cpx.parameters.timelimit.set(60)
    cpx.parameters.barrier.qcpconvergetol.set(1e-12)
    cpx.set_results_stream(None)
    cpx.set_warning_stream(None)
    cpx.solve()

    w_rMV = cpx.solution.get_values()
    # Round near-zero portfolio weights
    w_rMV = np.array(w_rMV)
    w_rMV[w_rMV<1e-6] = 0
    w_optimal = w_rMV / np.sum(w_rMV)

    allocated_money = w_optimal*portfolio_value
    x_optimal = np.floor(allocated_money/cur_prices)
    transaction_fee = np.dot(cur_prices, abs(x_optimal-x_init))*0.005
    cash_optimal = portfolio_value - np.dot(x_optimal,cur_prices) - transaction_fee

    return x_optimal, cash_optimal
```

A rounding procedure is designed the same as in Assignment 1for all strategies except the “Buy and Hold” strategy as it holds the initial portfolio unchanged. *np.floor()* function is used to round the unit of each asset down to the nearest integer.

A validation procedure is contained in loops when performing these strategies:

```python
if cash[strategy][period-1]<0 :
    if cash[5,0]<0:
        portfolio_value = (np.dot(curr_positions,cur_prices) + curr_cash)*2
        # 200% long positon in the beginning for leveraged ERC
    else:
        portfolio_value = np.dot(curr_positions,cur_prices) + curr_cash
        #Portfolio value before the balance strategy

    weight = (cur_prices * x[strategy, period-1])/portfolio_value
    excess_cash = abs(cash[strategy, period-1])*weight
    excess_stock = np.ceil(excess_cash/cur_prices) # The units of stocks need to sell
    x[strategy, period-1] = x[strategy, period-1] - excess_stock # New optimal protfolio
    new_tran_fee = np.dot(cur_prices , abs(x[strategy, period-1]-curr_positions)) * 0.005

    if strategy ==5:
      #Interest cost related to LERC needs to be considered
        cash[strategy, period-1] = portfolio_value - np.dot(cur_prices,x[strategy][period-1]) - new_tran_fee - init_value*r_rf/6
    else:
        cash[strategy, period-1] = portfolio_value - np.dot(cur_prices,x[strategy][period-1]) - new_tran_fee
```

If the amount in cash account is negative, the portfolio needs to be re-evaluated by selling excess stocks. The excess stocks are calculated through the current weight multiply the negative amount in cash account, then divided by current prices. And using a *np.ceil()* function here is to avoid selling to less, so that the cash will be negative again. After that, the amount in cash account need to be rebalanced.

2 Analyze Results

Output for 12 periods 2020-2021

Initial portfolio value = $ 1000013.0

Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = $ 1000013.00, value end = $ 893956.82

Strategy "Equally Weighted Portfolio", value begin = $ 990894.69, value end = $ 892935.28

Strategy "Mininum Variance Portfolio", value begin = $ 992762.91, value end = $ 916118.56

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990064.14, value end = $ 922169.64

