WTS $\exp\left[-\frac{1}{2}\left(\left(\sum_{i=1}^{n}\phi(x_i-\theta)^2\right)+\tau(\theta-\theta_0)^2\right)\right]$ ①

$\propto \exp\left(-\frac{1}{2}(\tau+n\phi)\left(\theta-\frac{1}{\tau+n\phi}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)^2\right)$ as a function of θ ②

① $= \exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n}\phi(x_i^2-2x_i\theta+\theta^2)+\tau(\theta^2-2\theta\theta_0+\theta_0^2)\right)\right]$

$= \exp\left[-\frac{1}{2}\left(\phi\sum_{i=1}^{n}x_i^2-2\phi\theta\sum_{i=1}^{n}x_i+n\phi\theta^2+\tau\theta^2-2\theta\tau\theta_0+\tau\theta_0^2\right)\right]$

$= \exp\left[-\frac{1}{2}\left(\tau\theta^2+n\phi\theta^2-2\theta\tau\theta_0-2\phi\theta\sum_{i=1}^{n}x_i+\phi\sum_{i=1}^{n}x_i^2+\tau\theta_0^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\tau\theta^2+n\phi\theta^2-2\theta\tau\theta_0-2\phi\theta\sum_{i=1}^{n}x_i\right)\right]$ in terms of θ we perceive ϕ, τ, θ_0, and $\sum_{i=1}^{n}x_i^2$ as constants

$= \exp\left[-\frac{1}{2}\left((\tau+n\phi)\theta^2-2\theta\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)\right]$

② $= \exp\left[-\frac{1}{2}(\tau+n\phi)\left(\theta^2-\frac{2\theta}{\tau+n\phi}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)+\frac{1}{(\tau+n\phi)^2}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left((\tau+n\phi)\theta^2-2\theta\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)\right]$ in terms of θ

Therefore, ① is proportional to ② as a function of θ.