Contents

1 Data Structure

1.1 BIT_range_update

```
struct BIT {
  long long M[N], A[N];
  BIT() {
    memset(M, 0, sizeof M);
    memset(A, 0, sizeof A);
  }
  void update(int i, long long mul, long long add) {
    while (i < N) {
      M[i] += mul;
      A[i] += add;
      i |= (i + 1);
    }
  }
  void upd(int l, int r, long long x) {
    update(l, x, -x * (l - 1));
    update(r, -x, x * r);
  }
  long long query(int i) {
    long long mul = 0, add = 0;
    int st = i;
    while (i >= 0) {
      mul += M[i];
      add += A[i];
      i = (i & (i + 1)) - 1;
    }
    return (mul * st + add);
  }
  long long query(int l, int r) {
    return query(r) - query(l - 1);
  }
} t;
```

1.2 CHT

```
/**
 *   Lines should be added non-increasing order of m for minimizing
 Non-decreasing order of m for maximizing
 *   Intersection point of two lines (m1,c1) , (m2,c2) is
 x = (c2-c1)/(m1-m2)
 **/
ll M[MAX] , C[MAX];
struct CHT {
  int len , cur;
  void init() {
    len = 0 ,cur = 0;
  }
  /// returns true if line[len-1] is unnecessary when we add line(nm,nc)
  inline bool isBad(ll nm,ll nc) {
    return (
    (C[len-1]-C[len-2])/(double)(M[len-2]-M[len-1]) >= (nc-C[len-2])/(double)(M[len-2]-nm) );
    //return ( (C[len-1]-C[len-2])*(M[len-2]-nm) >= (M[len-2]-M[len-1])*(nc-C[len-2]) );
  }
  inline void addLine(ll nm,ll nc) {
    if(len == 0) M[len] = nm , C[len] = nc , ++len;
    else if( M[len-1] == nm ) {
      if(C[len-1] <= nc) return; /// <= to minimize, >= to maximize
      else C[len-1] = nc;
    }
    else {
      while(len >= 2 && isBad(nm,nc)) --len;
      M[len] = nm , C[len] = nc , ++len;
    }
  }
  inline ll getY(int id , ll x) {
    return ( M[id]*x + C[id] );
  }
  inline ll sortedQuery( ll x ) {
    if(cur >= len ) cur = len-1;
    while( cur < len-1 && getY(cur+1,x) >= getY(cur,x) ) cur++; /// <= to minimize, >= to maximize
    return getY(cur,x);
  }
  inline ll TS( ll x ) {
    int low = 0, high = len-1 , mid ;
    while( high - low > 1 ) {
      mid = low + high >> 1;
      if(getY(mid,x) < getY(mid+1,x)) low = mid + 1; /// > to minimize , < to maximize
      else high = mid;
    }
    return max(getY(low,x),getY(high,x)); /// adjust min/max
  }
};
CHT cht;
cht.init();
```

1.3 Implicit Treap

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
/***
    Treap as Interval Tree(1 based) With Insert and Remove Operation at any position
    The key(BST Value) is not explicitly stored and determined in the runtime.
    That's why called implicit treap
***/
typedef struct node{
    int prior, sz;
    ll val; ///value stored in the array
    ll sum; ///whatever info you want to maintain in segment tree for each node
    ll lazy; ///whatever lazy update you want to do
    struct node *l, *r, *p;
} node;
typedef node* pnode;
pnode Treap;
inline int getSize(pnode t) { return t ? t->sz : 0; }
inline ll get_sum(pnode t) { return t ? t->sum : 0; }
inline void lazyUpdate(pnode t){
    if(!t or !t->lazy) return;
    t->val += t->lazy;
    t->sum += t->lazy * getSize(t);
    if(t->l) t->l->lazy += t->lazy;
    if(t->r) t->r->lazy += t->lazy;
    t->lazy=0;
}
/// operation of segment tree and size,parent update
inline void operation(pnode t) {
    if(!t) return;
    lazyUpdate(t->l); lazyUpdate(t->r);
    ///imp:propagate lazy before combining t->l,t->r;
    t->sz = getSize(t->l) + 1 + getSize(t->r);
    t->sum = get_sum(t->l) + t->val + get_sum(t->r); /// updateing sum
    if(t->l) t->l->p = t;
    if(t->r) t->r->p = t;
}
/// The subarray[1:pos] is saved in node l, the rest in r
/// add --> Number of nodes that are not in t's subtree and has index less that t
void split(pnode t, pnode &l, pnode &r, int pos, int add = 0){
    if(!t) return void( l = r = NULL) ;
    lazyUpdate(t);
    int curr_pos = add + getSize(t->l)+1;
    if(curr_pos <= pos) split(t->r, t->r, r, pos, curr_pos), l = t;
    else split(t->l, l, t->l, pos, add), r = t;
    operation(t);
}
void merge(pnode &t, pnode l, pnode r){
    lazyUpdate(l); lazyUpdate(r);
    if(!l or !r) t = l ? l : r;
    else if(l->prior > r->prior) merge(l->r, l->r, r), t = l;
    else merge(r->l, l, r->l), t = r;
    operation(t);
}
pnode newNode(ll val){
    pnode ret = (pnode)malloc(sizeof(node));
    ret->prior = rand();
    ret->sz = 1;
    ret->val = ret->sum = val;
    ret->lazy = 0;
    ret->p = ret->l = ret->r = NULL;
    return ret;
}
///changes the value of the node at position id to val
inline void point_update(pnode &t, int id, ll val) {
    int sz = getSize(t->l);
    if( sz == (id-1) ) {
        t->val = val;
        pnode cur = t;
        while(cur != NULL) operation(cur), cur = cur->p;
    }
    else if(sz < (id-1) ) point_update(t->r, id-sz-1, val);
    else point_update(t->l, id, val);
}
/***
    * changes the value of the node at position id to val
    * Slower
    * Parent er track na rakhle use kora lagte pare
```

```
    void point_update(pnode &t, int id, ll val){
        pnode L, mid, R;
        split(t, L, mid, id-1);
        split(mid, t, R, 1);
        t->val = val;
        merge(mid, L, t);
        merge(t, mid, R);
    }
***/
/// deletes the node at position id
void Remove(pnode &t, int id){
    pnode L, mid, R, X;
    split(t, L, mid, id-1);
    split(mid, X, R, 1);
    delete X;
    merge(t, L, R);
}
/// inserts a node at position id having array value = val
void Insert(pnode &t, int id, ll val){
    pnode L, R, mid;
    pnode it = newNode(val);
    split(t, L, R, id-1);
    merge(mid, L, it);
    merge(t, mid, R);
}
/// add val to all the nodes [i:j]
void range_update(pnode t, int i, int j, ll val){
    pnode L, M, R;
    split(t, L, M, i-1);
    split(M, t, R, j-i+1);
    t->lazy += val;
    merge(M, L, t);
    merge(t, M, R);
}
/// range query [i:j]
ll range_query(pnode t, int i, int j){
    pnode L, M, R;
    split(t, L, M, i-1);
    split(M, t, R, j-i+1);
    ll ans = t->sum;
    merge(M, L, t);
    merge(t, M, R);
    return ans;
}
/// Freeing memory after each test case
void Delete(pnode &t){
    if(!t) return;
    if(t->l) Delete(t->l);
    if(t->r) Delete(t->r);
    delete(t);
    t = NULL;
}
ll ara[11];
int main(){
    ///creating a treap to use it as an interval tree of ara (1 based)
    int n = 10;
    for(int i=1; i<=n; i++) merge(Treap,Treap,newNode(ara[i]));
    Delete(Treap);  /// Deleting when work done
    return 0;
}

/*
/// Maximum contiguous sum merging
void operation(pnode t){
    if(!t)return;
    t->sum = get_sum(t->l) + t->val + get_sum(t->r);
    t->res = max( max(get_res(t->l), get_res(t->r)), max(0, get_rsum(t->l)) + t->val + max(0, get_lsum(t->r)));
    t->lsum = max(max(0,get_lsum(t->r)) + t->val + get_sum(t->l),get_lsum(t->l));
    t->rsum = max(get_sum(t->r) + t->val + max(0,get_rsum(t->l)),get_rsum(t->r));
}
*/
```

1.4 dynamic_cht

```
//add lines with -m and -b and return -ans to
//make this code work for minimums.(not -x)
const ll inf = -(1LL << 62);
struct line {
  ll m, b;
  mutable function<const line*() > succ;
  bool operator < (const line& rhs) const {
    if (rhs.b != inf) return m < rhs.m;
    const line* s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};
struct CHT : public multiset<line> {
  bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y -> m == z -> m && y -> b <= z -> b;
    }
    auto x = prev(y);
    if (z == end()) return y -> m == x -> m && y -> b <= x -> b;
    return 1.0 * (x -> b - y -> b) * (z -> m - y -> m) >= 1.0 * (y -> b - z -> b) * (y -> m - x -> m);
  }
  void add(ll m, ll b) {
    auto y = insert({ m, b });
    y->succ = [ = ] { return next(y) == end() ? 0 : &*next(y); };
    if (bad(y)) {
      erase(y);
      return;
    }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }
  ll query(ll x) {
    auto l = *lower_bound((line) {
      x, inf
    });
    return l.m * x + l.b;
  }
};
CHT* cht;
ll a[N], b[N];
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n;
  cin >> n;
  for(int i = 0; i < n; i++) cin >> a[i];
  for(int i = 0; i < n; i++) cin >> b[i];
  cht = new CHT();
  cht -> add(-b[0], 0);
  ll ans = 0;
  for(int i = 1; i < n; i++) {
    ans = -cht -> query(a[i]);
    cht -> add(-b[i], -ans);
  }
  cout << ans << nl;
  return 0;
}
```

1.5 gp_hash_table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct chash {
  int operator()(int x) const { return x ^ RANDOM; }
};
gp_hash_table<key, int, chash> table;
```

1.6 mos_algo

```
struct data{
  int l, r, id, bn;
  data() {}
  data(int _l, int _r, int _id){
    l = _l, r = _r, id = _id;
    bn = l / block_sz;
  }
  bool operator < (const data& other) const{
    if (bn != other.bn) return (bn < other.bn);
    return ((bn & 1) ? (r < other.r) : (r > other.r));
  }
}
int curL = 0, curR = -1;
for(int i = 0; i < Q.sz; i++){
  while(curL > Q[i].L){
    curL--; add(curL);
  }
  while(curR < Q[i].R){
    curR++; add(curR);
  }
  while(curL < Q[i].L){
    remove(curL); curL++;
  }
  while(curR > Q[i].R){
    remove(curR); curR--;
  }
}
```

1.7 ordered_set

```
#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update
```

```cpp
using namespace std;
using namespace __gnu_pbds;
/// we can replace int with other data types
/// If the data type is user defined, we need to define less operator for that type
typedef tree<
    int ,
    null_type ,
    less < int > , // "less_equal<int>," for multiset
    rb_tree_tag,
    tree_order_statistics_node_update > ordered_set;
/// ordered_set has become a data type, OS is an ordered_set
ordered_set OS;
```

1.8 persistant_segtree

```cpp
/** Persistent Segment Tree using static Array
  Point Update, Range Sum
  Initialize ncnt to 0 in every test case **/
const int MAX = 100010;
int ncnt = 0;
struct node {
  int sum;
  int left,right;
  node() {}
  node(int val) {
    sum = val;
    left = right = -1;
  }
} tree[ ? ];
/// input araay
int ara[MAX];
/// root nodes for all versions
int version[MAX];
void build(int n,int st,int ed) {
  if (st==ed) {
    tree[n] = node(ara[st]);
    return;
  }
  int mid = (st+ed) / 2;
  tree[n].left = ++ncnt;
  tree[n].right = ++ncnt;
  build(tree[n].left, st, mid);
  build(tree[n].right, mid+1, ed);
  tree[n].sum = tree[tree[n].left].sum + tree[tree[n].right].sum;
}
void update(int prev,int cur,int st,int ed,int id, int val)
{
  if (id > ed or id < st) return;
  if (st == ed) {
    tree[cur] = node(val);
    return;
  }
  int mid = (st+ed) / 2;
  if (id <= mid) {
    tree[cur].right = tree[prev].right;
    tree[cur].left = ++ncnt;
    update(tree[prev].left,tree[cur].left, st, mid, id, val);
  }
  else {
    tree[cur].left = tree[prev].left;
    tree[cur].right = ++ncnt;
    update(tree[prev].right, tree[cur].right, mid+1, ed, id, val);
  }
  tree[cur].sum = tree[tree[cur].left].sum + tree[tree[cur].right].sum;
}
int query(int n,int st,int ed,int i,int j){
  if(st>=i && ed<=j) return tree[n].sum;
  int mid = (st+ed)/2;
  if(mid<i) return query(tree[n].right,mid+1,ed,i,j);
  else if(mid>=j) return query(tree[n].left,st,mid,i,j);
  else return query(tree[n].left,st,mid,i,j) + query(tree[n].right,mid+1,ed,i,j);
}
int main() {
  int n,q,l,r,k;
  sii(n,q);
  version[0] = ++ncnt;
  build(version[0],1,n);
  version[1] = ++ncnt;
  update(version[0],version[1],1,n,id,val);
  query(version[0],1,n,id,id);
  query(version[1],1,n,id,id);
  return 0;
}
```

1.9 segment_tree

```cpp
int ara[MAX];
struct node {
  int sum;
} tree[4 * MAX];
int lazy[4 * MAX];
node Merge(node a, node b) {
  node ret;
  ret.sum = a.sum + b.sum;
  return ret;
}
void lazyUpdate(int n, int st, int ed) {
  if(lazy[n] != 0){
    tree[n].sum += ((ed - st + 1) * lazy[n]);
    if(st != ed){
      lazy[2 * n] += lazy[n];
      lazy[2 * n + 1] += lazy[n];
    }
    lazy[n] = 0;
  }
}
void build(int n, int st, int ed) {
  lazy[n] = 0;
  if(st == ed){
    tree[n].sum = ara[st];
    return;
  }
  int mid = (st + ed) / 2;
  build(2 * n, st, mid);
  build(2 * n + 1, mid + 1, ed);
  tree[n] = Merge(tree[2 * n], tree[2 * n + 1]);
}
void update(int n, int st, int ed, int i, int j, int v) {
  lazyUpdate(n, st, ed);
  if(st > j or ed < i) return;
  if(st >= i and ed <= j){
    lazy[n] += v;
    lazyUpdate(n, st, ed);
    return;
  }
  int mid = (st + ed) / 2;
  update(2 * n, st, mid, i, j, v);
  update(2 * n + 1, mid+1, ed, i, j, v);
  tree[n] = Merge(tree[2 * n], tree[2 * n + 1]);
}
node query(int n, int st, int ed, int i, int j) {
  lazyUpdate(n, st, ed);
  if(st >= i and ed <= j) return tree[n];
  int mid = (st + ed) / 2;
  if(mid < i) return query(2 * n + 1, mid + 1, ed, i, j);
  else if(mid >= j) return query(2 * n, st, mid, i, j);
  else return Merge(query(2 * n, st, mid, i, j), query(2 * n + 1, mid + 1, ed, i, j));
}
```

1.10 sparse_table

```cpp
int st[K + 1][MAXN];
void build() {
  std::copy(array.begin(), array.end(), st[0]);
  for (int i = 1; i <= K; i++)
    for (int j = 0; j + (1 << i) <= N; j++)
      st[i][j] = f(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
}
```

2 Geometry

2.1 2D Primitive

2.1.1 Angle

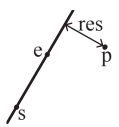
A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

```cpp
/* Usage:
 *  vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
 *  int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
 *  // sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i
*/
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
```

```cpp
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) < make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ? make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

2.1.2 Line Distance

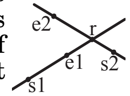
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.




