MiniKanren Semantics for Programs Transformation

No Author Given

No Institute Given

1 Syntax

Supplementary syntax categories:

$$
\begin{aligned}
\mathcal{X} &= x_1,\, x_2, \dots && \text{(syntactic variables)} \\
\mathcal{S} &= \sigma_1,\, \sigma_2, \dots && \text{(semantic variables)} \\
\mathcal{R}^n &= r_1^n, r_2^n, \dots && \text{(predicate names indexed by the arity n)}
\end{aligned}
$$

Terms:

$$
\begin{aligned}
\mathcal{T}(\mathcal{V}) &= \mathcal{V} \,|\, C^n\, t_1, \dots, t_n && \text{(terms parameterized by variable type)} \\
\mathcal{T}_{\mathcal{X}} &= \mathcal{T}(\mathcal{X}) && \text{(terms which contain only syntactic variables)} \\
\mathcal{T}_{\mathcal{S}} &= \mathcal{T}(\mathcal{S}) && \text{(terms which contain only syntactic variables)}
\end{aligned}
$$

Goals:

$$
\begin{aligned}
\mathcal{G} = {} & t_1 \equiv t_2 && \text{(unification)} \\
| \; & g_1 \wedge g_2 && \text{(conjunction)} \\
| \; & g_1 \vee g_2 && \text{(disjunction)} \\
| \; & \underline{\texttt{fresh}}\;\; x \;\; g && \text{(fresh logical variable binder)} \\
| \; & \mathcal{R}^n\, t_1, \dots, t_n && \text{(relations)}
\end{aligned}
$$

Definitions:

$$
\mathcal{D}^n = \lambda x_1, \dots, x_n . g \;\; \text{(relation definition)}
$$

States:

$$
\begin{aligned}
I &= \mathcal{X} \to \mathcal{T}_{\mathcal{S}} && \text{(syntax variables interpretation)} \\
\Sigma &= \mathcal{S} \to \mathcal{T}_{\mathcal{S}} && \text{(substitutions)} \\
\Delta &= 2^{\mathcal{S}} && \text{(the set of used semantic variables)} \\
\mathfrak{S} &= I \times \Sigma \times \Delta && \text{(states)} \\
\Gamma &= \mathcal{R}^n \to \mathcal{D}^n && \\
\Gamma &\vdash \mathfrak{s} \xrightarrow{g} \bar{\mathfrak{s}} && \text{(semantics of the goal g)}
\end{aligned}
$$

2 Semantics

$$\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{t_1 \equiv t_2} [],\ mgu\,(\iota\, t_1)\sigma\ (\iota\, t_2)\sigma = \bot$$

$$\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{t_1 \equiv t_2} (\iota, mgu\,(\iota\, t_1)\sigma\ (\iota\, t_2)\sigma, \delta)$$

$$\frac{\Gamma \vdash (\iota', \sigma, \delta') \xrightarrow{g} \bar{\mathfrak{s}}}{\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{\underline{\texttt{fresh}}\ x\ g} \bar{\mathfrak{s}}},\ \iota' = \iota[x \leftarrow s], \delta' = \delta \cup \{x\}, x \notin \delta$$

$$\frac{\Gamma \vdash (\iota', \sigma, \delta) \xrightarrow{g} \bar{\mathfrak{s}}}{\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{r^n\ t_1 \dots t_n} \bar{\mathfrak{s}}},\ \iota' = \iota[x_i \leftarrow t_i\iota], \Gamma\, r^n = \lambda x_1 \dots x_n . g$$

$$\begin{array}{c}
\Box\iota\ :\ \mathcal{T_X} \to \mathcal{T_S} \\
x\iota = \iota(x) \\
\mathcal{C}^n\, t_1 \dots t_n = \mathcal{C}^n\,(t_1\iota) \dots (t_n\iota)
\end{array}$$

$$\frac{\Gamma \vdash \mathfrak{s} \xrightarrow{g_1} \bar{\mathfrak{s}_1} \quad \Gamma \vdash \mathfrak{s} \xrightarrow{g_2} \bar{\mathfrak{s}_2}}{\Gamma \vdash \mathfrak{s} \xrightarrow{g_1 \vee g_2} \bar{\mathfrak{s}_1} \oplus \bar{\mathfrak{s}_2}}$$

$$\frac{\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{g_1} \bar{\mathfrak{s}_1} \quad \Gamma \vdash \bar{\mathfrak{s}_1} \xrightarrow{g_2\iota} \bar{\mathfrak{s}_2}}{\Gamma \vdash (\iota, \sigma, \delta) \xrightarrow{g_1 \wedge g_2} \bar{\mathfrak{s}_2}}$$

$$\frac{\Gamma \vdash \mathfrak{s} \xrightarrow{g} \bar{\mathfrak{s}_1} \quad \Gamma \vdash \bar{\mathfrak{s}} \xrightarrow{g} \bar{\mathfrak{s}_2}}{\Gamma \vdash \mathfrak{s} : \bar{\mathfrak{s}} \xrightarrow{g} \bar{\mathfrak{s}_1} \oplus \bar{\mathfrak{s}_2}}$$