Medina Martínez Brayan Alejandro Serie 02 Parte 1.

1. Para cada una de las funciones indicadas determina

$$f(2);\ f\left(\frac{1}{2}\right) : f\left(-\frac{2}{3}\right) : f(0)$$

a) $f(x) = 3x^2 - 2x^3 + x^4 - 5. \longrightarrow f(2).$

$f(2) = 3(2)^2 - 2(2)^3 + (2)^4 - 5.$
$= 3(4) - 2(8) + 16 - 5$
$= 12 - 16 + 16 - 5 = 7.$

$f(x) = 3x^2 - 2x^3 + x^4 - 5 \longrightarrow f\left(\frac{1}{2}\right).$

$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 - 5.$
$= 3\left(\frac{1}{4}\right) - 2\left(\frac{1}{8}\right) + \frac{1}{16} - 5$
$= \frac{3}{4} - \frac{1}{4} + \frac{1}{16} - \frac{20}{4} = -\frac{18}{4} + \frac{1}{16} = -\frac{9}{2} + \frac{1}{16}.$
$= \left(\frac{8}{8}\right)\left(-\frac{9}{2}\right) + \frac{1}{16} = -\frac{72}{16} + \frac{1}{16} = -\frac{71}{16}$

$f(x) = 3x^2 - 2x^3 + x^4 - 5 \longrightarrow f\left(-\frac{2}{3}\right)$

$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 - 2\left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^4 - 5.$
$= 3\left(\frac{4}{9}\right) - 2\left(-\frac{8}{27}\right) + \left(\frac{16}{81}\right) - 5.$
$= \frac{4}{3} + \frac{16}{27} + \frac{16}{81} - \frac{405}{81} = \frac{27}{27}\left(\frac{4}{3}\right) + \frac{3}{3}\left(\frac{16}{27}\right) + \frac{16}{81} - \frac{405}{81}$
$= \frac{108}{81} + \frac{48}{81} + \frac{16}{81} - \frac{408}{81} = -\frac{283}{81}$

1

$f(x) = 3x^2 - 2x^3 + x^4 - 5 \longrightarrow f(\sqrt{2})$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2(\sqrt{2})^3 + (\sqrt{2})^4 - 5$

$f(\sqrt{2}) = 3(2) - 2(2)^{\frac{3}{2}} \; 2^{\frac{4}{2}} - 5$

$= 6 - 5.65 + 4 - 5 = -0.65$

$f(x) = 3x^2 - 2x^3 + x^4 - 5 \longrightarrow f(0)$

$f(0) = 3(0)^2 - 2(0)^3 + (0)^4 - 5 = -5$

b) - $f(x) = \begin{cases} x^2 - 3x & \text{si } x \in [-4, 1) \\ \frac{x}{2} + 5 & \text{si } x \in [1, 5] \end{cases} \longrightarrow (2)$

$f(x) = \begin{cases} 4 - 6 & \text{si } x \in [-4, 1) \\ \frac{2}{2} + 5 & \text{si } x \in [1, 5] \end{cases}$

$f(x) = \begin{cases} -2 & \text{si } x \in [-4, 1) \\ \boxed{6} & \text{si } x \in [1, 5] \end{cases} \rightarrow$

- $f(\frac{1}{2})$

$(\frac{1}{2})^2 - 3(\frac{1}{2})$ si $x \in [-4, 1)$

$\left(\frac{\frac{1}{2}}{\frac{2}{1}}\right) + 5$ si $x \in [1, 5]$

$\frac{1}{4} - \frac{6}{4}$ si $x \in [-4, 1)$ $= \boxed{-\frac{5}{4}}$

$\frac{1}{4} + \frac{20}{4}$ si $x \in [1, 5]$ $= \frac{21}{4}$

b) - $f(-\frac{2}{3})$.

$$f(\tfrac{1}{2})\begin{cases}\left(-\frac{2}{3}\right)^2-3\left(-\frac{2}{3}\right) & \text{si } x\in[-4,1)\\ \left(\dfrac{-\frac{2}{3}}{\frac{2}{1}}\right)+5 & \text{si } x\in[5,1]\end{cases}$$

$$f(\tfrac{1}{2})=\begin{cases}\frac{4}{9}+\frac{18}{9} & \text{si } x\in[-4,1) & =\boxed{\frac{22}{9}}\\ -\frac{2}{6}+\frac{30}{6} & \text{si } x\in[5,1] & =-\frac{11}{3}\end{cases}$$

