

Lengstu vaxandi hlutrunur (LIS)

Bergur Snorrason

1. febrúar 2022

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- ▶ Sjáum núna hvernig við getum gert það með kvikri betun.

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- ▶ Það er oft talað um að finna „lengstu vaxandi hlutrununa“ þó það geti verið margar jafna langar „lengstu vaxandi hlutrunur“.

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- ▶ Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \leq k < x \\ a_k \leq a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```
6 int v[MAXN], d[MAXN];
7 int dp_lookup(int x)
8 {
9     if (d[x] != -1) return d[x];
10    if (x == 0) return 1;
11    int i;
12    for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13        d[x] = max(d[x], dp_lookup(i) + 1);
14    return d[x];
15 }
16
17 int lis(int *a, int n)
18 {
19     int i, r = 1;
20     for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
21     for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
22     return r;
```

- ▶ Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- ▶ Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun.

- ▶ Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak hlutrunu (a_1, \dots, a_x) af lengd y .

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- ▶ Við fáum því

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- ▶ Grunntilvikin eru til að auðvelda útfærslu.
- ▶ Lengd lengstu vaxandi hlutrunanna er þá stærst ℓ þannig að $f(n, \ell)$ er ekki ∞ .

```

6 int v[MAXN], d[MAXN][MAXN];
7 int dp_lookup(int x, int y)
8 {
9     if (x == -1) return y == 0 ? -(1 << 30) : (1 << 30);
10    if (d[x][y] != -1) return d[x][y];
11    if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
12    return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
13 }
14
15 int lis(int *a, int n)
16 {
17     int i, j, x;
18     for (i = 0; i < n; i++) v[i] = a[i];
19     for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
20     for (x = n; dp_lookup(n - 1, x) == (1 << 30); x--);
21     return x;
22 }

```

- ▶ Við erum með tvívítt stöðurúm af stærð $(n + 1) \cdot (n + 1)$ og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- ▶ Heildar tímaflækjan er því $\mathcal{O}(n^2)$.
- ▶ En þetta er sama tímaflækja og við fengum áðan.
- ▶ Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

► Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- Sjáum að fallgildi $f(x, y)$ ákvarðast af $f(x-1, y)$ og $f(x-1, y-1)$.
- Svo við getum fyllt í töfluna í lestrarröð.


```

1 #include <stdio.h>
2 #include <string.h>
3 #define MAXN 1001
4 int min(int a, int b) { return a > b ? b : a; }
5
6 int lis(int *a, int *b, int n)
7 {
8     int i, j, x, y;
9     int d[n + 1][n + 1];
10    for (i = 0; i < n + 1; i++) d[0][i] = (1 << 30);
11    for (i = 0; i < n + 1; i++) d[i][0] = -(1 << 30);
12    for (i = 1; i < n + 1; i++)
13    {
14        for (j = 0; j < n + 1; j++)
15        {
16            if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
17            else d[i][j] = min(d[i - 1][j], a[i - 1]);
18        }
19        for (int ii = 0; ii < n + 1; ii++) { for (int jj = 0; jj < n + 1; jj++)
20            printf("\n\n");
21        }
22        for (x = n; d[n][x] == (1 << 30); x--);
23        for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
24        return x;
25    }
26
27 int main()
28 {
29     int n, m, i, j, x, l;
30     scanf("%d", &n);
31     int a[n], b[n];
32     for (i = 0; i < n; i++) scanf("%d", &a[i]);
33     l = lis(a, b, n);
34
35     printf("Lend lengstu vaxandi hlutruna er %d\n", l);
36     for (i = 0; i < l; i++) printf("%d ", b[i]);
37     printf("\n");
38

```

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- ▶ Heildar tímaflækjan er sú sama og áðan, svo hún er $\mathcal{O}(n^2)$.
- ▶ Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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-INF	1	6	INF	INF	INF	INF	INF	INF	INF
-INF	1	5	INF	INF	INF	INF	INF	INF	INF
-INF	1	5	8	INF	INF	INF	INF	INF	INF
-INF	1	4	8	INF	INF	INF	INF	INF	INF
-INF	1	4	7	INF	INF	INF	INF	INF	INF
-INF	1	4	7	9	INF	INF	INF	INF	INF

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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

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- ▶ Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrst skrefinu.
- ▶ En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```

1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <string.h>
4 #include <time.h>
5 #include <assert.h>
6 #define MAXN 1001
7 int max(int a, int b) { return a < b ? b : a; }
8
9 int lis(int *a, int *b, int n)
10 {
11     int i, j, x, y;
12     int d[n + 1], e[n];
13     for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -(1 << 30) : (1 << 30);
14     for (i = 0; i < n; i++)
15     {
16         int r = 0, s = n + 1;
17         while (r < s)
18         {
19             int m = (r + s) / 2;
20             if (d[m] <= a[i]) r = m + 1;
21             else s = m;
22         }
23         d[r] = a[i], e[i] = d[r - 1];
24     }
25     for (x = n; d[x] == (1 << 30); x--);
26     for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--)
27         if (a[i] == y) y = e[i], b[j--] = a[i];
28     return x;
29 }
30
31 int v[MAXN], d[MAXN];
32 int dp_lookup(int x)
33 {
34     if (d[x] != -1) return d[x];
35     if (x == 0) return 1;
36     int i;
37     for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
38         d[x] = max(d[x], dp_lookup(i) + 1);

```