Strategy "Equal Risk Contributions Portfolio", value begin = $ 991847.27, value end = $ 901792.10

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 983449.58, value end = $ 797159.19

Strategy "Robust Optimization Portfolio", value begin = $ 992215.86, value end = $ 917710.09

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39

Strategy "Equally Weighted Portfolio", value begin = $ 931120.96, value end = $ 862089.46

Strategy "Mininum Variance Portfolio", value begin = $ 955861.61, value end = $ 850844.71

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 962134.02, value end = $ 1017296.68

Strategy "Equal Risk Contributions Portfolio", value begin = $ 939764.09, value end = $ 857807.29

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 871648.26, value end = $ 702559.01

Strategy "Robust Optimization Portfolio", value begin = $ 958703.48, value end = $ 925609.97

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = $ 937916.75, value end = $ 913415.30

Strategy "Equally Weighted Portfolio", value begin = $ 830851.37, value end = $ 933896.07

Strategy "Mininum Variance Portfolio", value begin = $ 826534.53, value end = $ 853621.88

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 974443.97, value end = $ 1175848.26

Strategy "Equal Risk Contributions Portfolio", value begin = $ 828653.28, value end = $ 920253.88

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 638054.98, value end = $ 827533.79

Strategy "Robust Optimization Portfolio", value begin = $ 888794.07, value end = $ 964589.68

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = $ 905419.70, value end = $ 994693.42

Strategy "Equally Weighted Portfolio", value begin = $ 927494.95, value end = $ 1060461.80

Strategy "Mininum Variance Portfolio", value begin = $ 855945.60, value end = $ 981091.34

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1219727.54, value end = $ 1607178.91

Strategy "Equal Risk Contributions Portfolio", value begin = $ 916666.25, value end = $ 1051129.60

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 815987.77, value end = $ 1093481.65

Strategy "Robust Optimization Portfolio", value begin = $ 968767.04, value end = $ 1102565.17

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18

Strategy "Equally Weighted Portfolio", value begin = $ 1068070.99, value end = $ 998970.15

Strategy "Mininum Variance Portfolio", value begin = $ 982845.15, value end = $ 942326.63

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1641167.27, value end = $ 1554392.71

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1058706.50, value end = $ 996300.55

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1104953.90, value end = $ 976119.22

Strategy "Robust Optimization Portfolio", value begin = $ 1107627.62, value end = $ 1060873.37

Period 6: start date 11/02/2020, end date 12/31/2020

Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.74

