2 Typing Rules

Notations

- Γ : Typing environment that maps identifiers to types.
- R : Return type context (a stack of return types).
- E : Set of declared thrown types.
- L : Loop context.
- μ : Reassignability map that maps identifier to either var (reassignable) or const (non-reassignable).

Literals

$$\langle integer \rangle : \text{Int} \quad \text{(T-Int)}$$

$$\langle real \rangle : \text{Int} \quad \text{(T-Real)}$$

$$\langle string \rangle : \text{string} \quad \text{(T-String)}$$

$$\texttt{true} : \text{Bool} \quad \text{(T-True)}$$

$$\texttt{false} : \text{Bool} \quad \text{(T-False)}$$

Variables

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad \text{(T-Var)}$$

Subtyping

$$S <: S \quad \text{(S-Refl)}$$

$$\frac{S <: U \quad U <: T}{S <: T} \quad \text{(S-Trans)}$$

Subsumption

$$\frac{\Gamma \vdash e : S \quad S <: T}{\Gamma \vdash e : T} \quad \text{(T-Sub)}$$

Function

$$\frac{\Gamma, x : T_{arg};\ \mathrm{R} :: T_{ret};\ \mathrm{E} = \{T_{err}\};\ \mathrm{L} = \emptyset \vdash S : \mathrm{Void}}{\Gamma; \mathrm{R}; \mathrm{E}; \mathrm{L} \vdash \texttt{fn}(x : T_{arg}) \to T_{ret}\ \texttt{throw}\ T_{err}\ \texttt{do}\ S\ \texttt{end} : T_{arg} \to T_{ret}\ \mathrm{throw}\ T_{err}} \quad \text{(T-Fn)}$$

$$\frac{\begin{array}{l}\Gamma, f : T_{arg} \to T_{ret}\ \mathrm{throw}\ T_{err}, x : T_{arg};\ \mathrm{R} :: T_{ret};\ \mathrm{E} = \{T_{err}\};\ \mathrm{L} = \emptyset;\ \mu[f \mapsto \mathrm{const}] \\ \vdash S : \mathrm{Void} \quad f \notin \mathrm{dom}(\Gamma)\end{array}}{\Gamma; \mathrm{R}; \mathrm{E}; \mathrm{L}; \mu \vdash \texttt{fn}\ f(x : T_{arg}) \to T_{ret}\ \texttt{throw}\ T_{err}\ \texttt{do}\ S\ \texttt{end} : \mathrm{Void}} \quad \text{(T-FnDef)}$$

$$\frac{\Gamma;\ \mathrm{R} :: T_{ret} \vdash e : T_{ret} \quad \mathrm{R} \neq \emptyset}{\Gamma;\ \mathrm{R} \vdash \texttt{return}\ e : \mathrm{Void}} \quad \text{(T-Return)}$$

$$\frac{\Gamma \vdash f : T_{arg} \to T_{ret}\ \mathrm{throw}\ T_{err} \quad \Gamma \vdash e : T_{arg}}{\Gamma; E \cup \{T_{err}\} \vdash f(e) : T_{ret}} \quad \text{(T-App)}$$

Variable Definitions

$$\frac{\Gamma; \mu \vdash e : T \quad x \notin \mathrm{dom}(\Gamma)}{\Gamma, x : T;\ \mu[x \mapsto \mathrm{var}] \vdash \texttt{let}\ x = e : \mathrm{Void}} \quad \text{(T-Let)}$$

$$\frac{\Gamma; \mu \vdash e : T \quad x \notin \mathrm{dom}(\Gamma)}{\Gamma, x : T;\ \mu[x \mapsto \mathrm{const}] \vdash \texttt{const}\ x = e : \mathrm{Void}} \quad \text{(T-Const)}$$

Sequence

$$\frac{\Gamma \vdash S_1 : \mathrm{Void} \quad \Gamma \vdash S_2 : \mathrm{Void}}{\Gamma \vdash S_1\ ;\ S_2 : \mathrm{Void}} \quad \text{(T-Sequence)}$$

Assignment

$$\frac{\Gamma(x) = T;\ \mu(x) = \mathrm{var}, \Gamma; \mu \vdash e : S \quad S <: T}{\Gamma; \mu \vdash x = e : \mathrm{Void}} \quad \text{(T-Assign)}$$

