Lengstu vaxandi hlutrunur (LIS)

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- Við höfum nú þegar skoðað hvernig finna megi lengstu vaxandi hlutrunur með tæmandi leit.
- Sjáum núna hvernig við getum gert það með kvikri bestun.

Hlutrunur

- Látum $(a_1, \ldots, a_n)$ vera runu af tölum.
- Runa $(b_1, \ldots, b_k)$ er *hlutruna* rununnar ef til eru heiltölur $j_1 < \cdots < j_k$ þannig að $a_{j_m} = b_m$.
- Runan $(1, 2, 3, 4, 5)$ hefur, til dæmis, hlutrunurnar $(1, 2, 5)$, $(1, 3, 5)$ og (4).
- En aftur á móti eru $(2, 1)$ og $(1, 6)$ ekki hlutrunur.
- Við viljum nú, fyrir gefna runu, finna lengdina á lengstu vaxandi hlutrunum rununnar.
- Það er oft talað um að finna „lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar „lengstu vaxandi hlutrunur".

- Látum $(a_1, \ldots, a_n)$ vera n talna runu.
- Látum $f(x)$ tákna lengd lengstu vaxandi hlutrunu $(a_1, \ldots, a_x)$, sem endar í a_x.
- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max\limits_{\substack{1 \le k < x \\ a_k \le a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```
 6 int v[MAXN], d[MAXN];
 7 int dp_lookup(int x)
 8 {
 9     if (d[x] != -1) return d[x];
10     if (x == 0) return 1;
11     int i;
12     for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13         d[x] = max(d[x], dp_lookup(i) + 1);
14     return d[x];
15 }
16
17 int lis(int *a, int n)
18 {
19     int i, r = 1;
20     for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
21     for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
22     return r;
23 }
```

- Við erum með einvítt stöðurúm af stærð n og hver uppfærsla er í $\mathcal{O}(\, n\,)$.
- Heildartímaflækjan er því $\mathcal{O}(n^2)$.
- Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

- Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak vaxandi hlutrunu $(a_1, \ldots, a_x)$ af lengd y.
- Við fáum því

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Grunntilvikin eru til að auðvelda útfærlsu.
- Lengd lengstu vaxandi hlutrunanna er þá stærsta ℓ þannig að $f(n, \ell) < \infty$.

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 7 int v[MAXN], d[MAXN][MAXN];
 8 int dp_lookup(int x, int y)
 9 {
10     if (x == -1) return y == 0 ? -INF : INF;
11     if (d[x][y] != -1) return d[x][y];
12     if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
13     return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
14 }
15
16 int lis(int *a, int n)
17 {
18     int i, j, x;
19     for (i = 0; i < n; i++) v[i] = a[i];
20     for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
21     for (x = n; dp_lookup(n - 1, x) == INF; x--);
22     return x;
23 }
```

- Við erum með tvívítt stöðurúm af stærð $(n+1) \cdot (n+1)$ og hver uppfærsla er í $\mathcal{O}(\,1\,)$.
- Heildartímaflækjan er því $\mathcal{O}(n^2)$.
- En þetta er sama tímaflækja og við fengum áðan.
- Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

- Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1,y)$ og $f(x-1,y-1)$.
- Svo við getum fyllt í töfluna í lestrarröð.