Strategy "Equally Weighted Portfolio", value begin = $ 1007808.61, value end = $ 1194012.24

Strategy "Mininum Variance Portfolio", value begin = $ 950760.65, value end = $ 1005493.44

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1552993.10, value end = $ 1790420.23

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1004193.05, value end = $ 1172323.89

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 988229.86, value end = $ 1335368.77

Strategy "Robust Optimization Portfolio", value begin = $ 1067935.28, value end = $ 1198451.39

Period 7: start date 01/04/2021, end date 02/26/2021

Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.08

Strategy "Equally Weighted Portfolio", value begin = $ 1180471.57, value end = $ 1266871.29

Strategy "Mininum Variance Portfolio", value begin = $ 1003517.51, value end = $ 974690.65

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1738681.39, value end = $ 1853487.58

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1159889.66, value end = $ 1210418.76

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1305523.27, value end = $ 1409942.74

Strategy "Robust Optimization Portfolio", value begin = $ 1185524.37, value end = $ 1182662.93

Period 8: start date 03/01/2021, end date 04/30/2021

Strategy "Buy and Hold", value begin = $ 957791.35, value end = $ 1019731.32

Strategy "Equally Weighted Portfolio", value begin = $ 1297231.52, value end = $ 1398504.17

Strategy "Mininum Variance Portfolio", value begin = $ 975010.32, value end = $ 1087612.27

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1901879.65, value end = $ 2061659.43

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1233317.88, value end = $ 1339555.01

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1453040.26, value end = $ 1672639.90

Strategy "Robust Optimization Portfolio", value begin = $ 1192448.71, value end = $ 1320171.35

Period 9: start date 05/03/2021, end date 06/30/2021

Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85

Strategy "Equally Weighted Portfolio", value begin = $ 1397378.73, value end = $ 1458925.47

Strategy "Mininum Variance Portfolio", value begin = $ 1087428.12, value end = $ 1076272.11

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2053129.04, value end = $ 2016341.16

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1338548.16, value end = $ 1367064.94

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1666326.40, value end = $ 1724809.65

Strategy "Robust Optimization Portfolio", value begin = $ 1319127.06, value end = $ 1316878.08

Period 10: start date 07/01/2021, end date 08/31/2021

Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.19

Strategy "Equally Weighted Portfolio", value begin = $ 1466325.72, value end = $ 1517393.74

Strategy "Mininum Variance Portfolio", value begin = $ 1076304.20, value end = $ 1086084.64

