$$\int \frac{dx}{\left[a^2+x^2\right]^{3/2}}$$

la $\quad x = a\sinh u \qquad u = \sinh^{-1}\frac{x}{a}$

$\quad dx = a\cosh u \, du$

$$= \int \frac{a\cosh u \, du}{\left[a^2(1+\sinh^2 u)\right]^{3/2}}$$

$$= \frac{1}{a^2}\int \frac{\cosh u \, du}{\cosh^3 u}$$

$$= \frac{1}{a^2}\int \frac{du}{\cosh^2 u}$$

$$= \frac{1}{a^2}\tanh u$$

$$= \frac{1}{a^2}\tanh \sinh^{-1}\frac{x}{a}$$

$$= \frac{x}{a^3}\frac{1}{\cosh \sinh^{-1}\frac{x}{a}}$$

Har at

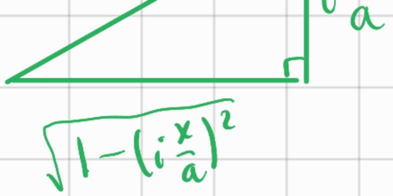
$$\cosh x = \cos ix$$

$$\sinh x = -i\sin ix \quad \Rightarrow \sinh^{-1}x = -i\sin^{-1}(ix)$$

$$= \frac{x}{a^3}\left[\cos\left(i\left(-i\sin^{-1}\left(i\frac{x}{a}\right)\right)\right)\right]^{-1}$$

$$x \left[\quad \right]^{-1}$$

$$= \frac{x}{a^3}\left[\cos\left(\sin^{-1}\left(i\frac{x}{a}\right)\right)\right]$$



$$= \frac{x}{a^3}\left[\sqrt{1-\left(i\frac{x}{a}\right)^2}\right]^{-1}$$

$$= \frac{x}{a^2}\,\frac{1}{\sqrt{a^2+x^2}}$$