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# AS PROJECT

by

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PGP DSBA

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## Problem 1

### Problem Statement:

Salary is hypothesized to depend on educational qualification and occupation. To understand the dependency, the salaries of 40 individuals [SalaryData.csv] are collected and each person's educational qualification and occupation are noted. Educational qualification is at three levels, High school graduate, Bachelor, and Doctorate. Occupation is at four levels, Administrative and clerical, Sales, Professional or specialty, and Executive or managerial. A different number of observations are in each level of education – occupation combination.

[Assume that the data follows a normal distribution. In reality, the normality assumption may not always hold if the sample size is small.]

### Information of Salary Dataset:

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 40 entries, 0 to 39
Data columns (total 3 columns):
#   Column      Non-Null Count  Dtype
---  -
0   Education    40 non-null     object
1   Occupation    40 non-null     object
2   Salary       40 non-null     int64
dtypes: int64(1), object(2)
```

Figure 1.1: Salary Dataset Info

1.1 State the null and the alternate hypothesis for conducting one-way ANOVA for both Education and Occupation individually.

|                   |  |   |
|-------------------|--|---|
| Approach followed | <ol style="list-style-type: none"> <li>Identified the different population for independent factors Education and Occupation using 'Categorical' function from pandas library.</li> <li>Stated null and alternate hypothesis for factors Education and Occupation to conduct one-way Anova.</li> </ol>  |   |
| Output            | <pre>In [10]: DF.Education = pd.Categorical(DF.Education)           DF.Education.value_counts</pre>  | <pre>Name: Education, dtype: category Categories (3, object): [' Bachelors', ' Doctorate', ' HS-grad']&gt;</pre>                            |
|                   | Figure 1.1.1: Categories in Education  |   |
|                   | <pre>In [9]: DF.Occupation = pd.Categorical(DF.Occupation)           DF.Occupation.value_counts</pre>  | <pre>Name: Occupation, dtype: category Categories (4, object): [' Adm-clerical', ' Exec-managerial', ' Prof-specialty', ' Sales']&gt;</pre> |
|                   | Figure 1.1.2: Categories in Occupation   |   |
| Inference         | <p><b>Null and alternate hypothesis for Education:</b></p> <p><math>H_0: \mu_a = \mu_b = \mu_c = \dots = \mu_k</math> (All population(categories within Education that is Bachelors, Doctorate and High-School Graduate) means are equal)</p> <p><math>H_a: \mu_a \neq \mu_b \neq \mu_c \neq \dots \neq \mu_k</math> (Not all population means are equal, or least one pair is unequal)</p> <p><b>Null and alternate hypothesis for Occupation:</b></p> <p><math>H_0: \mu_a = \mu_b = \mu_c = \dots = \mu_k</math> (All population(categories within occupation that is Administrative and clerical, Executive and managerial, Professional, Sales) means are equal)</p> <p><math>H_a: \mu_a \neq \mu_b \neq \mu_c \neq \dots \neq \mu_k</math> (Not all population means are equal, or least one pair is unequal)</p> |   |

1.2 Perform one-way ANOVA for Education with respect to the variable 'Salary'. State whether the null hypothesis is accepted or rejected based on the ANOVA results.

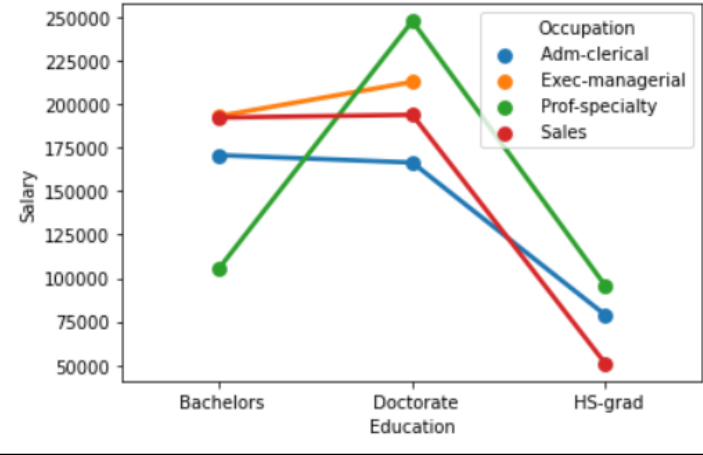
|                   |  |
|-------------------|--|
| Approach followed | <div><div></div><div><div></div><div><div></div><div></div></div><div><div></div><div></div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> 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|-------------------|--|

### 1.3 Perform one-way ANOVA for variable Occupation with respect to the variable 'Salary'. State whether the null hypothesis is accepted or rejected based on the ANOVA results.

|                   |  |              |              |          |          |   |        |               |     |              |              |          |          |          |      |              |              |     |     |
|-------------------|--|--------------|--------------|----------|----------|---|--------|---------------|-----|--------------|--------------|----------|----------|----------|------|--------------|--------------|-----|-----|
| Approach followed | <div><div></div><div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div></div><div><ol style="list-style-type: none"><li>1. One way ANOVA identifies effect of one variable in this case occupation. Using the ols function from statsmodels library the ANOVA table is generated for Occupation with respect to Salary.</li><li>2. The ratio of Variance between samples to variance within sample denoted by F determines whether null hypothesis is rejected/accepted.</li><li>3. The default confidence level (alpha = 0.05) is 95%.</li></ol></div></div>  |              |              |          |          |   |        |               |     |              |              |          |          |          |      |              |              |     |     |
| Output            | <div><div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div></div><div><pre>In [12]: formula = 'Salary ~ C(Occupation)' model = ols(formula, DF).fit() aov_table = anova_lm(model) print(aov_table)</pre></div><div><table><tr><td></td><td>df</td><td>sum_sq</td><td>mean_sq</td><td>F</td><td>PR(&gt;F)</td></tr><tr><td>C(Occupation)</td><td>3.0</td><td>1.125878e+10</td><td>3.752928e+09</td><td>0.884144</td><td>0.458508</td></tr><tr><td>Residual</td><td>36.0</td><td>1.528092e+11</td><td>4.244701e+09</td><td>NaN</td><td>NaN</td></tr></table></div></div> <div>Figure 1.3.1: ANOVA Table output for Occupation</div> |              | df           | sum_sq   | mean_sq  | F | PR(>F) | C(Occupation) | 3.0 | 1.125878e+10 | 3.752928e+09 | 0.884144 | 0.458508 | Residual | 36.0 | 1.528092e+11 | 4.244701e+09 | NaN | NaN |
|                   | df   | sum_sq       | mean_sq      | F        | PR(>F)   |   |        |               |     |              |              |          |          |          |      |              |              |     |     |
| C(Occupation)     | 3.0  | 1.125878e+10 | 3.752928e+09 | 0.884144 | 0.458508 |   |        |               |     |              |              |          |          |          |      |              |              |     |     |
| Residual          | 36.0   | 1.528092e+11 | 4.244701e+09 | NaN      | NaN      |   |        |               |     |              |              |          |          |          |      |              |              |     |     |
| Inference         | <div>As it can be seen from the results above F value is greater than 0.05 hence the null hypothesis for Occupation is accepted. That is to say population means for Occupation are equal.</div>   |              |              |          |          |   |        |               |     |              |              |          |          |          |      |              |              |     |     |

### 1.5 What is the interaction between the two treatments? Analyze the effects of one variable on the other (Education and Occupation) with the help of an interaction plot.

|                          |  |
|--------------------------|--|
| <b>Approach followed</b> | With the help of pointplot, an interaction plot between Education and Occupation is generated. This will help to analyze the effects between both factors. |
|--------------------------|--|

|                     |  |
|---------------------|--|
| Output of pointplot | <pre>&lt;AxesSubplot:xlabel='Education', ylabel='Salary'&gt;</pre>  <p><b>Figure 1.5.1: Interaction plot between Education and Occupation</b></p>  |
| Inference           | <p>As can be seen from the pointplot,</p> <ol style="list-style-type: none"> <li>1. For all occupations except Executive and Managerial, high school graduates have less salary than Bachelors and Doctorates.</li> <li>2. A professional or specialist with bachelors has almost a similar salary as a high-school graduate.</li> <li>3. Occupations of Administrative and clerical, Sales have same similar salary for Bachelors and Doctorate.</li> <li>4. An Executive and Managerial with Doctorate has slightly high salary than a person with Bachelors. This implies that a person with Doctorate will earn much more as a Professional or Specialist than a person in Sales or Executive and Managerial.</li> <li>5. Education does not have major effect on Occupation, it though has on Salary. A person with at least a Bachelors degree can earn a higher salary quite similar to a Doctorate in some occupations.</li> </ol> |