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2015231.05, value end = $ 2121342.65

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1373011.59, value end = $ 1422376.92

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1732922.18, value end = $ 1834654.17

Strategy "Robust Optimization Portfolio", value begin = $ 1319569.84, value end = $ 1357322.72

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41

Strategy "Equally Weighted Portfolio", value begin = $ 1513162.50, value end = $ 1563073.98

Strategy "Mininum Variance Portfolio", value begin = $ 1080554.78, value end = $ 1056714.10

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2102071.12, value end = $ 2144059.16

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1416715.11, value end = $ 1430946.19

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1818791.85, value end = $ 1848091.02

Strategy "Robust Optimization Portfolio", value begin = $ 1348781.09, value end = $ 1319397.64

Period 12: start date 11/01/2021, end date 12/31/2021

Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35

Strategy "Equally Weighted Portfolio", value begin = $ 1584442.29, value end = $ 1646238.26

Strategy "Mininum Variance Portfolio", value begin = $ 1054119.02, value end = $ 1048179.24

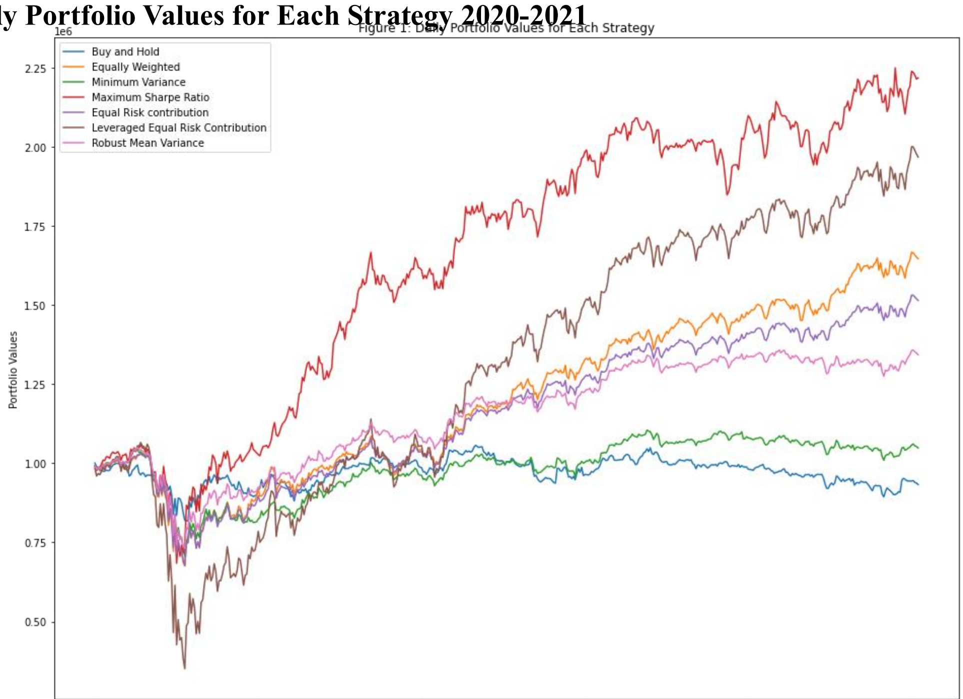
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2113205.80, value end = $ 2217514.96

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1440679.16, value end = $ 1490789.25

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1864026.18, value end = $ 1967134.72

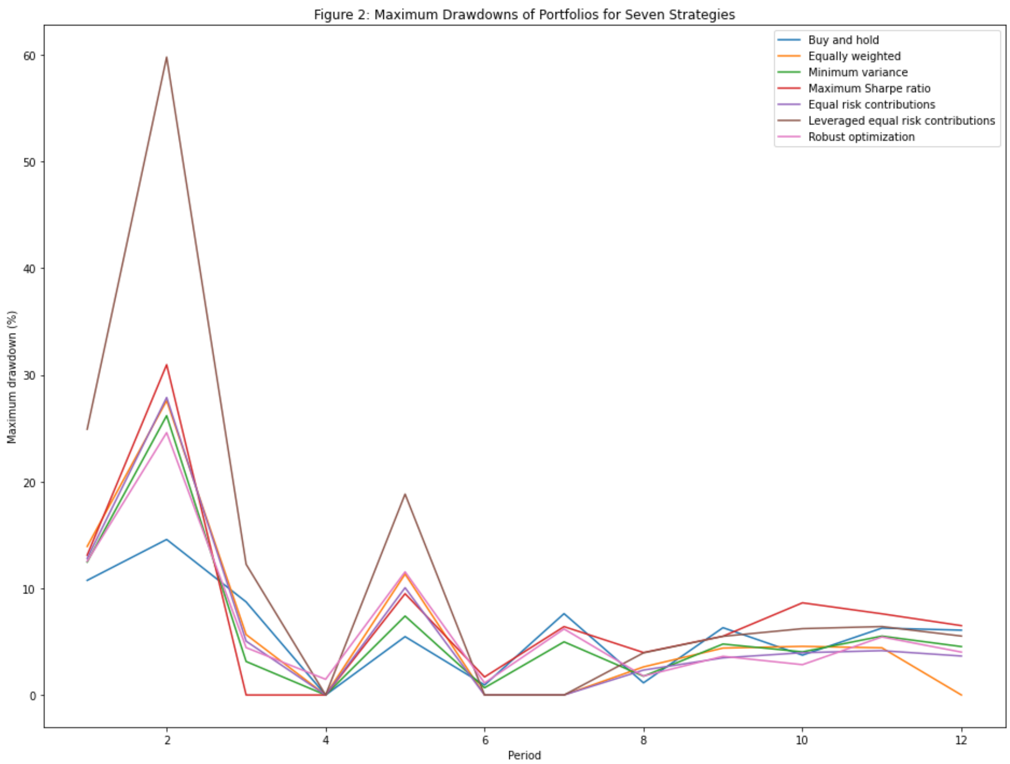
Strategy "Robust Optimization Portfolio", value begin = $ 1313198.55, value end = $ 1342326.32

Daily Portfolio Values for Each Strategy 2020-2021

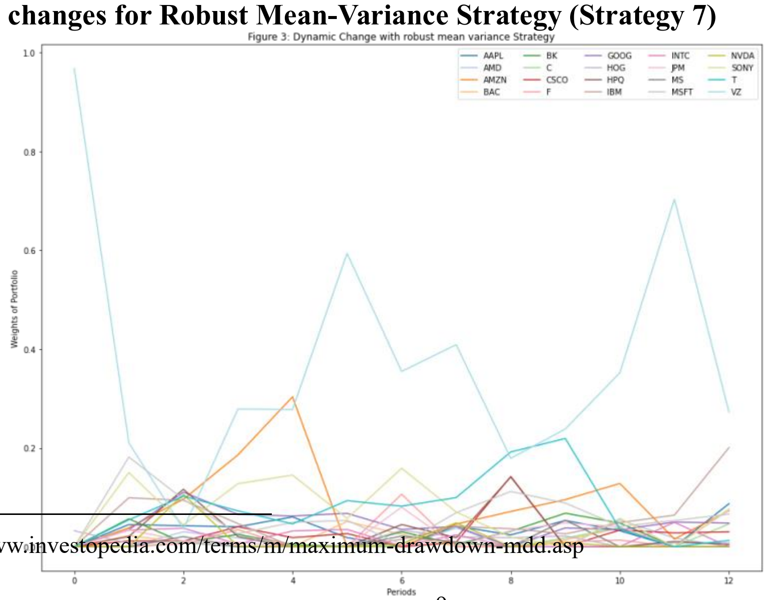


Maximum Drawdowns of Portfolios for Seven Strategies

Maximum drawdowns maximum observed loss from a peak to a trough of a portfolio before a new peak is attained.[1] Below is the graph of maximum drawdowns of the portfolios for each period of all strategies:

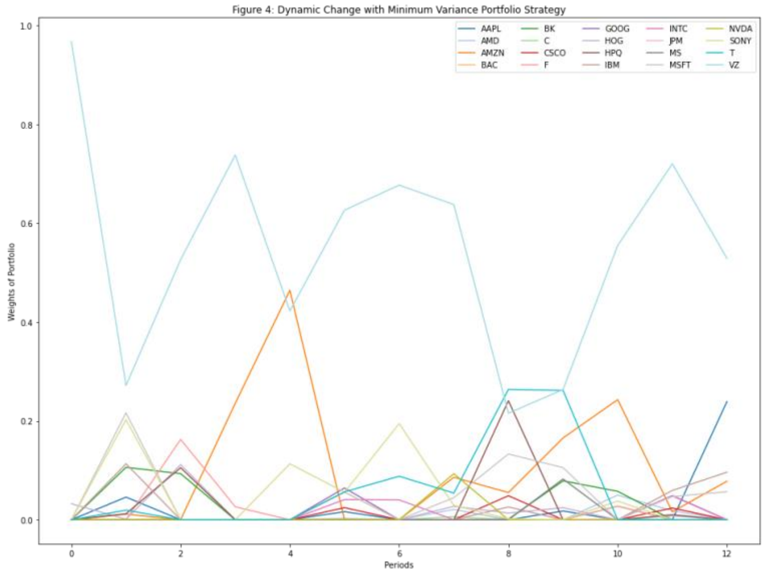


Dynamic changes for Robust Mean-Variance Strategy (Strategy 7)

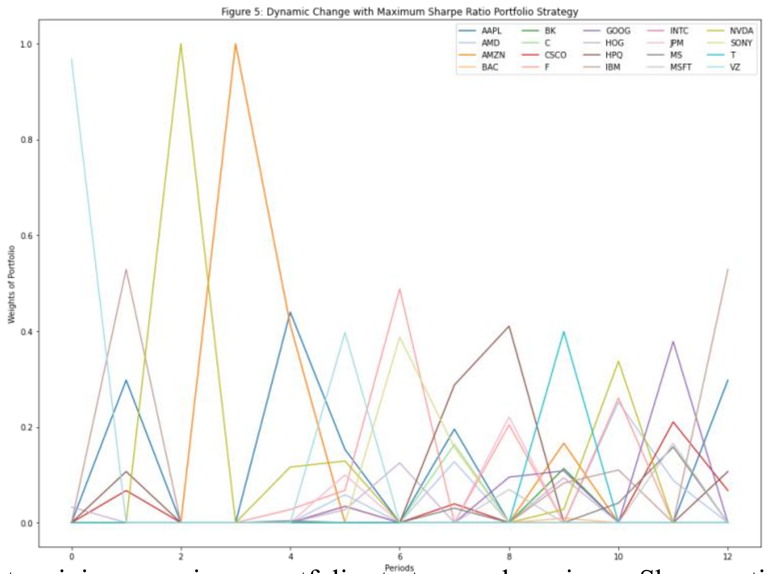


[1] https://www.investopedia.com/terms/m/maximum-drawdown-mdd.asp

Dynamic Change for Minimum Variance Portfolio Strategy (Strategy 3)



Dynamic Change for Maximum Sharpe Ratio Portfolio Strategy (Strategy 4)



Compared to minimum variance portfolio strategy and maximum Sharpe ratio portfolio strategy, the Robust mean-variance portfolio strategy did reduce the trading. As can see from the graphs, for minimum variance portfolio strategy and maximum Sharpe ratio portfolio strategy, in several periods the weight of one of the 20 assets approached to 1. But for Robust mean-variance portfolio strategy, this situation did not happen except for the initial portfolio.

