XXTitle

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XXTITLE

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Dedication

Abstract

Útdráttur á ensku sem er að hámarki 250 orð.

Útdráttur

Hér kemur útdráttur á íslensku sem er að hámarki 250 orð. Reynið að koma útdráttum á eina blaðsíðu en ef tvær blaðsíður eru nauðsynlegar á seinni blaðsíða útdráttar að hefjast á oddatölusíðu (hægri síðu).

Preface

Formála má sleppa og skal þá fjarlægja þessa blaðsíðu. Formáli skal hefjast á oddatölu blaðsíðu og nota skal Section Break (Odd Page).

Ekki birtist blaðsíðutal á þessum fyrstu síðum ritgerðarinnar en blaðsíðurnar teljast með og hafa áhrif á blaðsíðutal sem birtist með rómverskum tölum fyrst á efnisyfirliti.

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Abbreviations

Í þessum kafla mega koma fram listar yfir skammstafanir og/eða breytuheiti. Gefið kaflanum nafn við hæfi, t.d. Skammstafanir eða Breytuheiti. Þessum kafla má sleppa ef hans er ekki þörf.
The section could be titled: Glossary, Variable Names, etc.

Acknowledgments

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1. Introduction

2. Preliminaries

2.1. Functional analysis

2.1.1. Hahn-Banach

2.1.2. Riesz representation theorem

3. Rudin-Carleson theorem

4. F. and M. Riesz theorem

5. A generalization of the Rudin-Carleson theorem

This borrows from Bishop (reference TODO).

Theorem 1 (General Rudin-Carleson theorem)**.** *Let X be a compact Hausdorff space, $V = (C(X), \|\cdot\|_\infty)$, B be a closed subspace of $C(X)$, $B^\perp$ be the annihilating measures for B, S be a closed subset of X, and f be a continues function on S. If $\int_S f d\mu = 0$ holds for all $\mu \in B^\perp$ then there exists a function $F \in B$ such that $F = f$ on S.*

Proof. Since f is continuous and S is a closed subset of a compact set, and therefore also compact, f is bounded. So we can, with out loss of generality, assume that $|f| < r < 1$ on S. Let U_r be the subset of B defined by $U_r = \{g;\ \|g\| < r\}$ and ϕ be the mapping from B to $C(S)$ that sends a member of B to its restriction on S. It suffices to show that $f \in \phi(U_r)$. Let's first show that $f \in \overline{\phi(U_r)} =: V_r$, by assuming otherwise, and showing it leads to a contradiction.

We now assume $f \notin V_r$. By Hahn-Banach (TODO ref) we can define a bounded linear functional α, such that $|\alpha(f)| > 1$ and $|\alpha(h)| < 1$, for $h \in V_r$. We can then define a measure μ_1 by the Riesz-representation theorem (TODO ref) that fulfills

$$\alpha(g) = \int g d\mu_1$$

for all $g \in C(S)$. We will refer to the associated functional on B by $\beta(g) = \phi(\alpha(g))$. Since $\phi(g) \in V_r$ for all $g \in U_r$ we have that

$$\beta(g) = \alpha(\phi(g)) < 1,$$

for all $g \in U_r$, due to the construction of α. From this we get

$$\begin{aligned}
\|\beta\| &= \sup\{|\beta(g)|;\ |g| < 1\} \\
&= \sup\{(1/r)|\beta(g)|;\ |g| < r\} \\
&< \sup\{(1/r);\ |g| < r\} \\
&= 1/r.
\end{aligned}$$

□

A. Annað