Lambda Calculus

Abstract Syntax

$$X = \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \dots\} \text{ — variables}$$

$$C = \{\mathtt{X}, \mathtt{Y}, \mathtt{Z}, \dots\} \text{ — constructors}$$

$$\begin{array}{rcl} \Lambda = & X & \text{— variable} \\ \mid & C & \text{— constructor} \\ \mid & \lambda X.\Lambda & \text{— abstraction} \\ \mid & \Lambda\Lambda & \text{— application} \end{array}$$

Normal Order Reductions (Big-Step)

Substitution:

$$x[z \leftarrow B] = \begin{cases} B & , \quad z = x \\ x & , \quad z \neq x \end{cases}$$

$$X[x \leftarrow B] = X$$

$$(M\ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])$$

$$(\lambda z.B)[x \leftarrow A] = \begin{cases} \lambda z.B & , \quad z = x \\ \lambda z.(B[x \leftarrow A) & , \quad z \neq x \end{cases}$$

Reduction rules:

$$x \to x,\ x \in X \cup C \qquad \text{[Var]}$$

$$\frac{A \to A'}{\lambda x.A \to \lambda x.A'} \qquad \text{[Abs]}$$

$$\frac{A[x \leftarrow N] \to K}{(\lambda x.A)\ N \to K} \qquad \text{[Red]}$$

$$\frac{M \to \lambda x.A,\ A[x \leftarrow N] \to K}{M\ N \to K} \qquad [\textsc{App}]$$

$$\frac{M \to M',\ N \to N'}{M\ N \to M'\ N'},\ M' \neq \lambda x.A \qquad [\textsc{Arg}]$$