Compare Trading Strategies and discuss performance

Among the three new strategies, "Leveraged equal risk contributions" portfolio strategy has the highest portfolio values in Figure 1. "Equal risk contributions" portfolio strategy and "Robust mean-variance optimization" portfolio strategy share a similar trend, but "Equal risk contributions" portfolio strategy has higher value than "Robust mean-variance optimization" portfolio strategy.

If compare all seven strategies together, from the figure 1, “Maximum Sharpe ratio” portfolio strategy produces the best results in portfolio re-balancing. It outperforms all three new strategies from day 100 to the end. Therefore, I will select “Maximum Sharpe ratio” portfolio strategy to manage my own portfolio.

3 Test Trading Strategies for Years 2008 and 2009

Output for 12 periods 2008-2009

Initial portfolio value = $ 385097.15

Period 1: start date 01/02/2008, end date 02/29/2008

Strategy "Buy and Hold", value begin = $ 385097.15, value end = $ 325918.34

Strategy "Equally Weighted Portfolio", value begin = $ 381647.56, value end = $ 326844.46

Strategy "Mininum Variance Portfolio", value begin = $ 383262.06, value end = $ 327184.08

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 381265.54, value end = $ 332652.59

Strategy "Equal Risk Contributions Portfolio", value begin = $ 381903.45, value end = $ 331094.88

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 377340.51, value end = $ 272160.19

Strategy "Robust Optimization Portfolio", value begin = $ 382119.97, value end = $ 324691.49

Period 2: start date 03/03/2008, end date 04/30/2008

Strategy "Buy and Hold", value begin = $ 325807.08, value end = $ 349997.20

Strategy "Equally Weighted Portfolio", value begin = $ 321976.31, value end = $ 354688.12

Strategy "Mininum Variance Portfolio", value begin = $ 322739.67, value end = $ 365796.04

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 325785.77, value end = $ 344234.61

Strategy "Equal Risk Contributions Portfolio", value begin = $ 326429.75, value end = $ 362023.59

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 259617.08, value end = $ 332973.94

Strategy "Robust Optimization Portfolio", value begin = $ 323118.80, value end = $ 347591.67

Period 3: start date 05/01/2008, end date 06/30/2008

Strategy "Buy and Hold", value begin = $ 357929.49, value end = $ 322881.56

Strategy "Equally Weighted Portfolio", value begin = $ 366286.80, value end = $ 308856.09

Strategy "Mininum Variance Portfolio", value begin = $ 373203.13, value end = $ 351565.73

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 349025.01, value end = $ 312522.62

Strategy "Equal Risk Contributions Portfolio", value begin = $ 372432.09, value end = $ 324778.62

