Lambda Calculus

Abstract Syntax

$X = \{\texttt{x}, \texttt{y}, \texttt{z}, \ldots\}$ — variables

$C = \{\texttt{X}, \texttt{Y}, \texttt{Z}, \ldots\}$ — constructors

$$
\begin{array}{llll}
\Lambda = & X & \text{— variable} \\
& | \quad C & \text{— constructor} \\
& | \quad \lambda X.\Lambda & \text{— abstraction} \\
& | \quad \Lambda\Lambda & \text{— application}
\end{array}
$$

Head Reductions (Big-Step)

Substitution:

$$
x[z \leftarrow B] = \left\{ \begin{array}{lll} B & , & z = x \\ x & , & z \neq x \end{array} \right.
$$

$$
X[x \leftarrow B] = X
$$

$$
(M\ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])
$$

$$
(\lambda z.B)[x \leftarrow A] = \left\{ \begin{array}{lll} \lambda z.B & , & z = x \\ \lambda z.(B[x \leftarrow A) & , & z \neq x \end{array} \right.
$$

Reduction rules:

$$
x \to x,\ x \in X \cup C \tag{[VAR]}
$$

$$
\frac{A \to A'}{\lambda x.A \to \lambda x.A'} \tag{[ABS]}
$$

$$
\frac{M \to \lambda x.A,\ A[x \leftarrow N] \to K}{M\ N \to K} \tag{[RED]}
$$

$$\frac{M \to M'}{M\ N \to M'\ N}, \ M' \neq \lambda x.A \qquad\qquad [\textsc{App}]$$