

INVENTORY MODEL

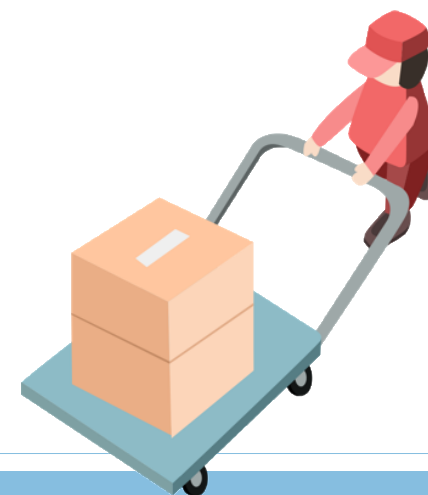
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Part A





Problem 1 Statement

Inventory



Target:

- ✓ Trace of model
(Graphs [10 Days])
 - ✓ Event lists
 - ✓ Demanding
 - ✓ Inventory level
 - ✓ Revenue
 - ✓ Profit
- ✓ Whether make profit in a month? & Average Net Profit

- ✓ Customer Appearance: Poisson Process with rate 8
- ✓ Demanding: Discrete Random Variable ($D=1,2,3,4$)
- ✓ If inventory level $<$ lower limit \rightarrow Replenishment

Delivery: 2h





Nomenclatures



Time & Counter

L: Delivery time

t: Current time

T: Limit time

d: Counter of delivery

i: Counter of events

Event_type: Type of events



Quantity

s: Minimum inventory level

S: Maximum inventory level

y: Quantity of goods ordered but not arrived

w: Sales volume



Cost & Loss

C: Cost price **c**: Cost price/unit

H: Storing cost **h**: Store price/unit

Dv: Delivery cost

Loss: What should have been earned is not earned



Events list

t_0 : The arrival time of one customer

t_1 : The arrival time of product



Target

D: Demanding/ customer

x: Current inventory level

R: Revenue **r**: sell price/unit

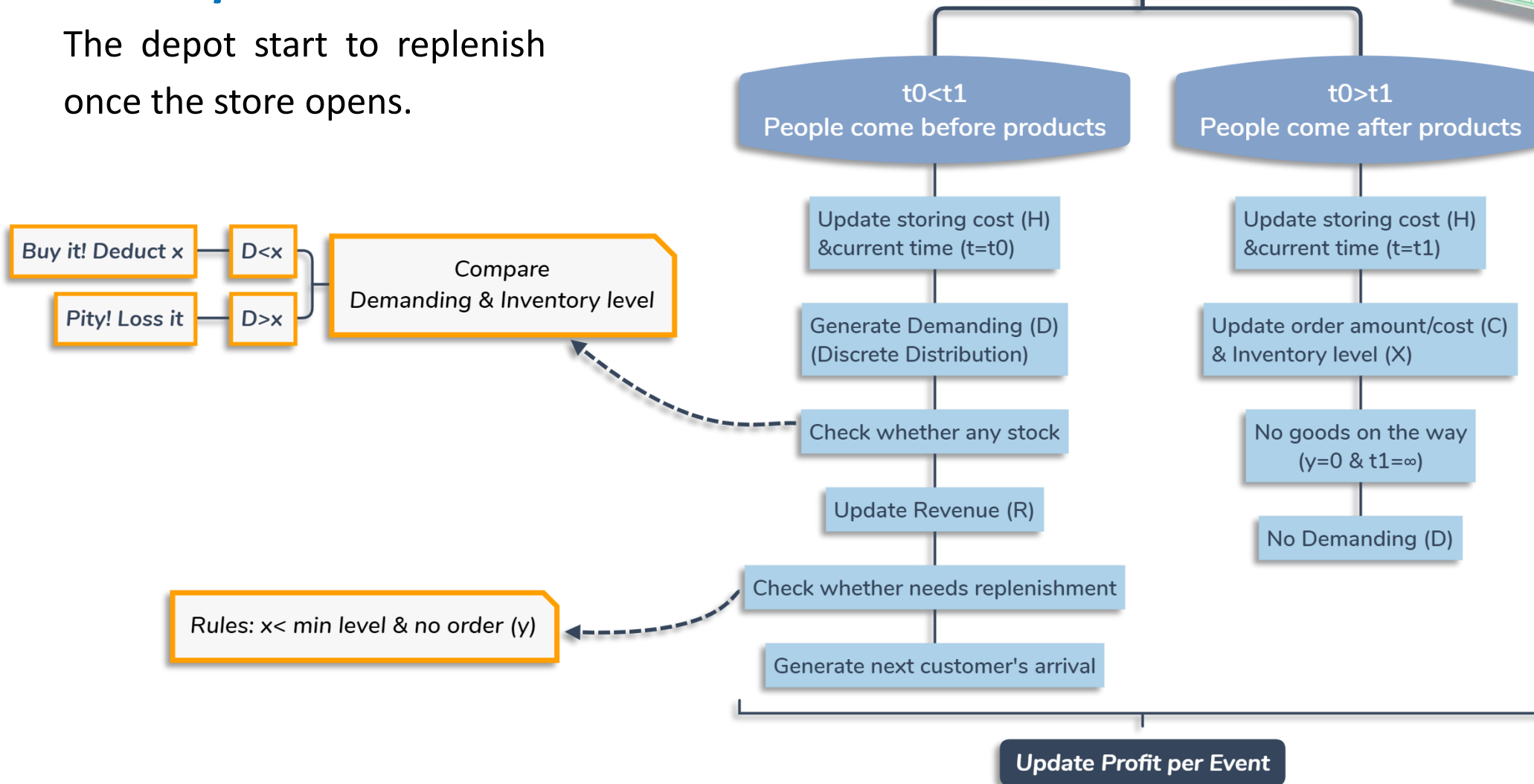
Profit: Net Profit



Algorithm

Assumption

The depot start to replenish once the store opens.