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 351547.28, value end = $ 253757.33

Strategy "Robust Optimization Portfolio", value begin = $ 356132.96, value end = $ 327792.25

Period 4: start date 07/01/2008, end date 08/29/2008

Strategy "Buy and Hold", value begin = $ 324349.75, value end = $ 326489.53

Strategy "Equally Weighted Portfolio", value begin = $ 309311.08, value end = $ 315781.58

Strategy "Mininum Variance Portfolio", value begin = $ 351908.45, value end = $ 356330.19

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 324819.19, value end = $ 314804.81

Strategy "Equal Risk Contributions Portfolio", value begin = $ 324434.81, value end = $ 328429.91

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 250162.73, value end = $ 258282.68

Strategy "Robust Optimization Portfolio", value begin = $ 327525.41, value end = $ 331431.79

Period 5: start date 09/02/2008, end date 10/31/2008

Strategy "Buy and Hold", value begin = $ 333252.73, value end = $ 274022.75

Strategy "Equally Weighted Portfolio", value begin = $ 316557.88, value end = $ 231328.43

Strategy "Mininum Variance Portfolio", value begin = $ 348427.44, value end = $ 269132.55

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 306071.66, value end = $ 228989.56

Strategy "Equal Risk Contributions Portfolio", value begin = $ 328196.32, value end = $ 246982.86

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 254926.81, value end = $ 89801.50

Strategy "Robust Optimization Portfolio", value begin = $ 327401.67, value end = $ 255338.26

Period 6: start date 11/03/2008, end date 12/31/2008

Strategy "Buy and Hold", value begin = $ 282342.11, value end = $ 305967.56

Strategy "Equally Weighted Portfolio", value begin = $ 229919.92, value end = $ 198801.74

Strategy "Mininum Variance Portfolio", value begin = $ 269515.24, value end = $ 248309.15

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 226272.66, value end = $ 175168.42

Strategy "Equal Risk Contributions Portfolio", value begin = $ 246166.81, value end = $ 218166.02

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 85254.89, value end = $ 28645.30

Strategy "Robust Optimization Portfolio", value begin = $ 257350.98, value end = $ 247805.42

Period 7: start date 01/02/2009, end date 02/27/2009

Strategy "Buy and Hold", value begin = $ 313366.90, value end = $ 258275.19

Strategy "Equally Weighted Portfolio", value begin = $ 207280.81, value end = $ 169875.32

Strategy "Mininum Variance Portfolio", value begin = $ 256358.59, value end = $ 244247.95

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 174538.91, value end = $ 145312.37

Strategy "Equal Risk Contributions Portfolio", value begin = $ 226737.53, value end = $ 195549.23

