

## Expressions: Abstract Syntax

$X = \{\mathbf{x}, \mathbf{y}, \mathbf{z}, \dots\}$  — variables

$\otimes = \{+, -, *, /, \%, <, <=, >, >=, ==, !=, \&\&, ||\}$  — binary operators

$\mathcal{E} = \mathbb{Z} \mid X \mid \mathcal{E} \otimes \mathcal{E}$

## Big-Step Operational Semantics (Strict)

$s : X \rightarrow \mathbb{Z}$  — partial function from variables to integers (state)

$s, \mathbf{x} s \mathbf{xx} \in X$  [VAR]

$s, z z z \in \mathbb{Z}$  [CONST]

$$\frac{s \xrightarrow{A} a, s \xrightarrow{B} b}{s \xrightarrow{A \otimes B} a \oplus b}$$
 [BINOP]

$\otimes$	$a \oplus b$
$+$	$a + b$
$-$	$a - b$
$*$	$a \times b$
$/$	$a/b, b \neq 0$
$\%$	$a \bmod b, b \neq 0$
$<$	$\begin{cases} 1 & , & a \leq b \\ 0 & , & a > b \end{cases}$
$\leq$	$\begin{cases} 1 & , & a \leq b \\ 0 & , & a > b \end{cases}$
$>$	$\begin{cases} 1 & , & a > b \\ 0 & , & a \leq b \end{cases}$
$\geq$	$\begin{cases} 1 & , & a \geq b \\ 0 & , & a < b \end{cases}$
$==$	$\begin{cases} 1 & , & a = b \\ 0 & , & a \neq b \end{cases}$
$!=$	$\begin{cases} 1 & , & a \neq b \\ 0 & , & a = b \end{cases}$
$\&\&$	$\bar{a} \vee \bar{b}$
$\ \$	$\bar{a} \wedge \bar{b}$

$$\bar{x} = \begin{cases} true & , & x = 1 \\ false & , & x = 0 \end{cases}$$

### Semantic Function

$$\llbracket \bullet \rrbracket : \mathcal{E} \mapsto (X \rightarrow \mathbb{Z}) \rightarrow \mathbb{Z}$$

$$\llbracket E \rrbracket s = z \iff s \xrightarrow{E} z$$