1. A discrete random variable X with

$$\mathbb{P}(X = x) = \gamma(x)\theta^x/c(\theta), x = 0, 1, 2, \dots,$$

where $\gamma(x) \geq 0, \theta > 0$, and $c(\theta) = \sum_{x=0}^{\infty} \gamma(x)\theta^x$, is called a random variable with a power series distribution. Show that

   (a) $\{\gamma(x)\theta^x/c(\theta) : \theta > 0\}$ is an exponential family;

   (b) if $X_1, X_2, \dots, X_n$ is a random sample with a power seires distribution $\gamma(x)\theta^x/c(\theta)$, then $\sum_{i=1}^n X_i$ has the power series distribution $\gamma_n(x)\theta^x/[c(\theta)]^n$, where $\gamma_n(x)$ is the coefficient of θ^x in the power series expansion of $[c(\theta)]^n$.

2. Let $X_1, \cdots, X_n$ be a random sample from the pdf

$$f(x|\theta) = \frac{1}{\theta}e^{-(x-\theta)/\theta}, \quad \theta < x < \infty, 0 < \theta < \infty.$$

   (a) Find a statistic that is minimal sufficient for θ.

   (b) Show whether the minimal sufficient statistic in (a) is complete.

3. Let (X_i, Y_i) be i.i.d. bivariate normal samples with $\mathsf{E}[X_i] = \mathsf{E}[Y_i] = 0$, $\mathrm{Var}(X_i) = \mathrm{Var}(Y_i) = 1$, and $\mathrm{cov}(X_i, Y_i) = \theta \in (-1, 1)$.

   (a) Find a minimal sufficient statistic for θ.

   (b) Show whether the minimal sufficient statistic in (a) is complete or not.

   (c) Prove that $T_1 = \sum_{i=1}^n X_i^2$ and $T_2 = \sum_{i=1}^n Y_i^2$ are both ancillary, but (T_1, T_2) is not ancillary.

4. Let $X_1, \cdots, X_n$ be a random sample from a uniform distribution on the interval $(\theta, 2\theta)$, $\theta > 0$. Find a minimal sufficient statistic for θ. Is the statistic complete?

5. Let $X_1, \cdots, X_n$ be a random sample from the normal distribution $N(\theta, 2)$ when $\theta = 0$ and the normal distribution $N(\theta, 1)$ when $\theta \neq 0$. Show that the sample mean $\bar{X}$ is a complete statistic for θ but it is not a sufficient statistic for θ.

6. Let $X_1, X_2, \dots, X_n$ be i.i.d. Normal$(0, \sigma^2)$. Compute $\mathsf{E}[S]$, where

$$S = \frac{(\sum_{i=1}^n X_i)^2}{\sum_{i=1}^n X_i^2}$$

[Hint: Use Basu's theorem.]