

# Lengstu vaxandi hlutrunur (LIS)

Bergur Snorrason

10. febrúar 2022

- ▶ Við höfum nú þegar skoðað hvernig finna megí lengstu vaxandi hlutrunur með tæmandi leit.

- ▶ Við höfum nú þegar skoðað hvernig finna meggi lengstu vaxandi hlutrunur með tæmandi leit.
- ▶ Sjáum núna hvernig við getum gert það með kvikri bestun.

# Hlutrúður

- ▶ Látum  $(a_1, \dots, a_n)$  vera runu af tölum.

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- ▶ Runa  $(b_1, \dots, b_k)$  er *hlutruna* rununnar ef til eru heiltölur  $j_1 < \dots < j_k$  þannig að  $a_{j_m} = b_m$ .

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- ▶ Runan  $(1, 2, 3, 4, 5)$  hefur, til dæmis, hlutrunurnar  $(1, 2, 5)$ ,  $(1, 3, 5)$  og  $(4)$ .

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- ▶ En aftur á móti eru  $(2, 1)$  og  $(1, 6)$  ekki hlutrunur.

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- ▶ Runan  $(1, 2, 3, 4, 5)$  hefur, til dæmis, hlutrúnurnar  $(1, 2, 5)$ ,  $(1, 3, 5)$  og  $(4)$ .
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- ▶ Við viljum nú, fyrir gefna runu, finna lengdina á lengstu vaxandi hlutrúnum rununnar.



# Hlutrunur

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- ▶ Runan  $(1, 2, 3, 4, 5)$  hefur, til dæmis, hlutrunurnar  $(1, 2, 5)$ ,  $(1, 3, 5)$  og  $(4)$ .
- ▶ En aftur á móti eru  $(2, 1)$  og  $(1, 6)$  ekki hlutrunur.
- ▶ Við viljum nú, fyrir gefna runu, finna lengdina á lengstu vaxandi hlutrunum rununnar.
- ▶ Það er oft talað um að finna „lengstu vaxandi hlutrununa“ þó það geti verið margar jafn langar „lengstu vaxandi hlutrunur“.

- ▶ Látum  $(a_1, \dots, a_n)$  vera  $n$  talna runu.

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- ▶ Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \leq k < x \\ a_k \leq a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```

6  int v[MAXN], d[MAXN];
7  int dp_lookup(int x)
8  {
9      if (d[x] != -1) return d[x];
10     if (x == 0) return 1;
11     int i;
12     for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13         d[x] = max(d[x], dp_lookup(i) + 1);
14     return d[x];
15 }
16
17 int lis(int *a, int n)
18 {
19     int i, r = 1;
20     for (i = 0; i < n; i++) d[i] = -1, v[i] = a[i];
21     for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
22     return r;
23 }

```

- ▶ Við erum með einvítt stöðurúm af stærð  $n$  og hver uppfærsla er í  $\mathcal{O}(\quad)$ .

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- ▶ Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

- ▶ Látum nú  $f(x, y)$  tákna minnsta mögulega aftasta stak vaxandi hlutrunu  $(a_1, \dots, a_x)$  af lengd  $y$ .

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- ▶ Við fáum því

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- ▶ Grunntilvikin eru til að auðvelda útfærslu.
- ▶ Lengd lengstu vaxandi hlutrunanna er þá stærsta  $\ell$  þannig að  $f(n, \ell) < \infty$ .

```

7 int v[MAXN], d[MAXN][MAXN];
8 int dp_lookup(int x, int y)
9 {
10     if (x == -1) return y == 0 ? -INF : INF;
11     if (d[x][y] != -1) return d[x][y];
12     if (v[x] < dp_lookup(x - 1, y - 1)) return d[x][y] = dp_lookup(x - 1, y);
13     return d[x][y] = min(dp_lookup(x - 1, y), v[x]);
14 }
15
16 int lis(int *a, int n)
17 {
18     int i, j, x;
19     for (i = 0; i < n; i++) v[i] = a[i];
20     for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = -1;
21     for (x = n; dp_lookup(n - 1, x) == INF; x--);
22     return x;
23 }

```

- ▶ Við erum með tvívítt stöðurúm af stærð  $(n + 1) \cdot (n + 1)$  og hver uppfærsla er í  $\mathcal{O}(\quad)$ .



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- ▶ En þetta er sama tímaflækja og við fengum áðan.
- ▶ Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

► Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x - 1, y), & \text{ef } a_x < f(x - 1, y - 1), \\ \min(f(x - 1, y), a_x), & \text{annars.} \end{cases}$$

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- Sjáum að fallgildi  $f(x, y)$  ákvarðast af  $f(x-1, y)$  og  $f(x-1, y-1)$ .

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- Sjáum að fallgildi  $f(x, y)$  ákvarðast af  $f(x-1, y)$  og  $f(x-1, y-1)$ .
- Svo við getum fyllt í töfluna í lestrarröð.



```

7 int lis(int *a, int *b, int n)
8 {
9     int i, j, x, y;
10    int d[n + 1][n + 1];
11    for (i = 0; i < n + 1; i++) d[0][i] = INF;
12    for (i = 0; i < n + 1; i++) d[i][0] = -INF;
13    for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
14    {
15        if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
16        else d[i][j] = min(d[i - 1][j], a[i - 1]);
17    }
18    for (x = n; d[n][x] == INF; x--);
19    for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
20    return x;
21 }

```

- ▶ Heildartímaflækjan er sú sama og áðan, svo hún er  $\mathcal{O}(\quad)$ .

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- ▶ Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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- ▶ Tökum eftir að við þurfum í raun alltaf bara að geyma síðustu línuna sem við kláruðum.

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- ▶ Annað sem við sjáum er að línurnar eru allar vaxandi.

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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

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- ▶ En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```

10 int lis(int *a, int *b, int n)
11 {
12     int i, j, x, y;
13     int d[n + 1], e[n];
14     for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
15     for (i = 0; i < n; i++)
16     {
17         int r = 0, s = n + 1;
18         while (r < s)
19         {
20             int m = (r + s)/2;
21             if (d[m] <= a[i]) r = m + 1;
22             else s = m;
23         }
24         d[r] = a[i], e[i] = d[r - 1];
25     }
26     for (x = n; d[x] == INF; x--);
27     for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
28         y = e[i], b[j--] = a[i];
29     return x;
30 }

```



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