1. Let $X_1, \ldots, X_n$ be i.i.d. sample from Normal$(\theta, a\theta)$, where θ is unknown.
   (a) Find the $1-\alpha$ confidence set for a that is obtained from the LRT of $H_0 : a = a_0$ versus $H_1 : a \neq a_0$.
   (b) Assume that $a = 1$ is known, show that $(\bar{X} - \theta)/\sqrt{a\theta/n}$ is pivot and derive a $1-\alpha$ confidence set for θ.
   (c) Assume that $a = 1$ is known, show that $(\bar{X} - \theta)/\sqrt{S^2/n}$ is pivot and derive a $1-\alpha$ confidence set for θ.
   (d) Assume that $a = 1$ is known, show that $(n-1)S^2/\theta$ is pivot and derive a $1-\alpha$ confidence set for θ.

2. Let X be a single observation from density $f(x) = \theta x^{\theta-1}, 0 \leq x \leq 1$.
   (a) Let $Y = -\log(X)^{-1}$, evaluate the confidence coefficient of the set $[y/2, y]$.
   (b) Find a pivotal quantity and use it to construct a confidence interval having the same confidence coefficient as in part (a).
   (c) Compare the length from part (a) and (b).

3. Let $X_1, \ldots, X_n$ be i.i.d. sample from Normal(θ, σ^2).
   (a) If σ^2 is known, find a minimum value of n such that a $1-\alpha$ confidence interval for θ will have length no more than $\delta > 0$.
   (b) If σ^2 is unknown, find a minimum value of n such that a $1-\alpha$ confidence interval for θ will have expected length no more than $\delta > 0$.
   (c) Compare the length from part (a) and (b).

4. Let $X_1, ..., X_n$ be iid Normal(θ, θ^2). Give or describe four asymptotic confidence intervals for θ.

5. *A variance stabilizing approach.* Let $X_1, ..., X_n$ be i.i.d. from a Poisson distribution with mean θ, and let $\widehat{\theta}_n = \bar{X}_n$ be the maximum likelihood estimator of θ.
   (a) Find a function $g : [0, \infty) \to \mathbb{R}$ such that
   $$Z_n = \sqrt{n}[g(\widehat{\theta}_n) - g(\theta)] \Rightarrow N(0, 1).$$
   (b) Find a $1-\alpha$ asymptotic confidence interval for θ based on the approximate pivot Z_n.