

# Lambda Calculus

## Abstract Syntax

$X = \{x, y, z, \dots\}$  — variables

$C = \{X, Y, Z, \dots\}$  — constructors

|             |                      |               |
|-------------|----------------------|---------------|
| $\Lambda =$ | $X$                  | — variable    |
|             | $C$                  | — constructor |
|             | $\lambda X. \Lambda$ | — abstraction |
|             | $\Lambda \Lambda$    | — application |

## Normal Order Reductions (Small-Step)

Substitution:

$$x[z \leftarrow B] = \begin{cases} B & , \quad z = x \\ x & , \quad z \neq x \end{cases}$$

$$X[x \leftarrow B] = X$$

$$(M \ N)[x \leftarrow A] = (M[x \leftarrow A])(N[x \leftarrow A])$$

$$(\lambda z. B)[x \leftarrow A] = \begin{cases} \lambda z. B & , \quad z = x \\ \lambda z. (B[x \leftarrow A]) & , \quad z \neq x \end{cases}$$

Reduction rules:

$$\frac{A \rightarrow A'}{\lambda x. A \rightarrow \lambda x. A'} \quad [\text{ABS}]$$

$$(\lambda x. A) \ B \rightarrow A[x \leftarrow B] \quad [\text{RED}]$$

$$\frac{M \rightarrow M'}{M \ N \rightarrow M' \ N}, \quad M \neq \lambda x. A, \quad M \text{ is not in a normal form} \quad [\text{APP}]$$

$$\frac{N \rightarrow N'}{M \ N \rightarrow M \ N'}, \text{ } M \text{ is in a normal form} \quad [\text{ARG}]$$