Expressions: Abstract Syntax

$$X = \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \dots\} \text{ — variables}$$

$$\otimes = \{\mathtt{+},\ \mathtt{-},\ \mathtt{*},\ \mathtt{/},\ \mathtt{\%},\ \mathtt{<},\ \mathtt{<=},\ \mathtt{>},\ \mathtt{>=},\ \mathtt{==},\ \mathtt{!=},\ \mathtt{\&\&},\ \mathtt{||}\} \text{ — binary operators}$$

$$\mathcal{E} = \mathbb{Z} \mid X \mid \mathcal{E} \otimes \mathcal{E}$$

Small-Step Operational Semantics (Non-Strict)

$$s : X \to \mathbb{Z} \text{ — partial function from variables to integers (state)}$$

$$s \xrightarrow{\mathtt{x}} s\,\mathtt{x},\ \mathtt{x} \in X \qquad [\textsc{Var}]$$

$$s \xrightarrow{z} z,\ z \in \mathbb{Z} \qquad [\textsc{Const}]$$

$$\frac{s \xrightarrow{A} A'}{s \xrightarrow{A \otimes B} A' \otimes B},\ A \notin \mathbb{Z} \qquad [\textsc{Binop\_Left}]$$

$$\frac{s \xrightarrow{B} B'}{s \xrightarrow{a \otimes B} a \otimes B'},\ \begin{array}{cc} a \in \mathbb{Z} & \wedge \\ B \notin \mathbb{Z} & \wedge \\ (\otimes \neq \text{"||"} \vee a \neq 1) & \wedge \\ (\otimes \neq \text{"\&\&"} \vee a \neq 0) & \end{array} \qquad [\textsc{Binop\_Right}]$$

$$s \xrightarrow{a \otimes B} a,\ \begin{array}{cc} (\otimes = \text{"||"} \wedge a = 1) & \vee \\ (\otimes = \text{"\&\&"} \wedge a = 0) & \end{array} \qquad [\textsc{Binop\_Left\_NS}]$$

$$s \xrightarrow{a \otimes b} a \oplus b,\ a, b \in \mathbb{Z} \qquad [\textsc{Binop}]$$

$\otimes$	$a \oplus b$
$+$	$a + b$
$-$	$a - b$
$*$	$a \times b$
$/$	$a/b,\ b \neq 0$
$\%$	$a \bmod b,\ b \neq 0$
$<$	$\begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases}$
$<=$	$\begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases}$
$>$	$\begin{cases} 1 & , \quad a > b \\ 0 & , \quad a \leq b \end{cases}$
$>=$	$\begin{cases} 1 & , \quad a \geq b \\ 0 & , \quad a < b \end{cases}$
$==$	$\begin{cases} 1 & , \quad a = b \\ 0 & , \quad a \neq b \end{cases}$
$!=$	$\begin{cases} 1 & , \quad a \neq b \\ 0 & , \quad a = b \end{cases}$
$\&\&$	$\overline{a} \vee \overline{b}$
$\vert\vert$	$\overline{a} \wedge \overline{b}$

$$\overline{x} = \begin{cases} true & , \quad x = 1 \\ false & , \quad x = 0 \end{cases}$$

Semantic Function

$$[\![\bullet]\!] : \mathcal{E} \mapsto (X \to \mathbb{Z}) \to \mathbb{Z}$$

$$[\![E]\!]\, s = z \iff s \xrightarrow{E} z$$