

# Lambda Calculus

## Abstract Syntax

$X = \{x, y, z, \dots\}$  – variables

$C = \{X, Y, Z, \dots\}$  – constructors

|             |                              |               |
|-------------|------------------------------|---------------|
| $\Lambda =$ | $X$                          | – variable    |
|             | $  \quad C$                  | – constructor |
|             | $  \quad \lambda X. \Lambda$ | – abstraction |
|             | $  \quad \Lambda \Lambda$    | – application |

## Call by Name (Big-Step)

Substitution:

$$x[z \leftarrow B] = \begin{cases} B & , \quad z = x \\ x & , \quad z \neq x \end{cases}$$

$$X[x \leftarrow B] = X$$

$$(M \ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])$$

$$(\lambda z. B)[x \leftarrow A] = \begin{cases} \lambda z. B & , \quad z = x \\ \lambda z. (B[x \leftarrow A]) & , \quad z \neq x \end{cases}$$

Reduction rules:

$$x \rightarrow x, \quad x \in X \cup C \quad \text{[VAR]}$$

$$\frac{M \rightarrow \lambda x. A, \quad A[x \leftarrow N] \rightarrow K}{M \ N \rightarrow K} \quad \text{[RED]}$$

$$\frac{M \rightarrow M'}{M \ N \rightarrow M' \ N}, \quad M' \neq \lambda x. A \quad \text{[APP]}$$