# HW#1: Warm-up (6%)

*CS241 Fall 2020*

## Due: Friday, September 11 at 11:59:59pm

## Group Policy: Individual, no collaboration is allowed.

## Overview

- This assignment aims to refresh your understanding about math concepts essential to data structures and Algorithms. All concepts are already covered in your high school or in your prerequisite courses. In particular, serials and induction covered in the second half of the HW will be reviewed on 09/01.
- You have 8 questions in this assignment. Please complete all of them.
- Please show your solution in a **step-by-step** fashion. Missing steps will result in losing points.

## Submission Instructions

- **Please do not email me your homework. Please enroll into CS241FALL@Gradescope(Entry Code:9B6JYN) to turn in all your assignments. If it is not on Gradescope, then it receives zero.**
- If you prefer to print the document out and write down answers with pen and paper,
    - Please write neatly. Illegible handwriting could lead to significant loss of points.
    - Once you are ready to submit your homework, please scan your homework and save the file as **"YourFullName_CS241Fall2020_HW1.pdf"**
    - Upload your homework to Gradescope.

  or
- If you prefer to do your homework digitally,
    - choose "File"-> "Make a copy" to create a copy of this google document under **your google account**, and write your answers in your own copy because you do not have permission to edit this document in place.
        - If you type your answers in google document, you can use "Insert"->"Equation" to enter mathematical equations. Once you are ready to submit your homework, choose "File" -> "Download" -> "PDF Document" to save your homework locally as a pdf file.
        - If you use a graphic tablet pen or tablet stylus pen to do free-hand writing, please also make sure your answers are legible. Illegible handwriting could lead to significant loss of points.
    - Please try to keep the current formatting of one-question-per-page. There should be plenty of space for you to write your answers. Thanks for your collaboration in helping me with streaming grading!
    - Rename your homework file to "**YourFullName_CS241Fall2020_HW1.pdf**" and upload the file through Gradescope.

1 [15 points]. Without a calculator, which of the following are equal? Why or why not?

i. $5^3$ and $3^5$ Not Equal

ii. $8^2$ and $4^3$ Equal

iii. $16^{lg(4)}$ and $4^{lg(16)}$ Equal.

(i) $5^3 = 5 \cdot 5 \cdot 5 = 125$

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

(ii) $8^2 = 8 \cdot 8 = 64$

$4^3 = 4 \cdot 4 \cdot 4 = 64$

(iii) $16^{\log_2 4} = 2^{(4)(\log_2 4)} = (2^{\log_2 4})^4 = 4^4 = 256$

$4^{\log_2 16} = 2^{(2)(\log_2 16)} = (2^{\log_2 16})^2 = 16^2 = 256$

**2 [20 points].** [Compare the growth rates] You can use a calculator or google spreadsheet to complete this question. Google spreadsheet is recommended.

Given a function $y = nlg(n)$ with the data points as show below,

| $n$ | $y = nlg(n)$ | $y = n^2$ | $y = n$ | $y = lg(n)$ | $y = 2^n$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 2 |
| 2 | 2 | 4 | 2 | 1 | 4 |
| 4 | 8 | 16 | 4 | 2 | 16 |
| 8 | 4 | 64 | 8 | 3 | 256 |
| 16 | 64 | 256 | 16 | 4 | 65536 |
| 32 | 160 | 1024 | 32 | 5 | 4294967296 |
| 64 | 384 | 4096 | 64 | 6 | 1.84467E19 |

A. Complete the above table for all functions.

B. Based on the table from question A), calculate the growth rates for all data points and fill out the table as shown below. How do define calculate growth rate? If you have two data points $f(n) = y$ and $f(n+1) = y'$, then we define the growth rate at $n$ is $y' - y$. A sample growth rate of $y = n^2$ when $n = 4$ has been prefilled for your reference.

| *growth rate* | $y = nlg(n)$ | $y = n^2$ | $y = n$ | $y = lg(n)$ | $y = 2^n$ |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 1 | 2 |
| 2 | 2.754888 | 5 | 1 | 0.584962 | 4 |
| 4 | 3.609640 | 9 | 1 | 0.321928 | 16 |
| 8 | 4.529325 | 17 | 1 | 0.169925 | 256 |
| 16 | 5.486868 | 33 | 1 | 0.087463 | 65536 |
| 32 | 6.465006 | 65 | 1 | 0.044394 | 4294967296 |
| 64 | 7.453908 | 129 | 1 | 0.022368 | 1.84467E19 |

C. Based on the table from B), sort the order of growth of all functions in table B) from fastest to slowest.

$y=2^n$, $y=n^2$, $y=nlg(n)$, $y=n$, $y=lg(n)$.

