**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**A)** Theworkbeing after 10 min, so the avenge time increase from 45 min to 55min so μ

= 55 and б = 8 and X = 60

For the normal distribution

Z = (X – μ)/ б

= (60 - 55) / 8

= 0.625

From python P (0.625)

from scipy. stats import norm

nd=norm (45,8) # mean, Sd

x1 = nd.cdf (50)

1-nd.cdf (50)

**P (X > 50) = 0.26598552904870054**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**A)** Given that

Mean (*μ*) = 38

SD (*σ* )= 6

***(A)*** X1 = 38; X2 = 44

Z score = (X1 – μ)/ б

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % = 137 out of 400

Z score = (X2 – μ)/ б

Z score for 44 = (44 - 38)/6 = 1 => 84.13 %

=> People above 44 age = 100 - 84.13 = 15.87% = 63    out of 400

Hence People b/w 38 & 44 age = 84.13 - 50 = 34.13 % =137 out 400

Hence More employees at the processing center are older than 44 than

between 38 and 44. is F**ALSE**

***(B)*** Z score for 30 = (30 - 38)/6 = -1.33 = 9.15 % = 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees – **TRUE**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**A)** As we know that if X ∼ N (µ1, σ12), and Y ∼ N (µ2, σ22) are two independents random

variables

Then X1 + X2 ∼ N (µ1 + µ2, σ12 + σ22), and X1 – X2 ∼ N (µ1 − µ2, σ12+ σ22).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X

and Y, Then Z ∼ N (aµ1 + bµ2, a^2 σ12 + b^2 σ22).

Therefore, in the question 2X1 ~ N (2µ, 4 σ2)

And X1+X2 ~ N (µ + µ, σ2 + σ2) ~ N (2 µ, 2 σ22)

2X1-(X1+X2) = N (4µ, 6 σ2)

**2X1 ~ N (2µ, 4 σ2)**

**X1+X2 ~ N (4µ, 6 σ2)**

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**A) Step-by-step explanation:**

Given:  p(a<x<b) = 0.99,

mean(μ) =100,

Standard Deviation(σ2) = 20

**To Find**: Identify symmetric values for the standard normal distribution such that the area

enclosed is 0.99

From the above details, we have to excluded area of 0.005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z (0.5) = stats.norm.ppf (0.005) = -2.576

Z value at 99.5 percentile is given as

Z (99.5) = stats.norm.ppf (0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576) + 100= 151.5

**Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**A) (A) RANGE is (99.008134,980.991896) in millions**

**(B) 5th percentile of the profit (in millions of rupees) is 170.0**

**(C) FIRST DIVISION**

**i.e.: p (division 1 in marking loss) = 0.04779**

**p (division 2 in marking loss) = 0.04005**