- $f(\sqrt{2})$

$$f(\sqrt{2})=\begin{cases}(\sqrt{2})^2-3(\sqrt{2}) & \text{si } x\in[-4,1)\\ \frac{\sqrt{2}}{2}+5 & \text{si } x\in[5,1]\end{cases}$$

$$f(\sqrt{2})=\begin{cases}2-4.2426 & \text{si } x\in[-4,1) & =-2.2426\\ 0.7071+5 & \text{si } x\in[5,1] & =\boxed{5.7071}\end{cases}$$

- $f(0)$

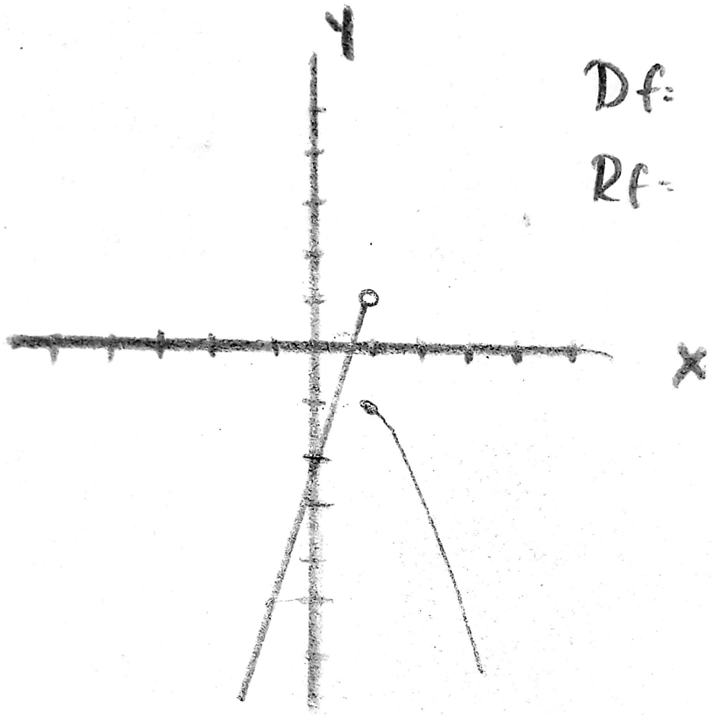
$$f(0):\begin{cases}x^2-3(x) & \text{si } x\in[-4,1)\\ \frac{x}{2}+5 & \text{si } x\in[5,1]\end{cases}$$

$$f(0):\begin{cases}(0)^2-3(0) & \text{si } x\in[-4,1) & =\boxed{0}\\ \frac{0}{2}+5 & \text{si } x\in[5,1] & =5\end{cases}$$

2. Determina el dominio, recorrido y gráfica de las siguientes funciones explícitas.

a)
$$f(x) = \begin{cases} 3x - 2 & \text{si } x < 1 \\ -x^2 & \text{si } x \geq 1 \end{cases}$$

$\to$ Recta
$\to$ Parábola



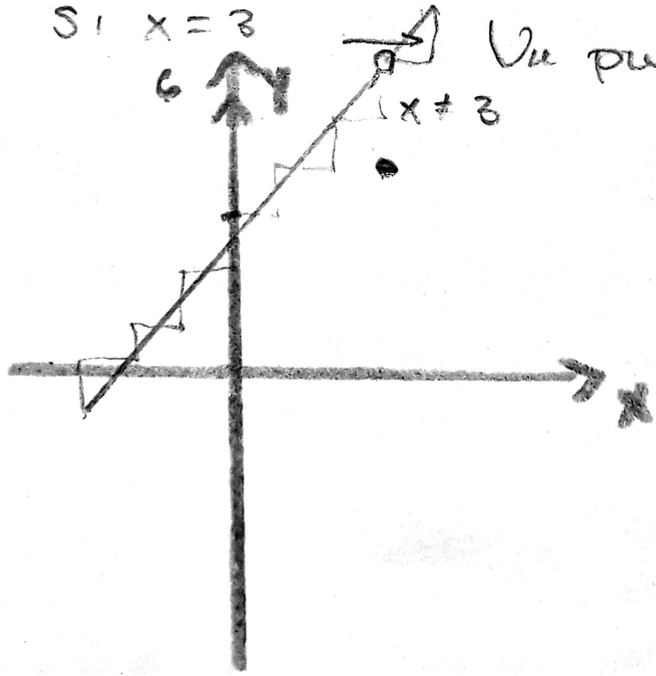
Df: $\mathbb{R}$.

