Lambda Calculus

Abstract Syntax

$X = \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \dots\}$ — variables

$C = \{\mathtt{X}, \mathtt{Y}, \mathtt{Z}, \dots\}$ — constructors

$$\begin{array}{rcll} \Lambda = & X & \text{— variable} \\ \mid & C & \text{— constructor} \\ \mid & \lambda X.\Lambda & \text{— abstraction} \\ \mid & \Lambda\Lambda & \text{— application} \end{array}$$

Normal Order Reductions (Small-Step)

Substitution:

$$x[z \leftarrow B] = \begin{cases} B & , \quad z = x \\ x & , \quad z \neq x \end{cases}$$

$$X[x \leftarrow B] = X$$

$$(M\ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])$$

$$(\lambda z.B)[x \leftarrow A] = \begin{cases} \lambda z.B & , \quad z = x \\ \lambda z.(B[x \leftarrow A) & , \quad z \neq x \end{cases}$$

Reduction rules:

$$\frac{A \to A'}{\lambda x.A \to \lambda x.A'} \qquad [\textsc{Abs}]$$

$$(\lambda x.A)\ B \to A[x \leftarrow B] \qquad [\textsc{Red}]$$

$$\frac{M \to M'}{M\ N \to M'\ N},\ M \neq \lambda x.A,\ M \text{ is not in a normal form} \qquad [\textsc{App}]$$

$$\frac{N \to N'}{M\ N \to M\ N'},\ M \text{ is in a normal form} \qquad [\textsc{Arg}]$$