Scope

$$\frac{\Gamma \vdash S : \mathrm{Void}}{\Gamma \vdash \texttt{do}\ S\ \texttt{end}\ : \mathrm{Void}} \quad \text{(T-Scope)}$$

If

$$\frac{\Gamma \vdash e : \mathrm{Bool} \quad \Gamma \vdash S_1 : \mathrm{Void} \quad \Gamma \vdash S_2 : \mathrm{Void}}{\Gamma \vdash \texttt{if}\ e\ \texttt{do}\ S_1\ \texttt{end else do}\ S_2\ \texttt{end} : \mathrm{Void}} \quad \text{(T-IfElse)}$$

$$\frac{\Gamma \vdash e_1 : \mathrm{Bool} \quad \Gamma \vdash e_2 : T_1 \quad \Gamma \vdash e_3 : T_2}{\Gamma \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : \mathrm{lub}(T_1, T_2)} \quad \text{(T-IfThen)}$$

Match

$$\frac{\Gamma, p \vdash g : \text{Bool} \quad \Gamma, p \vdash e : T_r}{\Gamma \vdash \texttt{case}\ p\ \texttt{if}\ g\ \Rightarrow e : T_r} \quad \text{(T-Case)}$$

$$\frac{\Gamma \vdash e : T_s \quad \forall i.\ \Gamma, x : T_s \vdash \texttt{case}\ p_i\ \texttt{if}\ g_i \Rightarrow e_i : T_i}{\Gamma \vdash \texttt{match}\ e\ \texttt{as}\ x\ \texttt{case}\ p_i\ \texttt{if}\ g_i \Rightarrow e_i \ldots \texttt{end} : \text{lub}(T_1, \ldots, T_n)} \quad \text{(T-MatchExpr)}$$

$$\frac{\Gamma, p \vdash g : \text{Bool} \quad \Gamma, p \vdash S : \text{Void}}{\Gamma \vdash \texttt{case}\ p\ \texttt{if}\ g\ \texttt{do}\ S\ \texttt{end} : \text{Void}} \quad \text{(T-CaseStmt)}$$

$$\frac{\Gamma \vdash e : T_s \quad \forall i.\ \Gamma, x : T_s \vdash \texttt{case}\ p_i\ \texttt{if}\ g_i\ \texttt{do}\ S_i\ \texttt{end} : \text{Void}}{\Gamma \vdash \texttt{match}\ e\ \texttt{as}\ x\ \texttt{case}\ p_i\ \texttt{if}\ g_i\ \texttt{do}\ S_i\ \texttt{end} \ldots \texttt{end} : \text{Void}} \quad \text{(T-MatchStmt)}$$

While

$$\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma;\ \text{L}, \ell \vdash S : \text{Void}}{\Gamma;\ \text{L} \vdash \texttt{while}\ e\ \texttt{do}\ S\ \texttt{end} : \text{Void}} \quad \text{(T-While)}$$

Break/Continue

$$\frac{\text{L} \neq \emptyset}{\Gamma;\ \text{L} \vdash \texttt{break} : \text{Void}} \quad \text{(T-Break)}$$

$$\frac{\text{L} \neq \emptyset}{\Gamma;\ \text{L} \vdash \texttt{continue} : \text{Void}} \quad \text{(T-Continue)}$$

Throw

$$\frac{\Gamma \vdash e : T_{err} \quad T_{err} \in \text{E}}{\Gamma \vdash \texttt{throw}\ e : \text{Void}} \quad \text{(T-Throw)}$$

Try

$$\frac{\Gamma \vdash e : T_1 \quad \Gamma \vdash e_{\text{def}} : T_2}{\Gamma \vdash \texttt{try}\ e\ \texttt{else}\ e_{\text{def}} : \text{lub}(T_1, T_2)} \quad \text{(T-TryElse)}$$

$$\frac{\Gamma \vdash S : \text{Void} \quad \forall i.\ \Gamma, x_i : T_i \vdash S_i : \text{Void}}{\Gamma \vdash \texttt{try}\ S\ \texttt{catch}\ T_i\ \texttt{as}\ x_i\ \texttt{do}\ S_i\ \texttt{end}\ \ldots\ \texttt{end} : \text{Void}} \quad \text{(T-TryCatch)}$$