Rf: $(-\infty, 1)$.

b
$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{si } x \neq 3 \\ 4 & \text{si } x = 3 \end{cases}$$

$\to \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = y\,(x+3)$ Recta.

$\to$ Un punto en $(4, 3)$.



Df: $\mathbb{R}$

Rf: $\mathbb{R} - \{6\}$.

2. c)

$$f(x) = \begin{cases} 1 - 4x - x^2 & si \quad -3 < x < -2 \rightarrow \text{Parábola} \\ x^2 + 4x + 2 & si \quad -2 < x < 0 \rightarrow \text{Parábola} \\ 2 & si \quad 0 \leq x \leq 2 \end{cases}$$

Desarollamos

$y = 1 - 4x - x^2$

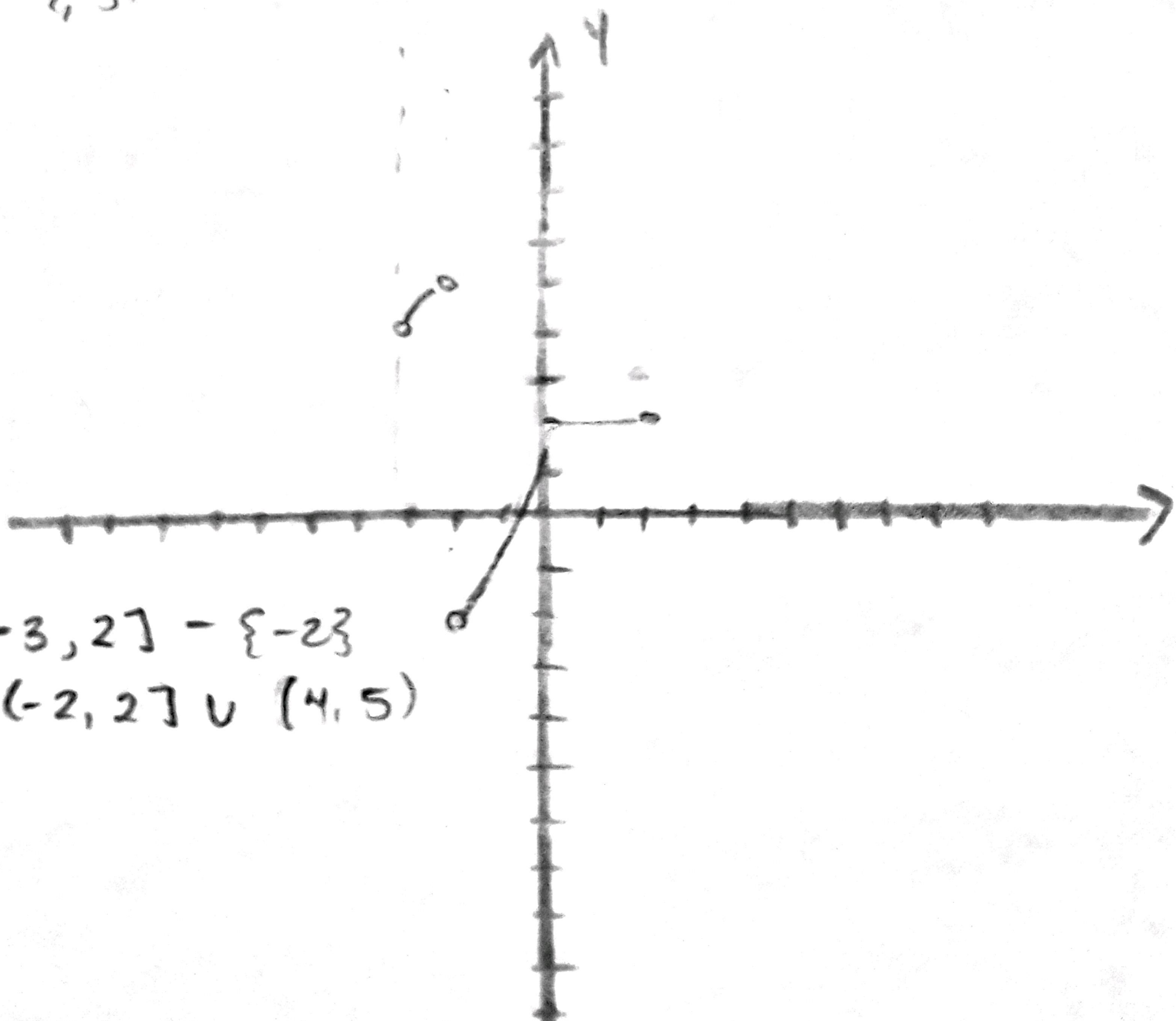
$(y-1) = -(x^2+4x+4)$

$(y-5) = -(x+2)^2$

$V(-2, 5)$

$$\boxed{\begin{array}{l} y = x^2 + 4x + 2 \\ (y+2) = (x+2)^2 \end{array}} \quad V(-2,-2)$$