Matlab Code

1st . Set initial values

```
%Time:
T=30;                %limit time
t=[];t(1)=0;t(2)=0;  %current time
L=2/24;              %Delivery time
t0=-(1/8)*log(rand(1)); %The arrival time of one customer(initial value)
t1=t0+2/24;          %The arrival time of product(initial value)
```

```
%Quantity:
x=[];x(1)=0;x(2)=0;  %Current inventory level
D=[];D(1)=0;         %Demanding/ customer
s=10;S=120;          %Minimum & Maximum inventory level
y=0;                 %Quantity of goods ordered but not arrived
```

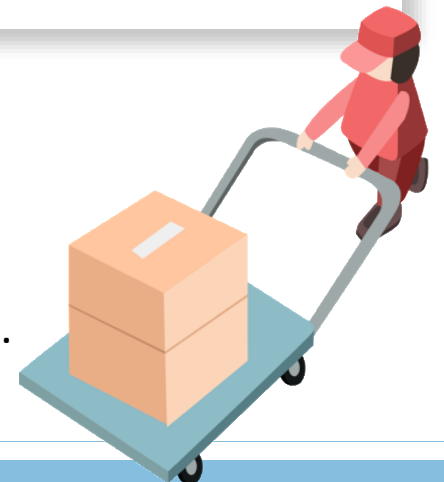
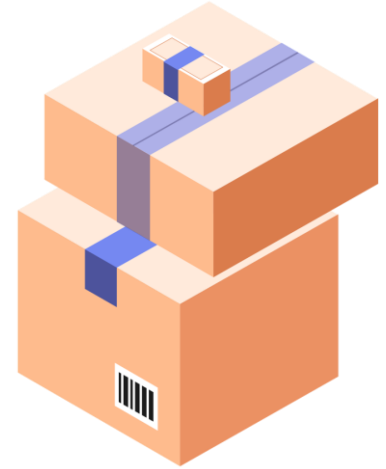
```
%Cost price
r=12;                %sell price/unit
h=0.5;               %store price/unit
c=5;                 %cost price/unit
Dv=10;               %Delivery cost/ride
```

```
%Cost series
H=[];H(1)=0;         %Storing cost
C=[];C(1)=0;         %Cost price
Loss=[];              %Money that should have been earned is not earned
R=[];R(1)=0;         %Revenue
Profit=[];Profit(1)=0; %Net Profit
```

```
%Counters
i=2;                  %Counter of event
d=0;                  %Counter of delivery
```

Assumption

The store start to replenish once the first customer comes.

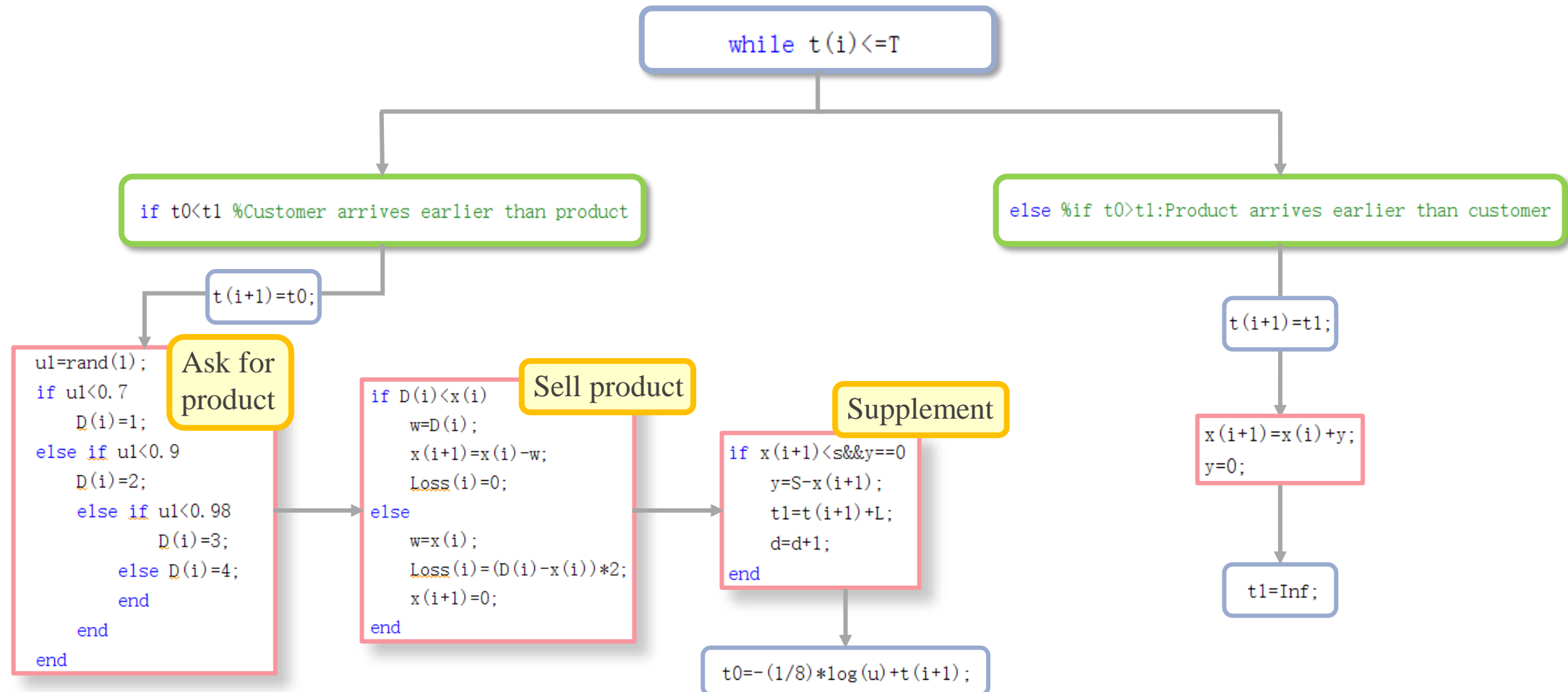




Matlab Code



2nd. Build the structure





Matlab Code

```
if t0<t1 %Customer arrives earlier than product
```

```
t(i+1)=t0;
```

```
ul=rand(1);  
if ul<0.7  
    D(i)=1;  
else if ul<0.9  
    D(i)=2;  
else if ul<0.98  
    D(i)=3;  
else D(i)=4;  
end  
end  
end
```

- **Generate demanding:**
discrete random variable

| | | | | |
|---|-----|-----|------|------|
| N | 1 | 2 | 3 | 4 |
| p | 0.7 | 0.2 | 0.08 | 0.02 |

```
if D(i)<x(i)  
    w=D(i);  
    x(i+1)=x(i)-w;  
    Loss(i)=0;  
else  
    w=x(i);  
    Loss(i)=(D(i)-x(i))*2;  
    x(i+1)=0;  
end
```

