Lengstu vaxandi hlutrunur (LIS)

Bergur Snorrason

4. febrúar 2022

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- Sjáum núna hvernig við getum gert það með kvikri bestun.

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- Það er oft talað um að finna „lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar „lengstu vaxandi hlutrunur".

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- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max\limits_{\substack{1 \leq k < x \\ a_k \leq a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```c
int v[MAXN], d[MAXN];
int dp_lookup(int x)
{
    if (d[x] != -1) return d[x];
    if (x == 0) return 1;
    int i;
    for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
        d[x] = max(d[x], dp_lookup(i) + 1);
    return d[x];
}

int lis(int *a, int n)
{
    int i, r = 1;
    for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
    for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
    return r;
}
```

- Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

- Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak vaxandi hlutrunu $(a_1, \ldots, a_x)$ af lengd y.

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- Við fáum því

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- Grunntilvikin eru til að auðvelda útfærlsu.
- Lengd lengstu vaxandi hlutrunanna er þá stærsta ℓ þannig að $f(n,\ell) < \infty$.

```
int v[MAXN], d[MAXN][MAXN];
int dp_lookup(int x, int y)
{
    if (x == -1) return y == 0 ? -(1 << 30) : (1 << 30);
    if (d[x][y] != -1) return d[x][y];
    if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
    return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
}

int lis(int *a, int n)
{
    int i, j, x;
    for (i = 0; i < n; i++) v[i] = a[i];
    for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
    for (x = n; dp_lookup(n - 1, x) == (1 << 30); x--);
    return x;
}
```

- Við erum með tvívítt stöðurúm af stærð $(n+1) \cdot (n+1)$ og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

- Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

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- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1,y)$ og $f(x-1,y-1)$.

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- Sjáum að fallgildi $f(x, y)$ ákvarðast af $f(x-1, y)$ og $f(x-1, y-1)$.
- Svo við getum fyllt í töfluna í lestrarröð.

```c
int lis(int *a, int *b, int n)
{
    int i, j, x, y;
    int d[n + 1][n + 1];
    for (i = 0; i < n + 1; i++) d[0][i] = (1 << 30);
    for (i = 0; i < n + 1; i++) d[i][0] = -(1 << 30);
    for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
    {
        if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
        else d[i][j] = min(d[i - 1][j], a[i - 1]);
    }
    for (x = n; d[n][x] == (1 << 30); x--);
    for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
    return x;
}
```

- Heildartímaflækjan er sú sama og áðan, svo hún er $\mathcal{O}(\quad)$.

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- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
  -INF    3    6  INF  INF  INF  INF  INF  INF  INF
  -INF    2    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    6  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5  INF  INF  INF  INF  INF  INF  INF
  -INF    1    5    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    8  INF  INF  INF  INF  INF  INF
  -INF    1    4    7  INF  INF  INF  INF  INF  INF
> -INF    1    4    7    9  INF  INF
```

```
3, 6, 2, 1, 5, 8, 4, 7, 9
                        ^

                                                      |
  -INF   INF   INF   INF   INF   INF   INF   INF   INF   INF
  -INF     3   INF   INF   INF   INF   INF   INF   INF   INF
  -INF     3     6   INF   INF   INF   INF   INF   INF   INF
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  -INF     1     6   INF   INF   INF   INF   INF   INF   INF
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                                                                             |
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- En afhverju gildir þetta?

Afhverju er allar línurnar raðaðar

- Rifjum upp að

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f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}
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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

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- Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.
- En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```c
int lis(int *a, int *b, int n)
{
    int i, j, x, y;
    int d[n + 1], e[n];
    for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -(1 << 30) : (1 << 30);
    for (i = 0; i < n; i++)
    {
        int r = 0, s = n + 1;
        while (r < s)
        {
            int m = (r + s)/2;
            if (d[m] <= a[i]) r = m + 1;
            else s = m;
        }
        d[r] = a[i], e[i] = d[r - 1];
    }
    for (x = n; d[x] == (1 << 30); x--);
    for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
        y = e[i], b[j--] = a[i];
    return x;
}
```

- Nú er forritið einföld `for`-lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.

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- Helmingunarleitin er $\mathcal{O}(\log n)$.
- Heildartímaflækan er því $\mathcal{O}(n \log n)$.