**1.6 Perform a two-way ANOVA based on the Education and Occupation (along with their interaction Education\*Occupation) with the variable 'Salary'. State the null and alternative hypotheses and state your results. How will you interpret this result?**

|                   |  |
|-------------------|--|
| Approach followed | <ol style="list-style-type: none"> <li>1. Two-way ANOVA helps to understand the interaction between two independent variables in this case Education and Occupation and, also the influence one variable on another.</li> <li>2. 3 pair of the hypothesis is defined for a two-way ANOVA.</li> <li>3. One for each of the factors and one for the interaction between two factors in this case Education and Occupation.</li> </ol>  |
| Hypothesis        | <p><math>H_0: \mu_a = \mu_b = \mu_c = \dots = \mu_k</math> (All population(categories within Education that is Bachelors, Doctorate and High-School Graduate) means are equal)</p> <p><math>H_a: \mu_a \neq \mu_b \neq \mu_c \neq \dots \neq \mu_k</math> (Not all population means are equal, or least one pair is unequal)</p><br><p><math>H_0: \mu_a = \mu_b = \mu_c = \dots = \mu_k</math> (All population(categories within occupation that is Administrative and clerical, Executive and managerial, Professional, Sales) means are equal)</p> <p><math>H_a: \mu_a \neq \mu_b \neq \mu_c \neq \dots \neq \mu_k</math> (Not all population means are equal, or least one pair is unequal)</p><br><p><math>H_0</math>: There is no interaction between Education and Occupation.</p> <p><math>H_a</math>: There is interaction between Education and Occupation.</p> |

|                            |   |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
|----------------------------|---|--------------|--------------|-----------|---------|---|---|--------------|-----|--------------|--------------|-----------|--|---------------|-----|--------------|--------------|----------|--|----------------------------|-----|--------------|--------------|----------|--|----------|------|--------------|--------------|-----|--|--|--------|--------------|--------------|---------------|--------------|----------------------------|--------------|----------|-----|
| Output                     | <pre>In [13]: formula = 'Salary ~ C(Education) + C(Occupation) + C(Education):C(Occupation)' model = ols(formula, DF).fit() aov_table = anova_lm(model) print(aov_table)</pre> <table><tr><td></td><td>df</td><td>sum_sq</td><td>mean_sq</td><td>F</td><td>\</td></tr><tr><td>C(Education)</td><td>2.0</td><td>1.026955e+11</td><td>5.134773e+10</td><td>72.211958</td><td></td></tr><tr><td>C(Occupation)</td><td>3.0</td><td>5.519946e+09</td><td>1.839982e+09</td><td>2.587626</td><td></td></tr><tr><td>C(Education):C(Occupation)</td><td>6.0</td><td>3.634909e+10</td><td>6.058182e+09</td><td>8.519815</td><td></td></tr><tr><td>Residual</td><td>29.0</td><td>2.062102e+10</td><td>7.110697e+08</td><td>NaN</td><td></td></tr></table><br><table><tr><td></td><td>PR(&gt;F)</td></tr><tr><td>C(Education)</td><td>5.466264e-12</td></tr><tr><td>C(Occupation)</td><td>7.211580e-02</td></tr><tr><td>C(Education):C(Occupation)</td><td>2.232500e-05</td></tr><tr><td>Residual</td><td>NaN</td></tr></table> |              | df           | sum_sq    | mean_sq | F | \ | C(Education) | 2.0 | 1.026955e+11 | 5.134773e+10 | 72.211958 |  | C(Occupation) | 3.0 | 5.519946e+09 | 1.839982e+09 | 2.587626 |  | C(Education):C(Occupation) | 6.0 | 3.634909e+10 | 6.058182e+09 | 8.519815 |  | Residual | 29.0 | 2.062102e+10 | 7.110697e+08 | NaN |  |  | PR(>F) | C(Education) | 5.466264e-12 | C(Occupation) | 7.211580e-02 | C(Education):C(Occupation) | 2.232500e-05 | Residual | NaN |
|                            | df  | sum_sq       | mean_sq      | F         | \       |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Education)               | 2.0   | 1.026955e+11 | 5.134773e+10 | 72.211958 |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Occupation)              | 3.0   | 5.519946e+09 | 1.839982e+09 | 2.587626  |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Education):C(Occupation) | 6.0   | 3.634909e+10 | 6.058182e+09 | 8.519815  |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| Residual                   | 29.0  | 2.062102e+10 | 7.110697e+08 | NaN       |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
|                            | PR(>F)  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Education)               | 5.466264e-12  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Occupation)              | 7.211580e-02  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| C(Education):C(Occupation) | 2.232500e-05  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| Residual                   | NaN   |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
|                            | <b>Figure 1.6.1: Two-way ANOVA table</b>  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |
| Inference                  | <p>As can be seen above, at 95% confidence value of alpha is 0.05</p> <ol style="list-style-type: none"><li>1. P-value for Education is less than 0.05, hence null hypothesis is rejected.</li><li>2. P-value for Occupation is more than 0.05, hence null hypothesis is accepted.</li><li>3. P-value for interaction between Education and Occupation is less than 0.05, hence null hypothesis is rejected. That to say there is no interaction between factors Education and Occupation.</li></ol>  |              |              |           |         |   |   |              |     |              |              |           |  |               |     |              |              |          |  |                            |     |              |              |          |  |          |      |              |              |     |  |  |        |              |              |               |              |                            |              |          |     |

### 1.7 Explain the business implications of performing ANOVA for this particular case study.

|               |  |
|---------------|--|
| <b>Answer</b> | <p>Based on observations on the dataset and ANOVA test following are business implications:</p> <ol style="list-style-type: none"> <li>1. It is important to complete high school graduation to get a job in majority of the occupations.</li> <li>2. The study reveals that degree of education matters when it comes to earning a higher salary.</li> <li>3. With the Bachelors degree a person can earn as much as a person with Doctorate degree in some occupations.</li> <li>4. With Doctorate degree a person can earn much more than a person with Bachelors but only in few specific occupations. This shows that it's important to understand what degree of Education can put you on higher salary range in a specific occupation as the requirement for all the occupations is not similar.</li> </ol> |
|---------------|--|

## Problem 2:

### Problem Statement:

The dataset Education - Post 12th Standard.csv contains information on various colleges. You are expected to do a Principal Component Analysis for this case study according to the instructions given. The data dictionary of the 'Education - Post 12th Standard.csv' can be found in the following file: Data Dictionary.xlsx.

### Information of Education Dataset:

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 777 entries, 0 to 776
Data columns (total 18 columns):
#   Column                Non-Null Count  Dtype
---  -
0   Names                 777 non-null    object
1   Apps                 777 non-null    int64
2   Accept               777 non-null    int64
3   Enroll              777 non-null    int64
4   Top10perc           777 non-null    int64
5   Top25perc           777 non-null    int64
6   F.Undergrad         777 non-null    int64
7   P.Undergrad         777 non-null    int64
8   Outstate            777 non-null    int64
9   Room.Board          777 non-null    int64
10  Books                777 non-null    int64
11  Personal             777 non-null    int64
12  PhD                 777 non-null    int64
13  Terminal            777 non-null    int64
14  S.F.Ratio           777 non-null    float64
15  perc.alumni         777 non-null    int64
16  Expend              777 non-null    int64
17  Grad.Rate           777 non-null    int64
dtypes: float64(1), int64(16), object(1)
```

