Lambda Calculus

Abstract Syntax

$X = \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \dots\}$ — variables

$C = \{\mathtt{X}, \mathtt{Y}, \mathtt{Z}, \dots\}$ — constructors

$$\begin{array}{rcll} \Lambda = & X & \text{— variable} \\ \mid & C & \text{— constructor} \\ \mid & \lambda X.\Lambda & \text{— abstraction} \\ \mid & \Lambda\Lambda & \text{— application} \end{array}$$

Head-Linear Reductions (Small-Step)

$$x : A, \Gamma;\ \Delta \vdash x \to A \qquad [\textsc{Var}]$$

$$\frac{\Gamma;\ N \cdot \Delta \vdash M \to M'}{\Gamma;\ \Delta \vdash M\ N \to M'\ N} \qquad [\textsc{App}]$$

$$\frac{x : B, \Gamma;\ \Delta \vdash A \to A'}{\Gamma;\ B \cdot \Delta \vdash \lambda x.A \to \lambda x.A'} \qquad [\textsc{Abs-Empty}]$$

$$\frac{\Gamma;\ \epsilon \vdash A \to A'}{\Gamma;\ \epsilon \vdash \lambda x.A \to \lambda x.A'} \qquad [\textsc{Abs}]$$