Lenses

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1 Introduction

This module 'Lenses' summarizes and extends your basic knowledge about lenses. After this module you can proceed with the modules 'Telescopes' or 'Grinding lenses'. When you completed all these modules you should be able to make your own telescope with the help of the module 'Making your own telescope' and explain how it works.

2 Ray diagram

This sections briefly summarises the knowledge of lenses from key stage 3 and 4.

One can use the thin lens formula

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o} \tag{2.1}$$

to calculate the focal length f, the image distance i, or the object distance o when the other two variables are known.[1]

The equation for linear magnification

$$M = -\frac{i}{o} = \frac{\text{Height image}}{\text{Height object}} \tag{2.2}$$

can be used to calculate the magnification. But also one of the four variables if the other three are known.

The image of an object can be constructed using three special construction lines in the ray diagram:

- One ray of light straight through the centre of the lens without refraction.
- One ray of light travelling parallel to the main axis of the lens before the lens that gets refracted towards and passes through the focal point after the lens.
- One ray of light that passes through the focal point before the lens and continues parallel to the main axis after the lens.

In figure 2.1 all distances and lines are shown. Notice that points are indicated using capitals while distances are in lower case.

[1] You might have used different symbols to denote the distance to the object and image: v instead of i and u instead of o.

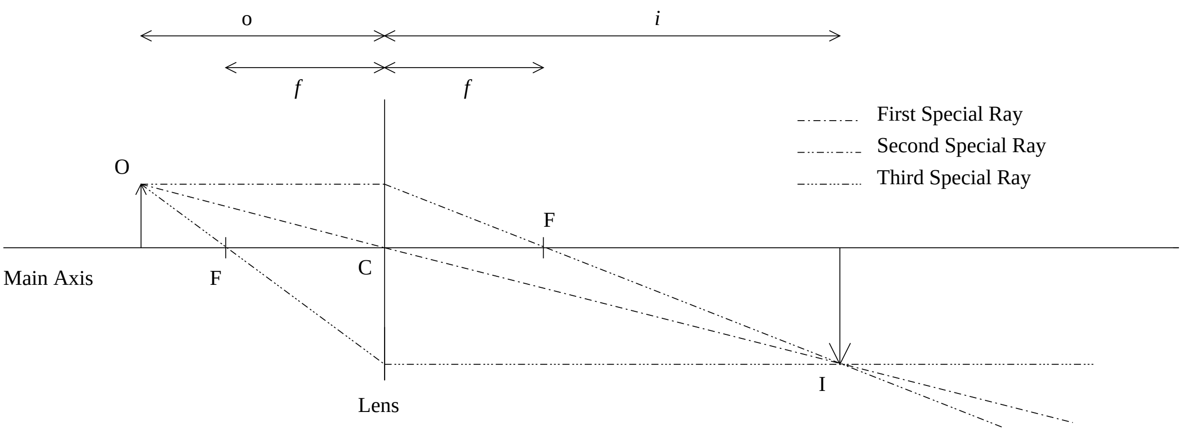

Figure 2.1: Constructing a ray diagram using three special lines.

C: Optical centre (Centre of the lens)

F: Focal point

O: Point of light, the object radiates (or reflects) light at every point

I: Image point

The three lines intersect each other at the some point some distance behind the lens. Here we find our image of the object. Do you remember where the image of the object is when the lines do not intersect after the lens?

3 How a lens works

Lenses work because of interaction between light and matter. When there is no matter present light will move with the speed of light in vacuum c.[2] The unit of distance, the metre, is defined according to the speed of light, therefore the speed of light is by definition exactly $c = 299792458\ m\ s^{-1}$.

From experience we know that light only travels through transparent materials. Inside the material the light will always move slower than in vacuum. In water the speed is roughly 3/4 of the speed of light (in vacuum), in glass 2/3. The difference in speed is the cause of refraction.

[2] Sometimes only 'speed of light' is mentioned in texts, from the context one needs to deduce if this is the speed of light in vacuum or not.

