Lengstu vaxandi hlutrunur

Bergur Snorrason

9. febrúar 2023

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- Sjáum núna hvernig við getum gert það með kvikri bestun.

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- Það er oft talað um að finna „lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar „lengstu vaxandi hlutrunur".

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- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \leq k < x \\ a_k \leq a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```
 6  int v[MAXN], d[MAXN];
 7  int dp_lookup(int x)
 8  {
 9      if (d[x] != -1) return d[x];
10      if (x == 0) return 1;
11      int i;
12      for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13          d[x] = max(d[x], dp_lookup(i) + 1);
14      return d[x];
15  }
16
17  int lis(int *a, int n)
18  {
19      int i, r = 1;
20      for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
21      for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
22      return r;
23  }
```

- Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í $\mathcal{O}(\ \ )$.

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- ▶ Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

▶ Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak vaxandi hlutrunu $(a_1, \ldots, a_x)$ af lengd y.

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- ▶ Við fáum því

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f(x, y) = \begin{cases}
-\infty, & \text{ef } x = 0 \text{ og } y = 0, \\
\infty, & \text{annars, ef } x = 0, \\
f(x - 1, y), & \text{ef } a_x < f(x - 1, y - 1), \\
\min(f(x - 1, y), a_x), & \text{annars.}
\end{cases}
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- ▶ Grunntilvikin eru til að auðvelda útfærlsu.

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- Grunntilvikin eru til að auðvelda útfærlsu.
- Lengd lengstu vaxandi hlutrunanna er þá stærsta ℓ þannig að $f(n, \ell) < \infty$.

```
 7  int v[MAXN], d[MAXN][MAXN];
 8  int dp_lookup(int x, int y)
 9  {
10      if (x == -1) return y == 0 ? -INF : INF;
11      if (d[x][y] != -1) return d[x][y];
12      if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
13      return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
14  }
15
16  int lis(int *a, int n)
17  {
18      int i, j, x;
19      for (i = 0; i < n; i++) v[i] = a[i];
20      for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
21      for (x = n; dp_lookup(n - 1, x) == INF; x--);
22      return x;
23  }
```

► Við erum með tvívítt stöðurúm af stærð $(n+1) \cdot (n+1)$ og hver uppfærsla er í $\mathcal{O}(\ )$.

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- ▶ En þetta er sama tímaflækja og við fengum áðan.

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- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

► Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1,y)$ og $f(x-1,y-1)$.

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- Sjáum að fallgildi $f(x, y)$ ákvarðast af $f(x - 1, y)$ og $f(x - 1, y - 1)$.
- Svo við getum fyllt í töfluna í lestrarröð.

```
7  int lis(int *a, int *b, int n)
8  {
9      int i, j, x, y, d[n + 1][n + 1];
10     for (i = 0; i < n + 1; i++) d[0][i] = INF;
11     for (i = 0; i < n + 1; i++) d[i][0] = -INF;
12     for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
13     {
14         if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
15         else d[i][j] = min(d[i - 1][j], a[i - 1]);
16     }
17     for (x = n; d[n][x] == INF; x--);
18     for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
19     return x;
20 }
```

► Heildartímaflækjan er sú sama og áðan, svo hún er $\mathcal{O}(\quad)$.

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- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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> -INF     1     4     7     9   INF   INF
```

```
3, 6, 2, 1, 5, 8, 4, 7, 9
                            ^
                                          |
    -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
    -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
    -INF    3    6  INF  INF  INF  INF  INF  INF  INF
    -INF    2    6  INF  INF  INF  INF  INF  INF  INF
    -INF    1    6  INF  INF  INF  INF  INF  INF  INF
    -INF    1    5  INF  INF  INF  INF  INF  INF  INF
    -INF    1    5    8  INF  INF  INF  INF  INF  INF
    -INF    1    4    8  INF  INF  INF  INF  INF  INF
    -INF    1    4    7  INF  INF  INF  INF  INF  INF
>   -INF    1    4    7    9  INF  INF
```

```
3, 6, 2, 1, 5, 8, 4, 7, 9
                            ^
                                              |
    -INF   INF   INF   INF   INF   INF   INF   INF   INF   INF
    -INF     3   INF   INF   INF   INF   INF   INF   INF   INF
    -INF     3     6   INF   INF   INF   INF   INF   INF   INF
    -INF     2     6   INF   INF   INF   INF   INF   INF   INF
    -INF     1     6   INF   INF   INF   INF   INF   INF   INF
    -INF     1     5   INF   INF   INF   INF   INF   INF   INF
    -INF     1     5     8   INF   INF   INF   INF   INF   INF
    -INF     1     4     8   INF   INF   INF   INF   INF   INF
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                          ^
                                                  |
    -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
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                          ^

                                                        |
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                              ^

                                                  |
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- Annað sem við sjáum er að línurnar eru allar vaxandi.

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- En afhverju gildir þetta?

Afhverju er allar línurnar raðaðar

- Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x - 1, y), & \text{ef } a_x < f(x - 1, y - 1), \\ \min(f(x - 1, y), a_x), & \text{annars.} \end{cases}$$

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▶ Rifjum upp að

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▶ Svo $f(x-1, y-1) \leq f(x, y) < f(x-1, y)$.

▶ En þá fæst líka að $f(x, y) < f(x-1, y) \leq f(x, y+1)$.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

- Við getum því lýst uppfærslu y-tu línunnar með:

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  - Látum $f(x, y) = a_x$.

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- Í neðansækinu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

- ▶ Við getum því lýst uppfærslu y-tu línunnar með:
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- ▶ Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.
- ▶ En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```
10  int lis(int *a, int *b, int n)
11  {
12      int i, j, x, y, d[n + 1], e[n];
13      for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
14      for (i = 0; i < n; i++)
15      {
16          int r = -1, s;
17          for (s = n + 1; s >= 1; s /= 2)
18              while (r + s < n + 1 && d[r + s] < a[i]) r += s;
19          d[r + 1] = a[i], e[i] = d[r];
20      }
21      for (x = n; d[x] == INF; x--);
22      for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
23          y = e[i], b[j--] = a[i];
24      return x;
25  }
```

► Nú er forritið einföld `for`-lykkja af lengd *n* sem framkvæmir helmingunarleit í hverri ítrun.

▶ Nú er forritið einföld `for` -lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.

▶ Helmingunarleitin er $\mathcal{O}(\quad)$.

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- Helmingunarleitin er $\mathcal{O}(\log n)$.
- Heildartímaflækan er því $\mathcal{O}($ $)$.

- Nú er forritið einföld `for`-lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.
- Helmingunarleitin er $\mathcal{O}(\log n)$.
- Heildartímaflækan er því $\mathcal{O}(n \log n)$.