Figure 2.1: Education Dataset Info

### Summary of Education Dataset:

|             | count | mean         | std         | min    | 25%    | 50%    | 75%     | max     |
|-------------|-------|--------------|-------------|--------|--------|--------|---------|---------|
| Apps        | 777.0 | 3001.638353  | 3870.201484 | 81.0   | 776.0  | 1558.0 | 3624.0  | 48094.0 |
| Accept      | 777.0 | 2018.804376  | 2451.113971 | 72.0   | 604.0  | 1110.0 | 2424.0  | 26330.0 |
| Enroll      | 777.0 | 779.972973   | 929.176190  | 35.0   | 242.0  | 434.0  | 902.0   | 6392.0  |
| Top10perc   | 777.0 | 27.558559    | 17.640364   | 1.0    | 15.0   | 23.0   | 35.0    | 96.0    |
| Top25perc   | 777.0 | 55.796654    | 19.804778   | 9.0    | 41.0   | 54.0   | 69.0    | 100.0   |
| F.Undergrad | 777.0 | 3699.907336  | 4850.420531 | 139.0  | 992.0  | 1707.0 | 4005.0  | 31643.0 |
| P.Undergrad | 777.0 | 855.298584   | 1522.431887 | 1.0    | 95.0   | 353.0  | 967.0   | 21836.0 |
| Outstate    | 777.0 | 10440.669241 | 4023.016484 | 2340.0 | 7320.0 | 9990.0 | 12925.0 | 21700.0 |
| Room.Board  | 777.0 | 4357.526384  | 1096.696416 | 1780.0 | 3597.0 | 4200.0 | 5050.0  | 8124.0  |
| Books       | 777.0 | 549.380952   | 165.105360  | 96.0   | 470.0  | 500.0  | 600.0   | 2340.0  |
| Personal    | 777.0 | 1340.642214  | 677.071454  | 250.0  | 850.0  | 1200.0 | 1700.0  | 6800.0  |
| PhD         | 777.0 | 72.660232    | 16.328155   | 8.0    | 62.0   | 75.0   | 85.0    | 103.0   |
| Terminal    | 777.0 | 79.702703    | 14.722359   | 24.0   | 71.0   | 82.0   | 92.0    | 100.0   |
| S.F.Ratio   | 777.0 | 14.089704    | 3.958349    | 2.5    | 11.5   | 13.6   | 16.5    | 39.8    |
| perc.alumni | 777.0 | 22.743887    | 12.391801   | 0.0    | 13.0   | 21.0   | 31.0    | 64.0    |
| Expend      | 777.0 | 9660.171171  | 5221.768440 | 3186.0 | 6751.0 | 8377.0 | 10830.0 | 56233.0 |
| Grad.Rate   | 777.0 | 65.463320    | 17.177710   | 10.0   | 53.0   | 65.0   | 78.0    | 118.0   |

Figure 2.2: Education Dataset Summary



## 2.1 Perform Exploratory Data Analysis [both univariate and multivariate analysis to be performed]. What insight do you draw from the EDA?

|                                       |  |
|---------------------------------------|--|
| <b>Approach followed</b>              | <ol style="list-style-type: none"> <li>1. For univariate analysis histplot and boxplot of each column is generated.</li> <li>2. For multivariate analysis pairplot and heat map of the entire dataset is generated.</li> <li>3. Inference is built for each factor from plots for univariate and multivariate.</li> </ol>  |
| <b>Output for Univariate Analysis</b> | <div data-bbox="336 365 1481 1025"> </div> <p data-bbox="515 1037 1305 1070"><b>Figure 2.1.1: Histplot of different factors from Education Dataset</b></p> <div data-bbox="336 1115 1481 1787"> </div> <p data-bbox="515 1798 1305 1832"><b>Figure 2.1.2: Boxplot of different factors from Education Dataset</b></p>  |
| <b>Inference</b>                      | <p>Following inferences are derived from Univariate Analysis:</p> <ol style="list-style-type: none"> <li>1. Majority of the features are right/left distributed. Only few features like top 10 and top 25 percentage, room board, outstate, graduation rate are normally distributed.</li> <li>2. Only top 25 percentage has no outliers, rest of the features have outliers present.</li> </ol> |

## Output for Multivariate Analysis

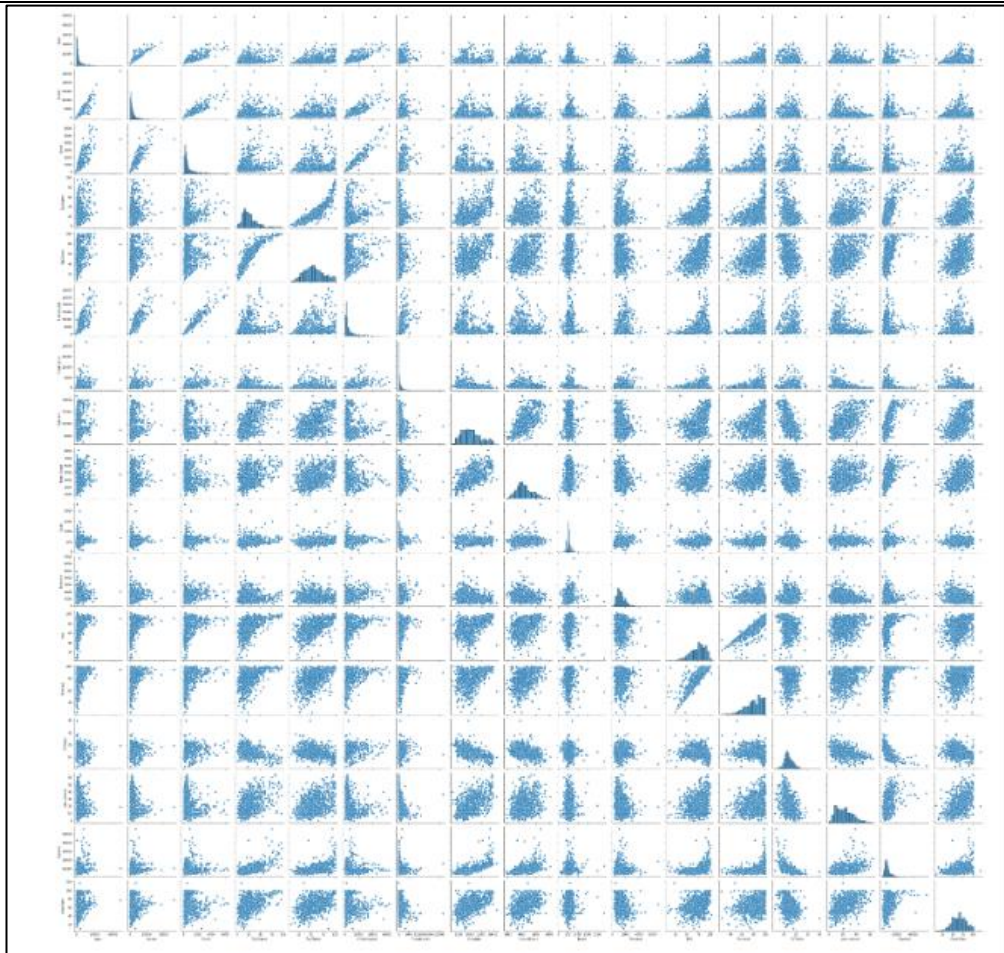
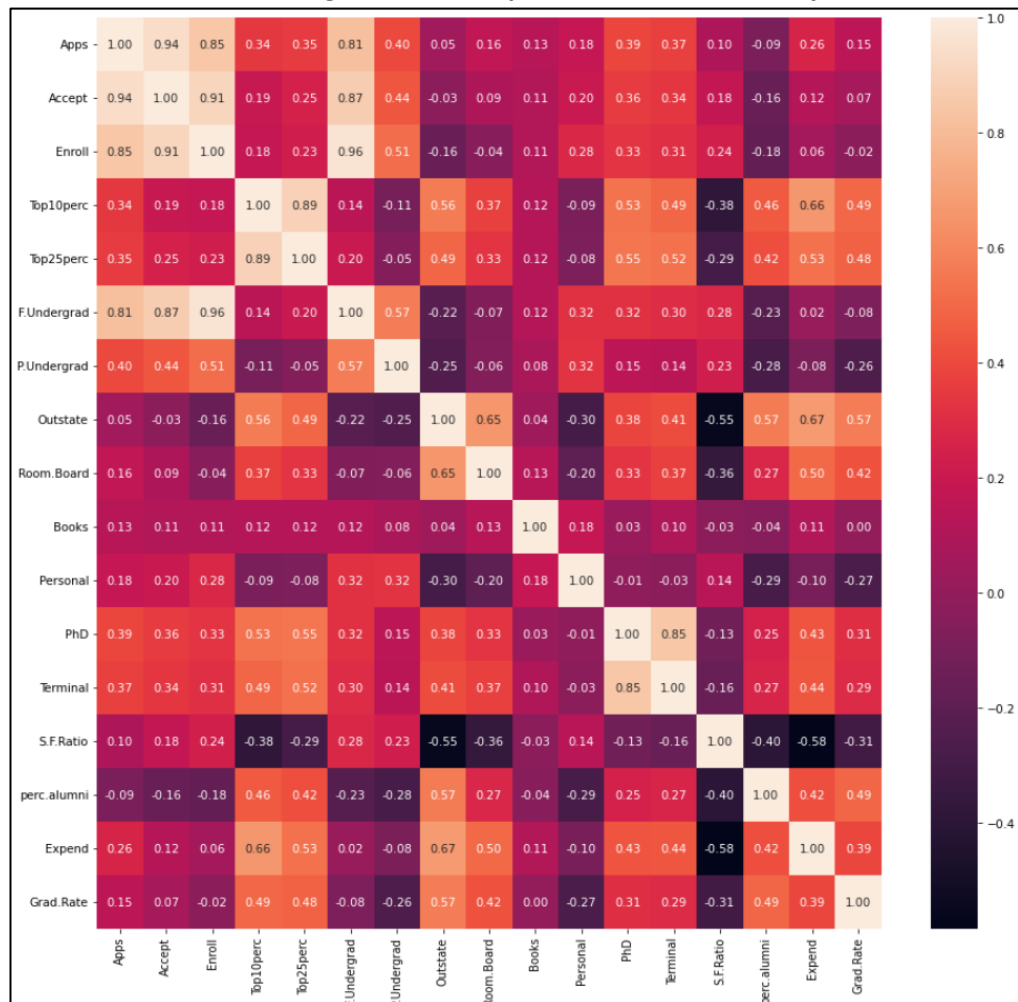