$y = 2$



$D_f: (-3, 2] - \{-2\}$

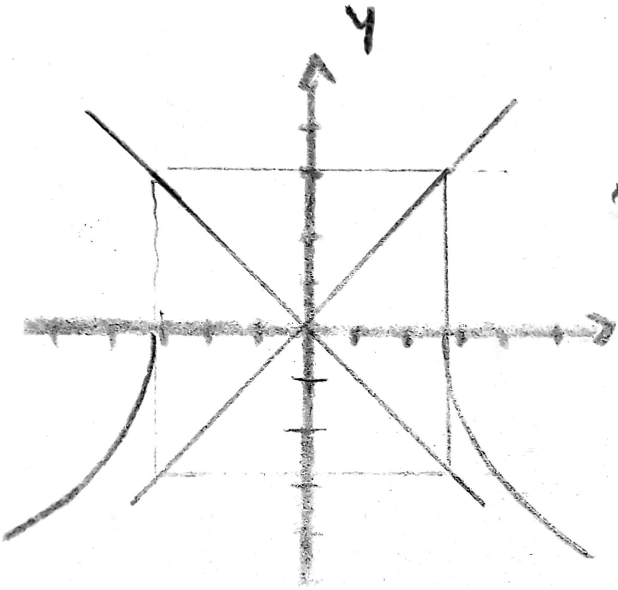
$R_f: (-2, 2] \cup (4, 5)$

2. d) $f(x) = -\sqrt{x^2-9}$

$y = -\sqrt{x^2-9} \longrightarrow y^2 = x^2-9$

$(-1)(-x^2+y^2=) - 9.(-1)$

$x^2-y^2=9 \longrightarrow \frac{x^2}{9}-\frac{y^2}{9}=1 \longrightarrow$ $\boxed{\text{Hiperbola}}$



Df: $(-\infty,-3] \cup [3,\infty)$

Rf $= (-\infty, 0]$

e) $f(x) = \sqrt{x^2-3x-4} \Rightarrow$ Hiperbola

$y^2 = x^2-3x-4$

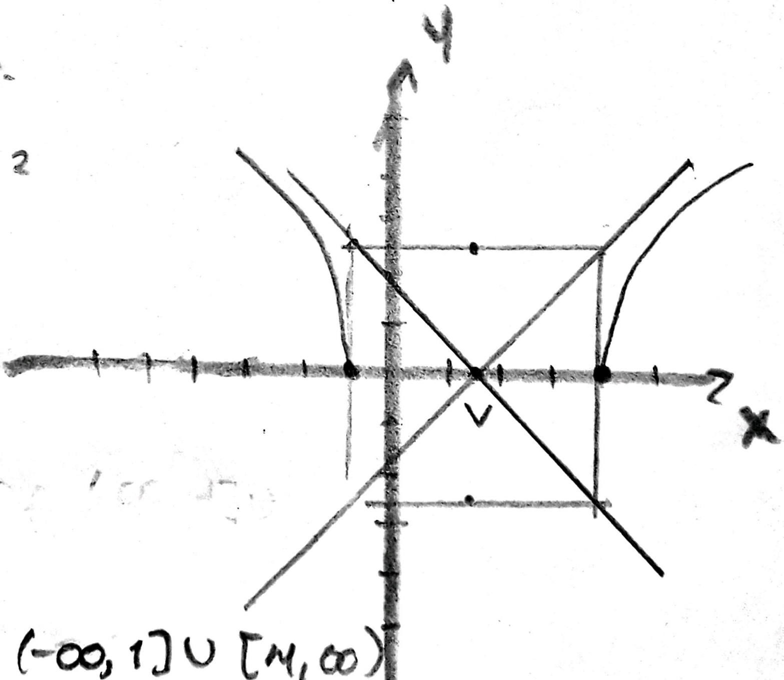
$y^2+4 = x^2-3x+\left(\frac{3}{2}\right)^2$