```cpp
#include "Point.h"
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

2.1.3 Line Intersection

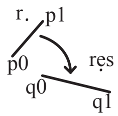
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.




```cpp
/* Usage:
 *  auto res = lineInter(s1,e1,s2,e2);
 *  if (res.first == 1)
 *    cout << "intersection point at " << res.second << endl;
 */
#pragma once
#include "Point.h"
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

2.1.4 Linear Transformation

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.




```cpp
#include "Point.h"
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1, const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

2.1.5 On Segment

```cpp
/* Description: Returns true iff p lies on the line segment from s to e.
 * Use \texttt{(segDist(s,e,p)<=epsilon)} instead when using Point<double>.
 */
#include "Point.h"
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

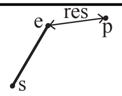
2.1.6 Point Sort

```cpp
// sort the points in counterclockwise order that starts from the half line x0,y0.
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair <ll, ll> point;
#define x first
#define y second
int main() {
  int n; cin >> n;
  vector <point> p(n);
  for (auto &it : p) scanf("%lld %lld", &it.x, &it.y);
  sort(p.begin(), p.end(), [] (point a, point b) {
    return atan2l(a.y, a.x) < atan2l(b.y, b.x);
  });
  for (auto it : p) printf("%lld %lld\n", it.x, it.y);
  return 0;
}
```

2.1.7 Point

```cpp
// Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
 typedef Point P;
 T x, y;
 explicit Point(T x=0, T y=0) : x(x), y(y) {}
 bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
 bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
 P operator+(P p) const { return P(x+p.x, y+p.y); }
 P operator-(P p) const { return P(x-p.x, y-p.y); }
 P operator*(T d) const { return P(x*d, y*d); }
 P operator/(T d) const { return P(x/d, y/d); }
 T dot(P p) const { return x*p.x + y*p.y; }
 T cross(P p) const { return x*p.y - y*p.x; }
 T cross(P a, P b) const { return (a-*this).cross(b-*this); }
 T dist2() const { return x*x + y*y; }
 double dist() const { return sqrt((double)dist2()); }
 // angle to x-axis in interval [-pi, pi]
 double angle() const { return atan2(y, x); }
 P unit() const { return *this/dist(); } // makes dist()=1
 P perp() const { return P(-y, x); } // rotates +90 degrees
 P normal() const { return perp().unit(); }
 // returns point rotated 'a' radians ccw around the origin
 P rotate(double a) const {
  return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
 friend ostream& operator<<(ostream& os, P p) {
  return os << "(" << p.x << "," << p.y << ")"; }
};
```

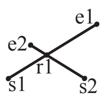
2.1.8 Segment Distance



Returns the shortest distance between point p and the line segment from point s to e.

```
/* Usage:
 *  Point<double> a, b(2,2), p(1,1);
 *  bool onSegment = segDist(a,b,p) < 1e-10;
 * Status: tested
 */
#pragma once
#include "Point.h"
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

2.1.9 Segment Intersection



If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

```
/* Usage:
 * vector<P> inter = segInter(s1,e1,s2,e2);
 * if (sz(inter)==1)
 *   cout << "segments intersect at " << inter[0] << endl;
 * Status: stress-tested, tested on kattis:intersection
 */
#include "Point.h"
#include "OnSegment.h"
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

2.1.10 Side Of

Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

```
/* Usage:
 *  bool left = sideOf(p1,p2,q)==1;
 * Status: tested
 */
#include "Point.h"
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

2.2 3D

2.2.1 3D Convex Hull

```
#include <bits/stdc++.h>
#define ll long long
#define sz(x) ((int) (x).size())
#define all(x) (x).begin(), (x).end()
#define vi vector<int>
#define pii pair<int, int>
#define rep(i, a, b) for(int i = (a); i < (b); i++)
using namespace std;
template<typename T>
using minpq = priority_queue<T, vector<T>, greater<T>>;
typedef long double ftype;
struct pt3 {
  ftype x, y, z;
  pt3(ftype x = 0, ftype y = 0, ftype z = 0) : x(x), y(y), z(z) {}
  pt3 operator-(const pt3 &o) const {
    return pt3(x - o.x, y - o.y, z - o.z);
  }
  pt3 cross(const pt3 &o) const {
    return pt3(y * o.z - z * o.y, z * o.x - x * o.z, x * o.y - y * o.x);
  }
  ftype dot(const pt3 &o) const {
    return x * o.x + y * o.y + z * o.z;
  }
};
// A face is represented by the indices of its three points a, b, c.
// It also stores an outward-facing normal vector q
struct face {
  int a, b, c;
  pt3 q;
};
// modify this depending on the coordinate sizes in your use case
const ftype EPS = 1e-9;
vector<face> hull3(const vector<pt3> &p) {
  int n = sz(p);
  assert(n >= 3);
  vector<face> f;
  // Consider an edge (a->b) dead if it is not a CCW edge of some current face
  // If an edge is alive but not its reverse, this is an exposed edge.
  // We should add new faces on the exposed edges.
  vector<vector<bool>> dead(n, vector<bool>(n, true));
  auto add_face = [&](int a, int b, int c) {
    f.push_back({a, b, c, (p[b] - p[a]).cross(p[c] - p[a])});
    dead[a][b] = dead[b][c] = dead[c][a] = false;
  };
  // Initialize the convex hull of the first 3 points as a
  // triangular disk with two faces of opposite orientation
  add_face(0, 1, 2);
  add_face(0, 2, 1);
  rep(i, 3, n) {
    // f2 will be the list of faces invisible to the added point p[i]
    vector<face> f2;
    for(face &F : f) {
      if((p[i] - p[F.a]).dot(F.q) > EPS) {
        // this face is visible to the new point, so mark its edges as dead
        dead[F.a][F.b] = dead[F.b][F.c] = dead[F.c][F.a] = true;
      }else {
        f2.push_back(F);
      }
    }
    // Add a new face for each exposed edge.
    // Only check edges of alive faces for being exposed.
    f.clear();
    for(face &F : f2) {
      int arr[3] = {F.a, F.b, F.c};
      rep(j, 0, 3) {
        int a = arr[j], b = arr[(j + 1) % 3];
        if(dead[b][a]) {
          add_face(b, a, i);
        }
      }
    }
    f.insert(f.end(), all(f2));
  }
  return f;
}
```

2.2.2 Point3D

Class to handle points in 3D space.T can be e.g. double or long long.

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

2.2.3 Polyhedron Volume

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

2.2.4 Spherical Distance

Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

2.3 Circle

2.3.1 Circle Intersection

Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

```
#include "Point.h"
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

2.3.2 Circle Polygon Intersection

Returns the area of the intersection of a circle with a ccw polygon.
Time: $O(n)$

```
#include "Point.h"
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

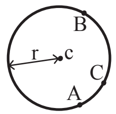
2.3.3 Circle Tangents

Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

```
#include "Point.h"
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

2.3.4 CircumCircle

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.




```
#include "Point.h"
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

2.4 Polygon

2.4.1 Hull Diameter

Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

```
#include "Point.h"
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

2.4.2 Line Hull Intersection

Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:

$(-1,-1)$ if no collision,

$(i,-1)$ if touching the corner i,

(i,i) if along side $(i,i+1)$,

(i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $O(\log n)$

```
#include "Point.h"
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

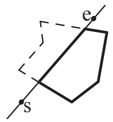
2.4.3 Polygon Center

Returns the center of mass for a polygon.
Time: $O(n)$

```
#include "Point.h"
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

2.4.4 Polygon Cut

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.




```
/* Usage:
 *  vector<P> p = ...;
 *  p = polygonCut(p, P(0,0), P(1,0));
 * Status: tested but not extensively
 */
#include "Point.h"
#include "lineIntersection.h"
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

2.5 Closest Pair

Finds the closest pair of points.
Time: $O(n\log n)$

```
#include "Point.h"
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

2.6 Convex Hull

```
typedef long long ll;
typedef pair <ll, ll> point;
#define x first
#define y second
inline ll area (point a, point b, point c) {
  return (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
}
vector <point> convexHull (vector <point> p) {
  int n = p.size(), m = 0;
  if (n < 3) return p;
  vector <point> hull(n + n);
  sort(p.begin(), p.end());
  for (int i = 0; i < n; ++i) {
    while (m > 1 and area(hull[m - 2], hull[m - 1], p[i]) <= 0) --m;
    hull[m++] = p[i];
  }
  for (int i = n - 2, j = m + 1; i >= 0; --i) {
    while (m >= j and area(hull[m - 2], hull[m - 1], p[i]) <= 0) --m;
    hull[m++] = p[i];
  }
  hull.resize(m - 1); return hull;
}
```

2.7 Minimum Enclosing Circle

```
// Expected runtime: O(n)
// Solves Gym 102299J
#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
typedef pair <ld, ld> point;
#define x first
#define y second
point operator + (const point &a, const point &b) {
  return point(a.x + b.x, a.y + b.y);
}
point operator - (const point &a, const point &b) {
  return point(a.x - b.x, a.y - b.y);
}
point operator * (const point &a, const ld &b) {
  return point(a.x * b, a.y * b);
}
point operator / (const point &a, const ld &b) {
  return point(a.x / b, a.y / b);
}
const ld EPS = 1e-8;
const ld INF = 1e20;
const ld PI = acosl(-1);
inline ld dist (point a, point b) {
  return hypotl(a.x - b.x, a.y - b.y);
}
inline ld sqDist (point a, point b) {
  return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) *
   (a.y - b.y);
}
inline ld dot (point a, point b) {
  return a.x * b.x + a.y * b.y;
}
inline ld cross (point a, point b) {
  return a.x * b.y - a.y * b.x;
}
inline ld cross (point a, point b, point c) {
  return cross(b - a, c - a);
}
inline point perp (point a) {
  return point(-a.y, a.x);
}
// circle through 3 points
pair <point, ld> getCircle (point a, point b, point
 c) {
  pair <point, ld> ret;
  ld den = (ld) 2 * cross(a, b, c);
  ret.x.x = ((c.y - a.y) * (dot(b, b) - dot(a, a))
   - (b.y - a.y) * (dot(c, c) - dot(a, a))) / den;
  ret.x.y = ((b.x - a.x) * (dot(c, c) - dot(a, a))
   - (c.x - a.x) * (dot(b, b) - dot(a, a))) / den;
  ret.y = dist(ret.x, a);
  return ret;
}
pair <point, ld> minCircleAux (vector <point> &s,
 point a, point b, int n) {
  ld lo = -INF, hi = INF;
  for (int i = 0; i < n; ++i) {
    auto si = cross(b - a, s[i] - a);
    if (fabs(si) < EPS) continue;
    point m = getCircle(a, b, s[i]).x;
    auto cr = cross(b - a, m - a);
    si < 0 ? hi = min(hi, cr) : lo = max(lo, cr);
  }
  ld v = 0 < lo ? lo : hi < 0 ? hi : 0;
  point c = (a + b) * 0.5 + perp(b - a) * v /
   sqDist(a, b);
  return {c, sqDist(a, c)};
}
pair <point, ld> minCircle (vector <point> &s,
 point a, int n) {
  random_shuffle(s.begin(), s.begin() + n);
  point b = s[0], c = (a + b) * 0.5;
  ld r = sqDist(a, c);
  for (int i = 1; i < n; ++i) {
    if (sqDist(s[i], c) > r * (1 + EPS)) {
      tie(c, r) = n == s.size() ? minCircle(s,
 s[i], i) : minCircleAux(s, a, s[i], i);
    }
  }
  return {c, r};
}
pair <point, ld> minCircle (vector <point> s) {
  assert(!s.empty());
  if (s.size() == 1) return {s[0], 0};
  return minCircle(s, s[0], s.size());
}
int n; vector <point> p;
int main() {
  cin >> n;
  while (n--) {
    double x, y;
    scanf("%lf %lf", &x, &y);
    p.emplace_back(x, y);
  }
  pair <point, ld> circ = minCircle(p);
  printf("%0.12f %0.12f %0.12f\n", (double)
    circ.x.x, (double) circ.x.y, (double) (0.5 *
    circ.y));
  return 0;
}
```

2.8 Point In Polygon