Figure 2.1.3: Pairplot for Mutivariate analysis



|           | Figure 2.1.3: Heatmap for multivariate analysis  |
|-----------|--|
| Inference | <p>Following inferences are derive from Multivariate analysis:</p> <ol style="list-style-type: none"> <li>1. Application feature is highly positively correlated with application accepted, students enrolled and full-time graduates.</li> <li>2. There is negative correlation between application and percentage of alumni. This shows not all students are part of alumni of their college or university.</li> <li>3. The application with top 10, 25 of higher secondary class, outstate, room board, books, personal, PhD, terminal, S.F ratio, expenditure and Graduation ratio are positively correlated.</li> </ol> |

## 2.2 Is scaling necessary for PCA in this case? Give justification and perform scaling.

|        |  |
|--------|--|
| Answer | <div><div></div><div><div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><d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## 2.3 Comment on the comparison between the covariance and the correlation matrices from this data.[on scaled data]

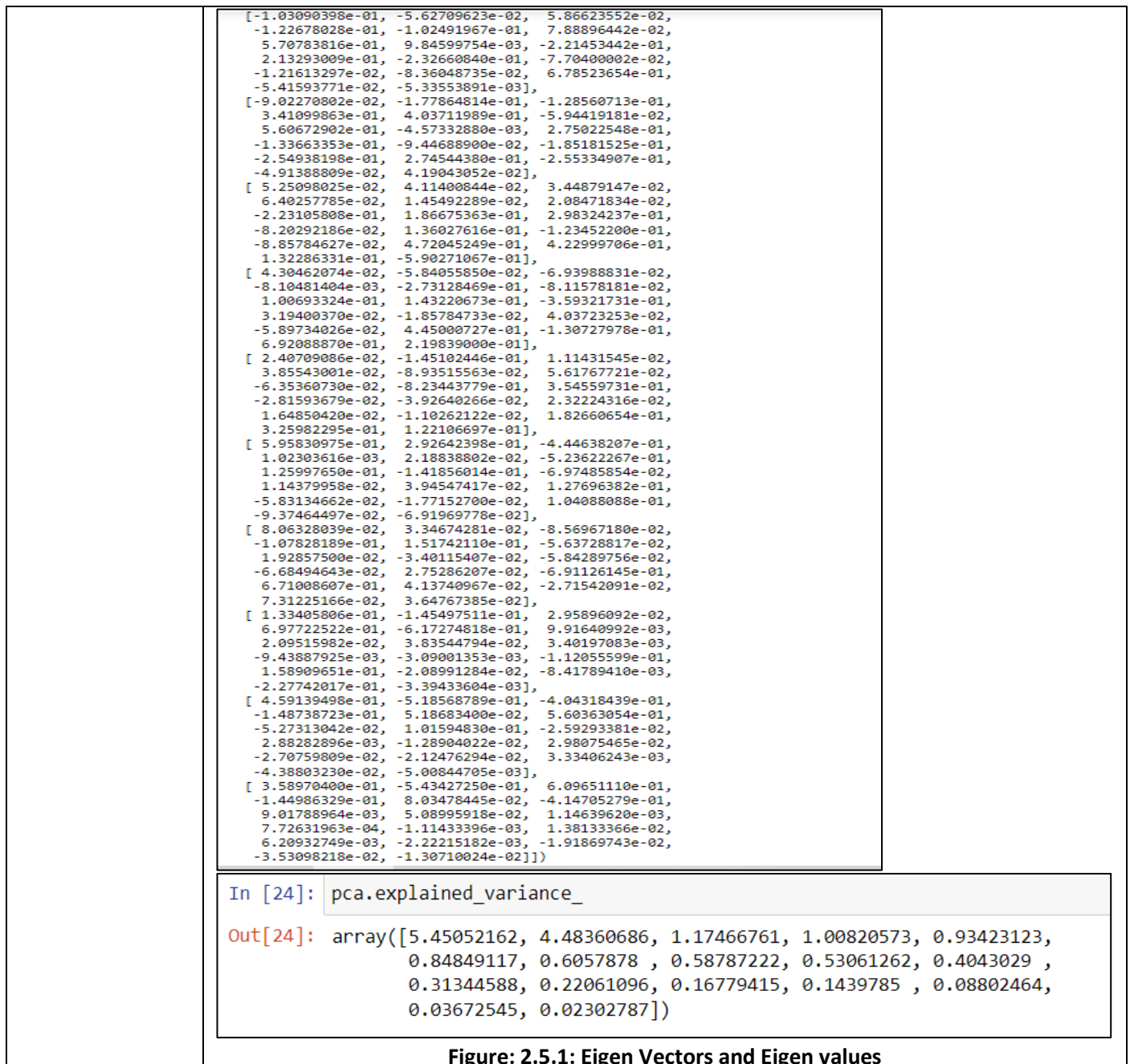
| Answer                           | Co-variance and correlation helps to understand the relation between dimensions and the dependency within them. Positive and negative values of co-variance & correlation indicate whether they are directly/inversely proportional.   |           |           |           |           |             |             |             |             |           |            |           |           |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
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| Output of Covariance             | <div>Covariance Matrix<br/>%\$ [[ 1.00128866 0.94466636 0.84791332 0.33927032 0.35209304 0.81554018<br/>0.3987775 0.05022367 0.16515151 0.13272942 0.17896117 0.39120081<br/>0.36996762 0.09575627 -0.09034216 0.2599265 0.14694372<br/>[ 0.94466636 1.00128866 0.91281145 0.19269493 0.24779465 0.87534985<br/>0.44183938 -0.02578774 0.09101577 0.11367165 0.20124767 0.35621633<br/>0.3380184 0.17645611 -0.16019604 0.12487773 0.06739929<br/>[ 0.84791332 0.91281145 1.00128866 0.18152715 0.2270373 0.96588274<br/>0.51372977 -0.1556777 -0.04028353 0.11285614 0.28129148 0.33189629<br/>0.30867133 0.23757707 -0.18102711 0.06425192 -0.02236983<br/>[ 0.33927032 0.19269493 0.18152715 1.00128866 0.89314445 0.1414708<br/>-0.10549205 0.5630552 0.37195909 0.1190116 -0.09343665 0.53251337<br/>0.49176793 -0.38537048 0.45607223 0.6617651 0.49562711<br/>[ 0.35209304 0.24779465 0.2270373 0.89314445 1.00128866 0.19970167<br/>-0.05364569 0.49002449 0.33191707 0.115676 -0.08091441 0.54656564<br/>0.52542506 -0.29500852 0.41840277 0.52812713 0.47789622<br/>[ 0.81554018 0.87534985 0.96588274 0.1414708 0.19970167 1.00128866<br/>0.57124738 -0.21602002 -0.06897917 0.11569867 0.31760831 0.3187472<br/>0.30040557 0.28006379 -0.22975792 0.01867565 -0.07887464<br/>[ 0.3987775 0.44183938 0.51372977 -0.10549205 -0.05364569 0.57124738<br/>1.00128866 -0.25383901 -0.06140453 0.08130416 0.32029384 0.14930637<br/>0.14208644 0.23283016 -0.28115421 -0.08367612 -0.25733218<br/>[ 0.05022367 -0.02578774 -0.1556777 0.5630552 0.49002449 -0.21602002<br/>-0.25383901 1.00128866 0.65509951 0.03890494 -0.29947232 0.38347594<br/>0.40850895 -0.55553625 0.56699214 0.6736456 0.57202613<br/>[ 0.16515151 0.09101577 -0.04028353 0.37195909 0.33191707 -0.06897917<br/>-0.06140453 0.65509951 1.00128866 0.12812787 -0.19968518 0.32962651<br/>0.3750222 -0.36309504 0.27271444 0.50238599 0.42548915<br/>[ 0.13272942 0.11367165 0.11285614 0.1190116 0.115676 0.11569867<br/>0.08130416 0.03890494 0.12812787 1.00128866 0.17952581 0.0269404<br/>0.10008351 -0.03197042 -0.04025955 0.11255393 0.00106226<br/>[ 0.17896117 0.20124767 0.28129148 -0.09343665 -0.08091441 0.31760831<br/>0.32029384 -0.29947232 -0.19968518 0.17952581 1.00128866 -0.01094989<br/>-0.03065256 0.13652054 -0.2863366 -0.090801804 -0.26969106<br/>[ 0.39120081 0.35621633 0.33189629 0.53251337 0.54656564 0.3187472<br/>0.14930637 0.38347594 0.32962651 0.0269404 -0.01094989 1.00128866<br/>0.85068186 -0.13069832 0.24932955 0.43331936 0.30543094<br/>[ 0.36996762 0.3380184 0.30867133 0.49176793 0.52542506 0.30040557<br/>0.14208644 0.40850895 0.3750222 0.10008351 -0.03065256 0.85068186<br/>1.00128866 -0.16031027 0.26747453 0.43936469 0.28990033<br/>[ 0.09575627 0.17645611 0.23757707 -0.38537048 -0.29500852 0.28006379<br/>0.23283016 -0.55553625 -0.36309504 -0.03197042 0.13652054 -0.13069832<br/>-0.16031027 1.00128866 -0.4034484 -0.5845844 -0.30710565<br/>[ -0.09034216 -0.16019604 -0.18102711 0.45607223 0.41840277 -0.22975792<br/>-0.28115421 0.56699214 0.27271444 -0.04025955 -0.2863366 0.24932955<br/>0.26747453 -0.4034484 1.00128866 0.41825001 0.49153016<br/>[ 0.2599265 0.12487773 0.06425192 0.6617651 0.52812713 0.01867565<br/>-0.08367612 0.6736456 0.50238599 0.11255393 -0.090801804 0.43331936<br/>0.43936469 -0.5845844 0.41825001 1.00128866 0.39084571<br/>[ 0.14694372 0.06739929 -0.02236983 0.49562711 0.47789622 -0.07887464<br/>-0.25733218 0.57202613 0.42548915 0.00106226 -0.26969106 0.30543094<br/>0.28990033 -0.30710565 0.49153016 0.39084571 1.00128866]]</div>   |           |           |           |           |             |             |             |             |           |            |           |           |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |  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| Figure 2.3.1: Covariance Matrix  |  |           |           |           |           |             |             |             |             |           |            |           |           |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Output of Correlation            | <table><tr><th></th><th>Apps</th><th>Accept</th><th>Enroll</th><th>Top10perc</th><th>Top25perc</th><th>F.Undergrad</th><th>P.Undergrad</th><th>Outstate</th><th>Room.Board</th><th>Books</th><th>Personal</th><th>PhD</th><th>Termin</th></tr><tr><td>Apps</td><td>1.000000</td><td>0.943451</td><td>0.846822</td><td>0.338834</td><td>0.351640</td><td>0.814491</td><td>0.398264</td><td>0.050159</td><td>0.164939</td><td>0.132559</td><td>0.178731</td><td>0.390697</td><td>0.369491</td></tr><tr><td>Accept</td><td>0.943451</td><td>1.000000</td><td>0.911637</td><td>0.192447</td><td>0.247476</td><td>0.874223</td><td>0.441271</td><td>-0.025755</td><td>0.090899</td><td>0.113525</td><td>0.200989</td><td>0.355758</td><td>0.337583</td></tr><tr><td>Enroll</td><td>0.846822</td><td>0.911637</td><td>1.000000</td><td>0.181294</td><td>0.226745</td><td>0.964640</td><td>0.513069</td><td>-0.155477</td><td>-0.040232</td><td>0.112711</td><td>0.280929</td><td>0.331469</td><td>0.308271</td></tr><tr><td>Top10perc</td><td>0.338834</td><td>0.192447</td><td>0.181294</td><td>1.000000</td><td>0.891995</td><td>0.141289</td><td>-0.105356</td><td>0.562331