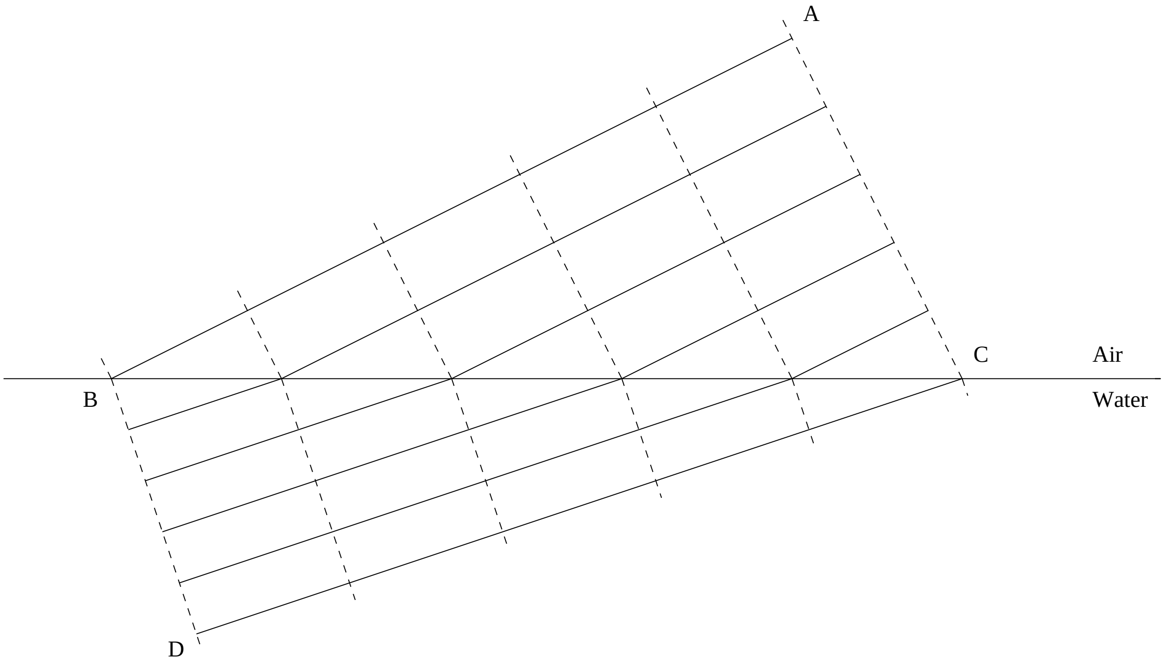

Figure 3.1: Refraction at an air/water interface.

Exercise 1 : In the module 'Mirrors' the Huygens principle was introduced. Explain what happens to the distance between the wave fronts when a beam of light goes from water to vacuum (where no matter is present).

In figure 3.1 two large triangles are shown, ΔABC and ΔBCD. Clearly the segment (BC) is present in both triangles. We know from the module 'Mirrors' that the wave front is always perpendicular to the wave beam: $\angle \mathrm{BAC} = \angle \mathrm{BDC} = 90°$

Exercise 2 : Explain how to determine the value of $\sin(\angle \mathrm{ABC})$.

Exercise 3 : Explain how to determine the value of $\sin(\angle \mathrm{BCD})$.

Exercise 4 : Show that:

$$\frac{\sin(\angle ABC)}{\sin(\angle BCD)} = \frac{(AC)}{(BD)} \tag{3.1}$$

Exercise 5 : Explain than the ratio $\frac{(AC)}{(BD)}$ is equal to the ratio between the speeds of light.

The ration between the speeds of light is called the refractive index, in this case $n_{air \to water}$.

Exercise 6 : Show that the angles ∠ABC and ∠BCD are equal to the angle of incidence i (between the incident ray and the surface normal) and the angle of refraction r (between the refracted ray and the surface normal).

Exercise 7 : Explain the following formula[3]:

$$n = \frac{\sin(i)}{\sin(r)} \tag{3.2}$$

4 Construction of refraction at plane and spherical surfaces

Drawing many wave fronts and beams can be very cumbersome, we will therefore use two different construction techniques to draw refraction at plane and spherical surfaces. These techniques allow us to follow a single ray of light; ray tracing.

Take a look at figure 4.1. In this figure there are a couple of numbers. These relate to the following steps in ray tracing:

1. Draw the surface between both materials (interface), in this case air and glass.
2. Draw the incident ray.
3. Draw the surface normal at the intersection of the incident ray and the surface. Remember that the surface normal is always perpendicular to the surface.
4. Draw a circle with the intersection as its centre. We are free to choose the radius of the circle, in this case we chose 1.5 cm.
5. Draw a second concentric circle (having the same midpoint) with a radius N times larger than the previous circle, in this case 2.25 cm.

[3]The formula is known as Snell's Law, named after the Dutch scientist Willebrord Snel van Royen (1580-1626). In his time Latin names where the norm so he is better known as Snellius. The relationship he described was already known but he was the first to capture it in a mathematical notation. We do not use Snellius his original notation but a modified form by René Descartes, therefore the French call the law Descartes Law.