$y^2+\left(\frac{3}{2}\right)^2+4 = \left(x-\frac{3}{2}\right)^2$

$y^2+\frac{25}{4} = \left(x-\frac{3}{2}\right)^2$

$-\left(x-\frac{3}{2}\right)^2+y^2 = -\frac{25}{4}$

$\frac{\left(x-\frac{3}{2}\right)}{\frac{25}{4}} - \frac{y^2}{\frac{25}{4}} = 1$



$V\left(\frac{3}{2}, 0\right)$

$\boxed{a: \frac{5}{2}}$ $\boxed{b, \frac{5}{2}}$

Df: $(-\infty,1] \cup [4,\infty)$

Rf: $[0,\infty)$

2. f).

$$f(x) = \begin{cases} 2 - \frac{1}{3}\sqrt{9-(x+2)^2} & \text{si } x < 1 \\ 3 - (x-2)^2 & \text{si } x \geq 1. \end{cases}$$

$$y = 2 - \frac{1}{3}\sqrt{9-(x+2)^2}$$

$$(y-2)^2 = -\frac{1}{3}\left(9-(x+2)^2\right)$$

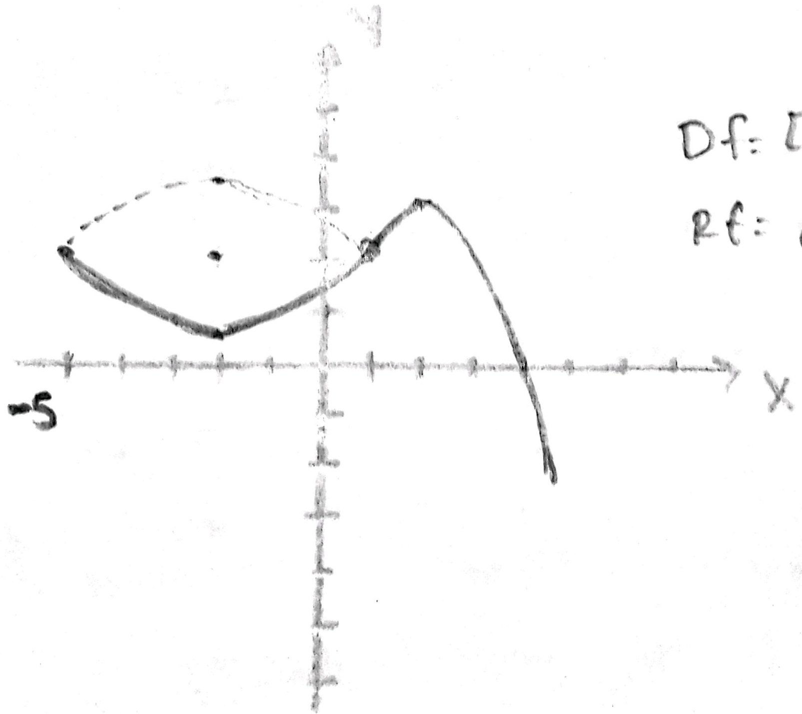
$$-3(y-2)^2 = 9-(x+2)^2$$

- $\frac{(x+2)^2}{9} - \frac{3(y-2)^2}{9} = \frac{9}{9} \longrightarrow \frac{(x+2)^2}{9} - \frac{(y-2)^2}{3} = 1$

$c(-2,2)$

- $y = 3-(x-2)^2 \longrightarrow (y-3) = -(x-2)^2$

$v(2,3)$



$Df: [-5, \infty)$

$Rf: (-\infty, 3]$

2. g)

$$f(x): \begin{cases} 2-(x-1)^2 & si \quad x<2 \\ -2+2\sqrt{1-(x-3)^2} & si \quad x\geq 2 \end{cases}$$

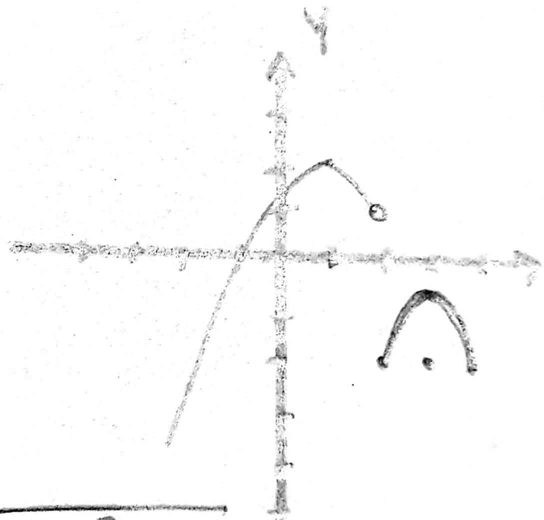
$y = 2-(x-2)^2$

- $(y-2) = -(x-1)^2 \qquad v\,(1,2)$

- $y = -2+2\sqrt{1-(x-3)^2}$

$(y+2)^2 = 2\,(1-(x-3))^2 \longrightarrow \dfrac{(y+2)^2}{2} + \dfrac{(x-3)^2}{1} = 1$