</td><td>0.371480</td><td>0.118858</td><td>-0.093316</td><td>0.531828</td><td>0.491135</td></tr><tr><td>Top25perc</td><td>0.351640</td><td>0.247476</td><td>0.226745</td><td>0.891995</td><td>1.000000</td><td>0.199445</td><td>-0.053577</td><td>0.489394</td><td>0.331490</td><td>0.115527</td><td>-0.080810</td><td>0.545862</td><td>0.524749</td></tr><tr><td>F.Undergrad</td><td>0.814491</td><td>0.874223</td><td>0.964640</td><td>0.141289</td><td>0.199445</td><td>1.000000</td><td>0.570512</td><td>-0.215742</td><td>-0.068890</td><td>0.115550</td><td>0.317200</td><td>0.318337</td><td>0.300019</td></tr><tr><td>P.Undergrad</td><td>0.398264</td><td>0.441271</td><td>0.513069</td><td>-0.105356</td><td>-0.053577</td><td>0.570512</td><td>1.000000</td><td>-0.253512</td><td>-0.061326</td><td>0.081200</td><td>0.319882</td><td>0.149114</td><td>0.141906</td></tr><tr><td>Outstate</td><td>0.050159</td><td>-0.025755</td><td>-0.155477</td><td>0.562331</td><td>0.489394</td><td>-0.215742</td><td>-0.253512</td><td>1.000000</td><td>0.654256</td><td>0.038855</td><td>-0.299087</td><td>0.382982</td><td>0.407996</td></tr><tr><td>Room.Board</td><td>0.164939</td><td>0.090899</td><td>-0.040232</td><td>0.371480</td><td>0.331490</td><td>-0.068890</td><td>-0.061326</td><td>0.654256</td><td>1.000000</td><td>0.127963</td><td>-0.199428</td><td>0.329202</td><td>0.374540</td></tr><tr><td>Books</td><td>0.132559</td><td>0.113525</td><td>0.112711</td><td>0.118858</td><td>0.115527</td><td>0.115550</td><td>0.081200</td><td>0.038855</td><td>0.127963</td><td>1.000000</td><td>0.179295</td><td>0.026906</td><td>0.099955</td></tr><tr><td>Personal</td><td>0.178731</td><td>0.200989</td><td>0.280929</td><td>-0.093316</td><td>-0.080810</td><td>0.317200</td><td>0.319882</td><td>-0.299087</td><td>-0.199428</td><td>0.179295</td><td>1.000000</td><td>-0.010936</td><td>-0.030611</td></tr><tr><td>PhD</td><td>0.390697</td><td>0.355758</td><td>0.331469</td><td>0.531828</td><td>0.545862</td><td>0.318337</td><td>0.149114</td><td>0.382982</td><td>0.329202</td><td>0.026906</td><td>-0.010936</td><td>1.000000</td><td>0.849587</td></tr><tr><td>Terminal</td><td>0.369491</td><td>0.337583</td><td>0.308274</td><td>0.491135</td><td>0.524749</td><td>0.300019</td><td>0.141904</td><td>0.407983</td><td>0.374540</td><td>0.099955</td><td>-0.030611</td><td>0.849587</td><td>1.000000</td></tr><tr><td>S.F.Ratio</td><td>0.095633</td><td>0.176229</td><td>0.237271</td><td>-0.384875</td><td>-0.294629</td><td>0.279703</td><td>0.232531</td><td>-0.554821</td><td>-0.362628</td><td>-0.031929</td><td>0.136345</td><td>-0.130530</td><td>-0.160106</td></tr><tr><td>perc.alumni</td><td>-0.090226</td><td>-0.159990</td><td>-0.180794</td><td>0.455485</td><td>0.417864</td><td>-0.229462</td><td>-0.280792</td><td>0.566262</td><td>0.272363</td><td>-0.040208</td><td>-0.285968</td><td>0.249009</td><td>0.267111</td></tr><tr><td>Expend</td><td>0.259592</td><td>0.124717</td><td>0.064169</td><td>0.660913</td><td>0.527447</td><td>0.018852</td><td>-0.083568</td><td>0.672779</td><td>0.501739</td><td>0.112409</td><td>-0.097892</td><td>0.432762</td><td>0.438755</td></tr><tr><td>Grad.Rate</td><td>0.146755</td><td>0.067313</td><td>-0.022341</td><td>0.494989</td><td>0.477281</td><td>-0.078773</td><td>-0.257001</td><td>0.571290</td><td>0.424942</td><td>0.001061</td><td>-0.269344</td><td>0.305038</td><td>0.289525</td></tr></table> |           | Apps      | Accept    | Enroll    | Top10perc   | Top25perc   | F.Undergrad | P.Undergrad | Outstate  | Room.Board | Books     | Personal  | PhD | Termin | Apps | 1.000000 | 0.943451 | 0.846822 | 0.338834 | 0.351640 | 0.814491 | 0.398264 | 0.050159 | 0.164939 | 0.132559 | 0.178731 | 0.390697 | 0.369491 | Accept | 0.943451 | 1.000000 | 0.911637 | 0.192447 | 0.247476 | 0.874223 | 0.441271 | -0.025755 | 0.090899 | 0.113525 | 0.200989 | 0.355758 | 0.337583 | Enroll | 0.846822 | 0.911637 | 1.000000 | 0.181294 | 0.226745 | 0.964640 | 0.513069 | -0.155477 | -0.040232 | 0.112711 | 0.280929 | 0.331469 | 0.308271 | Top10perc | 0.338834 | 0.192447 | 0.181294 | 1.000000 | 0.891995 | 0.141289 | -0.105356 | 0.562331 | 0.371480 | 0.118858 | -0.093316 | 0.531828 | 0.491135 | Top25perc | 0.351640 | 0.247476 | 0.226745 | 0.891995 | 1.000000 | 0.199445 | -0.053577 | 0.489394 | 0.331490 | 0.115527 | -0.080810 | 0.545862 | 0.524749 | F.Undergrad | 0.814491 | 0.874223 | 0.964640 | 0.141289 | 0.199445 | 1.000000 | 0.570512 | -0.215742 | -0.068890 | 0.115550 | 0.317200 | 0.318337 | 0.300019 | P.Undergrad | 0.398264 | 0.441271 | 0.513069 | -0.105356 | -0.053577 | 0.570512 | 1.000000 | -0.253512 | -0.061326 | 0.081200 | 0.319882 | 0.149114 | 0.141906 | Outstate | 0.050159 | -0.025755 | -0.155477 | 0.562331 | 0.489394 | -0.215742 | -0.253512 | 1.000000 | 0.654256 | 0.038855 | -0.299087 | 0.382982 | 0.407996 | Room.Board | 0.164939 | 0.090899 | -0.040232 | 0.371480 | 0.331490 | -0.068890 | -0.061326 | 0.654256 | 1.000000 | 0.127963 | -0.199428 | 0.329202 | 0.374540 | Books | 0.132559 | 0.113525 | 0.112711 | 0.118858 | 0.115527 | 0.115550 | 0.081200 | 0.038855 | 0.127963 | 1.000000 | 0.179295 | 0.026906 | 0.099955 | Personal | 0.178731 | 0.200989 | 0.280929 | -0.093316 | -0.080810 | 0.317200 | 0.319882 | -0.299087 | -0.199428 | 0.179295 | 1.000000 | -0.010936 | -0.030611 | PhD | 0.390697 | 0.355758 | 0.331469 | 0.531828 | 0.545862 | 0.318337 | 0.149114 | 0.382982 | 0.329202 | 0.026906 | -0.010936 | 1.000000 | 0.849587 | Terminal | 0.369491 | 0.337583 | 0.308274 | 0.491135 | 0.524749 | 0.300019 | 0.141904 | 0.407983 | 0.374540 | 0.099955 | -0.030611 | 0.849587 | 1.000000 | S.F.Ratio | 0.095633 | 0.176229 | 0.237271 | -0.384875 | -0.294629 | 0.279703 | 0.232531 | -0.554821 | -0.362628 | -0.031929 | 0.136345 | -0.130530 | -0.160106 | perc.alumni | -0.090226 | -0.159990 | -0.180794 | 0.455485 | 0.417864 | -0.229462 | -0.280792 | 0.566262 | 0.272363 | -0.040208 | -0.285968 | 0.249009 | 0.267111 | Expend | 0.259592 | 0.124717 | 0.064169 | 0.660913 | 0.527447 | 0.018852 | -0.083568 | 0.672779 | 0.501739 | 0.112409 | -0.097892 | 0.432762 | 0.438755 | Grad.Rate | 0.146755 | 0.067313 | -0.022341 | 0.494989 | 0.477281 | -0.078773 | -0.257001 | 0.571290 | 0.424942 | 0.001061 | -0.269344 | 0.305038 | 0.289525 |
