Lengstu vaxandi hlutrunur (LIS)

Bergur Snorrason

17. febrúar 2022

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- Sjáum núna hvernig við getum gert það með kvikri bestun.

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Hlutrunur

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- Runa $(b_1, \ldots, b_k)$ er *hlutruna* rununnar ef til eru heiltölur $j_1 < \cdots < j_k$ þannig að $a_{j_m} = b_m$.

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- Við viljum nú, fyrir gefna runu, finna lengdina á lengstu vaxandi hlutrunum rununnar.
- Það er oft talað um að finna „lengstu vaxandi hlutrununa" þó það geti verið margar jafn langar „lengstu vaxandi hlutrunur".

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- Þá fáum við

$$f(x) = \begin{cases} 1, & \text{ef } x = 1, \\ \max_{\substack{1 \le k < x \\ a_k \le a_x}} f(k) + 1, & \text{annars.} \end{cases}$$

```
 6  int v[MAXN], d[MAXN];
 7  int dp_lookup(int x)
 8  { // Finnur lengd lengstu vaxandi hlutruna a sem enda í staki x.
 9      if (d[x] != −1) return d[x];
10      if (x == 0) return 1;
11      int i;
12      for (d[x] = 1, i = 0; i < x; i++) if (v[i] <= v[x])
13          d[x] = max(d[x], dp_lookup(i) + 1);
14      return d[x];
15  }
16
17  int lis(int *a, int n)
18  { // Finnur lengd lengstu vaxandi hlutruna a.
19      int i, r = 1;
20      for (i = 0; i < n; i++) d[i] = −1, v[i] = a[i];
21      for (i = 0; i < n; i++) r = max(r, dp_lookup(i));
22      return r;
23  }
```

▶ Við erum með einvítt stöðurúm af stærð *n* og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- Við getum bætt þetta, en fyrst þurfum við að leysa þetta aftur með kvikri bestun, en annarri rakningarformúlu.

▶ Látum nú $f(x, y)$ tákna minnsta mögulega aftasta stak vaxandi hlutrunu $(a_1, \ldots, a_x)$ af lengd y.

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- Við fáum því

$$
f(x, y) = \begin{cases}
-\infty, & \text{ef } x = 0 \text{ og } y = 0, \\
\infty, & \text{annars, ef } x = 0, \\
f(x - 1, y), & \text{ef } a_x < f(x - 1, y - 1), \\
\min(f(x - 1, y), a_x), & \text{annars.}
\end{cases}
$$

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- Grunntilvikin eru til að auðvelda útfærlsu.

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- Grunntilvikin eru til að auðvelda útfærlsu.
- Lengd lengstu vaxandi hlutrunanna er þá stærsta ℓ þannig að $f(n, \ell) < \infty$.

```
 7  int v[MAXN], d[MAXN][MAXN];
 8  int dp_lookup(int x, int y)
 9  { // Finnur minnsta aftasta stak vaxandi hlutruna a[0:x] af lengd y.
10      if (x == −1) return y == 0 ? −INF : INF;
11      if (d[x][y] != −1) return d[x][y];
12      if (v[x] < dp_lookup(x − 1, y − 1)) return d[x][y] = dp_lookup(x − 1, y);
13      return d[x][y] = min(dp_lookup(x − 1, y), v[x]);
14  }
15
16  int lis(int *a, int n)
17  { // Finnur lengd lengstu vaxandi hlutruna a.
18      int i, j, x;
19      for (i = 0; i < n; i++) v[i] = a[i];
20      for (i = 0; i < n; i++) for (j = 0; j < n + 1; j++) d[i][j] = −1;
21      for (x = n; dp_lookup(n − 1, x) == INF; x−−);
22      return x;
23  }
```

- Við erum með tvívítt stöðurúm af stærð $(n + 1) \cdot (n + 1)$ og hver uppfærsla er í $\mathcal{O}(\quad)$.

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- En þetta er sama tímaflækja og við fengum áðan.

- ▶ Við erum með tvívítt stöðurúm af stærð $(n + 1) \cdot (n + 1)$ og hver uppfærsla er í $\mathcal{O}(1)$.
- ▶ Heildartímaflækjan er því $\mathcal{O}(n^2)$.
- ▶ En þetta er sama tímaflækja og við fengum áðan.
- ▶ Prófum að útfæra þetta neðansækið og sjáum hvað við fáum.

▶ Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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▶ Sjáum að fallgildi $f(x, y)$ ákvarðast af $f(x - 1, y)$ og $f(x - 1, y - 1)$.