$c\,(+3,-2)$



D_f: $(-\infty, 4]$

R_f: $(-\infty, 2]$

h) $$f(x): \begin{cases} 1+\sqrt{(x+1)^2+4} & si \quad x\leq -1 \\ (x-1)^2+1 & si \quad x>-1 \end{cases}$$

$y = 1+\sqrt{(x+1)^2+4}$

$(y-1)^2 = (x+1)^2+4$

$-(x+1)^2+(y-1)^2 = 4$

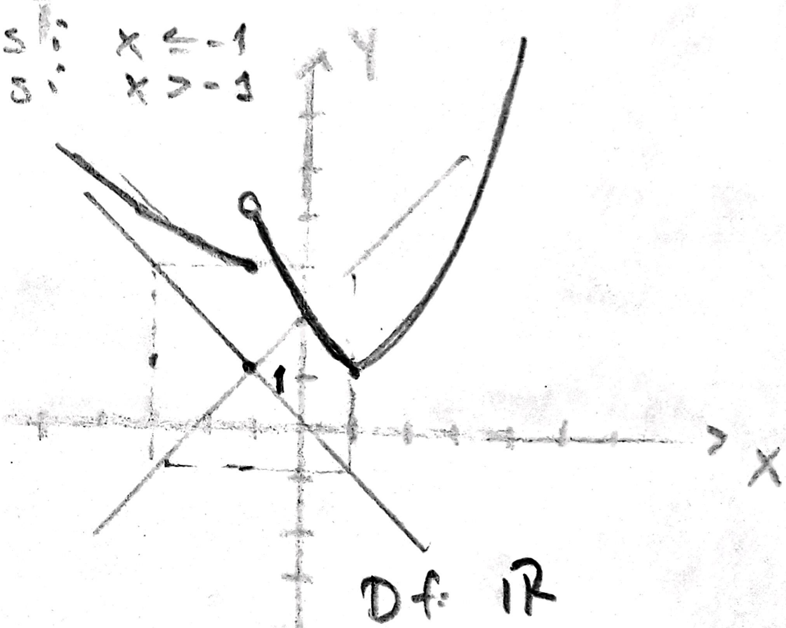
$$\boxed{-\frac{(x+1)^2}{4} + \frac{(y-1)^2}{4} = 1}$$

$c\,(-1,1)$

$y = (x-1)^2+1$

$$\boxed{(y-1) = (x-1)^2}$$

$V\,(1,1)$



D_f: $\mathbb{R}$

R_f: $[1, \infty)$

2. i)

$$f(x): \begin{cases} -3-\sqrt{4-(x+4)^2} & si \quad x<-2 \\ -x^2-2x-1 & si \quad x \geq -2 \end{cases}$$

- $y=-3-\sqrt{4-(x+4)^2}$

$(y+3)^2 = 4-(x+4)^2$

$(x+4)^2+(y+3)^2=4.$ $\quad c(-4,-3) \quad r=2$

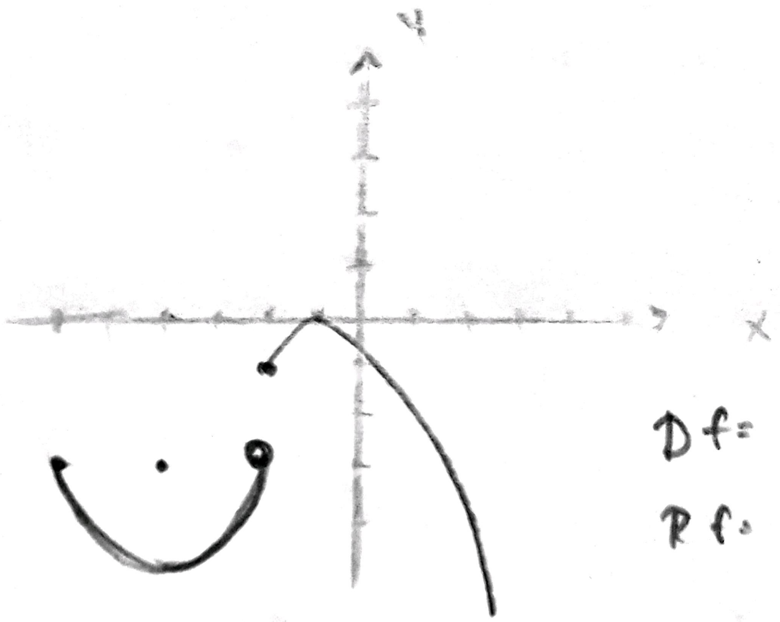
$$c: \begin{cases} 2\cos\theta - 4 \\ 2\operatorname{sen}\theta - 3 \end{cases}$$