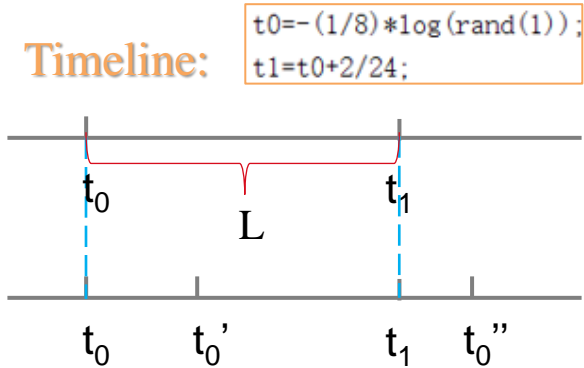
- **Compare demanding with current inventory:**
update state of inventory
calculate the loss

```
if x(i+1)<s&&y==0  
    y=S-x(i+1);  
    t1=t(i+1)+L;  
    d=d+1;  
end
```

Check if it is time to supplement:
define ordered quantity y
define the arrival time of order t_1
count deliver frequency d

```
u=rand(1);  
t0=-(1/8)*log(u)+t(i+1);
```

- **Generate new t_0**





Matlab Code

```
else %if  $t_0 > t_1$ : Product arrives earlier than customer
```

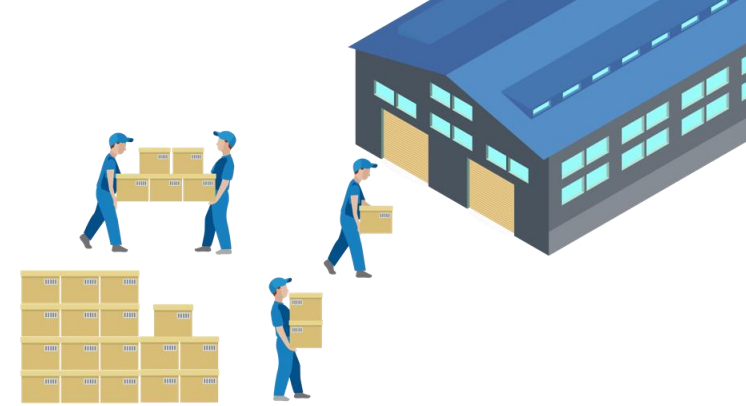
```
 $t(i+1) = t_1;$ 
```

```
 $x(i+1) = x(i) + y;$   
 $y = 0;$ 
```

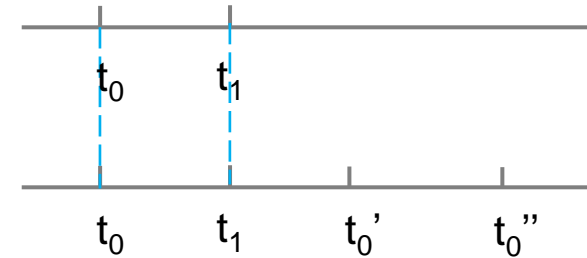
```
 $t_1 = \text{Inf};$ 
```

- Already supplement the products
- No waiting order

- Ensure that next time is $t_0 < t_1$



Timeline:





Matlab Code

3rd. Calculate the money



```
if t0<t1 %Customer arrives earlier than product
```

```
t(i+1)=t0;
```

```
u1=rand(1);  
if u1<0.7  
    D(i)=1;  
else if u1<0.9  
    D(i)=2;  
else if u1<0.98  
    D(i)=3;  
else D(i)=4;  
end  
end
```

```
if D(i)<x(i)  
    w=D(i);  
    x(i+1)=x(i)-w;  
    Loss(i)=0;  
else  
    w=x(i);  
    Loss(i)=(D(i)-x(i))*2;  
    x(i+1)=0;  
end
```

```
R(i)=R(i-1)+w*r-Loss(i);
```

```
if x(i+1)<s&& y==0  
    y=S-x(i+1);  
    t1=t(i+1)+L;  
    d=d+1;  
end
```

```
C(i)=C(i-1);
```

```
t0=-(1/8)*log(u)+t(i+1);
```

```
Profit(i)=R(i)-C(i)-H(i)-d*Dv;
```

```
else %if t0>t1:Product arrives earlier than customer
```

```
t(i+1)=t1;
```

```
x(i+1)=x(i)+y;  
y=0;
```

```
H(i)=H(i-1)+(t1-t(i))*x(i-1)*h;  
R(i)=R(i-1);  
C(i)=C(i-1)+c*y;
```

```
t1=Inf;
```



02

Result of Part A



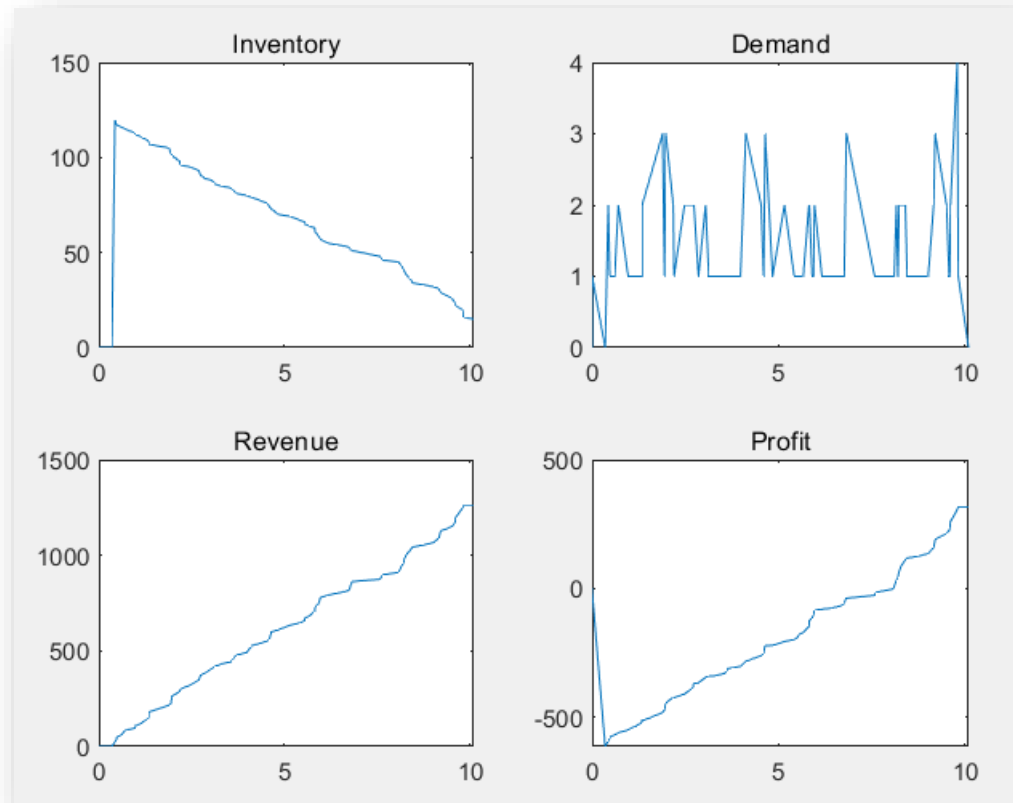


Matlab Code



Output

10 days:



| | 1 Time | 2 Demand | 3 Inventory_Level | 4 Revenue | 5 Profit | 6 Loss | 7 Event_type |
|----|-----------|-------------|----------------------|--------------|-------------|-----------|-----------------|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | t0 |
| 2 | 0 | 1 | 0 | -2 | -12 | 2 | t0 |
| 3 | 0.0098 | 0 | 0 | -2 | -612 | 0 | t1 |
| 4 | 0.0931 | 1 | 120 | 10 | -602.2688 | 0 | t0 |
| 5 | 0.1309 | 2 | 119 | 34 | -580.3434 | 0 | t0 |
| 6 | 0.1658 | 3 | 117 | 70 | -551.3949 | 0 | t0 |
| 7 | 0.2863 | 2 | 114 | 94 | -539.1152 | 0 | t0 |
| 8 | 0.4920 | 1 | 112 | 106 | -536.5940 | 0 | t0 |
| 9 | 0.6612 | 1 | 111 | 118 | -564.2773 | 0 | t0 |
| 10 | 1.3762 | 1 | 110 | 130 | -559.5936 | 0 | t0 |
| 11 | 1.5093 | 1 | 109 | 142 | -548.0720 | 0 | t0 |
| 12 | 1.5180 | 2 | 108 | 166 | -533.1491 | 0 | t0 |
| 13 | 1.6861 | 1 | 106 | 178 | -548.0477 | 0 | t0 |
| 14 | 2.1937 | 1 | 105 | 190 | -538.2322 | 0 | t0 |
| 15 | 2.2353 | 1 | 104 | 202 | -528.3945 | 0 | t0 |
| 16 | 2.2768 | 1 | 103 | 214 | -522.0897 | 0 | t0 |
| 17 | 2.3874 | 1 | 102 | 226 | -522.1154 | 0 | t0 |
| 18 | 2.6232 | 1 | 101 | 238 | -512.9977 | 0 | t0 |
| 19 | 2.6803 | 1 | 100 | 250 | -503.9436 | 0 | t0 |
| 20 | 2.7392 | 1 | 99 | 262 | -497.6589 | 0 | t0 |
| 21 | 2.8547 | 1 | 98 | 274 | -487.0382 | 0 | t0 |
| 22 | 2.8828 | 3 | 97 | 310 | -456.8662 | 0 | t0 |
| 23 | 3.0030 | 1 | 94 | 322 | -457.1360 | 0 | t0 |
| 24 | 3.2641 | 1 | 93 | 334 | -447.2962 | 0 | t0 |
| 25 | 3.3105 | 1 | 92 | 346 | -435.3160 | 0 | t0 |

Net Profit = 244.635283

Average Net Profit = 24.703882

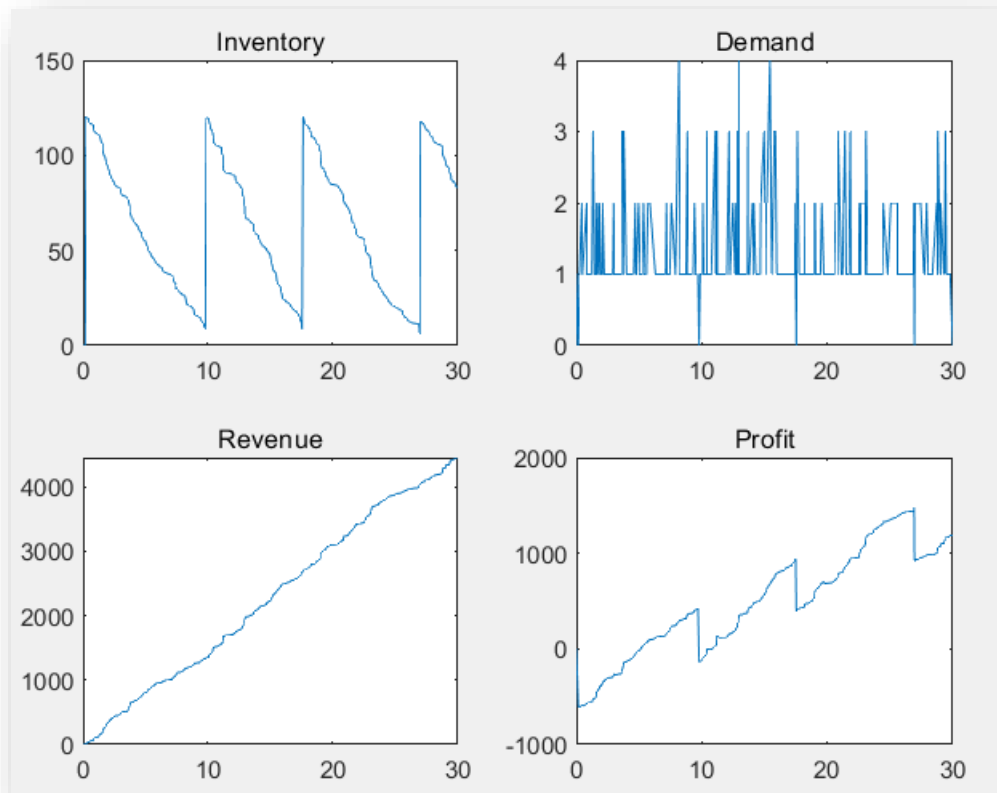


Matlab Code



Output

30 days:



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----|--------|--------|-----------------|---------|-----------|------|------------|
| | Time | Demand | Inventory_Level | Revenue | Profit | Loss | Event_type |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 t0 |
| 2 | 0 | 1 | 0 | -2 | -12 | 2 | 0 t0 |
| 3 | 0.1818 | 0 | 0 | -2 | -612 | 0 | t1 |
| 4 | 0.2651 | 1 | 120 | 10 | -609.5232 | 0 | t0 |
| 5 | 0.4238 | 1 | 119 | 22 | -604.2679 | 0 | t0 |
| 6 | 0.5372 | 1 | 118 | 34 | -599.7146 | 0 | t0 |
| 7 | 0.6634 | 1 | 117 | 46 | -591.3295 | 0 | t0 |
| 8 | 0.7252 | 1 | 116 | 58 | -583.2704 | 0 | t0 |
| 9 | 0.7931 | 2 | 115 | 82 | -561.3718 | 0 | t0 |
| 10 | 0.8297 | 1 | 113 | 94 | -564.3973 | 0 | t0 |
| 11 | 1.0956 | 1 | 112 | 106 | -565.8978 | 0 | t0 |
| 12 | 1.3367 | 1 | 111 | 118 | -558.5194 | 0 | t0 |
| 13 | 1.4200 | 1 | 110 | 130 | -548.7276 | 0 | t0 |
| 14 | 1.4601 | 2 | 109 | 154 | -536.5920 | 0 | t0 |
| 15 | 1.6778 | 1 | 107 | 166 | -525.8388 | 0 | t0 |
| 16 | 1.7011 | 1 | 106 | 178 | -518.5983 | 0 | t0 |
| 17 | 1.7909 | 3 | 105 | 214 | -493.3095 | 0 | t0 |
| 18 | 1.9949 | 3 | 102 | 250 | -465.3618 | 0 | t0 |
| 19 | 2.1528 | 1 | 99 | 262 | -455.7193 | 0 | t0 |
| 20 | 2.2005 | 2 | 98 | 286 | -435.3952 | 0 | t0 |
| 21 | 2.2755 | 1 | 96 | 298 | -432.1305 | 0 | t0 |
| 22 | 2.4575 | 1 | 95 | 310 | -426.3060 | 0 | t0 |
| 23 | 2.5875 | 3 | 94 | 346 | -395.9776 | 0 | t0 |
| 24 | 2.7081 | 1 | 91 | 358 | -386.7507 | 0 | t0 |
| 25 | 2.7691 | 1 | 90 | 370 | -379.2459 | 0 | t0 |
| 26 | 2.8690 | 2 | 89 | 394 | -360.2402 | 0 | t0 |

Net Profit = 1267.126943

Average Net Profit = 42.359962



03

Part B

A more complicated inventory system

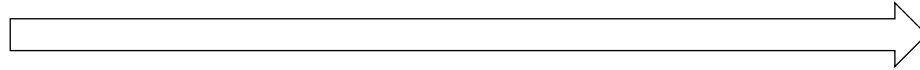
Calcium tablet





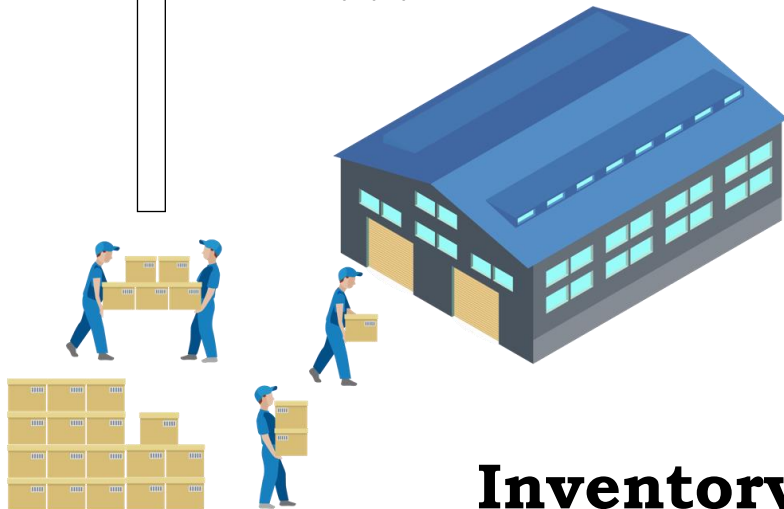
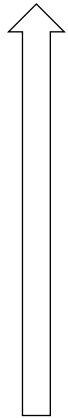
Depot

Cost \$\$\$

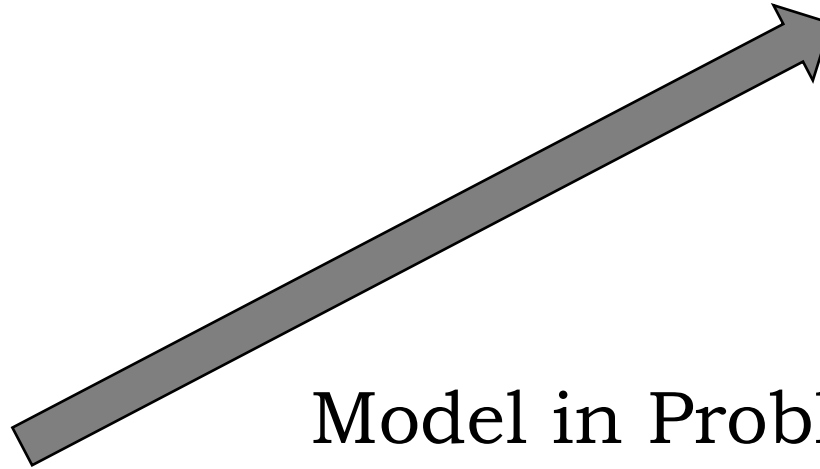


Store

Cost \$\$\$



Inventory



Model in Problem I



Idea introduction



Depot

Ordering policy

$s_2=50$ unit

$S_2=300$ unit

frequently orders, small amount

Delivery cost $K_1=5$ ¥

Delivery time $L_1=0.1$ Day

Ordering policy

$s_1 = 20$ unit

$S_1 = 100$ unit



Store

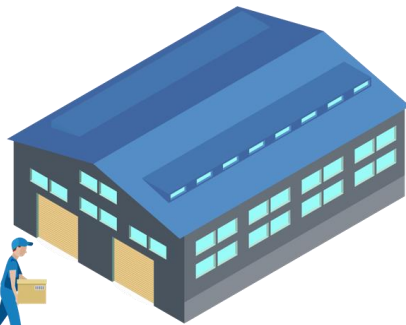


Delivery cost $K_2=5$ ¥

Delivery time $L_2=2$ Day

Holding cost per item per unit time : $h_2=0.5$ ¥

Not frequently orders, large amount



Inventory

Demand: D

Sell price per unit : $r = 200$ ¥

Cost price per unit : $c=50$ ¥

Loss due to shortage : $L (D, x)$

Holding cost per item per unit time : $h_1=2$ ¥

Idea Explanation



Time Period
30 Days

Case 1: ($X_2 < s_2$)
✗ Remaining Amount of Products at
Depot < low limit line of Depot

Case 2: ($X_2 > s_2$)
✓ Remaining Amount of Products at
Depot > low limit line of Depot

✗ Case 1: $t(i) < t_{\text{wait}} + L_2$ (Delivery time)
No product in Depot yet

✓ Case 2: $t(i) < t_{\text{wait}} + L_2$ (Delivery time)
Product in Depot

😬 (SAME AS Problem 1)