|                                  | Apps   | Accept    | Enroll    | Top10perc | Top25perc | F.Undergrad | P.Undergrad | Outstate    | Room.Board  | Books     | Personal   | PhD       | Termin    |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Apps                             | 1.000000   | 0.943451  | 0.846822  | 0.338834  | 0.351640  | 0.814491    | 0.398264    | 0.050159    | 0.164939    | 0.132559  | 0.178731   | 0.390697  | 0.369491  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Accept                           | 0.943451   | 1.000000  | 0.911637  | 0.192447  | 0.247476  | 0.874223    | 0.441271    | -0.025755   | 0.090899    | 0.113525  | 0.200989   | 0.355758  | 0.337583  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Enroll                           | 0.846822   | 0.911637  | 1.000000  | 0.181294  | 0.226745  | 0.964640    | 0.513069    | -0.155477   | -0.040232   | 0.112711  | 0.280929   | 0.331469  | 0.308271  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Top10perc                        | 0.338834   | 0.192447  | 0.181294  | 1.000000  | 0.891995  | 0.141289    | -0.105356   | 0.562331    | 0.371480    | 0.118858  | -0.093316  | 0.531828  | 0.491135  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Top25perc                        | 0.351640   | 0.247476  | 0.226745  | 0.891995  | 1.000000  | 0.199445    | -0.053577   | 0.489394    | 0.331490    | 0.115527  | -0.080810  | 0.545862  | 0.524749  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| F.Undergrad                      | 0.814491   | 0.874223  | 0.964640  | 0.141289  | 0.199445  | 1.000000    | 0.570512    | -0.215742   | -0.068890   | 0.115550  | 0.317200   | 0.318337  | 0.300019  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| P.Undergrad                      | 0.398264   | 0.441271  | 0.513069  | -0.105356 | -0.053577 | 0.570512    | 1.000000    | -0.253512   | -0.061326   | 0.081200  | 0.319882   | 0.149114  | 0.141906  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Outstate                         | 0.050159   | -0.025755 | -0.155477 | 0.562331  | 0.489394  | -0.215742   | -0.253512   | 1.000000    | 0.654256    | 0.038855  | -0.299087  | 0.382982  | 0.407996  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Room.Board                       | 0.164939   | 0.090899  | -0.040232 | 0.371480  | 0.331490  | -0.068890   | -0.061326   | 0.654256    | 1.000000    | 0.127963  | -0.199428  | 0.329202  | 0.374540  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Books                            | 0.132559   | 0.113525  | 0.112711  | 0.118858  | 0.115527  | 0.115550    | 0.081200    | 0.038855    | 0.127963    | 1.000000  | 0.179295   | 0.026906  | 0.099955  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Personal                         | 0.178731   | 0.200989  | 0.280929  | -0.093316 | -0.080810 | 0.317200    | 0.319882    | -0.299087   | -0.199428   | 0.179295  | 1.000000   | -0.010936 | -0.030611 |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| PhD                              | 0.390697   | 0.355758  | 0.331469  | 0.531828  | 0.545862  | 0.318337    | 0.149114    | 0.382982    | 0.329202    | 0.026906  | -0.010936  | 1.000000  | 0.849587  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Terminal                         | 0.369491   | 0.337583  | 0.308274  | 0.491135  | 0.524749  | 0.300019    | 0.141904    | 0.407983    | 0.374540    | 0.099955  | -0.030611  | 0.849587  | 1.000000  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| S.F.Ratio                        | 0.095633   | 0.176229  | 0.237271  | -0.384875 | -0.294629 | 0.279703    | 0.232531    | -0.554821   | -0.362628   | -0.031929 | 0.136345   | -0.130530 | -0.160106 |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| perc.alumni                      | -0.090226  | -0.159990 | -0.180794 | 0.455485  | 0.417864  | -0.229462   | -0.280792   | 0.566262    | 0.272363    | -0.040208 | -0.285968  | 0.249009  | 0.267111  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Expend                           | 0.259592   | 0.124717  | 0.064169  | 0.660913  | 0.527447  | 0.018852    | -0.083568   | 0.672779    | 0.501739    | 0.112409  | -0.097892  | 0.432762  | 0.438755  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Grad.Rate                        | 0.146755   | 0.067313  | -0.022341 | 0.494989  | 0.477281  | -0.078773   | -0.257001   | 0.571290    | 0.424942    | 0.001061  | -0.269344  | 0.305038  | 0.289525  |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Figure 2.3.2: Correlation Matrix |  |           |           |           |           |             |             |             |             |           |            |           |           |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |
| Inference                        | <p>As can be seen from above correlation matrix,</p> <ol style="list-style-type: none"><li>Application, acceptance, enrolled, full-time graduates are highly correlated.</li><li>High correlation visible in Top 10 and top 25 percentage.</li></ol>   |           |           |           |           |             |             |             |             |           |            |           |           |     |        |      |          |          |          |          |          |          |          |          |          |          |          |          |          |        |          |          |          |          |          |          |          |           |          |          |          |          |          |        |          |          |          |          |          |          |          |           |           |          |          |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |          |          |          |          |           |          |          |          |           |          |          |             |          |          |          |          |          |          |          |           |           |          |          |          |          |             |          |          |          |           |           |          |          |           |           |          |          |          |          |          |          |           |           |          |          |           |           |          |          |          |           |          |          |            |          |          |           |          |          |           |           |          |          |          |           |          |          |       |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |           |           |          |          |           |           |          |          |           |           |     |          |          |          |          |          |          |          |          |          |          |           |          |          |          |          |          |          |          |          |          |          |          |          |          |           |          |          |           |          |          |          |           |           |          |          |           |           |           |          |           |           |             |           |           |           |          |          |           |           |          |          |           |           |          |          |        |          |          |          |          |          |          |           |          |          |          |           |          |          |           |          |          |           |          |          |           |           |          |          |          |           |          |          |