```
// Test if a point is inside a convex polygon in
 O(lg n) time
// Solves SPOJ INOROUT
typedef long long ll;
typedef pair <ll, ll> point;
#define x first
#define y second
struct segment {
  point P1, P2;
  segment () {}
  segment (point P1, point P2) : P1(P1), P2(P2) {}
};
inline ll ccw (point A, point B, point C) {
  return (B.x - A.x) * (C.y - A.y) - (C.x - A.x) *
   (B.y - A.y);
}
inline bool pointOnSegment (segment S, point P) {
  ll x = P.x, y = P.y, x1 = S.P1.x, y1 = S.P1.y, x2
   = S.P2.x, y2 = S.P2.y;
  ll a = x - x1, b = y - y1, c = x2 - x1, d = y2 -
   y1, dot = a * c + b * d, len = c * c + d * d;
  if (x1 == x2 and y1 == y2) return x1 == x and y1
   == y;
  if (dot < 0 or dot > len) return 0;
  return x1 * len + dot * c == x * len and y1 * len
   + dot * d == y * len;
}
const int M = 17;
const int N = 10010;
struct polygon {
  int n; // n > 1
  point p[N]; // clockwise order
  polygon () {}
  polygon (int _n, point *T) {
    n = _n;
    for (int i = 0; i < n; ++i) p[i] = T[i];
  }
  bool contains (point P, bool strictlyInside) {
    int lo = 1, hi = n - 1;
    while (lo < hi){
      int mid = lo + hi >> 1;
      if (ccw(p[0], P, p[mid]) > 0) lo = mid + 1;
      else hi = mid;
    }
    if (ccw(p[0], P, p[lo]) > 0) lo = 1;
    if (!strictlyInside and
    pointOnSegment(segment(p[0], p[n - 1]), P))
    return 1;
    if (!strictlyInside and
    pointOnSegment(segment(p[lo], p[lo - 1]), P))
    return 1;
    if (lo == 1 or ccw(p[0], P, p[n - 1]) == 0)
    return 0;
    return ccw(p[lo], P, p[lo - 1]) < 0;
  }
};
```

2.9 sweep

```
const double EPS = 1E-9;
struct pt {
  double x, y;
};
struct seg {
  pt p, q;
  int id;
  double get_y(double x) const {
    if (abs(p.x - q.x) < EPS)
      return p.y;
    return p.y + (q.y - p.y) * (x - p.x) / (q.x -
    p.x);
  }
};
bool intersect1d(double l1, double r1, double l2,
 double r2) {
  if (l1 > r1)
    swap(l1, r1);
  if (l2 > r2)
    swap(l2, r2);
  return max(l1, l2) <= min(r1, r2) + EPS;
}
```

```cpp
int vec(const pt& a, const pt& b, const pt& c) {
  double s = (b.x - a.x) * (c.y - a.y) - (b.y -
   a.y) * (c.x - a.x);
  return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
}
bool intersect(const seg& a, const seg& b)
{
  return intersect1d(a.p.x, a.q.x, b.p.x, b.q.x) &&
    intersect1d(a.p.y, a.q.y, b.p.y, b.q.y) &&
    vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
    vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
}
bool operator<(const seg& a, const seg& b)
{
  double x = max(min(a.p.x, a.q.x), min(b.p.x,
   b.q.x));
  return a.get_y(x) < b.get_y(x) - EPS;
}
struct event {
  double x;
  int tp, id;
  event() {}
  event(double x, int tp, int id) : x(x), tp(tp),
   id(id) {}
  bool operator<(const event& e) const {
    if (abs(x - e.x) > EPS)
      return x < e.x;
    return tp > e.tp;
  }
};
set<seg> s;
vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
  return it == s.begin() ? s.end() : --it;
}
set<seg>::iterator next(set<seg>::iterator it) {
  return ++it;
}
pair<int, int> solve(const vector<seg>& a) {
  int n = (int)a.size();
  vector<event> e;
  for (int i = 0; i < n; ++i) {
    e.push_back(event(min(a[i].p.x, a[i].q.x), +1,
   i));
    e.push_back(event(max(a[i].p.x, a[i].q.x), -1,
   i));
  }
  sort(e.begin(), e.end());
  s.clear();
  where.resize(a.size());
  for (size_t i = 0; i < e.size(); ++i) {
    int id = e[i].id;
    if (e[i].tp == +1) {
      set<seg>::iterator nxt =
   s.lower_bound(a[id]), prv = prev(nxt);
      if (nxt != s.end() && intersect(*nxt, a[id]))
        return make_pair(nxt->id, id);
      if (prv != s.end() && intersect(*prv, a[id]))
        return make_pair(prv->id, id);
      where[id] = s.insert(nxt, a[id]);
    } else {
      set<seg>::iterator nxt = next(where[id]), prv
   = prev(where[id]);
      if (nxt != s.end() && prv != s.end() &&
   intersect(*nxt, *prv))
        return make_pair(prv->id, nxt->id);
      s.erase(where[id]);
    }
  }
  return make_pair(-1, -1);
}
```

3 Graph

3.1 2Sat

```cpp
/***
 * 1 based index for variables
 * F = (a op b) and (c op d) and ...... (y op z)
 a, b, c ... are the variables
 sat::satisfy() returns true if there is some
   assignment(True/False)
 for all the variables that make F = True
 * init() at the start of every case
 ***/
namespace sat{
  const int MAX = 200010; /// number of variables *
   2
  bool vis[MAX];
  vector <int> ed[MAX], rev[MAX];
  int n, m, ptr, dfs_t[MAX], ord[MAX], par[MAX];
  inline int inv(int x){
    return ((x) <= n ? (x + n) : (x - n));
  }
  /// Call init once
  void init(int vars){
    n = vars, m = vars << 1;
    for (int i = 1; i <= m; i++){
      ed[i].clear();
      rev[i].clear();
    }
  }
  /// Adding implication, if a then b ( a --> b )
  inline void add(int a, int b){
    ed[a].push_back(b);
    rev[b].push_back(a);
  }

  /// (a or b) is true --> OR(a,b)
  /// (a or b) is true --> OR(inv(a),b)
  /// (a or b) is true --> OR(a,inv(b))
  /// (a or b) is true --> OR(inv(a),inv(b))
  inline void OR(int a, int b){
    add(inv(a), b);
    add(inv(b), a);
  }
  /// same rule as or
  inline void AND(int a, int b){
    add(a, b);
    add(b, a);
  }
  /// same rule as or
  void XOR(int a,int b){
    add(inv(b), a);
    add(a, inv(b));
    add(inv(a), b);
    add(b, inv(a));
  }
  /// same rule as or
  inline void XNOR(int a, int b){
    add(a,b);
    add(b,a);
    add(inv(a), inv(b));
    add(inv(b), inv(a));
  }
  /// (x <= n) means forcing variable x to be true
  /// (x = n + y) means forcing variable y to be
   false
  inline void force_true(int x){
    add(inv(x), x);
  }
  inline void topsort(int s){
    vis[s] = true;
    for(int x : rev[s]) if(!vis[x]) topsort(x);
    dfs_t[s] = ++ptr;
  }
  inline void dfs(int s, int p){
    par[s] = p;
    vis[s] = true;
    for(int x : ed[s]) if (!vis[x]) dfs(x, p);
  }
  void build(){
    CLR(vis);
    ptr = 0;
    for(int i=m;i>=1;i--) {
      if (!vis[i]) topsort(i);
      ord[dfs_t[i]] = i;
    }
    CLR(vis);
    for (int i = m; i >= 1; i--){
      int x = ord[i];
      if (!vis[x]) dfs(x, x);
    }
  }
  /// Returns true if the system is 2-satisfiable
   and returns the solution (vars set to true) in
   vector res
  bool satisfy(vector <int>& res){
    build();
    CLR(vis);
    for (int i = 1; i <= m; i++){
      int x = ord[i];
      if (par[x] == par[inv(x)]) return false;
      if (!vis[par[x]]){
        vis[par[x]] = true;
        vis[par[inv(x)]] = false;
      }
    }
    res.clear();
    for (int i = 1; i <= n; i++){
      if (vis[par[i]]) res.push_back(i);
    }
    return true;
  }
}
```

3.2 Centroid_decomp

```cpp
vector <int> ed[MAX]; /// adjacency list of the
   input tree
bool isCentroid[MAX]; /// if the node is already a
   centroid of some part
int sub[MAX], cpar[MAX], clevel[MAX];
```

```cpp
int dis[20][MAX]; /// dis[i][j] = distance of node j from the root of the i'th level of decomposition
void calcSubTree(int s,int p) {
  sub[s] = 1;
  for(int x : ed[s]) {
    if(x == p or isCentroid[x]) continue;
    calcSubTree(x,s);
    sub[s] += sub[x];
  }
}
int nn; /// number of nodes in the part
int getCentroid(int s,int p) {
  for(int x : ed[s]) {
    if(!isCentroid[x] && x!=p && sub[x]>(nn/2))
    return getCentroid(x,s);
  }
  return s;
}
void setDis(int s, int from, int p, int lev) {
  dis[from][s] = lev;
  for(int x : ed[s]) {
    if(x == p or isCentroid[x] )  continue;
    setDis(x, from, s, lev+1);
  }
}
///complexity --> O(nlog(n))
void decompose(int s,int p,int lev) {
  calcSubTree(s,p);
  nn = sub[s];
  int c = getCentroid(s,p);
  setDis(c,lev,p,0);
  isCentroid[c] = true;
  cpar[c] = p;
  clevel[c] = lev;
  for(int x : ed[c]) {
    if(!isCentroid[x]) decompose(x,c,lev+1);
  }
}
int ans[MAX];
inline void update(int v) {
  int u = v;
  while(u!=-1) {
    ans[u] = min(ans[u], dis[clevel[u]][v]);
    u = cpar[u];
  }
}
inline int query(int v) {
  int ret = INF;
  int u = v;
  while(u != -1) {
    ret = min(ret, dis[clevel[u]][v]+ans[u]);
    u = cpar[u];
  }
  return ret;
}
int main() {
  decompose(1,-1,0);
  for(int i=1; i<=n; i++) ans[i] = INF;
  update(v);
  query(v));
  return 0;
}
```

3.3 articulation_point

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N = 1e5 + 10;
vector<int> g[N];
int vis[N], low[N], cut[N], now = 0, n, m;
void dfs(int u, int p) {
  low[u] = vis[u] = ++now; int ch = 0;
  for(int v : g[u]){
    if(v ^ p) {
      if(vis[v]) low[u] = min(low[u], vis[v]);
      else {
        ch++; dfs(v, u);
        low[u] = min(low[u], low[v]);
        if(p + 1 && low[v] >= vis[u]) cut[u] = 1;
        if(low[v] > vis[u]) {
          printf("Bridge %d -- %d\n", u, v);
        }
      }
    }
  } if(p == -1 && ch > 1) cut[u] = 1;
}
void ArticulationPointAndBridge() {
  now = 0;
  for(int i = 0; i < n; i++) {
    if(!vis[i]) dfs(i, -1);
  }
}
```

3.4 bcc

```cpp
// clear ed[] every test case
// tot -> total number of components
// bcc[i] contains the nodes of the i'th component
// any self loop or multiple edge?
const int MAX = ?;
vector <int> ed[MAX];
bool cut[MAX];
int tot, Time, low[MAX], st[MAX];
vector <int> bcc[MAX];
stack <int> S;
void popBCC(int s,int x) {
  cut[s] = 1;
  bcc[++tot].pb(s);
  while(bcc[tot].back() ^ x) {
    bcc[tot].pb(S.top());
    S.pop();
  }
}
void dfs(int s, int p = -1) {
  S.push(s);
  int ch = 0;
  st[s] = low[s] = ++Time;
  for(int x : ed[s]) {
    if(!st[x]) {
      ch++;
      dfs(x,s);
      low[s] = min(low[s],low[x]);
      if(p != -1 and low[x] >= st[s]) popBCC(s,x);
      else if(p == -1) if(ch > 1) popBCC(s,x);
    }
    else if(p != x) low[s] = min(low[s],st[x]);
  }
  if(p == -1 && ch > 1) cut[s] = 1;
}
void processBCC(int n) {
  for(int i=1;i<=n;i++) bcc[i].clear();
  CLR(st); CLR(cut);
  Time = tot = 0;
  for(int i=1; i<=n; i++) {
    if(!st[i]) {
      dfs(i,-1);
      if(!S.empty()) ++tot;
      while(!S.empty()) {
        bcc[tot].push_back(S.top());
        S.pop();
      }
    }
  }
}
```

3.5 bridge_tree

```cpp
/***
  1 based indexing
  call to processBridge(node,edges) generates bridge tree
  and the edge list of that is brTree
  Clear ed , isBridge , brTree per test case
 ***/
const int MAXN = ?;
const int MAXE = ?;
struct edges {
  int u,v;
} ara[MAXE];
vector <int> ed[MAXN]; /// actual graph
vector <int> isBridge[MAXN]; /// if the edge is a bridge, the entry will be 1
vector <int> brTree[MAXN]; /// edges of the bridge tree
bool vis[MAXN];
int st[MAXN], low[MAXN], Time = 0;
int cnum; /// number of nodes in bridge tree
int comp[MAXN];
void findBridge(int s,int par) {
  int i,x,child = 0,j;
  vis[s] = 1;
  Time++;
  st[s] = low[s] = Time;
  for(i=0; i<ed[s].size(); i++) {
    x = ed[s][i];
    if(!vis[x]) {
      child++;
      findBridge(x,s);
      low[s] = min(low[s],low[x]);
      if(low[x] > st[s]) {
        isBridge[s][i] = 1;
        j = lower_bound(ed[x].begin(),ed[x].end(),s)-ed[x].begin();
        isBridge[x][j] = 1;
      }
    }
    else if(par!=x)
      low[s] = min(low[s],st[x]);
  }
}
```

```
void dfs(int s) {
  int i,x;
  vis[s] = 1;
  comp[s] = cnum;
  for(i=0; i<ed[s].size(); i++) {
    if(!isBridge[s][i]) {
      x = ed[s][i];
      if(!vis[x]) dfs(x);
    }
  }
}
void processBridge(int n,int m) {
  CLR(vis);
  Time = 0;
  for(int i=1; i<=n; i++) if(!vis[i]) findBridge(i,-1);
  cnum = 0;
  CLR(vis);
  for(int i=1; i<=n; i++) {
    if(!vis[i]) {
      cnum++;
      dfs(i);
    }
  }
  n = cnum; ///number of nodes in the bridge tree
  for(int i=1; i<=m; i++) {
    if(comp[ara[i].u] != comp[ara[i].v]) {
      brTree[comp[ara[i].u]].pb(comp[ara[i].v]);
      brTree[comp[ara[i].v]].pb(comp[ara[i].u]);
    }
  }
}

int main() {
  int n,m,u,v;
  scanf("%d %d",&n,&m);
  for(int i=1; i<=m; i++) {
    sii(u,v);
    ed[u].pb(v);
    ed[v].pb(u);
    isBridge[u].pb(0);
    isBridge[v].pb(0);
    ara[i].u = u;
    ara[i].v = v;
  }
  for(int i=1; i<=n; i++) sort(all(ed[i]));
  processBridge(n,m);
  return 0;
}
```