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 43079.86, value end = $ -19487.58

Strategy "Robust Optimization Portfolio", value begin = $ 252811.99, value end = $ 225352.16

Period 8: start date 03/02/2009, end date 04/30/2009

Strategy "Buy and Hold", value begin = $ 248688.22, value end = $ 286368.72

Strategy "Equally Weighted Portfolio", value begin = $ 161656.31, value end = $ 259972.40

Strategy "Mininum Variance Portfolio", value begin = $ 234618.58, value end = $ 320243.85

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 138300.37, value end = $ 180257.17

Strategy "Equal Risk Contributions Portfolio", value begin = $ 187938.04, value end = $ 275897.31

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ -37665.32, value end = $ 136032.49

Strategy "Robust Optimization Portfolio", value begin = $ 217548.39, value end = $ 287079.77

Period 9: start date 05/01/2009, end date 06/30/2009

Strategy "Buy and Hold", value begin = $ 287805.37, value end = $ 285824.08

Strategy "Equally Weighted Portfolio", value begin = $ 259540.37, value end = $ 273180.79

Strategy "Mininum Variance Portfolio", value begin = $ 317686.80, value end = $ 321093.58

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 179161.68, value end = $ 184958.88

Strategy "Equal Risk Contributions Portfolio", value begin = $ 275422.53, value end = $ 285660.98

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 132217.01, value end = $ 152312.71

Strategy "Robust Optimization Portfolio", value begin = $ 285677.89, value end = $ 289680.40

Period 10: start date 07/01/2009, end date 08/31/2009

Strategy "Buy and Hold", value begin = $ 286766.63, value end = $ 298338.27

Strategy "Equally Weighted Portfolio", value begin = $ 272870.84, value end = $ 321652.22

Strategy "Mininum Variance Portfolio", value begin = $ 320795.39, value end = $ 342128.17

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 180452.90, value end = $ 195159.21

Strategy "Equal Risk Contributions Portfolio", value begin = $ 285660.33, value end = $ 323318.22

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 149423.76, value end = $ 222953.75

Strategy "Robust Optimization Portfolio", value begin = $ 289586.41, value end = $ 309752.83

Period 11: start date 09/01/2009, end date 10/30/2009

Strategy "Buy and Hold", value begin = $ 291703.36, value end = $ 290193.57

Strategy "Equally Weighted Portfolio", value begin = $ 310079.53, value end = $ 328196.46

Strategy "Mininum Variance Portfolio", value begin = $ 333944.07, value end = $ 350718.05

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 186730.10, value end = $ 186551.00

Strategy "Equal Risk Contributions Portfolio", value begin = $ 313850.46, value end = $ 332132.93