- $y=-x^2-2x-1$

$(y+1) = -(x^2+2x)$

$y+1-1 = -(x^2+2x+1) \longrightarrow y=-(x+1)^2$

$v(-1,0)$



$Df: [-6,\infty)$

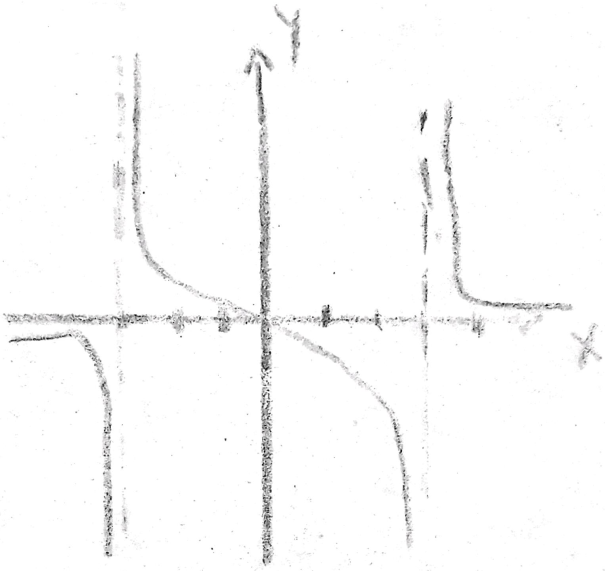
$Rf: (-\infty, 0].$

3. c)

$x^2y - 9y = x$

$y(x^2 - 9) = x$

$y = \dfrac{x}{x^2 - 9} \qquad x \neq -3, 3.$



d)