3.6 dinic

```
namespace dinic {
  using T = int;
  const T INF = 0x3f3f3f3f;
  const int MAXN = 5010;
  int n, src, snk, work[MAXN];
  T dist[MAXN];
  struct Edge{
    int to, rev_pos;
    T c, f;
  };
  vector <Edge> ed[MAXN];
  void init(int _n, int _src, int _snk) {
    n = _n, src = _src, snk = _snk;
    for(int i=1;i<=n;i++) ed[i].clear();
  }
  inline void addEdge(int u, int v, T c, T rc = 0) {
    Edge a = {v, (int)ed[v].size(), c, 0};
    Edge b = {u, (int)ed[u].size(), rc, 0};
    ed[u].push_back(a);
    ed[v].push_back(b);
  }
  bool dinic_bfs() {
    SET(dist);
    dist[src] = 0;
    queue <int> q;
    q.push(src);
    while(!q.empty()){
      int u = q.front(); q.pop();
      for(Edge &e : ed[u]){
        if(dist[e.to] == -1 and e.f < e.c) {
          dist[e.to] = dist[u] + 1;
          q.push(e.to);
        }
      }
    }
    return (dist[snk]>=0);
  }
  T dinic_dfs(int u, T fl){
    if (u == snk) return fl;
    for (; work[u] < (int)ed[u].size(); work[u]++) {
      Edge &e = ed[u][work[u]];
      if (e.c <= e.f) continue;
      int v = e.to;
      if (dist[v] == dist[u] + 1){
        T df = dinic_dfs(v, min(fl, e.c - e.f));
        if (df > 0){
          e.f += df;
          ed[v][e.rev_pos].f -= df;
          return df;
        }
      }
    }
    return 0;
  }
  T solve() {
    T ret = 0;
    while (dinic_bfs()) {
      CLR(work);
      while (T delta = dinic_dfs(src, INF)) ret += delta;
    }
    return ret;
  }
}

int main() {
  int n, m, u, v, c;
  cin >> n >> m;
  dinic::init(n, 1, n);
  while(m--) {
    cin >> u >> v >> c;
    dinic::addEdge(u, v, c, c);
  }
  cout << dinic::solve() << '\n';
  return 0;
}
```

3.7 dsu_on_tree

```
vector<int> *vec[maxn];
int cnt[maxn];
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p && sz[u] > mx)
           mx = sz[u], bigChild = u;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           dfs(u, v, 0);
    if(bigChild != -1)
        dfs(bigChild, v, 1), vec[v] = vec[bigChild];
    else
        vec[v] = new vector<int> ();
    vec[v]->push_back(v);
    cnt[ col[v] ]++;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           for(auto x : *vec[u]){
               cnt[ col[x] ]++;
               vec[v] -> push_back(x);
           }
    //now cnt[c] is the number of vertices in subtree of vertex v that has color c.
    // note that in this step *vec[v] contains all of the subtree of vertex v.
    if(keep == 0)
        for(auto u : *vec[v])
            cnt[ col[u] ]--;
}
```

3.8 euler_path

```
/*** 1 -based *****/
vector <int> ed[MAX+5], sltn;
int inDeg[MAX+5], outDeg[MAX+5];
bool vis[MAX+5];
void dfs(int nd) {
  vis[nd] = true; /// used to check the connectivity of the graph
  while(ed[nd].size()) {
    int v = ed[nd].back();
    ed[nd].pop_back();
    dfs(v);
  }
  sltn.pb(nd);
}
/// returns 0 if no Euler path or circuit exists
/// returns 1 if a Euler trail exists
/// returns 2 if a Euler circuit exists
int findEuler (int n) {
  int src , snk , ret = 1;
  bool found_src = false, found_snk = false;
  CLR(inDeg); CLR(outDeg);
  for(int u = 1; u <= n; u++) {
    for(int i = 0; i<ed[u].size(); i++) {
      int v = ed[u][i];
      outDeg[u]++;
      inDeg[v]++;
    }
  }
  int diff;
  for(int i = 1; i<=n; i++) {
    diff = outDeg[i] - inDeg[i];
```

```
    if(diff == 1) {
      if(found_src) return 0;
      found_src = true;
      src = i;
    }
    else if (diff == -1) {
      if(found_snk) return 0;
      found_snk = true;
      snk = i;
    }
    else if(diff != 0) return 0;
  }
  if(!found_src) {
    /// there actually exists a euler cycle. So you need to pick a random node with non-zero degrees.
    ret = 2;
    for(int i = 1 ; i <= n ; i++) {
      if( outDeg[i] ) {
        found_src = true;
        src = i;
        break;
      }
    }
  }
  if(!found_src) return ret; /// every node has out-degree 0
  CLR(vis);
  sltn.clear();
  dfs(src);
  for(int i = 1; i<=n; i++) {
    /// the underlying graph is not even weakly connected.
    if(outDeg[i] && !vis[i]) return 0;
  }
  /// printing path
  for(int i = (int)sltn.size()-1; i>=0; i--) printf("%d ",sltn[i]);
  puts("");
  return ret;
}
```

3.9 hld

```
int tt, tin[N], tout[N], sz[N], par[N][LG], hvc[N];
void dfs(int u, int p) {
  tin[u] = tt++, sz[u] = 1, par[u][0] = p;
  for (int j = 1; j < LG; ++j) {
    par[u][j] = par[par[u][j - 1]][j - 1];
  }
  int mx = 0;
  for (int &v : adj[u]) {
    if (v != p) {
      dfs(v, u);
      sz[u] += sz[v];
      if (sz[v] > mx) {
        mx = sz[v];
        hvc[u] = v;
      }
    }
  }
  tout[u] = tt - 1;
}
int ch_cnt, chno[N], chd[N], in[N], out[N];
void hld(int u, int p) {
  if (chd[ch_cnt] == -1) {
    chd[ch_cnt] = u;
  }
  chno[u] = ch_cnt, in[u] = tt++;
  if (hvc[u] != -1) {
    hld(hvc[u], u);
  }
  for (int &v : adj[u]) {
    if (v != p and v != hvc[u]) {
      ch_cnt++;
      hld(v, u);
    }
  }
  out[u] = tt - 1;
}
//pre req
adj[u].clear();
hvc[u] = -1;
tt = 0;
dfs(0, 0);
chd[ch] = -1;
ch_cnt = 0, tt = 0;
hld(0, 0);
```

3.10 hopcroft_karp

```
struct HopcroftKarp {
  const int N, M;
  std::vector<std::vector<int>> adj_left;
  std::vector<int> matchL, matchR;
  HopcroftKarp(int N, int M, const std::vector<std::pair<int, int>>& edge)
      : N(N), M(M), matchL(N, -1), matchR(M, -1), adj_left(N) {
    for (auto [l, r] : edge)
      adj_left[l].push_back(r);
  }
  int maxmatching() {
    int sz = 0;
    for (bool updated = true; updated;) {
      updated = false;
      static std::vector<int> root(N), prev(N), qq(N);
      static int qi, qj;
      // std::queue<int> q;
      qi = qj = 0;
      std::fill(root.begin(), root.end(), -1),
      std::fill(prev.begin(), prev.end(), -1);
      for (int i = 0; i < N; i++)
        if (matchL[i] == -1)
          qq[qj++] = i, root[i] = i, prev[i] = i;
      // q.push(i), root[i] = i;
      while (qi < qj) {
        int u = qq[qi++];
        // int u = q.front(); q.pop();
        if (matchL[root[u]] != -1) continue;
        for (int v : adj_left[u]) {
          if (matchR[v] == -1) {
            while (v != -1)
              matchR[v] = u, std::swap(matchL[u], v), u = prev[u];
            updated = true, sz++;
            break;
          }
          if (prev[matchR[v]] == -1)
            v = matchR[v], prev[v] = u, root[v] = root[u], qq[qj++] = v;
            // v = matchR[v], prev[v] = u, root[v] = root[u], q.push(v);
        }
      }
    }
    return sz;
  }
};
```

3.11 hungarian

```
// Given NN matrix A[i][j]. Calculate a permutation p[i] that minimize A[i][p[i]].
template <typename T>
pair <T, vector <int>> Hungarian (int n, int m, T c[N][N]) {
  vector <T> v(m), dist(m);
  vector <int> L(n, -1), R(m, -1);
  vector <int> index(m), prev(m);
  auto residue = [&] (int i, int j) {return c[i][j] - v[j];};
  iota(index.begin(), index.end(), 0);
  for (int f = 0; f < n; ++f) {
    for (int j = 0; j < m; ++j) {
      dist[j] = residue(f, j), prev[j] = f;
    }
    T w; int i, j, l, s = 0, t = 0;
    while (true) {
      if (s == t) {
        l = s, w = dist[index[t++]];
        for (int k = t; k < m; ++k) {
          j = index[k]; T h = dist[j];
          if (h <= w) {
            if (h < w) t = s, w = h;
            index[k] = index[t], index[t++] = j;
          }
        }
        for (int k = s; k < t; ++k) {
          j = index[k];
          if (R[j] < 0) goto augment;
        }
      }
      int q = index[s++], i = R[q];
      for (int k = t; k < m; ++k) {
        j = index[k];
        T h = residue(i, j) - residue(i, q) + w;
        if (h < dist[j]) {
          dist[j] = h, prev[j] = i;
          if (h == w) {
            if (R[j] < 0) goto augment;
            index[k] = index[t], index[t++] = j;
          }
        }
      }
    }
    augment:
    for (int k = 0; k < l; ++k) v[index[k]] += dist[index[k]] - w;
    do {
      R[j] = i = prev[j], swap(j, L[i]);
    } while (i ^ f);
  }
  T ret = 0;
  for (int i = 0; i < n; ++i) ret += c[i][L[i]];
```

```
    return {ret, L};
}
```

3.12 kuhn

```
/***
 * call init at the start of every test case
 * matchL[x] = y means node x of left side is matched to node y of right side
 * matchR[y] = x means node y of right side is matched to node x of left side
 * y is in G[x] if there is an edge between node x and node y
 Node x is in the left and node y is in the right side
 * worst case complexity V*E
 ***/
namespace bpm{
  const int L = 105;
  const int R = 105;
  vector <int> G[L];
  int matchR[R], matchL[L], vis[L], it;
  /// n = number of nodes in the left side
  void init(int n) {
    SET(matchL), SET(matchR), CLR(vis);
    it = 1;
    for(int i=1;i<=n;i++) G[i].clear();
  }
  inline void addEdge(int u,int v) { G[u].pb(v); }
  bool dfs(int s) {
    vis[s] = it;
    for(auto x : G[s]) {
      if( matchR[x] == -1 or (vis[matchR[x]] != it and dfs(matchR[x])) ) {
        matchL[s] = x; matchR[x] = s;
        return true;
      }
    }
    return false;
  }
  int solve() {
    int cnt = 0;
    for(int i=1;i<=n;i++) {
      if(dfs(i)) cnt++, it++;
    }
    return cnt;
  }
}
```

3.13 lca

```
// Initial sp_par with -1
int LCA(int a, int b)
{
    if(depth[a]<depth[b]) swap(a,b);
    for(int i=LG-1;i>=0;i--) if(sp_par[a][i]!=-1&&depth[sp_par[a][i]]>=depth[b]) a=sp_par[a][i];
    if(a==b) return a;
    for(int i=LG-1;i>=0;i--) if(sp_par[a][i]!=-1&&sp_par[b][i]!=-1&&sp_par[a][i]!=sp_par[b][i]) a=sp_par[a][i],b=sp_par[b][i];
    return sp_par[a][0];
}
```

3.14 mcmf

```
/**** 1 BASED NODE INDEXING. Comp : E * flow ***/
namespace mcmf {
  using T = int;
  const T INF = ?; // 0x3f3f3f3f or 0x3f3f3f3f3f3f3f3fLL
  const int MAX = ?; // maximum number of nodes

  int  n, src, snk;
  T dis[MAX], mCap[MAX];
  int par[MAX], pos[MAX];
  bool vis[MAX];
  struct Edge{
    int to, rev_pos;
    T cap, cost, flow;
  };
  vector <Edge> ed[MAX];
  void init(int _n, int _src, int _snk) {
    n = _n, src = _src, snk = _snk;
    for(int i=1;i<=n;i++) ed[i].clear();
  }
  void addEdge(int u, int v, T cap, T cost) {
    Edge a = {v, (int)ed[v].size(), cap, cost, 0};
    Edge b = {u, (int)ed[u].size(), 0, -cost, 0};
    ed[u].pb(a);
    ed[v].pb(b);
  }
  inline bool SPFA(){
    CLR(vis);
    for(int i=1; i<=n; i++) mCap[i] = dis[i] = INF;
    queue <int> q;
    dis[src] = 0;
    vis[src] = true; /// src is in the queue now
    q.push(src);
    while(!q.empty()){
      int u = q.front();
      q.pop();
      vis[u] = false; /// u is not in the queue now
      for(int i=0; i<(int)ed[u].size(); i++) {
        Edge &e = ed[u][i];
        int v = e.to;
        if(e.cap > e.flow && dis[v] > dis[u] + e.cost){
          dis[v] = dis[u] + e.cost;
          par[v] = u;
          pos[v] = i;
          mCap[v] = min(mCap[u],e.cap - e.flow);
          if(!vis[v]) {
            vis[v] = true;
            q.push(v);
          }
        }
      }
    }
    return (dis[snk] != INF);
  }
  inline pair <T, T> solve() {
    T F = 0, C = 0, f;
    int u, v;
    while(SPFA()){
      u = snk;
      f = mCap[u];
      F += f;
      while(u!=src){
        v = par[u];
        ed[v][pos[u]].flow += f; // edge of v-->u increases
        ed[u][ed[v][pos[u]].rev_pos].flow -= f;
        u = v;
      }
      C += dis[snk] * f;
    }
    return make_pair(F,C);
  }
}
```

