Lengstu vaxandi hlutrunur (LIS)

Bergur Snorrason

31. janúar 2022

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- Sjáum núna hvernig við getum gert það með kvikri betun.

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- Það er oft talað um að finna „lengstu vaxandi hlutrununa" þó það geti verið margar jafna langar „lengstu vaxandi hlutrunur".

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- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max\limits_{\substack{1 \leq k < x \\ a_k \leq a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```c
int v[MAXN], d[MAXN];
int dp_lookup(int x)
{
    if (d[x] != -1) return d[x];
    if (x == 0) return 1;
    int i;
    for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
        d[x] = max(d[x], dp_lookup(i) + 1);
    return d[x];
}

int lis(int *a, int n)
{
    int i;
    for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
    return dp_lookup(n - 1);
}
```

- Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun.

- Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak hlutrunu $(a_1, \ldots, a_x)$ af lengd y.

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- Við fáum því

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

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- Grunntilvikin eru til að auðvelda útfærlsu.
- Lengd lengstu vaxandi hlutrunanna er þá stærst ℓ þannig að $f(n, \ell)$ er ekki ∞.

```
int v[MAXN], d[MAXN][MAXN];
int dp_lookup(int x, int y)
{
    if (x == -1) return y == 0 ? -(1 << 30) : (1 << 30);
    if (d[x][y] != -1) return d[x][y];
    if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
    return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
}

int lis(int *a, int n)
{
    int i, j, x;
    for (i = 0; i < n; i++) v[i] = a[i];
    for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
    for (x = n; dp_lookup(n - 1, x) > n; x--);
    return x;
}
```

- Við erum með tvívítt stöðurúm af stærð $(n+1) \cdot (n+1)$ og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

- Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1,y)$ og $f(x-1,y-1)$.

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- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1,y)$ og $f(x-1,y-1)$.
- Svo við getum fyllt í töfluna í lestrarröð.

```c
#include <stdio.h>
#include <string.h>
#define MAXN 1001
int min(int a, int b) { return a > b ? b : a; }

int lis(int *a, int *b, int n)
{
    int i, j, x, y;
    int d[n + 1][n + 1];
    for (i = 0; i < n + 1; i++) d[0][i] = (1 << 30);
    for (i = 0; i < n + 1; i++) d[i][0] = -(1 << 30);
    for (i = 1; i < n + 1; i++)
    {
        for (j = 0; j < n + 1; j++)
        {
            if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
            else d[i][j] = min(d[i - 1][j], a[i - 1]);
        }
        for (int ii = 0; ii < n + 1; ii++) { for (int jj = 0; jj < n + 1; jj++)
        printf("\n\n");
    }
    for (x = n; d[n][x] > n; x--);
    for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
    return x;
}

int main()
{
    int n, m, i, j, x, l;
    scanf("%d", &n);
    int a[n], b[n];
    for (i = 0; i < n; i++) scanf("%d", &a[i]);
    l = lis(a, b, n);

    printf("Lend lengstu vaxandi hlutruna er %d\n", l);
    for (i = 0; i < l; i++) printf("%d ", b[i]);
    printf("\n");
```

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- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
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  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
  -INF    2    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    7  INF  INF  INF  INF  INF  INF
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  -INF    1    4    7  INF  INF  INF  INF  INF  INF
> -INF    1    4    7    9  INF  INF  INF
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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
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  -INF    1    4    7  INF  INF  INF  INF  INF  INF
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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
  -INF    2    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    7  INF  INF  INF  INF  INF  INF
> -INF    1    4    7    9  INF  INF  INF  INF
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3, 6, 2, 1, 5, 8, 4, 7, 9
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  -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
  -INF    2    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    6  INF  INF  INF  INF  INF  INF  INF
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  -INF    1    5    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    8  INF  INF  INF  INF  INF  INF
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3, 6, 2, 1, 5, 8, 4, 7, 9
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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
  -INF    2    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    8  INF  INF  INF  INF  INF  INF
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> -INF    1    4    7    9  INF  INF  INF  INF  INF
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-INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
-INF	3	INF	INF	INF	INF	INF	INF	INF	INF
-INF	3	6	INF	INF	INF	INF	INF	INF	INF
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-INF	1	6	INF	INF	INF	INF	INF	INF	INF
-INF	1	5	INF	INF	INF	INF	INF	INF	INF
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-INF	1	4	8	INF	INF	INF	INF	INF	INF
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- En afhverju gildir þetta?

Afhverju er allar línurnar raðaðar

- Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

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- Svo $f(x-1, y-1) \leq f(x, y) < f(x-1, y)$.
- En þá fæst líka að $f(x, y) < f(x-1, y) \leq f(x, y+1)$.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

- Við getum því lýst uppfærslu y-tu línunnar með:

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- Í neðansæknu útfærslunniframkvæmum við línulega leit í fyrst skrefinu.
- En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.