✓ Case 1: $X_1(i) > d(i)$
Product in Store Satisfy Demand

✗ Case 2: $X_1(i) < d(i)$
Product in Store Not Satisfy Demand

UPDATE

Update

Loss



Assumption

The store can supplement their storage until the depot have sufficient products

Details Explanation

MATLAB Code – Conditions Clarification

```
x1=0; s1=20; S1=100; r=200; L1=0.1; K1=5; h1=2; T=30;x2=0; s2=50; S2=300; L2=2; K2=5;
h2=0.5; c=50;
X1 = [x1]; % Remaining Amount of Products in Store
X2 = [x2]; % Remaining Amount of Products at Depot
d = [0]; % Demand of Each Customer
R = [0]; % Profit
t = []; % current time
t(1)=0;
i=1;
Loss=[];
cost=0;
y = 0; % the quantity of products have been ordered but not yet delivered
j=0;
t0 = -(1/8)*log(rand(1));
t1=t0+L1;
```

While $t < T$

- Case 1: $X_2 < s_2$ (Depot Lacks Products)
- Case 2: $X_2 > s_2$ (Depot doesn't Lack Products)



Case 1: $X2 < s2$ (Depot Lacks Products)

- %Record the time
- `t_wait = t(i);`
- %Case 1.1: If there is a demand from customers when the products heading for depot are still on the go
- `if t(i) < t_wait+L2`
- %Update the current time and storage at depot
 - `t(i+1) = t0;`
 - `X2(i+1)=X2(i);`
- %Customers Demand
 - `d(i) = randsample(4,1,true,[.7,.2,.08,.02]);`
- %Start considering the storage in the store

Case 1.1: Customer Purchases when Products Heading for Depot are Still on the Go

- %Check if the store has enough storage for customers
- if $X1(i) > d(i)$ %If yes
- %Update the storage and the profit, and no need to consider the loss
- $X1(i+1) = X1(i) - d(i);$
- $R(i+1) = R(i) + d(i) * r - (X1(i) * h1 - X2(i) * h2) * (t(i+1) - t0);$
- else %If no
- %Update the storage and the profit, and consider the loss
- $X1(i+1) = 0;$
- $Loss(i) = (d(i) - X1(i))^2;$
- $R(i+1) = R(i) + X1(i) * r - (X1(i) * h1 - X2(i) * h2) * (t(i+1) - t0) - Loss(i);$
- end

Case 1.2: Products Arrive at Depot

```
• %Case 1.2: If the products arrive at depot
• else
• %Update current time
•         t(i+1)=t(i);
•         t1=t(i)+L1;
• %Update the storage at depot and in the store
•         X2(i+1) = S2;
•         X1(i+1)=X1(i);
• %Update the profit
•         R(i+1) = R(i)-K2-c*(S2-X2(i));
• end
```

Case 2: $X2 > s2$ (Depot doesn't Lack Products)

Case 2.1: Customer arrives earlier than products

- 1 Check demand and inventory
- 2 Check if store needs replenishment

```
if X2(i) > s2 % Depot has enough inventory to supply the store
    if t0 < t1
        t(i+1) = t0; %update current time
        d(i) = randsample(4,1,true,[.7,.2,.08,.02]); %demand of customer
        X2(i+1) = X2(i);
        1 if d(i) < X1(i) %inventory level of store > customer demand, no loss
            w = d(i); %number of products sold to the customer
            X1(i+1) = X1(i) - w; %update inventory
            Loss(i) = 0;
        else %store can't provide the demanded products, Loss
            w = X1(i);
            Loss(i) = (d(i) - X1(i)) * 2;
            X1(i+1) = 0;
        end
        R(i+1) = R(i) + w * r - (X1(i) * h1 - X2(i) * h2) * (t(i+1) - t(i)); %profit

        2 if X1(i) < s1 && y == 0
            y = s1 - X1(i); %the quantity of products need to replenish
            X2(i+1) = X2(i) - y; %update depot's inventory level
            t1 = t(i+1) + L1; %arrival time of the replenished products
        end
    end
end
```

