CSE 569 Homework #1

Total 3 points

Due Tuesday, Sept. 24, 11:59PM.

Notes:

1. Submission of homework must be electronic. Most problems can be solved by hand. You can write down your solutions on paper and then take a picture of your hand-written sheets, and then upload your work onto Canvas. The due date/time will be strictly enforced. Late submissions will not be accepted by the system.
2. **If you have any questions on homework problems, you should post your question on the Canvas discussion board (under the Homework 1 Q & A), instead of sending emails to the instructor or TA. Questions will be answered there to avoid repetition. This also helps the entire class to stay on the same page whenever any clarification/correction is made.**
3. 0.5 point each for Q1 through Q6 (with a total of 3 points). Q7 is optional, and no point will be given.

Q1. (Problem 14 from the textbook; also refer to Problem 13, which we discussed during the class)

Consider the classification problem with rejection option.

(a) Use the results of Problem 13 to show that the following discriminant functions are optimal for such problems:

$$g_i(\mathrm{x}) = \begin{cases} p(\mathrm{x}|\omega_i)P(\omega_i) & i = 1, ..., c \\ \frac{\lambda_s - \lambda_r}{\lambda_s} \sum_{j=1}^{c} p(\mathrm{x}|\omega_j)P(\omega_j) & i = c+1. \end{cases}$$

(b) Plot these discriminant functions and the decision regions for the two-category one-dimensional case having

- $p(x|\omega_1) \sim N(1, 1)$,
- $p(x|\omega_2) \sim N(-1, 1)$,
- $P(\omega_1) = P(\omega_2) = 1/2$, and
- $\lambda_r/\lambda_s = 1/4$.

(c) Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1.

Q2. Consider a 1-dimensional 2-class classification problem with class-conditionals as follows:

$$p(x|\omega_1) = \begin{cases} 1, & x \in [0,\ 1] \\ 0, & otherwise \end{cases}, \qquad p(x|\omega_2) = \begin{cases} 2x, & x \in [0,\ 1] \\ 0, & otherwise \end{cases}$$

The priors are given as $P(\omega_1)$= $2/5$, $P(\omega_2)$= $3/5$.

(a) Find the decision rule for doing minimum error rate classification. Express the rule in the form of "Decide ω_1 if $x \in R_1$; Otherwise decide ω_2", where R_1 is to be found by you.

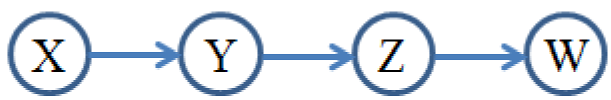
(b) Find the error rate in (a). R1 = 1/3

Q3. Consider a 1-dimensional, two-category classification problem, with prior probabilities $P(\omega_1) = 1/3$ and $P(\omega_2) = 2/3$. The class-conditional PDFs for the two classes are the normal densities $N(\mu_1,\sigma^2)$ and $N(\mu_2,\sigma^2)$, respectively. Note that these two PDFs have the same variance σ^2. $N(\mu, \sigma^2)$ denotes the normal density defined by

$$p(x|\mu) = \frac{1}{\sqrt{2\pi}\,\sigma} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

We further assume that the losses $\lambda_{11} = \lambda_{22} = 0$, but λ_{12} and λ_{21} are some positive values.
Find the optimal decision rule for classifying any feature point x.
[You need to present your rule in the form of "Deciding on ω_1 if $x \in R_1$; otherwise Deciding on ω_2", where R_1 needs to be explicitly defined in terms of the given parameters μ_1, μ_2, σ, λ_{12} and λ_{21}.]

Q4. Consider the following simple Bayesian network, where all the nodes are assumed to be binary random variables, i.e., X=x_0 or x_1 with certain probabilities, and similar notations will be used for Y, Z, and W.



This Bayesian network is fully specified if we are given the following (conditional) probabilities:
(for notational simplicity, we write $P(x_1)$ to mean $P(X=x_1)$, and so on)

$P(x_1) = 0.60$;
$P(y_1 \mid x_1) = 0.40$, $P(y_1 \mid x_0) = 0.30$;
$P(z_1 \mid y_1) = 0.25$, $P(z_1 \mid y_0) = 0.60$;
$P(w_1 \mid z_1) = 0.45$, $P(w_1 \mid z_0) = 0.30$;

(a) Suppose that X is measured and its value is x_1, compute the probability that we will obverse W having a value w_0, i.e., $P(w_0 \mid x_1)$.

(b) Suppose that W is measured and its value is w_1, compute the probability that we will obverse X having a value x_0, i.e., $P(x_0 \mid w_1)$. p(x0|w1) = p(w1|x0)*p(x0)/p(w1)

Q5. True-or-False: For a two-class classification problem using the minimum-error-rate rule, in general the decision boundary can take any form. However, if the underlying class-conditionals are Gaussian densities, then the decision boundary is linear (hyperplanes).

[ ] True [F] False

Brief explanation of your answer:

$\mu_1 < \mu_2$ $\quad \int_{-\infty}^{\frac{\mu_1+\mu_2}{2}} p(x|w_2)\,dx$

Q6. (From the textbook)

Let $p(x|\omega_i) \sim N(\mu_i, \sigma^2)$ for a two-category one-dimensional problem with $P(\omega_1) = P(\omega_2) = 1/2$.

(a) Show that the minimum probability of error is given by

$$P_e = \frac{1}{\sqrt{2\pi}} \int_a^\infty e^{-u^2/2}\, du,$$

where $a = |\mu_2 - \mu_1|/(2\sigma)$.

(b) Use the inequality

$$P_e = \frac{1}{\sqrt{2\pi}} \int_a^\infty e^{-t^2/2}\, dt \leq \frac{1}{\sqrt{2\pi}a} e^{-a^2/2}$$

to show that P_e goes to zero as $|\mu_2 - \mu_1|/\sigma$ goes to infinity.

(Optional) Q7. Part A. Consider the following *game*: Someone shows you three hats and tells you that the there is a prize in one of them. He asks you to choose one of the hats. You choose one hat and tell him which one you chose. He then lifts one of the hats you didn't choose and there is nothing under that hat. He then tells you that you can either stay with the hat you have originally chosen or switch to the other remaining hat. What should you do? Explain your answer.

Part B. (*Use this to help ensure your Part A is correct*). Design a computer-based experiment (i.e., write a computer program) to simulate the above game to verify your answer, by playing the game many times to obtain an averaged performance.