Уменьшение цены абстракции при типобезопасном встраивании реляционнного языка программирования в OCaml

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JetBrains Research

Языки программирования и компиляторы
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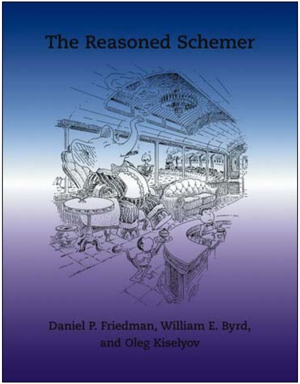
Ростов-на-Дону

Реляционное программирование на miniKanren

От программ-функций к программам-отношениям:

$$f : X \to Y \rightsquigarrow f^o \subseteq X \times Y$$



- Изначально DSL для Scheme/Racket с довольно минималистичной реализацией
- Семейство языков (μKanren, α-Kanren, cKanren, и т.д.)
- Встраивается как DSL в широкий набор языков (включая OCaml, Haskell, Scala, и т.д.)
- Daniel P. Friedman, William Byrd and Oleg Kiselyov. The Reasoned Schemer, The MIT Press, Cambridge, MA, 2005

Пример: реляционное слияние списков (OCaml/OCanren/miniKanren)

$\text{append} : \alpha \text{ list} \to \alpha \text{ list} \to \alpha \text{ list}$

$\text{append}^o \subseteq \alpha \text{ list} \times \alpha \text{ list} \times \alpha \text{ list}$

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В оригинальной реализации:

```
(define (appendᵒ xs ys xys)
  (conde
    [(≡ '() xs) (≡ ys xys)]
    [(fresh (h t tys)
       (≡ `(,h . ,t) xs)
       (≡ `(,h . ,tys) xys)
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Unification and refinement are virtually the main things to implement

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- the safety of unification/refinement implementation has to be justified separately;
- states must not escape their scope (otherwise the coherence between variable types and terms, stored in states, can be lost).

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Here:

- run — the only way to run goals;
- $\bar{n}$ — a numeral, describing the number of fresh variables, available for running the goal g; numerals can be manufactured quantum satis using the successor function, which is provided as well;
- $q_1, q_2 \dots q_n$ — these fresh variables;
- $a_1, a_2 \dots a_n$ — the streams of refined answers for the variables $q_1, q_2 \dots q_n$ respectively;
- h — a handler, which can make use of refined answers.

The framework guarantees, that variables are refined only in correct states.

Converting from- and to User Types

Injecting a user-type into logic domain and projecting the logical results back:

$$\begin{aligned} \uparrow_t &: t \to t^o \\ \downarrow_t &: t^o \to t \end{aligned}$$

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let rec (↑tree) t = ↑∀ ( fmap_tree_f (↑∀) (↑tree) t)
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$\texttt{let rec } (\uparrow_{\mathrm{tree}}) \texttt{ t = } \uparrow_\forall \texttt{ (} \mathit{fmap}_{\mathrm{tree}_f} \texttt{ (}\uparrow_\forall\texttt{) (}\uparrow_{\mathrm{tree}}\texttt{) t)}$

$\texttt{let rec } (\downarrow_{\mathrm{tree}}) \texttt{ l = } \mathit{fmap}_{\mathrm{tree}_f} \texttt{ (}\downarrow_\forall\texttt{) (}\downarrow_{\mathrm{tree}}\texttt{) (}\downarrow_\forall \texttt{ l)}$

Example

Current Implementation

- Repository: https://github.com/dboulytchev/OCanren
- Implements μKanren + disequality constraints
- Passes most of the original tests
- Outperforms μKanren on long queries

last

```
type 'a list = Nil | Cons of 'a * 'a list

let (_: int list) = Cons (1, Nil)
```