4 Math

4.1 FloorSum

```
// O(log a) sum^n floor(ax + b / c) = f
long long FloorSumAP(long long a, long long b, long long c, long long n){
  if(!a) return (b / c) * (n + 1);
  if(a >= c or b >= c) return ( ( n * (n + 1) ) / 2) * (a / c) + (n + 1) * (b / c) + FloorSumAP(a % c, b % c, c, n);
  long long m = (a * n + b) / c;
  return m * n - FloorSumAP(c, c - b - 1, a, m - 1);
}
// O(log a) sum^n x * floor(ax + b / c) = g, sum^n floro(ax + b / c)^2 = h
struct dat {
  long long f, g, h;
  dat(long long f = 0, long long g = 0, long long h = 0) : f(f), g(g), h(h) {};
};
long long mul(long long a, long long b){
  return (a * b) % MOD;
}
dat query(long long a, long long b, long long c, long long n){
  if(!a) return {mul(n + 1, b / c), mul(mul(mul(b / c, n), n + 1), inv2), mul(mul(n + 1, b / c), b /c)};
  long long f, g, h;
  dat nxt;
  if(a >= c or b >= c){
    nxt = query(a % c, b % c, c, n);
    f = (nxt.f + mul(mul(mul(n, n + 1), inv2), a / c) + mul(n + 1, b / c)) % MOD;
    g = (nxt.g + mul(a / c, mul(mul(n, n + 1), mul(2 * n + 1, inv6))) + mul(mul(b / c, mul(n, n + 1)), inv2)) % MOD;
    h = (nxt.h + 2 * mul(b / c, nxt.f) + 2 * mul(a / c, nxt.g) + mul(mul(a / c, a / c), mul(mul(n, n + 1), mul(2 * n + 1, inv6))) + mul(mul(b / c, b / c), n + 1) + mul(mul(a / c, b / c), mul(n, n + 1)) ) % MOD;
    return {f, g, h};
  }
  long long m = (a * n + b ) / c;
  nxt = query(c, c - b - 1, a, m - 1);
  f = (mul(m, n) - nxt.f) % MOD;
  g = mul( mul(m, mul(n, n + 1)) - nxt.h - nxt.f, inv2);
```

```
    h = (mul(n, mul(m, m + 1)) - 2 * nxt.g - 2 * nxt.f - f) % MOD;
    return {f, g, h};
}
```

4.2 NOD

```
N = input()
primes = array containing primes till 10^6
ans = 1
for all p in primes :
    if p*p*p > N:
        break
    count = 1
    while N divisible by p:
        N = N/p
        count = count + 1
    ans = ans * count
if N is prime:
    ans = ans * 2
else if N is square of a prime:
    ans = ans * 3
else if N != 1:
    ans = ans * 4
```

4.3 Pollard Rho

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned long long ull;
namespace Rho {
  ull mul (ull a, ull b, ull mod) {
    ll ret = a * b - mod * (ull) (1.L / mod * a * b);
    return ret + mod * (ret < 0) - mod * (ret >= (ll) mod);
  }
  ull bigMod (ull a, ull e, ull mod) {
    ull ret = 1;
    while (e) {
      if (e & 1) ret = mul(ret, a, mod);
      a = mul(a, a, mod), e >>= 1;
    }
    return ret;
  }
  bool isPrime (ull n) {
    if (n < 2 or n % 6 % 4 != 1) return (n | 1) == 3;
    ull a[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    ull s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull x : a) {
      ull p = bigMod(x % n, d, n), i = s;
      while (p != 1 and p != n - 1 and x % n and i--) p = mul(p, p, n);
      if (p != n - 1 and i != s) return 0;
    }
    return 1;
  }
  ull pollard (ull n) {
    auto f = [&] (ull x) {return mul(x, x, n) + 1;};
    ull x = 0, y = 0, t = 0, prod = 2, i = 1, q;
    while (t++ % 40 or __gcd(prod, n) == 1) {
      if (x == y) x = ++i, y = f(x);
      if ((q = mul(prod, max(x, y) - min(x, y), n))) prod = q;
      x = f(x), y = f(f(y));
    }
    return __gcd(prod, n);
  }
  vector <ull> factor (ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
  }
};
int t; ll n;
int main() {
  cin >> t;
  while (t--) {
    scanf("%lld", &n);
    vector <ull> facs = Rho::factor(n);
    sort(facs.begin(), facs.end());
    printf("%d", (int) facs.size());
    for (auto it : facs) printf(" %llu", it);
    puts("");
  }
  return 0;
}
```

4.4 catalan

```
//Recursive
const int MOD = ....
const int MAX = ....
int catalan[MAX];
void init() {
    catalan[0] = catalan[1] = 1;
    for (int i=2; i<=n; i++) {
        catalan[i] = 0;
        for (int j=0; j < i; j++) {
            catalan[i] += (catalan[j] * catalan[i-j-1]) % MOD;
            if (catalan[i] >= MOD) {
                catalan[i] -= MOD;
            }
        }
    }
}
//Analytical formula:
ans = ncr(2*n,n)-ncr(2*n,n-1)= ncr(2*n,n)/(n+1)
```

4.5 crt

```
//r[i][j]= inverse of p[i] modulo p[j]
//ans= x[0]+x[1]*p[0]+x[2]*(p[0]*p[1])+...+x[k-1]*(p[0]*p[1]*p[2]*...*p[k-2])
//ans %= ((p[0]*p[1]*p[2]*...*p[k-1])
for (int i = 0; i < k; ++i) {
    x[i] = a[i];
    for (int j = 0; j < i; ++j) {
        x[i] = r[j][i] * (x[i] - x[j]);

        x[i] = x[i] % p[i];
        if (x[i] < 0)
            x[i] += p[i];
    }
}
ll mul= p[0],res=x[0],tot=1;
F(i,0,k)tot *= p[i];
F(i,1,k){
 res+= x[i]*mul;
 res %= tot;
 mul *= p[i];
}
res %= mul;
return res;
```

4.6 derangement

```
int derangement(int n)
{
if(!n) return n;
if(n <= 2) return n-1;
return (n-1)*(derangement(n-1) + derangement(n-2));
}
```

4.7 diophantine

```
void print_solution(int a, int b, int c)
{
    int x, y;
    if (a == 0 && b == 0) {
        if (c == 0) {
            cout << "Infinite Solutions Exist" << endl;
        }
        else {
            cout << "No Solution exists" << endl;
        }
    }
    int gcd = gcd_extend(a, b, x, y);
    if (c % gcd != 0) {
        cout<< "No Solution exists"<< endl;
    }
    else {
        cout << "x = " << x * (c / gcd)<< ", y = " << y * (c / gcd)<< endl;
    }
}
```

4.8 discrete_log

```
// Returns minimum x for which a ^ x % m = b % m, a and m are coprime.
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 1ll * a) % m;
    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 1ll * a) % m;
    }
    for (int p = 1, cur = 1; p <= n; ++p) {
        cur = (cur * 1ll * an) % m;
        if (vals.count(cur)) {
```

```
            int ans = n * p - vals[cur];
            return ans;
        }
    }
    return -1;
}
```

4.9 factorial_mod_p

```
// O(log_p(n)) gives me n! % p for large n, p
int factmod(int n, int p) {
    vector<int> f(p);
    f[0] = 1;
    for (int i = 1; i < p; i++)
        f[i] = f[i-1] * i % p;
    int res = 1;
    while (n > 1) {
        if ((n/p) % 2)
            res = p - res;
        res = res * f[n%p] % p;
        n /= p;
    }
    return res;
}
```

4.10 fft

```
typedef complex<double> base;
#define PI acos(-1)
void fft(vector<base> &a, bool invert){
    int n = (int)a.size();
    for (int i = 1, j = 0; i < n; ++i){
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j)swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <<= 1){
        double ang = 2 * PI / len * (invert ? -1 :
 ↪  1);
        base wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len){
            base w(1);
            for (int j = 0; j < len / 2; ++j){
                base u = a[i + j], v = a[i + j +
 ↪  len / 2] * w;
                a[i + j] = u + v;
                a[i + j + len / 2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert) for (int i = 0; i < n; ++i) a[i] /=
 ↪  n;
}
void multiply(const vector<int> &a, const
 ↪  vector<int> &b, vector<int> &res){
    vector<base> fa(a.begin(), a.end()),
 ↪  fb(b.begin(), b.end());
    size_t n = 1;
    while (n < max(a.size(), b.size())) n <<= 1;
    n <<= 1;
    fa.resize(n), fb.resize(n);
    fft(fa, false), fft(fb, false);
    for (size_t i = 0; i < n; ++i) fa[i] *= fb[i];
    fft(fa, true); res.resize(n);
    for (size_t i = 0; i < n; ++i) res[i] =
 ↪  int(fa[i].real() + 0.5);
}
```

4.11 gauss_eliminition

```
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to
 ↪  be infinity or a big number
int gauss (vector < vector<double> > a,
 ↪  vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] /
 ↪  a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}
//modular
int gauss (vector < bitset<N> > a, int n, int m,
 ↪  bitset<N> & ans) {
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        for (int i=row; i<n; ++i)
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        if (! a[row][col])
            continue;
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row && a[i][col])
                a[i] ^= a[row];
            ++row;
    }
        // The rest of implementation is the same
 ↪  as above
}
//rank
const double EPS = 1E-9;
int compute_rank(vector<vector<double>> A) {
    int n = A.size();
    int m = A[0].size();
    int rank = 0;
    vector<bool> row_selected(n, false);
    for (int i = 0; i < m; ++i) {
        int j;
        for (j = 0; j < n; ++j) {
            if (!row_selected[j] && abs(A[j][i]) >
 ↪  EPS)
                break;
        }
        if (j != n) {
            ++rank;
            row_selected[j] = true;
            for (int p = i + 1; p < m; ++p)
                A[j][p] /= A[j][i];
            for (int k = 0; k < n; ++k) {
                if (k != j && abs(A[k][i]) > EPS) {
                    for (int p = i + 1; p < m; ++p)
                        A[k][p] -= A[j][p] *
 ↪  A[k][i];
                }
            }
        }
    }
    return rank;
}
```

4.12 gen_all_k_combs

```
vector<int> ans;
void gen(int n, int k, int idx, bool rev) {
    if (k > n || k < 0)
        return;
    if (!n) {
        for (int i = 0; i < idx; ++i) {
            if (ans[i])
                cout << i + 1;
        }
        cout << "\n";
        return;
    }
    ans[idx] = rev;
    gen(n - 1, k - rev, idx + 1, false);
    ans[idx] = !rev;
    gen(n - 1, k - !rev, idx + 1, true);
}
void all_combinations(int n, int k) {
    ans.resize(n);
    gen(n, k, 0, false);
}
```

4.13 matrix_expo

```
struct Matrix{
 int sz;
```

```
 vector<vector<long long> > mat;
 Matrix(vector<vector<long long> > m){
  sz=m.size();
  mat=m;
 }
 Matrix operator *(Matrix &other){
  Matrix product(mat);
  for(int i=0;i<sz;i++){
   for(int j=0;j<sz;j++)product.mat[i][j]=0;
  }
  for(int i=0;i<sz;i++){
   for(int j=0;j<sz;j++){
    for(int k=0;k<sz;k++)product.mat[i][k]=(product.mat[i][k]+mat[i][j]*other.mat[j][k])%mod;
   }
  }
  return product;
 }
};
Matrix expo(Matrix &m,ll n){
 Matrix res(m.mat);
 for(int i=0;i<m.sz;i++){
  for(int j=0;j<m.sz;j++){
   if(i==j)res.mat[i][i]=1;
   else res.mat[i][j]=0;
  }
 }
 while(n){
  if(n&1)res=res*m;
  n >>= 1;
  m=m*m;
 }
 return res;
}
```

4.14 next_lexicographical_k_comb

```
bool next_combination(vector<int>& a, int n) {
    int k = (int)a.size();
    for (int i = k - 1; i >= 0; i--) {
        if (a[i] < n - k + i + 1) {
            a[i]++;
            for (int j = i + 1; j < k; j++)
                a[j] = a[j - 1] + 1;
            return true;
        }
    }
    return false;
}
```

4.15 ntt