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 201582.56, value end = $ 237174.89

Strategy "Robust Optimization Portfolio", value begin = $ 302571.64, value end = $ 316425.62

Period 12: start date 11/02/2009, end date 12/31/2009

Strategy "Buy and Hold", value begin = $ 288596.05, value end = $ 323101.02

Strategy "Equally Weighted Portfolio", value begin = $ 329547.51, value end = $ 375597.87

Strategy "Mininum Variance Portfolio", value begin = $ 348121.53, value end = $ 390912.51

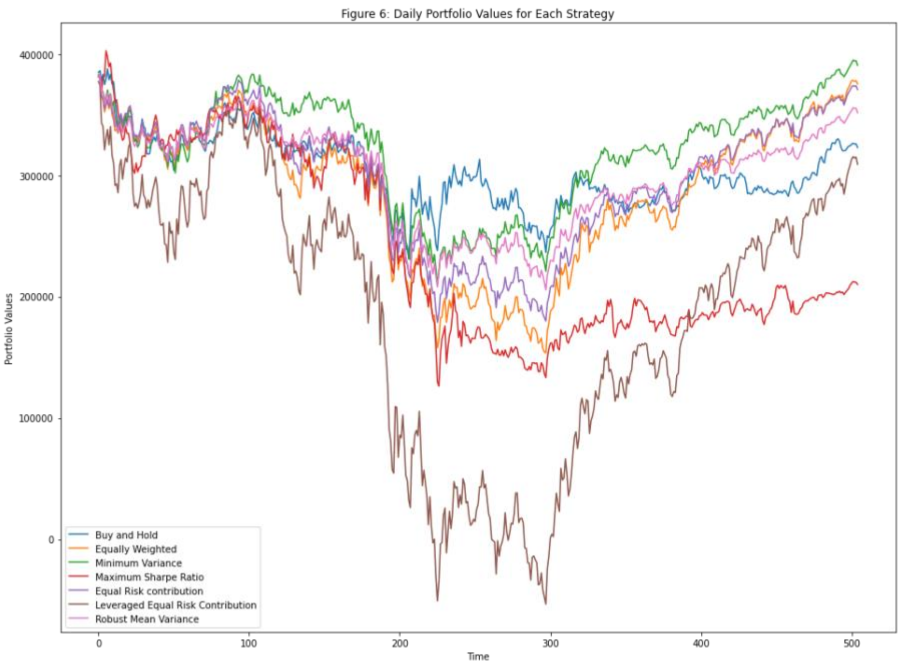
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 185608.02, value end = $ 210148.94

Strategy "Equal Risk Contributions Portfolio", value begin = $ 332762.51, value end = $ 370861.25

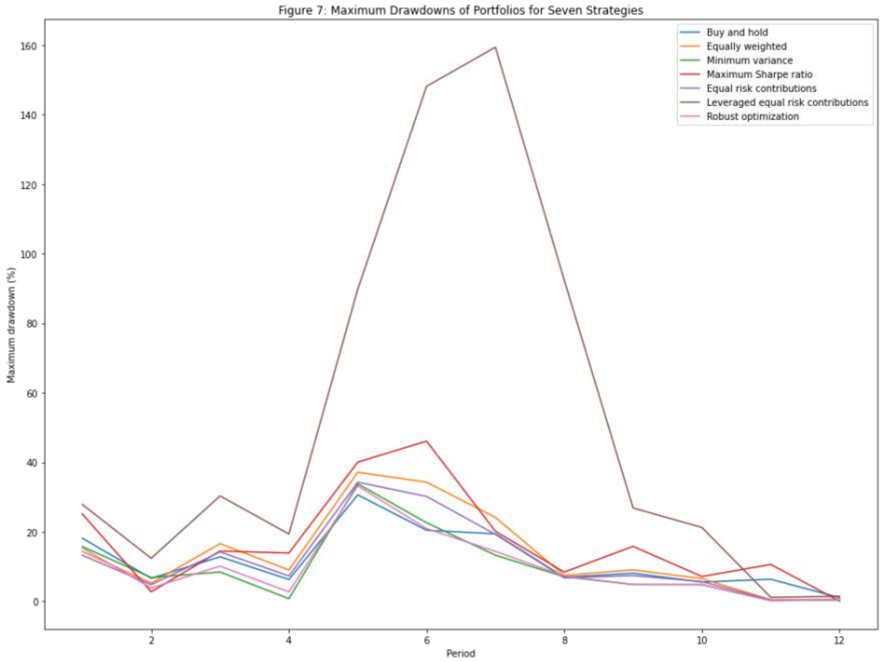
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 235509.65, value end = $ 309192.42

Strategy "Robust Optimization Portfolio", value begin = $ 314654.31, value end = $ 351784.05

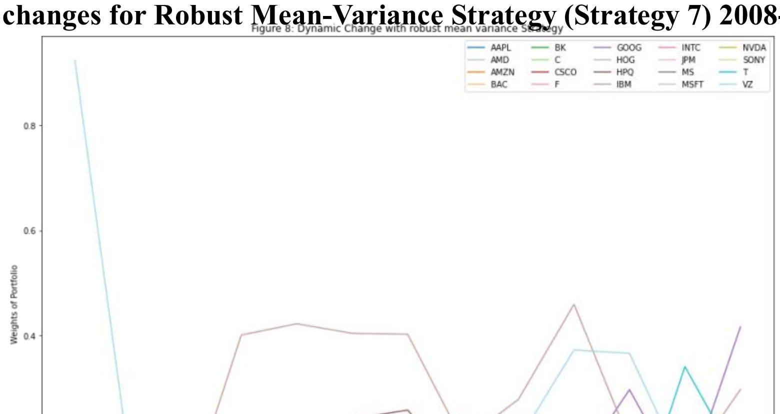
Daily Portfolio Values for Each Strategy 2008-2009



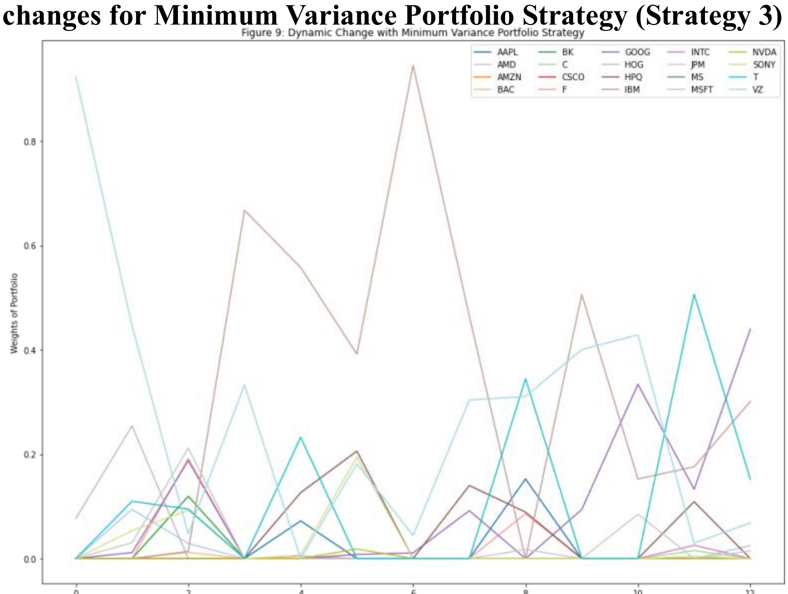
Maximum Drawdowns of Portfolios for Seven Strategies 2008-2009



Dynamic changes for Robust Mean-Variance Strategy (Strategy 7) 2008-2009



Dynamic changes for Minimum Variance Portfolio Strategy (Strategy 3) 2008-2009



Dynamic changes for Maximum Sharpe Ratio Portfolio Strategy (Strategy 4) 2008-2009

Compared to minimum variance portfolio strategy and maximum Sharpe ratio portfolio strategy, the Robust mean-variance portfolio strategy did reduce the trading for data in 2008-2009. For Robust mean-variance portfolio strategy, the weights of assets are all below 50% from period 1 to 12.

Compare Trading Strategies and discuss performance between 2020-2021 and 2008-2009

From the daily portfolio values figures, the trend in 2020-2021 and the trend in 2008-2009 are very different or say quite opposite. In Figure 1, "Maximum Sharpe Ratio" portfolio strategy and "Leverage Equal Risk Contributions" portfolio strategy are the two strategies with the best performance. Surprisingly, in 2008-2009, "Maximum Sharpe Ratio" portfolio strategy and "Leverage Equal Risk Contributions" portfolio strategy perform the worst, are the last two strategies. In Figure 1, "Minimum Variance" portfolio strategy and "Buy and hold" portfolio strategy give the worst performance with the lowest portfolio values. However, in 2008-2009, "Minimum Variance" portfolio strategy and "Buy and hold" portfolio strategy are quite steady and provide highest portfolio values.

For managing my own portfolio during 2008-2009, I would choose Minimum Variance Portfolio, since the value of this portfolio over 12 periods is the stable. This portfolio has the highest return in last a few periods.