## Lengstu vaxandi hlutrunur (LIS)

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<ul> <li>Við höfum nú þegar skoðað hvernig finna megi lengstu vaxandi</li> </ul>
hlutrunur með tæmandi leit.
Sjáum núna hvernig við getum gert það með kvikri bestun.

## Hlutrunur

- Látum  $(a_1, \ldots, a_n)$  vera runu af tölum.
- Runa  $(b_1, \ldots, b_k)$  er *hlutruna* rununnar ef til eru heiltölur  $j_1 < \cdots < j_k$  þannig að  $a_{j_m} = b_m$ .
- Runan (1,2,3,4,5) hefur, til dæmis, hlutrunurnar (1,2,5), (1,3,5) og (4).
- ▶ En aftur á móti eru (2,1) og (1,6) ekki hlutrunur.
- Við viljum nú, fyrir gefna runu, finna lengdina á lengstu vaxandi hlutrunum rununnar.
- Það er oft talað um að finna "lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar "lengstu vaxandi hlutrunur".

- Látum  $(a_1, \ldots, a_n)$  vera n talna runu.
- Látum f(x) tákna lengd lengstu vaxandi hlutrunu  $(a_1, \ldots, a_x)$ ,

Þá fáum við

sem endar í  $a_x$ .

 $f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \le k < x \\ a_k < a_k}} f(k) + 1, & \text{annars.} \end{cases}$ 

```
6 int v[MAXN], d[MAXN];
7 int dp lookup(int x)
8
  {
9
       if (d[x] != -1) return d[x];
10
       if (x == 0) return 1;
11
       int i;
12
       for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13
           d[x] = max(d[x], dp lookup(i) + 1);
14
       return d[x];
15 }
16
17
  int lis(int *a, int n)
18
```

for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];

for  $(i = 0; i < n; i++) r = max(r, dp_lookup(i));$ 

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int i. r = 1:

return r;

- ▶ Við erum með einvítt stöðurúm af stærð n og hver uppfærsla
- er í  $\mathcal{O}(n)$ .
- ► Heildartímaflækjan er því  $\mathcal{O}(n^2)$ .

▶ Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur

með kvikri bestun, en annarri rakningarformúlu.

- Látum nú f(x, y) tákna minnsta mögulega aftasta stak vaxandi hlutrunu  $(a_1, \ldots, a_x)$  af lengd y.
- Við fáum bví

- - $f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$
- Grunntilvikin eru til að auðvelda útfærlsu.
- ▶ Lengd lengstu vaxandi hlutrunanna er bá stærsta ℓ bannig að  $f(n,\ell)<\infty$ .

```
10     if (x == -1) return y == 0 ? -INF : INF;
11     if (d[x][y] != -1) return d[x][y];
12     if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
13     return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
14 }
15
16 int lis(int *a, int n)
17 {</pre>
```

for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;

7 int v[MAXN], d[MAXN][MAXN]; 8 int dp lookup(int x, int y)

int i, j, x;

return x:

for (i = 0; i < n; i++) v[i] = a[i];

for (x = n; dp lookup(n - 1, x) == INF; x--);

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- ▶ Við erum með tvívítt stöðurúm af stærð  $(n+1) \cdot (n+1)$  og hver uppfærsla er í  $\mathcal{O}(1)$ .
- Heildartímaflækjan er því  $\mathcal{O}(n^2)$ .
- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

Rifium upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Sjáum að fallgildi f(x, y) ákvarðast af f(x 1, y) og
- f(x-1, y-1).

Svo við getum fyllt í töfluna í lestrarröð.

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7 int lis(int *a, int *b, int n)
   {
9
       int i, j, x, y;
       int d[n + 1][n + 1];
       for (i = 0; i < n + 1; i++) d[0][i] = INF;
       for (i = 0; i < n + 1; i++) d[i][0] = -INF;
       for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
            \text{if } (a[i-1] < d[i-1][j-1]) \ d[i][j] = d[i-1][j]; \\
           else d[i][j] = min(d[i-1][j], a[i-1]);
       for (x = n; d[n][x] == INF; x--);
       for (i = 0; i < x; i++) b[i] = d[n-x+i+1][i+1];
19
       return x:
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- ▶ Heildartímaflækjan er sú sama og áðan, svo hún er  $\mathcal{O}(n^2)$ .
- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum

hvort við getum bætt tímaflækjuna.

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- Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.
- Annað sem við sjáum er að línurnar eru allar vaxandi.
- Að lokum sjáum við að hver lína er aðeins uppfærð einu sinni í

hverri ítrun.

En afhverju gildir betta?

## Afhverju er allar línurnar raðaðar

Rifjum upp að

$$f(x,y) = \left\{ \begin{array}{ll} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{array} \right.$$

- Þetta fæst með einfaldri þrepun.
- Fyrst línan er alltaf  $-\infty, \infty, \ldots, \infty$ , sem er röðuð.
- ► Gerum nú ráð fyrir að við séum í línu x og við þurfum að uppfæra y.
- ▶ Þá þarf  $a_x \ge f(x-1,y-1)$  því uppfærlsa á sér aðeins stað í síðasta tilfellinu.
  - ▶ En einnig barf  $a_x < f(x-1, y)$ .
  - ► Svo  $f(x-1, y-1) \le f(x, y) < f(x-1, y)$ .
  - ► En þá fæst líka að  $f(x,y) < f(x-1,y) \le f(x,y+1)$ .

## Afhverju uppfærum við bara einu sinni í hverri ítrun?

Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Gerum ráð fyrir að við séum í línu x og við þurfum að uppfæra y.
- Eins og við sáum áðan þá er  $f(x-1,y-1) \le a_x < f(x-1,y)$  nauðsynlegt skilyrði fyrir uppfærslu.
- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

## Afhverju uppfærum við bara einu sinni í hverri ítrun?

- ▶ Við getum því lýst uppfærslu y-tu línunnar með:
  - Finnum x bannig að  $f(x-1, y-1) \le a_x < f(x-1, y)$ .
  - Látum  $f(x,y) = a_x$ .
  - Uppfærum næstu línu.
- Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.
- En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```
10 int lis(int *a, int *b, int n)
11 {
12
       int i, j, x, y;
13
       int d[n + 1], e[n];
       for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
14
15
       for (i = 0: i < n: i++)
16
       {
17
           int r = 0, s = n + 1:
18
           while (r < s)
19
20
                int m = (r + s)/2;
21
                if (d[m] \le a[i]) r = m + 1;
22
                else s = m;
23
24
           d[r] = a[i], e[i] = d[r - 1];
25
       for (x = n; d[x] == INF; x--);
26
```

for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)

y = e[i], b[j--] = a[i];

27

28 29

30 }

return x;

- Nú er forritið einföld for-lykkja af lengd n sem framkvæmir
- helmingunarleit í hverri ítrun.

▶ Helmingunarleitin er  $\mathcal{O}(\log n)$ .

▶ Heildartímaflækan er því  $\mathcal{O}(n \log n)$ .