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 7 int lis(int *a, int *b, int n)
 8 {
 9     int i, j, x, y;
10     int d[n + 1][n + 1];
11     for (i = 0; i < n + 1; i++) d[0][i] = INF;
12     for (i = 0; i < n + 1; i++) d[i][0] = -INF;
13     for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
14     {
15         if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
16         else d[i][j] = min(d[i - 1][j], a[i - 1]);
17     }
18     for (x = n; d[n][x] == INF; x--);
19     for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
20     return x;
21 }
```

- Heildartímaflækjan er sú sama og áðan, svo hún er $\mathcal{O}(n^2)$.
- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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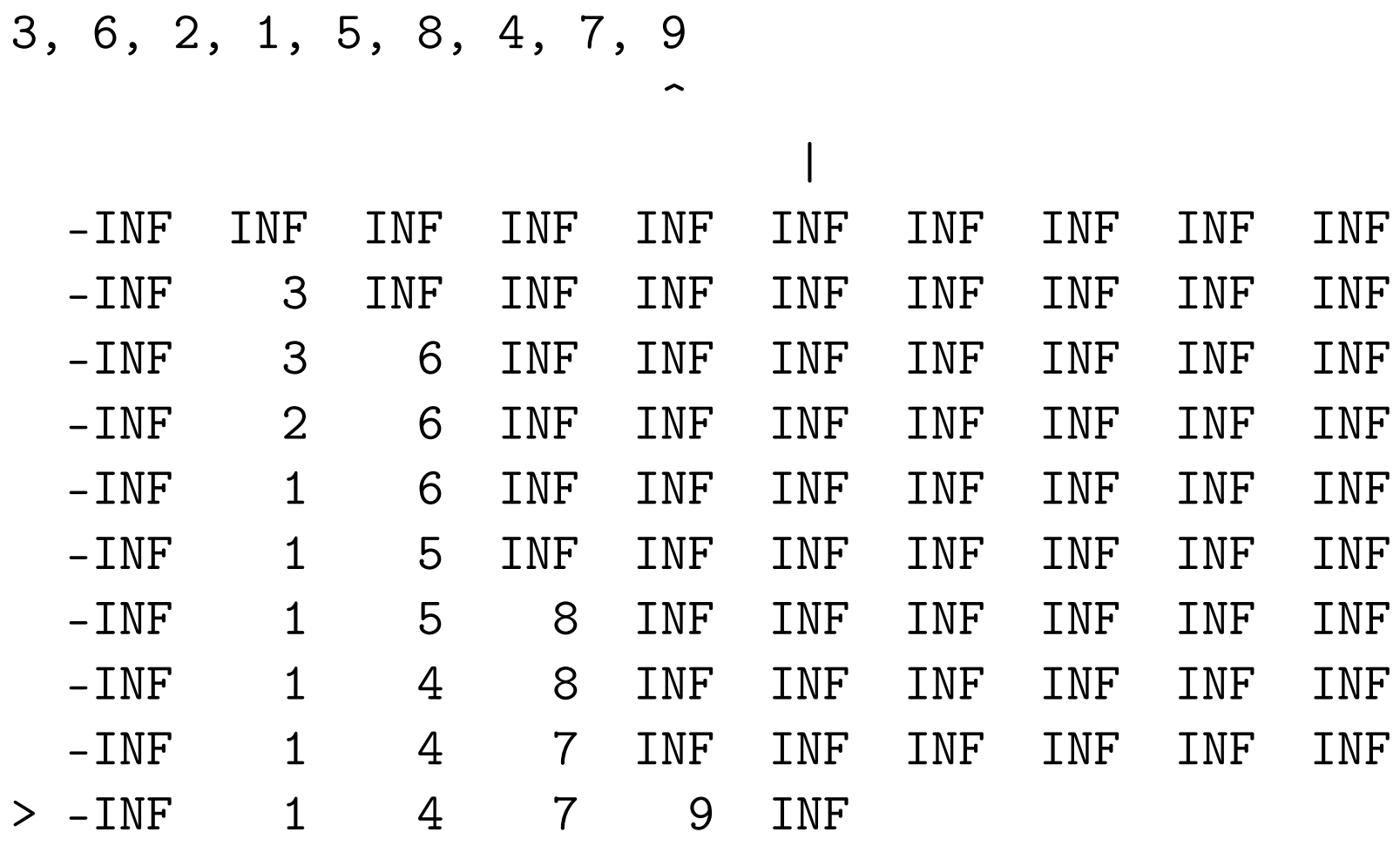
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- Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.
- Annað sem við sjáum er að línurnar eru allar vaxandi.
- Að lokum sjáum við að hver lína er aðeins uppfærð einu sinni í hverri ítrun.
- En afhverju gildir þetta?

Afhverju er allar línurnar raðaðar

- Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Þetta fæst með einfaldri þrepun.
- Fyrst línan er alltaf $-\infty, \infty, \ldots, \infty$, sem er röðuð.
- Gerum nú ráð fyrir að við séum í línu x og við þurfum að uppfæra y.
- Þá þarf $a_x \geq f(x-1,y-1)$ því uppfærlsa á sér aðeins stað í síðasta tilfellinu.
- En einnig þarf $a_x < f(x-1,y)$.
- Svo $f(x-1,y-1) \leq f(x,y) < f(x-1,y)$.
- En þá fæst líka að $f(x,y) < f(x-1,y) \leq f(x,y+1)$.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

- Rifjum upp að

$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1,y), & \text{ef } a_x < f(x-1,y-1), \\ \min(f(x-1,y), a_x), & \text{annars.} \end{cases}$$

- Gerum ráð fyrir að við séum í línu x og við þurfum að uppfæra y.
- Eins og við sáum áðan þá er $f(x-1,y-1) \leq a_x < f(x-1,y)$ nauðsynlegt skilyrði fyrir uppfærslu.
- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

- Við getum því lýst uppfærslu y-tu línunnar með:
  - Finnum x þannig að $f(x-1, y-1) \leq a_x < f(x-1, y)$.
  - Látum $f(x, y) = a_x$.
  - Uppfærum næstu línu.
- Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.
- En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```
10 int lis(int *a, int *b, int n)
11 {
12     int i, j, x, y;
13     int d[n + 1], e[n];
14     for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
15     for (i = 0; i < n; i++)
16     {
17         int r = 0, s = n + 1;
18         while (r < s)
19         {
20             int m = (r + s)/2;
21             if (d[m] <= a[i]) r = m + 1;
22             else s = m;
23         }
24         d[r] = a[i], e[i] = d[r - 1];
25     }
26     for (x = n; d[x] == INF; x--);
27     for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
28         y = e[i], b[j--] = a[i];
29     return x;
30 }
```

- Nú er forritið einföld `for`-lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.
- Helmingunarleitin er $\mathcal{O}(\log n)$.
- Heildartímaflækan er því $\mathcal{O}(n \log n)$.