$3xy - 6x + y - 2 = 0$

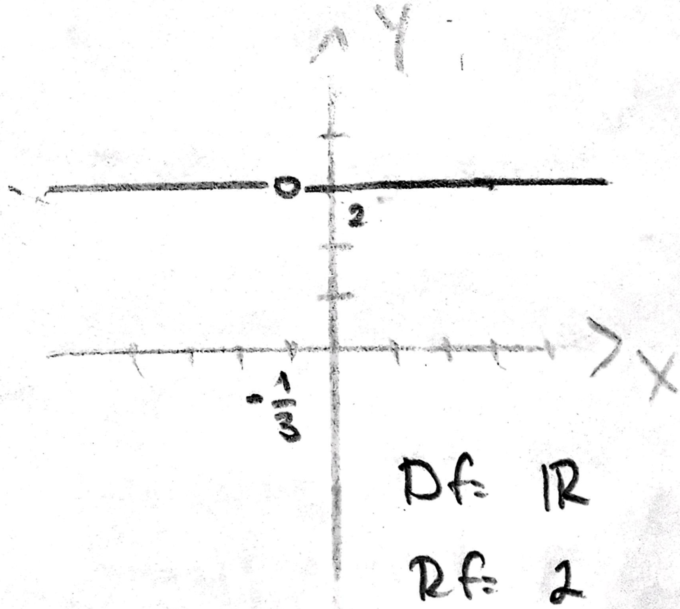
$3xy + y = 6x + 2$

$y(3x + 1) = 6x + 2$

$y = \dfrac{6x + 2}{3x + 1} \qquad x \neq -\frac{1}{3}$

$y = \dfrac{2\cancel{(3x+1)}}{\cancel{3x+1}}$

$y = 2$



Df: $\mathbb{R}$

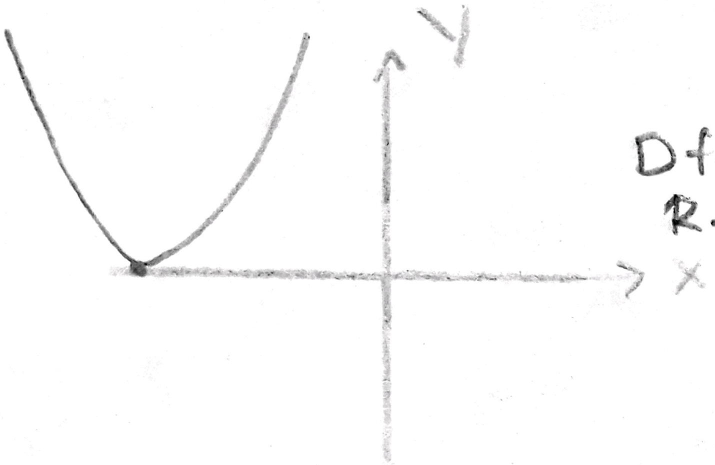
Rf: 2

1. Determine dominio, recorrido y grafica de las funciones parametricas

a) $f(x) = \begin{cases} x = t-5 \\ y = t^2 \end{cases}$ $\longrightarrow$ $t = x+5$.

$\longrightarrow$ $y = (x+5)^2$

$V(-5,0)$



$Df = \mathbb{R}$

$Rf = [0, \infty)$

1. b) $f(x): \begin{cases} x = \sqrt{t-4} \\ y = 3-2t \end{cases}$ $\longrightarrow$ $x^2+4 = t$

Sustituimos

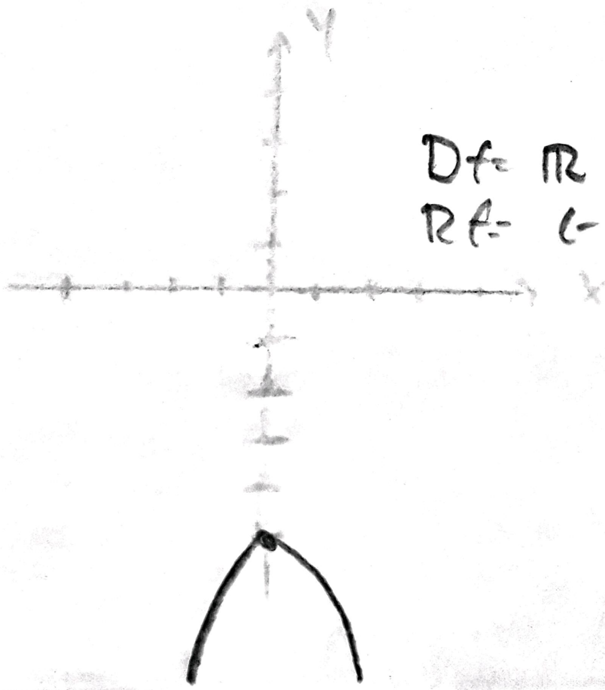
$y = 3-2(x^2+4)$

$y = 3-2x^2-8$

$y = -5-2x^2$

$(y+5) = -2x^2$

$v(0,-5)$



$Df = \mathbb{R}$

$Rf = (-\infty, -5]$

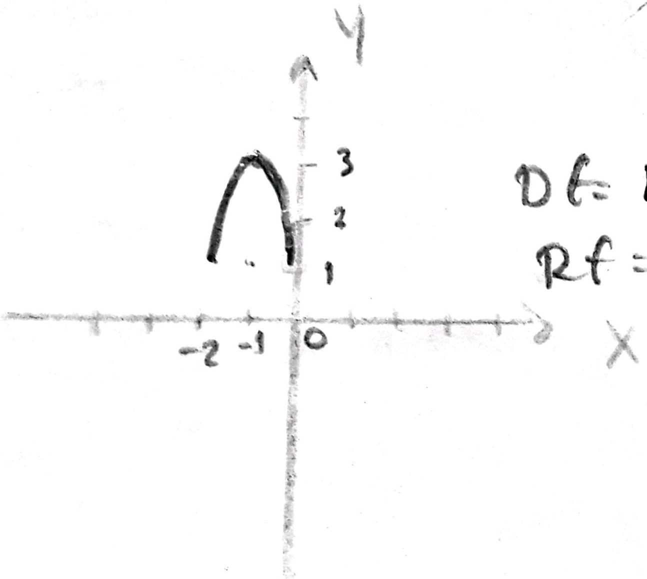
4.

c)

$$f(x): \begin{cases} x = \cos\theta - 1 \\ y = 2\,\text{sen}^2 + 1 \end{cases}$$

→ Parábola V.(−1, 1) abre hacia Y positivas.

Recorre hasta 2 en Y.



Df = [-2, 0]

Rf = [1, 3]

$\text{sen}^2 [1, 0].$

$2\,\text{sen}^2 [2, 0]$

$[0, 2] + 1 = [1, 3].$