```
const int mod = 998244353;
const int root = 15311432;
const int k = 1 << 23;
int root_1;
vector<int> rev;
void pre(int sz){
    root_1 = bigmod(root, mod - 2, mod);
    if (rev.size() == sz) return;
    rev.resize(sz);
    rev[0] = 0;
    int lg_n = __builtin_ctz(sz);
    for (int i = 1; i < sz; ++i)
    rev[i]=(rev[i>>1]>>1)|((i&1)<<(lg_n-1));
}
void fft(vector<int> &a, bool inv){
    int n = a.size();
    for (int i = 1; i < n - 1; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int len = 2; len <= n; len <<= 1) {
        int wlen = inv ? root_1 : root;
        for (int i = len; i < k; i <<= 1) wlen = 1ll * wlen * wlen % mod;
        for (int st = 0; st < n; st += len){
            int w = 1;
            for (int j = 0; j < len / 2; j++){
                int ev = a[st + j];
                int od = 1ll * a[st + j + len / 2] * w % mod;
                a[st + j] = ev + od < mod ? ev + od : ev + od - mod;
                a[st + j + len / 2] = ev - od >= 0 ? ev - od : ev - od + mod;
                w = 1ll * w * wlen % mod;
            }
        }
    }
    if (inv){
        int n_1 = bigmod(n, mod - 2, mod);
        for (int &x : a) x = 1ll * x * n_1 % mod;
    }
}
vector<int> mul(vector<int> &a, vector<int> &b){
    int n = a.size(), m = b.size(), sz = 1;
    while (sz < n + m - 1) sz <<= 1;
    vector<int> x(sz), y(sz), z(sz);
    for (int i = 0; i < sz; ++i){
        x[i] = i < n ? a[i] : 0;
        y[i] = i < m ? b[i] : 0;
    }
    pre(sz);fft(x, 0);fft(y, 0);
    for (int i = 0; i < sz; ++i) z[i] = 1ll * x[i] * y[i] % mod;
    fft(z, 1);z.resize(n + m - 1);
    return z;
}
```

4.16 seg_sieve

```
/*
Segmented Sieve
This code was for 1 <= a <= b <= 2^31-1
Change variable types appropriately.
*/
bool notPrime[ ? ];
void segmented_sieve(int a, int b)
{
    int p, f;
    mem(notPrime, 0);
    for (int i = 0; i < tot_prime; i++)
    {
        p = prime[i];
        if (a % p == 0)
            f = a;
        else
            f = (a - (a % p) + p);
        f = max(p * p, f);
        for (unsigned j = f; j <= b; j += p)
            notPrime[j - a] = true;
    }
    if (a == 1)
        notPrime[0] = 1;
}
```

4.17 stirling

```
/// Finds the number ways to put n balls into k indistinguishable boxes such
that no box is empty.
int stirling2(int n, int k)
{
 if(n < k)
 return 0;
 if(k == 1)
 return 1;
 if(dp[n][k] == dp[n][k])
 return dp[n][k];
 return dp[n][k] = stirling2(n-1,k-1) + stirling2(n-1,k)*k;
}
/// Finds the number of ways to put n elements into k cycles where no cycle
is empty
int stirling1(int n, int k)
{
 dp[n][k] = stirling1(n-1,k-1) + stirling(n-1,k)*n-1;
}
```

4.18 stirling_number_of_the_second_kind

```
// 1 / k! * sum (-1)^i nCr(k, i) * (k - i) ^ n
ll f(int n, int k) {
    ll res = 0;
    for (int i = 0; i < k; ++i) {
        if (i & 1) res = (res - nCr(k, i) * bp(k - i, n, mod) % mod + mod) % mod;
        else res = (res + nCr(k, i) * bp(k - i, n, mod) % mod) % mod;
    }
    if (res < 0) res += mod;
    return res * ifac[k] % mod;
}
```

4.19 totient

```
int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    phi[0] = 0;
```

```
    phi[1] = 1;
    for (int i = 2; i <= n; i++)
        phi[i] = i;
    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}
```

4.20 xor_basis

```
int basis[d]; // basis[i] keeps the mask of the
 ↪ vector whose f value is i
int sz;
void insertVector(int mask) {
 for (int i = 0; i < d; i++) {
  if ((mask & 1 << i) == 0) continue;
  if (!basis[i]) { // If there is no basis vector
   with the i'th bit set, then insert this vector
 ↪ into the basis
   basis[i] = mask;
   ++sz;
   return;
  }
  mask ^= basis[i]; // Otherwise subtract the basis
 ↪ vector from this vector
 }
}
```

5 Misc

5.1 DC_Optimization

```
void compute(int L, int R, int optL, int optR){
  if(L > R) return;
  int M = L + R >> 1;
  pair<ll, int> best(1LL << 60, -1);
  for(int k = optL; k <= min(M, optR); k++){
    best = min(best, {dp[prv][k] + C[k + 1][M], k});
  }
  dp[now][M] = best.ff;
  compute(L, M - 1, optL, best.ss);
  compute(M + 1, R, best.ss, optR);
}
```

5.2 Ordered Multiset

```
#include <bits/stdtr1c++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
/// Treap supporting duplicating values in set
/// Maximum value of treap * ADD must fit in long
 ↪ long
struct Treap{ /// hash = 96814
  int len;
  const int ADD = 1000010;
  const int MAXVAL = 1000000010;
  tr1::unordered_map <long long, int> mp; ///
 ↪ Change to int if only int in treap
  tree<long long, null_type, less<long long>,
    rb_tree_tag, tree_order_statistics_node_update>
 ↪ T;
  Treap(){
    len = 0;
    T.clear(), mp.clear();
  }
  inline void clear(){
    len = 0;
    T.clear(), mp.clear();
  }
  inline void insert(long long x){
    len++, x += MAXVAL;
    int c = mp[x]++;
    T.insert((x * ADD) + c);
  }
  inline void erase(long long x){
    x += MAXVAL;
    int c = mp[x];
    if (c){
      c--, mp[x]--, len--;
      T.erase((x * ADD) + c);
    }
  }
  /// 1-based index, returns the K'th element in
 ↪ the treap, -1 if none exists
  inline long long kth(int k){
    if (k < 1 || k > len) return -1;
    auto it = T.find_by_order(--k);
    return ((*it) / ADD) - MAXVAL;
  }
  /// Count of value < x in treap
  inline int count(long long x){
    x += MAXVAL;
    int c = mp[--x];
    return (T.order_of_key((x * ADD) + c));
  }
  /// Number of elements in treap
  inline int size(){
    return len;
  }
};
```

5.3 check

```
#!/bin/bash
g++ a.cpp -o a
g++ ac.cpp -o ac
g++ gen.cpp -o gen
for (( c=0 ; c<1000 ; c++ ))
do
 echo $c
 ./gen > inp
 ./a < inp > out1
 ./ac < inp > out2
 diff out1 out2
 if [ $? -ne 0 ]
 then
  echo "-----Input----"
  cat inp
  echo "------Output----"
  cat out
  echo "-----Accepted----"
  cat out1
 break
 fi
done
```

5.4 compile

```
alias rn='g++ -Wall -Wextra -pedantic -std=c++11
    -O2 -Wshadow -Wformat=2 -Wfloat-equal
    -Wconversion -Wlogical-op -Wshift-overflow=2
    -Wduplicated-cond -Wcast-qual -Wcast-align
    -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
    -D_FORTIFY_SOURCE=2 -fsanitize=address
    -fsanitize=undefined -fno-sanitize-recover
    -fstack-protector'
```

5.5 debug

```
struct debug {
#define contPrint { *this << "["; \
  int f = 0; for(auto it : x) { *this << (f?",
 ↪ ":""); *this << it; f = 1;} \
  *this << "]"; return *this;}
  ~debug(){cerr << endl;}
  template<class c> debug& operator<<(c x) {cerr <<
 ↪ x; return *this;}
  template<class c, class d>
    debug& operator<<(pair<c, d> x) {*this << "("
 ↪ << x.first << ", " << x.second << ")";
      return *this;}
  template<class c> debug& operator<<(vector<c> x)
 ↪ contPrint;
#undef contPrint
};
#define dbg(x) "[" << #x << ": " << x << "]  "
#define Wa() cerr << "[LINE: " << __LINE__ << "] ->
 ↪ "; debug() <<
#define FASTIO ios_base::sync_with_stdio(false);
 ↪ cin.tie(NULL);
```

5.6 pragma

```
// Pragmas
#pragma comment(linker, "/stack:200000000")
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx,avx2,fma")
```

5.7 random

```
// shuffle(v.begin(), v.end(),
 ↪ default_random_engine(rnd(1, 1000)));
mt19937 rng(chrono::steady_clock::now().time_since_
 ↪ epoch().count());
ll rnd(ll l, ll r) {
    return uniform_int_distribution<ll>(l, r) (rng);
}
```

5.8 vimrc

```
imap jk <Esc>
set nu
set mouse=a
set autoindent
set tabstop=4
```

```
set shiftwidth=4
set smartindent
set relativenumber
set laststatus=2
set hlsearch
let mapleader = " "
nnoremap <leader>s :w<Enter>
nnoremap <leader>y ggVG"+y<CR>
syntax on
vnoremap <leader>/ :s!^!//!<CR> :noh <CR>
vnoremap <leader>u :s!^//!!<CR>
nnoremap <leader>/ :s!^!//!<CR> :noh <CR>
nnoremap <leader>u :s!^//!!<CR>
```

6 Notes

6.1 Counting

1. **Lucas Theorem**
For non-negative integers m and n and a prime p, the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod p, \tag{1}$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \le n$.

2. **Stirling Numbers of the first kind**
$S(n,k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S(n,k) = (n-1) \cdot S(n-1,k) + S(n-1,k-1) \tag{2}$$

where, $S(0,0) = 1, S(n,0) = S(0,n) = 0$

$$\sum_{k=0}^{n} S(n,k) = n! \tag{3}$$

3. **Stirling Numbers of the second kind**
$S(n,k) \cdot k! =$ number of ways to color n nodes using colors from 1 to k such that each color is used at least once.
An r-associated Stirling number of the second kind is the number of ways to partition a set of n objects into k subsets, with each subset containing at least r elements. It is denoted by $S_r(n,k)$ and obeys the recurrence relation.

$$S_r(n+1,k) = kS_r(n,k) + \binom{n}{r-1} S_r(n-r+1,k-1) \tag{4}$$

4. **Bell Numbers**
Counts the number of partitions of a set.

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} \cdot B_k \tag{5}$$

$B_n = \sum_{k=0}^{n} S(n,k)$, where $S(n,k)$ is stirling number of second kind.

5. **Some identities**

Vandermonde's Identify: $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

Hockey-Stick Identify: $n, r \in N, n > r, \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

Involutions: permutations such that $p^2 =$ identity permutation. $a_0 = a_1 = 1$ and $a_n = a_{n-1} + (n-1)a_{n-2}$ for $n > 1$.

6.2 Fibonacci

Let A, B and n be integer numbers.

$$k = A - B$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \tag{6}$$

$$\sum_{i=0}^{n} F_i^2 = F_{n+1} F_n \tag{7}$$

$$\sum_{i=0}^{n} F_i F_{i+1} = F_{n+1}^2 - (-1)^n \tag{8}$$

$$\sum_{i=0}^{n} F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \tag{9}$$

$$GCD(F_m, F_n) = F_{GCD(m,n)} \tag{10}$$

$$\sum_{0 \le k \le n} \binom{n-k}{k} = Fib_{n+1} \tag{11}$$

$$\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1 \tag{12}$$

6.3 NumberTheory

$$\sum_{k=1}^{n} \frac{1}{\gcd(k,n)} = \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{1}{n} \sum_{d|n} d \cdot \phi(d) \tag{13}$$

$$\sum_{k=1}^{n} \frac{k}{\gcd(k,n)} = \frac{n}{2} \cdot \frac{1}{n} \cdot \sum_{d|n} d \cdot \phi(d) \tag{14}$$

$$\sum_{k=1}^{n} \frac{n}{\gcd(k,n)} = 2 * \sum_{k=1}^{n} \frac{k}{\gcd(k,n)} - 1, \text{ for, } n > 1 \tag{15}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} [\gcd(i,j) = 1] = \sum_{d=1}^{n} \mu(d) \left\lfloor \frac{n}{d} \right\rfloor^2 \tag{16}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \gcd(i,j) = \sum_{d=1}^{n} \phi(d) \left\lfloor \frac{n}{d} \right\rfloor^2 \tag{17}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} i \cdot j [\gcd(i,j) = 1] = \sum_{i=1}^{n} \phi(i) i^2 \tag{18}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{lcm}(i,j) = \sum_{l=1}^{n} \left(\frac{\left(1 + \left\lfloor \frac{n}{l} \right\rfloor\right)\left(\left\lfloor \frac{n}{l} \right\rfloor\right)}{2} \right)^2 = \sum_{d|l} \mu(d) l d \tag{19}$$

7 String

7.1 aho_corasick

```
/***
  Given n patterns and a text T, for every pattern
  you have to output the number of times that
↪   pattern
  appears in the text.
 * Don't forget to call build() after adding all
↪   the patterns
 to the Aho Corasick trie.
 * ans[i] contains the number of occurrences of the
↪   i'th pattern
 * link[x] = y means there is a suffix link from
↪   node x to node y
 * out_link[x] = y means we can go from x to y
↪   using the suffix links
 suppose the path is as follows : x , a , b, c,
↪   ..., y
 No pattern ends in node a, b, c, ... but some
↪   pattern ends at node y.
 * After a call to build(), the trie becomes a
↪   DAG(except node 0)
 next[x][c] = y means if we are currently at node x
↪   and the character
 c arrives, we will go to node y.