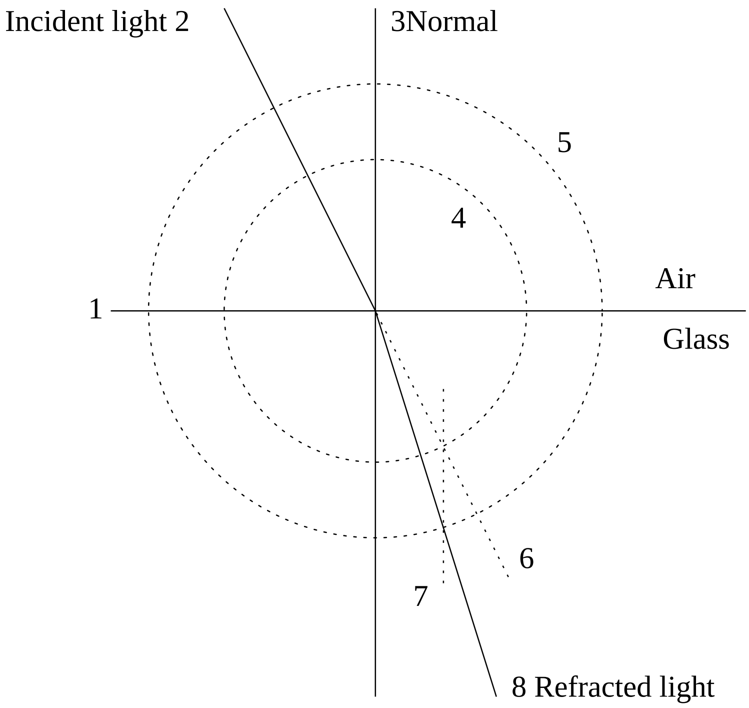

Figure 4.1: Refraction at a flat surface.

6. Extend the incident ray with a dotted line.
7. Draw a dotted line parallel to the surface normal starting at the intersection of the first circle (4) and the previous dotted line (6).
8. Draw the refracted line, starting at the end of the incident ray and through the intersection of the second circle (5) and the previous line (7).

Exercise 8 : A ray of light hits a diamond at an angle of $i = 25°$. Construct the refracted ray, $n_{air \to diamond} = 2.417$.

When constructing refractions at spherical surfaces we need three concentric circles. The procedure is now as follows (see figure 4.2):

1. Draw the main axis.
2. Draw the spherical surface. In this example we used $r = 3.6$ cm. Take care to mark the centre of the sphere.
3. Draw the incident ray.
4. Draw an auxiliary circle with a radius N (refractive index) times as large as the radius of the surface, in this example $r = 4.8$ cm.
5. Draw a second auxiliary circle with a radius N (refractive index) times as smaller than the radius of the surface, in this example $r = 2.7$ cm.