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$$f(x,y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

- Sjáum að fallgildi $f(x,y)$ ákvarðast af $f(x-1, y)$ og $f(x-1, y-1)$.
- Svo við getum fyllt í töfluna í lestrarröð.

```
 7  int lis(int *a, int *b, int n)
 8  { // Finnur eina af lengstu vaxandi hlutrunum a.
 9      int i, j, x, y;
10      int d[n + 1][n + 1];
11      for (i = 0; i < n + 1; i++) d[0][i] = INF;
12      for (i = 0; i < n + 1; i++) d[i][0] = -INF;
13      for (i = 1; i < n + 1; i++) for (j = 0; j < n + 1; j++)
14      {
15          if (a[i - 1] < d[i - 1][j - 1]) d[i][j] = d[i - 1][j];
16          else d[i][j] = min(d[i - 1][j], a[i - 1]);
17      }
18      for (x = n; d[n][x] == INF; x--);
19      for (i = 0; i < x; i++) b[i] = d[n - x + i + 1][i + 1];
20      return x;
21  }
```

- Heildartímaflækjan er sú sama og áðan, svo hún er $\mathcal{O}(\ \ )$.

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- Skoðum núna hvernig uppfærslan á töflunni fer fram og sjáum hvort við getum bætt tímaflækjuna.

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    -INF   3   6 INF INF INF INF INF INF INF
    -INF   2   6 INF INF INF INF INF INF INF
    -INF   1   6 INF INF INF INF INF INF INF
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    -INF    1    6  INF  INF  INF  INF  INF  INF  INF
    -INF    1    5  INF  INF  INF  INF  INF  INF  INF
    -INF    1    5    8  INF  INF  INF  INF  INF  INF
    -INF    1    4    8  INF  INF  INF  INF  INF  INF
    -INF    1    4    7  INF  INF  INF  INF  INF  INF
>   -INF    1    4    7    9  INF  INF
```

```
3, 6, 2, 1, 5, 8, 4, 7, 9
                          ^

                                        |
    -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
    -INF    3  INF  INF  INF  INF  INF  INF  INF  INF
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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
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   -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
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  -INF  INF  INF  INF  INF  INF  INF  INF  INF  INF
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- En afhverju gildir þetta?

Afhverju er allar línurnar raðaðar

▶ Rifjum upp að

$$f(x, y) = \begin{cases} -\infty, & \text{ef } x = 0 \text{ og } y = 0, \\ \infty, & \text{annars, ef } x = 0, \\ f(x-1, y), & \text{ef } a_x < f(x-1, y-1), \\ \min(f(x-1, y), a_x), & \text{annars.} \end{cases}$$

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- ▶ Þetta fæst með einfaldri þrepun.

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- En þá fæst líka að $f(x, y) < f(x - 1, y) \leq f(x, y + 1)$.

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- Þetta getur þó bara átt sér stað, í mesta lagi, einu sinni í röðuðum lista.

Afhverju uppfærum við bara einu sinni í hverri ítrun?

▶ Við getum því lýst uppfærslu y-tu línunnar með:

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- ▶ Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.

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- ▶ Í neðansæknu útfærslunni framkvæmum við línulega leit í fyrsta skrefinu.
- ▶ En þar sem línurnar eru allar raðaðar getum við í staðinn framkvæmt helmingunarleit.

```
10  int lis(int *a, int *b, int n)
11  { // Finnur eina af lengstu vaxandi hlutrunum a.
12      int i, j, x, y;
13      int d[n + 1], e[n];
14      for (i = 0; i < n + 1; i++) d[i] = i == 0 ? -INF : INF;
15      for (i = 0; i < n; i++)
16      {
17          int r = 0, s = n + 1;
18          while (r < s)
19          {
20              int m = (r + s)/2;
21              if (d[m] <= a[i]) r = m + 1;
22              else s = m;
23          }
24          d[r] = a[i], e[i] = d[r - 1];
25      }
26      for (x = n; d[x] == INF; x--);
27      for (i = n - 1, j = x - 1, y = d[x]; j >= 0; i--) if (a[i] == y)
28          y = e[i], b[j--] = a[i];
29      return x;
30  }
```

- Nú er forritið einföld `for`-lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.

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- Heildartímaflækan er því $\mathcal{O}($ $)$.

- ▶ Nú er forritið einföld `for`-lykkja af lengd n sem framkvæmir helmingunarleit í hverri ítrun.
- ▶ Helmingunarleitin er $\mathcal{O}(\log n)$.
- ▶ Heildartímaflækan er því $\mathcal{O}(n \log n)$.