## 2.4 Check the dataset for outliers before and after scaling. What insight do you derive here?

|        |  |
|--------|--|
| Answer | The outliers before applying the scaling and after scaling remains the same. This shows scaling does not do any treatment to outliers. It only normalises the data to a standard scale for better understanding. |
|--------|--|







## 2.6 Perform PCA and export the data of the Principal Component (eigenvectors) into a data frame with the original features

|        |  |
|--------|--|
| Answer | <ol style="list-style-type: none"> <li>1. Calculated the cumulative of all eigen vectors that summed to 100.</li> <li>2. Checked for variance up to 90%. Rejected the features if the percentage gap is more than 5%. In this case PCA is 5.</li> <li>3. Exported the data in dataframe as can be seen in below output.</li> </ol> |
|--------|--|

## Output

```
In [28]: tot = sum(eig_vals)
var_exp = [(i / tot) * 100 for i in sorted(eig_vals, reverse=True)]
cum_var_exp = np.cumsum(var_exp)
cum_var_exp
```

```
Out[28]: array([ 32.0206282 ,  58.36084263,  65.26175919,  71.18474841,
 76.67315352,  81.65785448,  85.21672597,  88.67034731,
 91.78758099,  94.16277251,  96.00419883,  97.30024023,
 98.28599436,  99.13183669,  99.64896227,  99.86471628,
100.      ])
```

```
In [36]: pca = PCA(n_components=5)
X_pca= pca.fit_transform(df_pca_scaled)
```

```
In [30]: pca.components_
```

```
Out[30]: array([[ 0.2487656 ,  0.2076015 ,  0.17630359,  0.35427395,  0.34400128,
 0.15464096,  0.0264425 ,  0.29473642,  0.24903045,  0.06475752,
-0.04252854,  0.31831287,  0.31705602, -0.17695789,  0.20508237,
 0.31890875,  0.25231565],
 [ 0.33159823,  0.37211675,  0.40372425, -0.08241182, -0.04477866,
 0.41767377,  0.31508783, -0.24964352, -0.13780888,  0.05634184,
 0.21992922,  0.05831132,  0.04642945,  0.24666528, -0.24659527,
-0.13168987, -0.16924053],
 [-0.06309177, -0.10124947, -0.08298583,  0.03505557, -0.02414803,
-0.06139256,  0.13968168,  0.04659896,  0.14896738,  0.67741165,
 0.49972111, -0.12702832, -0.06603759, -0.28984842, -0.14698927,
 0.22674391, -0.20806465],
 [ 0.28131048,  0.26781741,  0.16182682, -0.05154726, -0.10976653,
 0.10041227, -0.15855848,  0.13129135,  0.18499599,  0.08708922,
-0.23071057, -0.53472484, -0.51944301, -0.16118948,  0.01731422,
 0.07927351,  0.26912907],
 [ 0.00574198,  0.05578538, -0.05569409, -0.39543428, -0.42653376,
-0.04345363,  0.30238534,  0.22253215,  0.56091945, -0.12728881,
-0.22231105,  0.14016642,  0.20471965, -0.07938829, -0.21629741,
 0.07595799, -0.10926792]])
```

Figure: 2.6.1: PCA on dataset

```
In [37]: df_comp = pd.DataFrame(pca.components_, columns=list(df_pca_scaled))
df_comp.T
```

```
Out[37]:
```

|             | 0         | 1         | 2         | 3         | 4         |
|-------------|-----------|-----------|-----------|-----------|-----------|
| Apps        | 0.248766  | 0.331598  | -0.063092 | 0.281311  | 0.005741  |
| Accept      | 0.207602  | 0.372117  | -0.101249 | 0.267817  | 0.055786  |
| Enroll      | 0.176304  | 0.403724  | -0.082986 | 0.161827  | -0.055694 |
| Top10perc   | 0.354274  | -0.082412 | 0.035056  | -0.051547 | -0.395434 |
| Top25perc   | 0.344001  | -0.044779 | -0.024148 | -0.109767 | -0.426534 |
| F.Undergrad | 0.154641  | 0.417674  | -0.061393 | 0.100412  | -0.043454 |
| P.Undergrad | 0.026443  | 0.315088  | 0.139682  | -0.158558 | 0.302385  |
| Outstate    | 0.294736  | -0.249644 | 0.046599  | 0.131291  | 0.222532  |
| Room.Board  | 0.249030  | -0.137809 | 0.148967  | 0.184996  | 0.560919  |
| Books       | 0.064758  | 0.056342  | 0.677412  | 0.087089  | -0.127289 |
| Personal    | -0.042529 | 0.219929  | 0.499721  | -0.230711 | -0.222311 |
| PhD         | 0.318313  | 0.058311  | -0.127028 | -0.534725 | 0.140166  |
| Terminal    | 0.317056  | 0.046429  | -0.066038 | -0.519443 | 0.204720  |
| S.F.Ratio   | -0.176958 | 0.246665  | -0.289848 | -0.161189 | -0.079388 |
| perc.alumni | 0.205082  | -0.246595 | -0.146989 | 0.017314  | -0.216297 |
| Expend      | 0.318909  | -0.131690 | 0.226744  | 0.079273  | 0.075958  |
| Grad.Rate   | 0.252316  | -0.169241 | -0.208065 | 0.269129  | -0.109268 |

Figure: 2.6.2: Data in dataframe after PCA

**2.7 Write down the explicit form of the first PC (in terms of the eigenvectors. Use values with two places of decimals only). [hint: write the linear equation of PC in terms of eigenvectors and corresponding features]**

**Answer**

With the help of linear equation, following is the explicit form of PCA for 1<sup>st</sup> component:

|                                |   |
|--------------------------------|---|
|                                | <p>The Linear eq of 1 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Un dergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Pe rsonal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate}$ <p>Similarly, for the other components the explicit form is generated, refer to output below.</p>  |
| Output of explicit form of PCA | <pre> In [27]: for j in range(1, 18):           print('\n\nThe Linear eq of %s component: %j')           for i in range(0,17):               print('{:} * {:}'.format(np.round(pca.components_[0][i],2),df_pca_scaled.columns[i]),end=' + ')           print('\n') </pre> <p>The Linear eq of 1 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 2 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 3 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 4 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 5 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 6 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 7 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ <p>The Linear eq of 8 component:</p> $0.25 * \text{Apps} + 0.21 * \text{Accept} + 0.18 * \text{Enroll} + 0.35 * \text{Top10perc} + 0.34 * \text{Top25perc} + 0.15 * \text{F.Undergrad} + 0.03 * \text{P.Undergrad} + 0.29 * \text{Outstate} + 0.25 * \text{Room.Board} + 0.06 * \text{Books} + -0.04 * \text{Personal} + 0.32 * \text{PhD} + 0.32 * \text{Terminal} + -0.18 * \text{S.F.Ratio} + 0.21 * \text{perc.alumni} + 0.32 * \text{Expend} + 0.25 * \text{Grad.Rate} +$ |