**3 [10 points].**

A. For the following simple loop in Java, how many * are printed?

```
for(int i=0; i<1000; i++)
    for(int j=0; j< i; j++)
        System.out.println("*");
```

$\Rightarrow 1000$

Adding from 1 to 999

$i = 0 \Rightarrow 0$
$i = 1 \Rightarrow 1$
$i = 2 \Rightarrow 2$
$i = 3 \Rightarrow 3$
$\vdots$
$i = 999 \Rightarrow 999$
$i = 1000$ X

$$\frac{999(1+999)}{2}$$

$$= \frac{999(1000)}{2}$$

$$= 499{,}500$$

B. For the following simple loop in Java, how many * are printed?

```
for(int i=0; i<1000; i++)
    for(int j=0; j< i; j=j*2)
        System.out.println("*");
```

The printed of * is infinity.

when $i=1$, $j=0$, $0 \times 2 = 0$ it was always equal to zero and always less than one. It means * will printed infinity times.

**4 [15 points].** Without a calculator:

i. approximately how many megabytes (MB) is $2^{24}$ bytes,

$1 MB = 2^{20}$ bytes

$$\frac{2^{24}}{2^{20}} = 2^4 = \underline{16\ MB}$$

ii. how many gibibytes (GiB) is $2^{33}$ bytes,

$1 GB = 2^{30}$ bytes

$$\frac{2^{33}}{2^{30}} = 2^3 = 8\ GB.$$

iii. approximately how many kibibytes (KiB) is 4000 bytes?

$1 KB = 2^{10}$ bytes. $= 1024$ bytes

$$\frac{4000}{1024} = 3.9\ KB.$$

5 **[10 points].** [Serials] Using the formula we reviewed in class, what is the sum of the first 50 integers? What is the sum of the integers from 51 to 100?

The sum of first 50 integers:

$$\frac{50(1+50)}{2} = 1275$$

The Sum of the integers from 51 to 100:

$$\frac{50(51+100)}{2} = 3775$$

**6 [10 points].** Quickly approximate the following using formulas introduced in class:

i. $\sum_{k=0}^{30} k^2$

$$\sum_{k=0}^{n} k^d = \frac{n^{d+1}}{d+1}$$

$$\sum_{k=0}^{30} k^2 = \frac{30^{2+1}}{2+1} = \frac{30^3}{3} = \frac{27000}{3} = \boxed{9000}$$

ii. $\sum_{k=0}^{100} k^3$

$$\sum_{k=0}^{n} k^d = \frac{n^{d+1}}{d+1}$$

$$\sum_{k=0}^{100} k^3 = \frac{100^{3+1}}{3+1} = \frac{100^4}{4} = \frac{100{,}000{,}000}{4} = \boxed{25{,}000{,}000}$$

7 **[10 points]**. Approximately, what is the sum $\sum_{k=1}^{30} \frac{1}{2^k}$ ?

$$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

$$\sum_{k=1}^{30} \left(\tfrac{1}{2}\right)^k \qquad r=\tfrac{1}{2} \qquad n=29$$

$$= \left(\tfrac{1}{2}\right)\frac{1-\left(\tfrac{1}{2}\right)^{29+1}}{1-\tfrac{1}{2}} = \left(\tfrac{1}{2}\right)\frac{1-\left(\tfrac{1}{2}\right)^{30}}{1/2} = \tfrac{1}{2}\left(\frac{1-0}{1/2}\right) = \tfrac{1}{2}(2) = \boxed{1}.$$

$=$

$= -$

8 [10 points]. Show, using a proof by induction, that $\sum_{k=0}^{N} k2^k = 2 + (N-1)2^{(N+1)}$.

if $k=0$; $0(2)^0 = 0 \cdot 1 = 0$

$$\sum_{k=0}^{N} k2^k = 0 \cdot 2^0 + 1 \cdot 2^1 + 2 \cdot 2^2 \ldots + (N-1)2^{(N-1)} + N2^N$$

Since $n=0$, $\sum_{k=0}^{N} k2^k = 0$, then assume any ~~of~~ value of $N$ that ~~is~~ greater or equal to 0, which is $N = N+1$.

$$\sum_{k=0}^{N} k2^k = \sum_{k=0}^{N} k2^k + (N+1)(2^{N+1})$$

$$= 2 + (N-1)(2^{N+1}) + (N+1) \cdot (2^{N+1})$$

$$= 2 + (N-1)2^{(N+1)}.$$