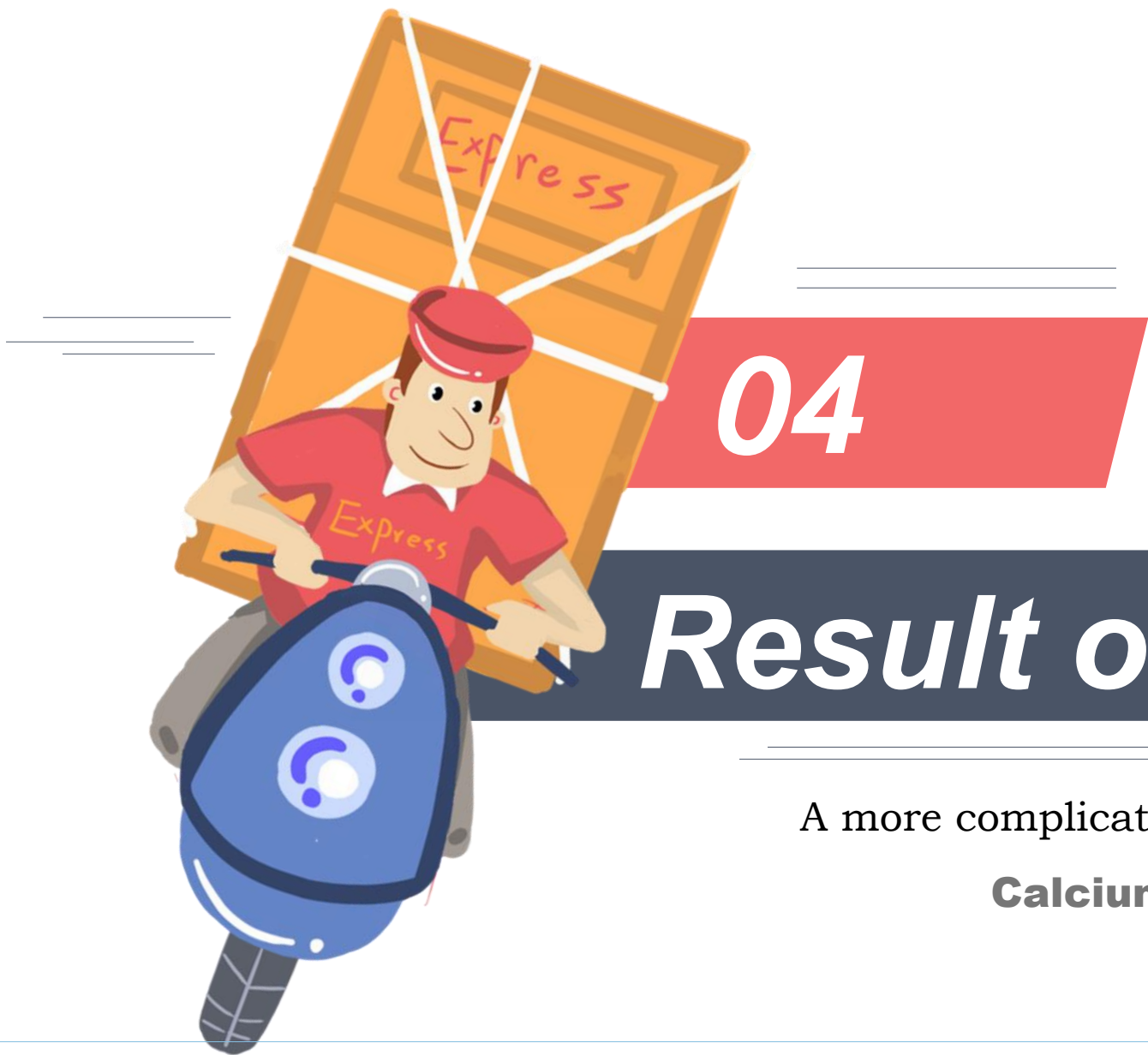

Case 2: $X2 > s2$ (Depot doesn't Lack Products)

Case 2.2: Products arrive earlier than customer

1 Check if store needs replenishment

2 Update inventory and profit

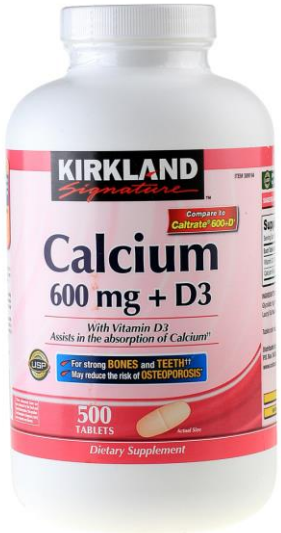
```
else %if products arrive earlier than customers(t0>t1)
    1 if X1(i)<s1 && y==0
        y=S1-X1(i); %the quantity of products need to replenish
        X2(i+1) = X2(i)-y; %update depot's inventory level
    else
        X2(i+1)=X2(i);
    end
    t(i+1)=t0+L1;
    R(i+1)=R(i);
    2 X1(i+1)=S1;
    cost = c*y;
    y=0;
    t1=Inf;
    d(i)=0;
    R(i+1) = R(i) - (X1(i) * h1 - X2(i) * h2)*(t(i+1)-t(i))-K1-cost;
end
```



Result of Part B

A more complicated inventory system

Calcium tablet





Output

T =

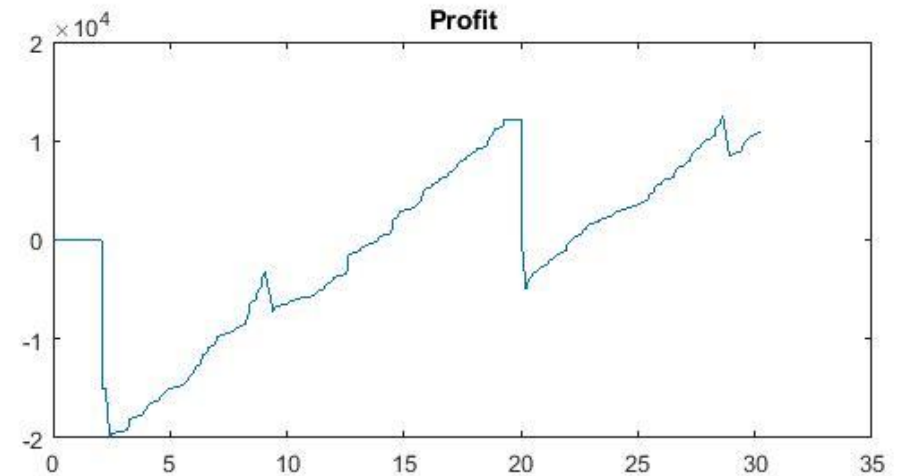
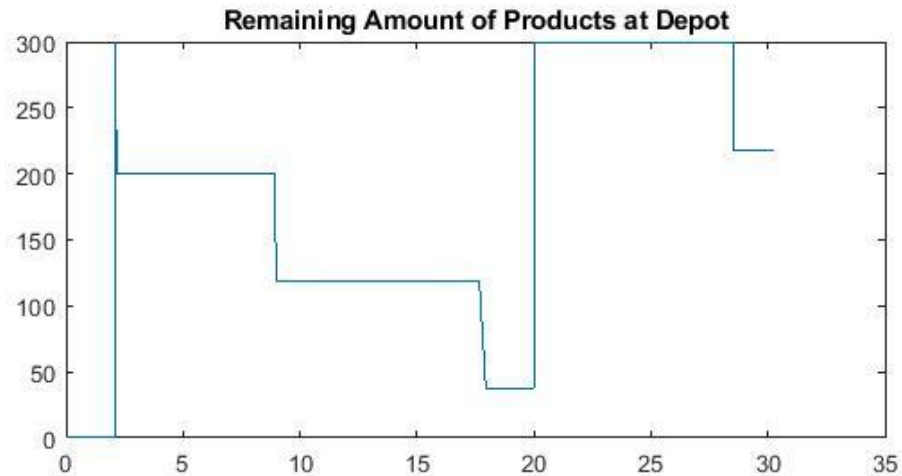
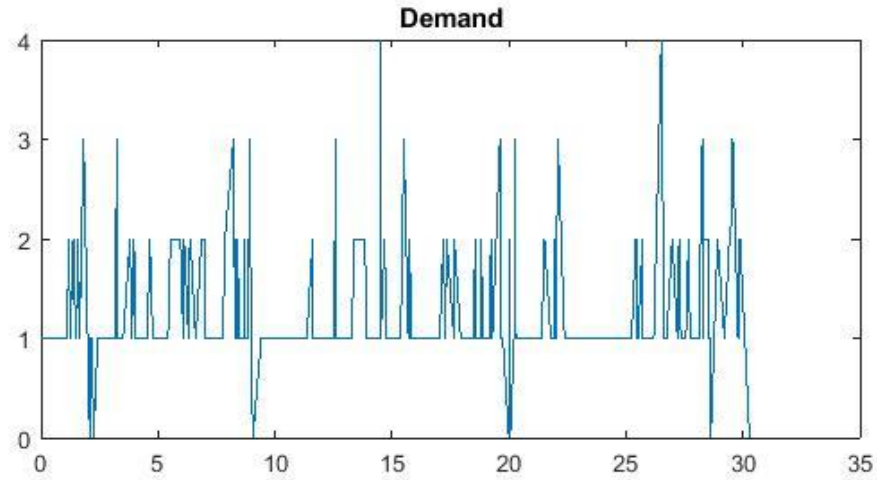
229×7 [table](#)