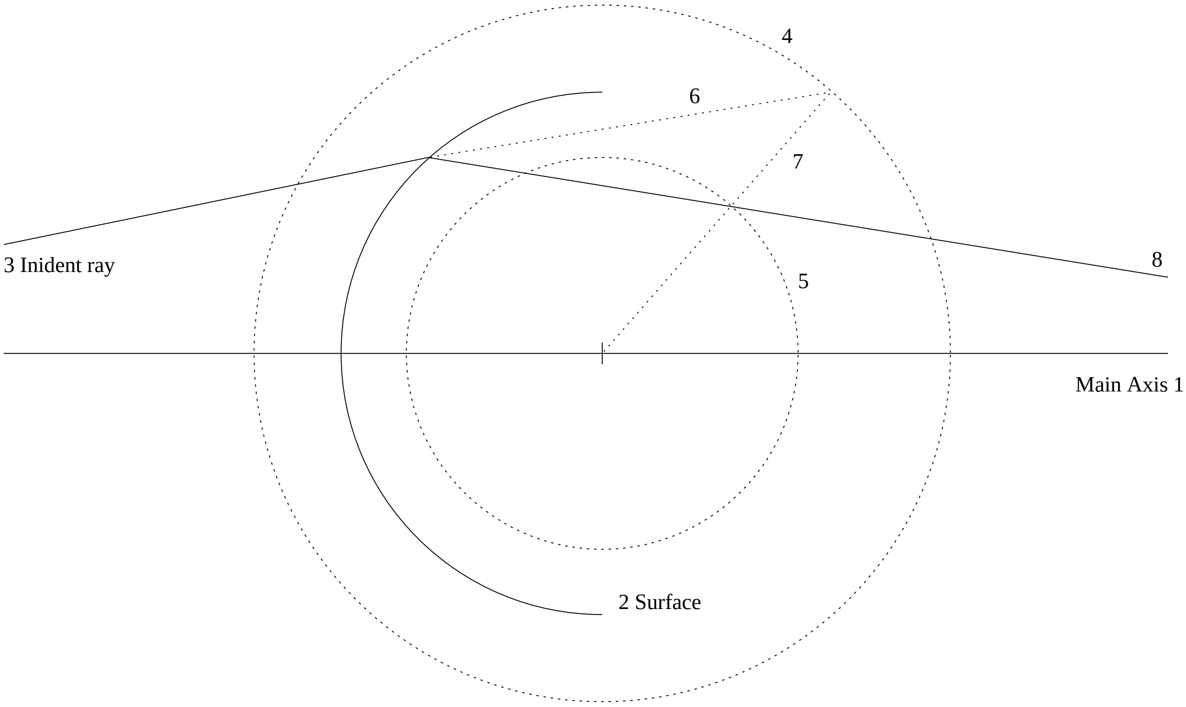

Figure 4.2: Refraction at a curved surface.

6. Continue the incident ray to the outermost circle (4) as if it was not refracted.
7. Draw a line from this intersection to the centre of the circles.
8. Draw the refracted ray from the reflecting surface towards the intersection of the previous line (7) and the inner circle (5).

Sometimes we are not interested in the exact path of light but only in the focal length of a lens. It that case we can use the Lens-Maker's Formula[4]:

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{4.1}$$

P is the power of the lens, a different way to report its focal length f, n is the refractive index, R_1 and R_2 are the radii of the curvature of the lens. Take care, convention dictates that lengths to the right are positive and to the left are negative. A lens with two convex sides (biconvex) has one positive radius R_1, from the surface to the centre, and one negative radius R_2, also from the surface to the centre.

If we know the dimensions and the refractive index of the lens we can calculate the focal length. A lens with $R_1 = 10.0$ cm, $R_2 = 5.0$ cm, and $n_{lens} = 1.50$ will have the

[4]Here we use an approximation of the Lens-Makers Formula only valid for thin lenses.

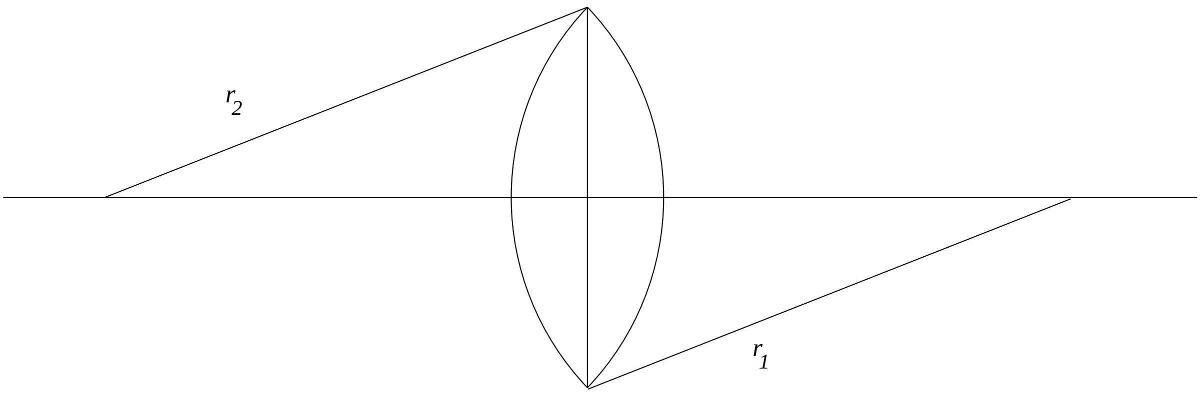

Figure 4.3: A lens.

following focal length:

$$\frac{1}{f} = (1.50 - 1)\left(\frac{1}{0.10} - \frac{1}{-0.050}\right)$$
$$\frac{1}{f} = (0.50)(10 + 20)$$
$$\frac{1}{f} = 15$$
$$f = \frac{1}{15}\ \text{m}$$

Exercise 9 : Calculate the focal length of plano-convex lens using the following parameters; $R_1 = \infty$, $R_2 = 4.0$ cm, and $n_{lens} = 2.0$.

Exercise 10 : Check your previous calculation by drawing the lens and two rays running parallel to the main axis placed at a distance of 1.0 and 2.0 cm.

Exercise 11 : Does the construction result in a clear focal point? If so, what is the focal distance?

Even if you work very carefully the focal point will not be a perfect spot. Our lens is not perfect. The specific lens error in this case is spherical aberration.