 * Suppose sum of the length of the characters is
↪   N. Text length is also
 at most N. If all the patterns are unique, total
↪   number of occurrences
 of all the patterns will not be more than " N
↪   sqrt(N) ".
 ***/
#include <bits/stdc++.h>
using namespace std;
const int N = ?; /// Total number of characters in
↪   pattern
const int A = ?; /// Alphabet size
struct AC {
  int nd, pt;
  int next[N][A], link[N], out_link[N], cnt[N],
↪   ans[N];
  vector <int> ed[N], out[N];
  AC(): nd(0), pt(0) { node(); }
  int node() {
    memset(next[nd], 0, sizeof next[nd]);
    link[nd] = out_link[nd] = cnt[nd] = 0;
    ed[nd].clear(), out[nd].clear();
    return nd++;
  }
  void clear() {
    nd = pt = 0;
    node();
  }
  inline int get(char c) { return c - 'a'; }
  void insert(const string &T) {
    int u = 0;
```

```
    for (char c : T) {
      if (!next[u][get(c)]) next[u][get(c)] =
↪   node();
      u = next[u][get(c)];
    }
    ans[pt] = 0;
    out[u].push_back(pt++);
  }
  void build() {
    queue <int> q;
    for (q.push(0); !q.empty(); ) {
      int u = q.front();
      q.pop();
      for (int c = 0; c < A; ++c) {
        int v = next[u][c];
        if (!v) next[u][c] = next[link[u]][c];
        else {
          link[v] = u ? next[link[u]][c] : 0;
          out_link[v] = out[link[v]].empty() ?
↪   out_link[link[v]] : link[v];
          ed[link[v]].push_back(v);
          q.push(v);
        }
      }
    }
  }
  void dfs(int s) {
    for(int x : ed[s]) dfs(x), cnt[s] += cnt[x];
    for(int e : out[s]) ans[e] = cnt[s];
  }
  void traverse(const string &S) {
    int u = 0;
    for (char c : S) {
      u = next[u][get(c)];
      cnt[u]++;
    }
    dfs(0);
  }
};
char str[1000010], pat[505];
int main() {
  //    freopen("in.txt","r",stdin);
  AC aho;
  int t,T;
  scanf("%d",&T);
  for(int t=1;t<=T;t++) {
    int n;
    scanf("%d",&n);
    scanf("%s",str);
    for(int i=1;i<=n;i++) {
      scanf("%s",pat);
      aho.insert(pat);
    }
    aho.build();
    aho.traverse(str);
    printf("Case %d:\n",t);
    for(int i=0;i<n;i++) {
      printf("%d\n",aho.ans[i]);
    }
    aho.clear();
  }
  return 0;
}
```

7.2 hash

```
struct Hash {
  struct base {
    string s; int b, mod;
    vector<int> hash, p;
    void init(string &_s, int _b, int _mod) { // b
↪   > 26, prime.
      s = _s; b = _b, mod = _mod;
      hash.resize(s.size());
      p.resize(s.size());
      hash[0] = s[0] - 'A' + 1; p[0] = 1;
      for(int i = 1; i < s.size(); ++i) {
        hash[i] = (ll) hash[i - 1] * b % mod;
        hash[i] += s[i] - 'A' + 1;
        if(hash[i] >= mod) hash[i] -= mod;
        p[i] = (ll) p[i - 1] * b % mod;
      }
    }
    int get(int l, int r) {
      int ret = hash[r];
      if(l) ret -= (ll) hash[l - 1] * p[r - l + 1]
↪   % mod;
      if(ret < 0) ret += mod;
      return ret;
    }
  } h[2];
  void init(string &s) {
    h[0].init(s, 29, 1e9+7);
    h[1].init(s, 31, 1e9+9);
  }
  pair<int, int> get(int l, int r) {
    return { h[0].get(l, r), h[1].get(l, r) };
  }
} H;
```

7.3 hash_segtree

```
/***
 * Everything is 0 based
 * Call precal() once in the program
 * Call update(1,0,n-1,i,j,val) to update the value
↪   of position
 i to j to val, here n is the length of the string
 * Call query(1,0,n-1,L,R) to get a node containing
↪   hash
 of the position [L:R]
 * Before any update/query
 - Call init(str) where str is the string to be
↪   hashed
 - Call build(1,0,n-1)
 ***/
#define INVALID_CHAR       -1
namespace strhash {
  int n;
  const int MAX = 100010;
  int ara[MAX];
  const int MOD[] = {1067737007, 1069815139};
  const int BASE[] = {982451653, 984516781};

  int BP[2][MAX], CUM[2][MAX];
  void init(char *str) {
    n = strlen(str);
    for(int i=0;i<n;i++) ara[i] = str[i]-'0'+1; ///
↪   scale str[i] if needed
  }
  void precal() {
    BP[0][0] = BP[1][0] = 1;
    CUM[0][0] = CUM[1][0] = 1;
    for(int i=1;i<MAX;i++) {
      BP[0][i] = ( BP[0][i-1] * (long long) BASE[0]
↪   ) % MOD[0];
      BP[1][i] = ( BP[1][i-1] * (long long) BASE[1]
↪   ) % MOD[1];
      CUM[0][i] = ( CUM[0][i-1] + (long long)
↪   BP[0][i] ) % MOD[0];
      CUM[1][i] = ( CUM[1][i-1] + (long long)
↪   BP[1][i] ) % MOD[1];
    }
  }
  struct node {
    int sz;
    int h[2];
    node() {}
  } tree[4*MAX];
  int lazy[4*MAX];
  inline void lazyUpdate(int n,int st,int ed) {
    if(lazy[n]!=INVALID_CHAR){
      tree[n].h[0] = (lazy[n] * (long long)
↪   CUM[0][ed-st]) % MOD[0];
      tree[n].h[1] = (lazy[n] * (long long)
↪   CUM[1][ed-st]) % MOD[1];
      if(st!=ed){
        lazy[2*n] = lazy[n];
        lazy[2*n+1] = lazy[n];
      }
      lazy[n] = INVALID_CHAR;
    }
  }
  inline node Merge(node a,node b) {
    node ret;
    ret.h[0] = ( ( a.h[0] * (long long) BP[0][b.sz]
↪   ) + b.h[0] ) % MOD[0];
    ret.h[1] = ( ( a.h[1] * (long long) BP[1][b.sz]
↪   ) + b.h[1] ) % MOD[1];
    ret.sz = a.sz + b.sz;
    return ret;
  }
  inline void build(int n,int st,int ed) {
    lazy[n] = INVALID_CHAR;
    if(st==ed) {
      tree[n].h[0] = tree[n].h[1] = ara[st];
      tree[n].sz = 1;
      return;
    }
    int mid = (st+ed)>>1;
    build(n+n,st,mid);
    build(n+n+1,mid+1,ed);
    tree[n] = Merge(tree[n+n],tree[n+n+1]);
  }
  inline void update(int n,int st,int ed,int i,int
↪   j,int v) {
    lazyUpdate(n,st,ed);
    if(st>j or ed<i) return;
    if(st>=i and ed<=j) {
      lazy[n] = v;
      lazyUpdate(n,st,ed);
      return;
```

```cpp
    }
    int mid = (st+ed)>>1;
    update(n+n,st,mid,i,j,v);
    update(n+n+1,mid+1,ed,i,j,v);
    tree[n] = Merge(tree[n+n],tree[n+n+1]);
  }
  inline node query(int n,int st,int ed,int i,int j){
    lazyUpdate(n,st,ed);
    if(st>=i and ed<=j) return tree[n];
    int mid = (st+ed)/2;
    if(mid<i) return query(n+n+1,mid+1,ed,i,j);
    else if(mid>=j) return query(n+n,st,mid,i,j);
    else return Merge(query(n+n,st,mid,i,j),query(n+n+1,mid+1,ed,i,j));
  }
}
```

7.4 kmp

```cpp
// returns the longest proper prefix array of pattern p
// where lps[i]=longest proper prefix which is also suffix of p[0...i]
vector<int> build_lps(string p) {
  int sz = p.size();
  vector<int> lps;
  lps.assign(sz + 1, 0);
  int j = 0;
  lps[0] = 0;
  for(int i = 1; i < sz; i++) {
    while(j >= 0 && p[i] != p[j]) {
      if(j >= 1) j = lps[j - 1];
      else j = -1;
    }
    j++;
    lps[i] = j;
  }
  return lps;
}
vector<int>ans;
// returns matches in vector ans in 0-indexed
void kmp(vector<int> lps, string s, string p) {
  int psz = p.size(), sz = s.size();
  int j = 0;
  for(int i = 0; i < sz; i++) {
    while(j >= 0 && p[j] != s[i])
      if(j >= 1) j = lps[j - 1];
      else j = -1;
    j++;
    if(j == psz) {
      j = lps[j - 1];
      // pattern found in string s at position i-psz+1
      ans.push_back(i - psz + 1);
    }
    // after each loop we have j=longest common suffix of s[0..i] which is also prefix of p
  }
}
```

7.5 manachar

```cpp
vector<int> d1(n);  // maximum odd length palindrome centered at i
                    // here d1[i]=the palindrome has d1[i]-1 right characters from i
                    // e.g. for aba, d1[1]=2;
for (int i = 0, l = 0, r = -1; i < n; i++) {
  int k = (i > r) ? 1 : min(d1[l + r - i], r - i);
  while (0 <= i - k && i + k < n && s[i - k] == s[i + k]) {
    k++;
  }
  d1[i] = k--;
  if (i + k > r) {
    l = i - k;
    r = i + k;
  }
}
vector<int> d2(n);  // maximum even length palindrome centered at i
                    // here d2[i]=the palindrome has d2[i]-1 right characters from i
                    // e.g. for abba, d2[2]=2;
for (int i = 0, l = 0, r = -1; i < n; i++) {
  int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
  while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k]) {
    k++;
  }
  d2[i] = k--;
  if (i + k > r) {
    l = i - k - 1;
    r = i + k ;
  }
}
```

7.6 palindromic_tree

```cpp
const int A = 26;
const int N = 300010;
char s[N]; long long ans;
int last, ptr, nxt[N][A], link[N], len[N], occ[N];
void feed (int at) {
  while (s[at - len[last] - 1] != s[at]) last = link[last];
  int ch = s[at] - 'a', temp = link[last];
  while (s[at - len[temp] - 1] != s[at]) temp = link[temp];
  if (!nxt[last][ch]) {
    nxt[last][ch] = ++ptr, len[ptr] = len[last] + 2;
    link[ptr] = len[ptr] == 1 ? 2 : nxt[temp][ch];
  }
  last = nxt[last][ch], ++occ[last];
}
int main() {
  len[1] = -1, len[2] = 0, link[1] = link[2] = 1, last = ptr = 2;
  scanf("%s", s + 1);
  for (int i = 1, n = strlen(s + 1); i <= n; ++i) feed(i);
  for (int i = ptr; i > 2; --i) ans = max(ans, len[i] * 1LL * occ[i]), occ[link[i]] += occ[i];
  printf("%lld\n", ans);
  return 0;
}
```

7.7 persistant_trie

```cpp
/***
 Given an array of size n, each value in array can be expressed using
 20 bits.
Query : L R K
max(a_i ^ K) for L <= i <= R
 ***/
const int MAX = 200010; /// maximum size of array
const int B = 19; /// maximum number of bits in a value - 1
int root[MAX], ptr = 0;
struct node {
  int ara[2], sum;
  node() {}
} tree[ MAX * (B+1) ];
void insert(int prevnode, int &curRoot, int val) {
  curRoot = ++ptr;
  int curnode = curRoot;
  for(int i = B; i >= 0; i--) {
    bool bit = val & (1 << i);
    tree[curnode] = tree[prevnode];
    tree[curnode].ara[bit] = ++ptr;
    tree[curnode].sum += 1;
    prevnode = tree[prevnode].ara[bit];
    curnode = tree[curnode].ara[bit];
  }
  tree[curnode] = tree[prevnode];
  tree[curnode].sum += 1;
}
int find_xor_max(int prevnode, int curnode, int x) {
  int ans = 0;
  for(int i = B; i >= 0; i--) {
    bool bit = x & (1 << i);
    if(tree[tree[curnode].ara[bit ^ 1]].sum > tree[tree[prevnode].ara[bit ^ 1]].sum) {
      curnode = tree[curnode].ara[bit ^ 1];
      prevnode = tree[prevnode].ara[bit ^ 1];
      ans = ans | (1 << i);
    }
    else {
      curnode = tree[curnode].ara[bit];
      prevnode = tree[prevnode].ara[bit];
    }
  }
  return ans;
}
void solve() {
  int n, q, L, R, K;
  cin >> n;
  for(int i=1;i<=n;i++) cin >> ara[i];
  for(int i=1;i<=q;i++) {
    cin >> L >> R >> K;
    cout << find_xor_max(root[L-1],root[R],K) << endl;
  }
}
```

7.8 suffix_array

```cpp
// Everything is 0-indexed
char s[N]; // Suffix array will be built for this string
int SA[N], iSA[N]; // SA is the suffix array, iSA[i] stores the rank of the i'th suffix
int cnt[N], nxt[N]; // Internal stuff
bool bh[N], b2h[N]; // Internal stuff
int lcp[N]; // Stores lcp of SA[i] and SA[i + 1]; lcp[n - 1] = 0
int lcpSparse[LOGN][N]; // lcpSparse[i][j] = min(lcp[j], ..., lcp[j - 1 + (1 << i)])
void buildSA(int n) {
  for (int i = 0; i < n; i++) SA[i] = i;
  sort(SA, SA + n, [](int i, int j) { return s[i] < s[j]; });
  for (int i = 0; i < n; i++) {
    bh[i] = i == 0 || s[SA[i]] != s[SA[i - 1]];
    b2h[i] = 0;
  }
  for (int h = 1; h < n; h <<= 1) {
    int tot = 0;
    for (int i = 0, j; i < n; i = j) {
      j = i + 1;
      while (j < n && !bh[j]) j++;
      nxt[i] = j; tot++;
    } if (tot == n) break;
    for (int i = 0; i < n; i = nxt[i]) {
      for (int j = i; j < nxt[i]; j++) iSA[SA[j]] = i;
      cnt[i] = 0;
    }
    cnt[iSA[n - h]]++;
    b2h[iSA[n - h]] = 1;
    for (int i = 0; i < n; i = nxt[i]) {
      for (int j = i; j < nxt[i]; j++) {
        int s = SA[j] - h;
        if (s < 0) continue;
        int head = iSA[s];
        iSA[s] = head + cnt[head]++;
        b2h[iSA[s]] = 1;
      }
      for (int j = i; j < nxt[i]; j++) {
        int s = SA[j] - h;
        if (s < 0 || !b2h[iSA[s]]) continue;
        for (int k = iSA[s] + 1; !bh[k] && b2h[k]; k++) b2h[k] = 0;
      }
    }
    for (int i = 0; i < n; i++) {
      SA[iSA[i]] = i;
      bh[i] |= b2h[i];
    }
  }
  for (int i = 0; i < n; i++) iSA[SA[i]] = i;
}
void buildLCP(int n) {
  for (int i = 0, k = 0; i < n; i++) {
    if (iSA[i] == n - 1) {
      k = 0;
      lcp[n - 1] = 0;
      continue;
    }
    int j = SA[iSA[i] + 1];
    while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
    lcp[iSA[i]] = k;
    if (k) k--;
  }
}
void buildLCPSparse(int n) {
  for (int i = 0; i < n; i++) lcpSparse[0][i] = lcp[i];
  for (int i = 1; i < LOGN; i++) {
    for (int j = 0; j < n; j++) {
      lcpSparse[i][j] = min(lcpSparse[i - 1][j],
      lcpSparse[i - 1][min(n - 1, j + (1 << (i - 1)))]);
    }
  }
}
pair<int, int> minLCPRange(int n, int from, int minLCP) {
  int r = from;
  for (int i = LOGN - 1; i >= 0; i--) {
    int jump = 1 << i;
    if (r + jump < n and lcpSparse[i][r] >= minLCP) r += jump;
  }
  int l = from;
  for (int i = LOGN - 1; i >= 0; i--) {
    int jump = 1 << i;
    if (l - jump >= 0 and lcpSparse[i][l - jump] >= minLCP) l -= jump;
  }
  return make_pair(l, r);
}
```

7.9 suffix_automata

