Lambda Calculus

Abstract Syntax

$X = \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \dots\}$ — variables

$C = \{\mathtt{X}, \mathtt{Y}, \mathtt{Z}, \dots\}$ — constructors

$$\begin{array}{rcll} \Lambda = & X & \text{— variable} \\ \mid & C & \text{— constructor} \\ \mid & \lambda X.\Lambda & \text{— abstraction} \\ \mid & \Lambda\Lambda & \text{— application} \end{array}$$

Simple Typing á la Curry

Types:

$$\mathcal{T} = A \mid \mathcal{T} \to \mathcal{T}$$

where A is a set of primitive types.

Environment $\Gamma : X \to \mathcal{T}$ is an assignment of types to (some) variables.

Typing judjements:

$$\Gamma \vdash x : \Gamma\, x \qquad \text{[Var]}$$

$$\frac{\Gamma, x : \tau \vdash M : \rho}{\Gamma \vdash \lambda x.M : \tau \to \rho} \qquad \text{[Abstr]}$$

$$\frac{\Gamma \vdash M : \tau \to \rho, \Gamma \vdash N : \tau}{\Gamma \vdash M\ N : \rho} \qquad \text{[App]}$$