| Time | Demand | Event_type | Inventory_Store | Inventory_Depot | Profit | Loss |
|---------|--------|------------|-----------------|-----------------|--------|------|
| 0 | 1 | t0 | 0 | 0 | 0 | 2 |
| 0.01072 | 1 | t0 | 0 | 0 | -2 | 2 |
| 0.32687 | 1 | t0 | 0 | 0 | -4 | 2 |
| 0.34989 | 2 | t0 | 0 | 0 | -6 | 4 |
| 0.38097 | 1 | t0 | 0 | 0 | -10 | 2 |
| 0.44672 | 1 | t0 | 0 | 0 | -12 | 2 |
| 0.56321 | 1 | t0 | 0 | 0 | -14 | 2 |
| 0.67075 | 1 | t0 | 0 | 0 | -16 | 2 |
| 0.7582 | 1 | t0 | 0 | 0 | -18 | 2 |
| 0.82961 | 2 | t0 | 0 | 0 | -20 | 4 |
| 0.84669 | 3 | t0 | 0 | 0 | -24 | 6 |
| 0.98218 | 4 | t0 | 0 | 0 | -30 | 8 |
| 0.98797 | 1 | t0 | 0 | 0 | -38 | 2 |
| 1.2188 | 1 | t0 | 0 | 0 | -40 | 2 |
| 1.393 | 2 | t0 | 0 | 0 | -42 | 4 |
| 1.5749 | 1 | t0 | 0 | 0 | -46 | 2 |



Plots for Demand, Storage, and Profit

30 Days





05

Conclusion





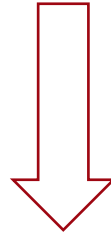
Conclusion



Comparison of the results of both questions

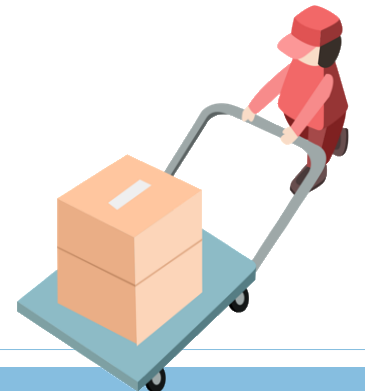
By setting the depot location,

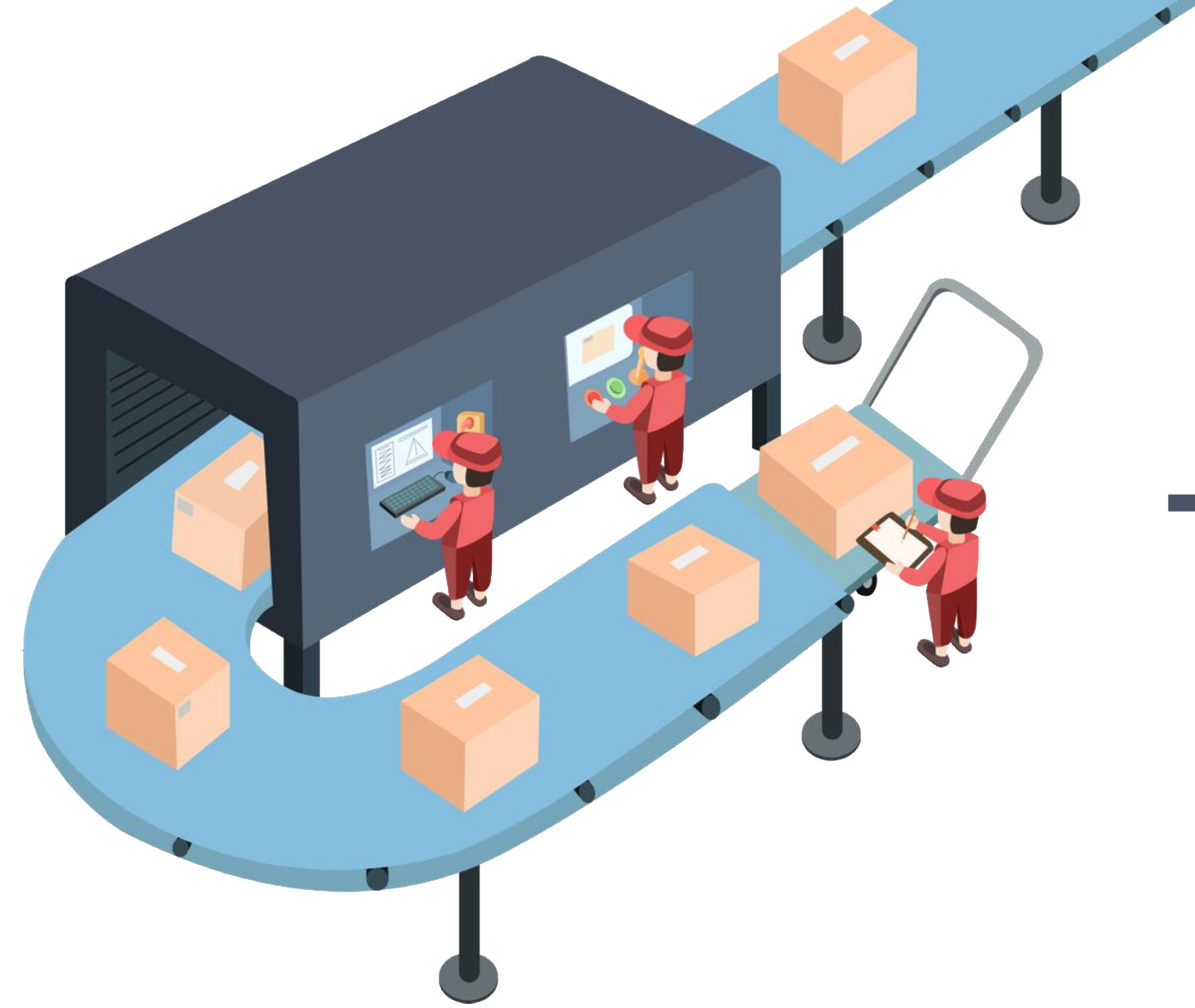
- The holding cost increase significantly
- Have to pay more delivery cost
- More likely to have loss



- Reduce the frequency of replenishment in depot
- Minimize the distance from depot to store and to inventory

Better logistics strategy by simulating the inventory changing process!





Thank you!

