

Lambda Calculus

Abstract Syntax

$X = \{x, y, z, \dots\}$ — variables

$C = \{X, Y, Z, \dots\}$ — constructors

$\Lambda =$	X	— variable
	$ \quad C$	— constructor
	$ \quad \lambda X. \Lambda$	— abstraction
	$ \quad \Lambda \Lambda$	— application

Weak Head Reductions/Call by Name (Small-Step)

Substitution:

$$x[z \leftarrow B] = \begin{cases} B & , \quad z = x \\ x & , \quad z \neq x \end{cases}$$

$$X[x \leftarrow B] = X$$

$$(M \ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])$$

$$(\lambda z. B)[x \leftarrow A] = \begin{cases} \lambda z. B & , \quad z = x \\ \lambda z. (B[x \leftarrow A]) & , \quad z \neq x \end{cases}$$

Reduction rules:

$$x \rightarrow x \quad \text{[VAR]}$$

$$X \rightarrow X \quad \text{[CNSTR]}$$

$$\lambda x. A \rightarrow \lambda x. A \quad \text{[ABS]}$$

$$(\lambda x.A) B \rightarrow A[x \leftarrow B] \quad [\text{RED}]$$

$$\frac{M \rightarrow M'}{M N \rightarrow M' N}, \quad M \neq \lambda x.A \quad [\text{APP}]$$