```cpp
/***
 * N = maximum possible string size
 * There won't be more that 2N - 1 nodes
 * There won't be more that 3N - 4 transitions
 * nodes are numbered from 0 to sz-1
 * scan sa::str
 * n = strlen(str)
 * call sa::build(n)
 * let's suppose sub_n represents the largest substring that is endpos equivalent to node n
 * cnt[i] = number of occurrences of sub_i in str
 * If terminal[i] = true, then sub_i is a suffix of str
 * There suffix link of node x to node y,
 Iff sub_y is the largest suffix of sub_x that is not endpos equivalent to node x.
***/
namespace sa{
  const int MAXN = 100005 << 1; /// 2 * maximum possible string size
  const int MAXC = 26; /// Size of the character set
  char str[MAXN];
  int n, sz, last; /// sz = number of nodes in the automaton( node indexing is 0 based)
  int len[MAXN], link[MAXN], ed[MAXN][MAXC], cnt[MAXN];
  bool terminal[MAXN];
  vector <int> G[MAXN];
  void init() {
    SET(ed[0]);
    len[0] = 0, link[0] = -1, sz = 1, last = 0, terminal[0] = false;
  }
  inline int scale(char c) { return c-'a'; }
  void extend(char c) {
    int cur = sz++;
    terminal[cur] = false;
    cnt[cur] = 1;
    SET(ed[cur]);
    len[cur] = len[last] + 1;
    int p = last;
    while (p != -1 && ed[p][c]==-1) {
      ed[p][c] = cur;
      p = link[p];
    }
    if (p == -1) link[cur] = 0;
    else {
      int q = ed[p][c];
      if (len[p] + 1 == len[q]) link[cur] = q;
      else {
        int clone = sz++;
        len[clone] = len[p] + 1;
        memcpy(ed[clone],ed[q],sizeof(ed[q]));
        link[clone] = link[q];
        while (p != -1 && ed[p][c] == q) {
          ed[p][c] = clone;
          p = link[p];
        }
        link[q] = link[cur] = clone;
        cnt[clone] = 0;
        terminal[clone] = false;
      }
    }
    last = cur;
  }
  /// needed to generate cnt[]
  void dfs(int s) {
    for(auto x : G[s]) dfs(x), cnt[s] += cnt[x];
  }
  void build() {
    init();
    int n = strlen(str);
    for(int i=0;i<n;i++) extend(scale(str[i]));
    /// construction of cnt[]
    for(int i=1;i<sz;i++) G[link[i]].pb(i);
    dfs(0);
    for(int i=0;i<sz;i++) G[i].clear();
    /// construction of terminal[]
    for(int i=last;i!=-1;i=link[i]) terminal[i] = true;
  }
}
```

7.10 trie

```cpp
#define N       200000 /// total number of characters given as input
#define S       26
int root,now;
```

```cpp
int nxt[N][S], cnt[N];
/// will be called from main
void init(){
  root = now = 1;
  CLR(nxt),CLR(cnt);
}
inline int scale(char ch) { return (ch - 'a'); }
inline void Insert(char s[],int sz){
  int cur = root, to;
  for(int i=0 ; i< sz ; i++){
    to = scale(s[i]) ;
    if( !nxt[cur][to] ) nxt[cur][to] = ++now;
    cur = nxt[cur][to];
  }
  cnt[cur]++;
}
inline bool Find(char s[],int sz){
  int cur = root, to;
  for(int i=0 ; i<sz ; i++){
    to = scale(s[i]) ;
    if( !nxt[cur][to] ) return false;
    cur = nxt[cur][to];
  }
  return (cnt[cur]!=0);
}
/// It's better to call the Delete() after checking if the
/// string we wanna delete actually exists in the trie
inline void Delete(char s[],int sz){
  int cur = root, to;
  for(int i=0 ; i<sz ; i++){
    to = scale(s[i]) ;
    cur = nxt[cur][to];
  }
  cnt[cur]--;
}
```

7.11 z_algo

```cpp
const int N = 100010;
char s[N];
int t, n, z[N];
int main() {
  scanf("%s", s);
  n = strlen(s), z[0] = n;
  int L = 0, R = 0;
  for (int i = 1; i < n; ++i) {
    if (i > R) {
      L = R = i;
      while (R < n && s[R - L] == s[R]) ++R;
      z[i] = R - L; --R;
    } else {
      int k = i - L;
      if (z[k] < R - i + 1) z[i] = z[k];
      else {
        L = i;
        while (R < n && s[R - L] == s[R]) ++R;
        z[i] = R - L; --R;
      }
    }
  }
}
```

8 Notes

8.1 Geometry

8.1.1 Triangles

Circumradius: $R = \frac{abc}{4A}$, Inradius: $r = \frac{A}{s}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

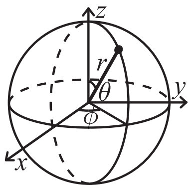
8.1.2 Quadrilaterals

With side lengths a,b,c,d, diagonals e,f, diagonals angle θ, area A and magic flux $F = b^2+d^2-a^2-c^2$:

$$4A = 2ef\cdot\sin\theta = F\tan\theta = \sqrt{4e^2f^2-F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac+bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

8.1.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \mathrm{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \mathrm{atan2}(y,x)$$

8.2 Sums

$1^2+2^2+3^2+\cdots+n^2 = \frac{n(2n+1)(n+1)}{6}$

$1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$

$1^4+2^4+3^4+\cdots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1}-1-\sum_{i=1}^{n}\left((i+1)^{m+1}-i^{m+1}-(m+1)i^m\right)\right]$

$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k n^{m+1-k}$

$\sum_{k=0}^{n} kx^k = (x-(n+1)x^{n+1}+nx^{n+2})/(x-1)^2$

8.3 Series

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots,(-\infty<x<\infty)$$

$$\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,(-1<x\le 1)$$

$$\sqrt{1+x} = 1+\frac{x}{2}-\frac{x^2}{8}+\frac{2x^3}{32}-\frac{5x^4}{128}+\ldots,(-1\le x\le 1)$$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots,(-\infty<x<\infty)$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots,(-\infty<x<\infty)$$

$$(x+a)^{-n} = \sum_{k=0}^{\infty}(-1)^k\binom{n+k-1}{k}x^k a^{-n-k}$$

8.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k\cdot(m^2-n^2),\ b = k\cdot(2mn),\ c = k\cdot(m^2+n^2),$$

with $m>n>0$, $k>0$, $m\perp n$, and either m or n even.

8.5 Number Theory

8.5.1 Primes

$p = 962592769$ is such that $2^{21}\mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p=2, a>2$, and there are $\phi(\phi(p^a))$ many. For $p=2, a>2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2\times\mathbb{Z}_{2^{a-2}}$.

8.5.2 Estimates

$\sum_{d|n} d = O(n\log\log n)$.

The number of divisors of n is at most around 100 for $n<5e4$, 500 for $n<1e7$, 2000 for $n<1e10$, 200 000 for $n<1e19$.

8.5.3 Perfect numbers

$n>1$ is called perfect if it equals sum of its proper divisors and 1. Even n is perfect iff $n = 2^{p-1}(2^p-1)$ and 2^p-1 is prime (Mersenne's). No odd perfect numbers are yet found.

8.5.4 Carmichael numbers

A positive composite n is a Carmichael number ($a^{n-1}\equiv 1 \pmod n$ for all a: $\gcd(a,n)=1$), iff n is square-free, and for all prime divisors p of n, $p-1$ divides $n-1$.

8.5.5 Mobius function

$\mu(1)=1$. $\mu(n)=0$, if n is not squarefree. $\mu(n)=(-1)^s$, if n is the product of s distinct primes. Let f, F be functions on positive integers. If for all $n\in N$, $F(n)=\sum_{d|n}f(d)$, then $f(n)=\sum_{d|n}\mu(d)F(\frac{n}{d})$, and vice versa. $\phi(n)=\sum_{d|n}\mu(d)\frac{n}{d}$. $\sum_{d|n}\mu(d)=1$.

If f is multiplicative, then $\sum_{d|n}\mu(d)f(d)=\prod_{p|n}(1-f(p))$, $\sum_{d|n}\mu(d)^2 f(d)=\prod_{p|n}(1+f(p))$.

8.5.6 Legendre symbol

If p is an odd prime, $a \in \mathbb{Z}$, then $\left(\frac{a}{p}\right)$ equals 0, if $p|a$; 1 if a is a quadratic residue modulo p; and -1 otherwise. Euler's criterion: $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod{p}$.

8.5.7 Jacobi symbol

If $n = p_1^{a_1} \cdots p_k^{a_k}$ is odd, then $\left(\frac{a}{n}\right) = \prod_{i=1}^{k} \left(\frac{a}{p_i}\right)^{k_i}$.

8.5.8 Primitive roots

If the order of g modulo m (min $n > 0$: $g^n \equiv 1 \pmod{m}$) is $\phi(m)$, then g is called a primitive root. If Z_m has a primitive root, then it has $\phi(\phi(m))$ distinct primitive roots. Z_m has a primitive root iff m is one of 2, 4, p^k, $2p^k$, where p is an odd prime. If Z_m has a primitive root g, then for all a coprime to m, there exists unique integer $i = \text{ind}_g(a)$ modulo $\phi(m)$, such that $g^i \equiv a \pmod{m}$. $\text{ind}_g(a)$ has logarithm-like properties: $\text{ind}(1) = 0$, $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$.
If p is prime and a is not divisible by p, then congruence $x^n \equiv a \pmod{p}$ has $\gcd(n, p-1)$ solutions if $a^{(p-1)/\gcd(n,p-1)} \equiv 1 \pmod{p}$, and no solutions otherwise. (Proof sketch: let g be a primitive root, and $g^i \equiv a \pmod{p}$, $g^u \equiv x \pmod{p}$. $x^n \equiv a \pmod{p}$ iff $g^{nu} \equiv g^i \pmod{p}$ iff $nu \equiv i \pmod{p}$.)

8.5.9 Discrete logarithm problem

Find x from $a^x \equiv b \pmod{m}$. Can be solved in $O(\sqrt{m})$ time and space with a meet-in-the-middle trick. Let $n = \lceil \sqrt{m} \rceil$, and $x = ny - z$. Equation becomes $a^{ny} \equiv ba^z \pmod{m}$. Precompute all values that the RHS can take for $z = 0, 1, \ldots, n-1$, and brute force y on the LHS, each time checking whether there's a corresponding value for RHS.

8.5.10 Pythagorean triples

Integer solutions of $x^2 + y^2 = z^2$ All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$ where $m > n, \gcd(m, n) = 1$ and $m \not\equiv n \pmod{2}$. All other triples are multiples of these. Equation $x^2 + y^2 = 2z^2$ is equivalent to $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$.

8.5.11 Postage stamps/McNuggets problem

Let a, b be relatively-prime integers. There are exactly $\frac{1}{2}(a-1)(b-1)$ numbers *not* of form $ax + by$ $(x, y \geq 0)$, and the largest is $(a-1)(b-1) - 1 = ab - a - b$.

8.5.12 Fermat's two-squares theorem

Odd prime p can be represented as a sum of two squares iff $p \equiv 1 \pmod{4}$. A product of two sums of two squares is a sum of two squares. Thus, n is a sum of two squares iff every prime of form $p = 4k + 3$ occurs an even number of times in n's factorization.

8.6 Permutations

8.6.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

8.6.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

8.6.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

8.6.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g $(g.x = x)$.
If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k)$$

8.7 Partitions and subsets

8.7.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

8.8 General purpose numbers

8.8.1 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

8.8.2 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

8.8.3 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

8.8.4 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

8.8.5 Bernoulli numbers

$\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$. $B_0 = 1, B_1 = -\frac{1}{2}$. $B_n = 0$, for all odd $n \neq 1$.

8.8.6 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

8.9 Inequalities

8.9.1 Titu's Lemma

For positive reals $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$,

$$\frac{{a_1}^2}{b_1} + \frac{{a_2}^2}{b_2} + \ldots + \frac{{a_n}^2}{b_n} \geq \frac{a_1 + a_2 + \ldots + {a_n}^2}{b_1 + b_2 + \ldots + b_n}$$

Equality holds if and only if $a_i = kb_i$ for a non-zero real constant k.

8.10 Games

8.10.1 Grundy numbers

For a two-player, normal-play (last to move wins) game on a graph (V, E): $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$, where $\text{mex}(S) = \min\{n \geq 0 : n \not\in S\}$. x is losing iff $G(x) = 0$.

8.10.2 Sums of games

- *Player chooses a game and makes a move in it* Grundy number of a position is xor of grundy numbers of positions in summed games.

- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them* A position is losing iff each game is in a losing position.

- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.

- *Player must move in all games, and loses if can't move in some game* A position is losing if any of the games is in a losing position.

8.10.3 Misère Nim

A position with pile sizes $a_1, a_2, \ldots, a_n \geq 1$, not all equal to 1, is losing iff $a_1 \oplus a_2 \oplus \cdots \oplus a_n = 0$ (like in normal nim.) A position with n piles of size 1 is losing iff n is *odd*.