# Lengstu vaxandi hlutrunur

Bergur Snorrason

February 19, 2024

► Við höfum nú þegar skoðað hvernig finna megi lengstu vaxandi hlutrunur með tæmandi leit.

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- ► Sjáum núna hvernig við getum gert það með kvikri bestun.

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- Það er oft talað um að finna "lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar "lengstu vaxandi hlutrunur".



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- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \le k < x \\ a_k \le a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```
6 int v[MAXN], d[MAXN];
7 int dp lookup(int x)
8
9
       if (d[x] != -1) return d[x];
10
       if (x == 0) return 1;
11
       int i;
12
       for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13
           d[x] = max(d[x], dp lookup(i) + 1);
14
       return d[x];
15 }
16
17 int lis(int *a, int n)
18
19
       int i, r = 1;
       for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
20
       for (i = 0; i < n; i++) r = max(r, dp lookup(i));
21
22
       return r;
23 }
```

▶ Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í  $\mathcal{O}($  ).

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- Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

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$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

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Grunntilvikin eru til að auðvelda útfærlsu.

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- Grunntilvikin eru til að auðvelda útfærlsu.
- ▶ Lengd lengstu vaxandi hlutrunanna er þá stærsta  $\ell$  þannig að  $f(n,\ell) < \infty$ .

```
7 int v[MAXN], d[MAXN][MAXN];
8 int dp lookup(int x, int y)
9 {
10
       if (x == -1) return y == 0 ? -INF : INF;
11
       if (d[x][y] != -1) return d[x][y];
       if (v[x] < dp \ lookup(x-1, y-1)) return d[x][y] = dp \ lookup(x-1, y);
12
       return d[x][y] = min(dp lookup(x - 1, y), v[x]);
13
14 }
15
16
  int lis(int *a, int n)
17
   {
18
       int i, j, x;
       for (i = 0; i < n; i++) v[i] = a[i];
19
20
       for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
21
       for (x = n; dp_lookup(n - 1, x) = INF; x--);
22
       return x:
23 }
```

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- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

► Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

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Sjáum að fallgildi f(x, y) ákvarðast af f(x - 1, y) og f(x - 1, y - 1).

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- Sjáum að fallgildi f(x, y) ákvarðast af f(x 1, y) og f(x 1, y 1).
- Svo við getum fyllt í töfluna í lestrarröð.

```
7 int lis(int *a, int *b, int n)
8
9
       int i, j, x, y, d[n + 1][n + 1];
10
       for (i = 0; i < n + 1; i++) d[0][i] = INF;
11
       for (i = 0; i < n + 1; i++) d[i][0] = -INF;
       for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
12
13
       {
           if (a[i-1] < d[i-1][j-1]) d[i][j] = d[i-1][j];
14
           else d[i][j] = min(d[i-1][j], a[i-1]);
15
16
17
       for (x = n; d[n][x] == INF; x--);
       for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
18
19
       return x:
20 }
```

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- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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► Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.

- ► Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.
- Annað sem við sjáum er að línurnar eru allar vaxandi.

- ► Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.
- Annað sem við sjáum er að línurnar eru allar vaxandi.
- Að lokum sjáum við að hver lína er aðeins uppfærð einu sinni í hverri ítrun.

- ► Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.
- Annað sem við sjáum er að línurnar eru allar vaxandi.
- Að lokum sjáum við að hver lína er aðeins uppfærð einu sinni í hverri ítrun.
- ► En afhverju gildir þetta?

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

► Rifjum upp að

$$f(x,y) = \left\{ \begin{array}{ll} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{array} \right.$$

Þetta fæst með einfaldri þrepun.

$$f(x,y) = \left\{ \begin{array}{ll} -\infty, & \text{ef } x=0 \text{ og } y=0, \\ \infty, & \text{annars, ef } x=0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y),a_x), & \text{annars.} \end{array} \right.$$

- Þetta fæst með einfaldri þrepun.
- lacktriangle Fyrst línan er alltaf  $-\infty,\infty,\ldots,\infty$ , sem er röðuð.

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- ▶ En þá fæst líka að  $f(x,y) < f(x-1,y) \le f(x,y+1)$ .

# Afhverju uppfærum við bara einu sinni í hverri ítrun?

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- Petta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

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- ► En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```
10 int lis(int *a, int *b, int n)
11 {
12
       int i, j, x, y, d[n + 1], e[n];
       for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
13
       for (i = 0; i < n; i++)
14
15
16
           int r = -1. s:
17
           for (s = n + 1; s >= 1; s /= 2)
18
               while (r + s < n + 1 \&\& d[r + s] < a[i]) r += s;
19
           d[r + 1] = a[i], e[i] = d[r];
20
       for (x = n; d[x] == INF; x--);
21
22
       for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
23
           y = e[i], b[j--] = a[i];
24
       return x;
25 }
```

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