

Expressions: Abstract Syntax

$$X = \{\mathbf{x}, \mathbf{y}, \mathbf{z}, \dots\} \text{ — variables}$$

$$\otimes = \{+, -, *, /, \%, <, <=, >, >=, ==, !=, \&\&, ||\} \text{ — binary operators}$$

$$\mathcal{E} = \mathbb{Z} \mid X \mid \mathcal{E} \otimes \mathcal{E}$$

Small-Step Operational Semantics (Non-Strict)

$$s : X \rightarrow \mathbb{Z} \text{ — partial function from variables to integers (state)}$$

$$s \xrightarrow{\mathbf{x}} s \ \mathbf{x}, \ \mathbf{x} \in X \quad [\text{VAR}]$$

$$s \xrightarrow{z} z, \ z \in \mathbb{Z} \quad [\text{CONST}]$$

$$\frac{s \xrightarrow{A} A'}{s \xrightarrow{A \otimes B} A' \otimes B}, \ A \notin \mathbb{Z} \quad [\text{BINOP_LEFT}]$$

$$\frac{s \xrightarrow{B} B'}{s \xrightarrow{a \otimes B} a \otimes B'}, \quad \begin{array}{l} a \in \mathbb{Z} \quad \wedge \\ B \notin \mathbb{Z} \quad \wedge \\ (\otimes \neq "||" \vee a \neq 1) \quad \wedge \\ (\otimes \neq "\&\&" \vee a \neq 0) \end{array} \quad [\text{BINOP_RIGHT}]$$

$$s \xrightarrow{a \otimes B} a, \quad \begin{array}{l} (\otimes = "||" \wedge a = 1) \quad \vee \\ (\otimes = "\&\&" \wedge a = 0) \end{array} \quad [\text{BINOP_LEFT_NS}]$$

$$s \xrightarrow{a \otimes b} a \oplus b, \ a, b \in \mathbb{Z} \quad [\text{BINOP}]$$

\otimes	$a \oplus b$
$+$	$a + b$
$-$	$a - b$
$*$	$a \times b$
$/$	$a/b, b \neq 0$
$\%$	$a \bmod b, b \neq 0$
$<$	$\begin{cases} 1 & , & a \leq b \\ 0 & , & a > b \end{cases}$
\leq	$\begin{cases} 1 & , & a \leq b \\ 0 & , & a > b \end{cases}$
$>$	$\begin{cases} 1 & , & a > b \\ 0 & , & a \leq b \end{cases}$
\geq	$\begin{cases} 1 & , & a \geq b \\ 0 & , & a < b \end{cases}$
$==$	$\begin{cases} 1 & , & a = b \\ 0 & , & a \neq b \end{cases}$
$!=$	$\begin{cases} 1 & , & a \neq b \\ 0 & , & a = b \end{cases}$
$\&\&$	$\bar{a} \vee \bar{b}$
$\ \$	$\bar{a} \wedge \bar{b}$

$$\bar{x} = \begin{cases} true & , & x = 1 \\ false & , & x = 0 \end{cases}$$

Semantic Function

$$\llbracket \bullet \rrbracket : \mathcal{E} \mapsto (X \rightarrow \mathbb{Z}) \rightarrow \mathbb{Z}$$

$$\llbracket E \rrbracket s = z \iff s \xrightarrow{E} z$$