The Linear eq of 9 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 10 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 11 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 12 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 13 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 14 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 15 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 16 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

The Linear eq of 17 component:  
 $0.25 * Apps + 0.21 * Accept + 0.18 * Enroll + 0.35 * Top10perc + 0.34 * Top25perc + 0.15 * F.Undergrad + 0.03 * P.Undergrad + 0.29 * Outstate + 0.25 * Room.Board + 0.06 * Books + -0.04 * Personal + 0.32 * PhD + 0.32 * Terminal + -0.18 * S.F.Ratio + 0.21 * perc.alumni + 0.32 * Expend + 0.25 * Grad.Rate +$

**Figure 2.7.1: Explicit form of PCA**

**2.8 Consider the cumulative values of the eigenvalues. How does it help you to decide on the optimum number of principal components? What do the eigenvectors indicate?**

**Answer**

After adding the Eigen Values, we will get sum of 100.  
 To decide the optimum number of principal components:  
 1. Check for cumulative variance up to 90%  
 2. The incremental value between the components should not be less than five percent.

The Eigen vectors or PC for this case study is five, we can understand how much each variable contributes to the principal components. In other words, we can also say weights attached to each variable. With this Eigen vectors we can understand which variable has more weightage and influences the dataset in the principal components. The PCA reduces the multi collinearity and with this reduced collinearity we can runs models and improved efficiency scores.

After PCA multicollinearity is reduced.

## Output

```
In [28]: tot = sum(eig_vals)
var_exp = [(i / tot) * 100 for i in sorted(eig_vals, reverse=True)]
cum_var_exp = np.cumsum(var_exp)
cum_var_exp
```

```
Out[28]: array([ 32.0206282 ,  58.36084263,  65.26175919,  71.18474841,
 76.67315352,  81.65785448,  85.21672597,  88.67034731,
 91.78758099,  94.16277251,  96.00419883,  97.30024023,
 98.28599436,  99.13183669,  99.64896227,  99.86471628,
100.      ])
```

```
In [36]: pca = PCA(n_components=5)
X_pca= pca.fit_transform(df_pca_scaled)
```

```
In [30]: pca.components_
```

```
Out[30]: array([[ 0.2487656 ,  0.2076015 ,  0.17630359,  0.35427395,  0.34400128,
 0.15464096,  0.0264425 ,  0.29473642,  0.24903045,  0.06475752,
-0.04252854,  0.31831287,  0.31705602, -0.17695789,  0.20508237,
 0.31890875,  0.25231565],
 [ 0.33159823,  0.37211675,  0.40372425, -0.08241182, -0.04477866,
 0.41767377,  0.31508783, -0.24964352, -0.13780888,  0.05634184,
 0.21992922,  0.05831132,  0.04642945,  0.24666528, -0.24659527,
-0.13168987, -0.16924053],
 [-0.06309177, -0.10124947, -0.08298583,  0.03505557, -0.02414803,
-0.06139256,  0.13968168,  0.04659896,  0.14896738,  0.67741165,
 0.49972111, -0.12702832, -0.06603759, -0.28984842, -0.14698927,
 0.22674391, -0.20806465],
 [ 0.28131048,  0.26781741,  0.16182682, -0.05154726, -0.10976653,
 0.10041227, -0.15855848,  0.13129135,  0.18499599,  0.08708922,
-0.23071057, -0.53472484, -0.51944301, -0.16118948,  0.01731422,
 0.07927351,  0.26912907],
 [ 0.00574198,  0.05578538, -0.05569409, -0.39543428, -0.42653376,
-0.04345363,  0.30238534,  0.22253215,  0.56091945, -0.12728881,
-0.22231105,  0.14016642,  0.20471965, -0.07938829, -0.21629741,
 0.07595799, -0.10926792]])
```

Figure 2.8.1: Output of PCA

```
In [33]: df_comp = pd.DataFrame(pca.components_, columns=list(df_pca_scaled))
df_comp
```

```
Out[33]:
```

|   | Apps      | Accept    | Enroll    | Top10perc | Top25perc | F.Undergrad | P.Undergrad | Outstate  | Room.Board | Books     | Personal  | PhD       | Terminal  | S.F.Ratio |
|---|-----------|-----------|-----------|-----------|-----------|-------------|-------------|-----------|------------|-----------|-----------|-----------|-----------|-----------|
| 0 | 0.248766  | 0.207602  | 0.176304  | 0.354274  | 0.344001  | 0.154641    | 0.026443    | 0.294736  | 0.249030   | 0.064758  | -0.042529 | 0.318313  | 0.317056  | -0.176957 |
| 1 | 0.331598  | 0.372117  | 0.403724  | -0.082412 | -0.044779 | 0.417674    | 0.315088    | -0.249644 | -0.137809  | 0.056342  | 0.219929  | 0.058311  | 0.046429  | 0.246665  |
| 2 | -0.063092 | -0.101249 | -0.082986 | 0.035056  | -0.024148 | -0.061393   | 0.139682    | 0.046599  | 0.148967   | 0.677412  | 0.499721  | -0.127028 | -0.066038 | -0.289848 |
| 3 | 0.281310  | 0.267817  | 0.161827  | -0.051547 | -0.109767 | 0.100412    | -0.158558   | 0.131291  | 0.184996   | 0.087089  | -0.230711 | -0.534725 | -0.519443 | -0.161189 |
| 4 | 0.005742  | 0.055785  | -0.055694 | -0.395434 | -0.426534 | -0.043454   | 0.302385    | 0.222532  | 0.560919   | -0.127289 | -0.222311 | 0.140166  | 0.204720  | -0.079388 |

```
In [34]: plt.figure(figsize=(15,7))
sns.heatmap(df_comp, cmap='rainbow', annot=True, fmt='.2f', yticklabels=['PC0', 'PC1', 'PC2', 'PC3', 'PC4'])
plt.show()
```

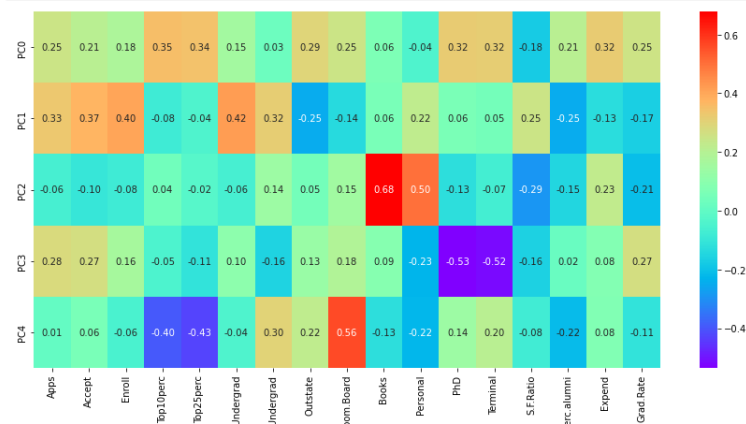


Figure 2.8.2: PCA Data in Data frame

**2.9 Explain the business implication of using the Principal Component Analysis for this case study. How may PCs help in the further analysis? [Hint: Write Interpretations of the Principal Components Obtained]**

**Answer**

Based on observation on dataset and PCA test, following are business implications:

|  |  |
|--|--|
|  | <ol style="list-style-type: none"><li>1. The Education dataset which has details about the multiple colleges and universities helps to study the various aspects of colleges, applications, fees, type of enrolments, etc.</li><li>2. The univariate and multivariate analysis helps to understand the data pattern for each variable and the correlation between variables.</li><li>3. From multivariate analysis, we saw that many variables are highly correlated.</li><li>4. With the help of scaling, data is brought down to one standard scale which gives better understanding of the dataset.</li><li>5. The Principal Component Analysis helps to reduce multicollinearity in dataset, which further helps to perform more analysis and derive results.</li><li>6. The PCA for this case study is 5 which shows the maximum variance of dataset.</li></ol> |
|--|--|