

2 Typing Rules

Notations

- Γ : Typing environment that maps identifiers to types.
- R : Return type context (a stack of return types).
- E : Set of declared thrown types.
- L : Loop context.
- μ : Reassignability map that maps identifier to either var (reassignable) or const (non-reassignable).

Literals

$$\langle integer \rangle : \text{Int} \quad (\text{T-INT})$$

$$\langle real \rangle : \text{Int} \quad (\text{T-REAL})$$

$$\langle string \rangle : \text{string} \quad (\text{T-STRING})$$

$$\text{true} : \text{Bool} \quad (\text{T-TRUE})$$

$$\text{false} : \text{Bool} \quad (\text{T-FALSE})$$

Variables

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad (\text{T-VAR})$$

Subtyping

$$S <: S \quad (\text{S-REFL})$$

$$\frac{S <: U \quad U <: T}{S <: T} \quad (\text{S-TRANS})$$

Subsumption

$$\frac{\Gamma \vdash e : S \quad S <: T}{\Gamma \vdash e : T} \quad (\text{T-SUB})$$

Function

$$\frac{\Gamma, x : T_{arg}; R :: T_{ret}; E = \{T_{err}\}; L = \emptyset \vdash S : \text{Void}}{\Gamma; R; E; L \vdash \text{fn}(x : T_{arg}) \rightarrow T_{ret} \text{ throw } T_{err} \text{ do } S \text{ end} : T_{arg} \rightarrow T_{ret} \text{ throw } T_{err}} \quad (\text{T-FN})$$

$$\frac{\begin{array}{l} \Gamma, f : T_{arg} \rightarrow T_{ret} \text{ throw } T_{err}, x : T_{arg}; R :: T_{ret}; E = \{T_{err}\}; L = \emptyset; \mu[f \mapsto \text{const}] \\ \vdash S : \text{Void} \quad f \notin \text{dom}(\Gamma) \end{array}}{\Gamma; R; E; L; \mu \vdash \text{fn } f(x : T_{arg}) \rightarrow T_{ret} \text{ throw } T_{err} \text{ do } S \text{ end} : \text{Void}} \quad (\text{T-FNDEF})$$

$$\frac{\Gamma; R :: T_{ret} \vdash e : T_{ret} \quad R \neq \emptyset}{\Gamma; R \vdash \text{return } e : \text{Void}} \quad (\text{T-RETURN})$$

$$\frac{\Gamma \vdash f : T_{arg} \rightarrow T_{ret} \text{ throw } T_{err} \quad \Gamma \vdash e : T_{arg}}{\Gamma; E \cup \{T_{err}\} \vdash f(e) : T_{ret}} \quad (\text{T-APP})$$

Variable Definitions

$$\frac{\Gamma; \mu \vdash e : T \quad x \notin \text{dom}(\Gamma)}{\Gamma, x : T; \mu[x \mapsto \text{var}] \vdash \text{let } x = e : \text{Void}} \quad (\text{T-LET})$$

$$\frac{\Gamma; \mu \vdash e : T \quad x \notin \text{dom}(\Gamma)}{\Gamma, x : T; \mu[x \mapsto \text{const}] \vdash \text{const } x = e : \text{Void}} \quad (\text{T-CONST})$$

Sequence

$$\frac{\Gamma \vdash S_1 : \text{Void} \quad \Gamma \vdash S_2 : \text{Void}}{\Gamma \vdash S_1 ; S_2 : \text{Void}} \quad (\text{T-SEQUENCE})$$

Assignment

$$\frac{\Gamma(x) = T; \mu(x) = \text{var}; \Gamma; \mu \vdash e : S \quad S <: T}{\Gamma; \mu \vdash x = e : \text{Void}} \quad (\text{T-ASSIGN})$$

Scope

$$\frac{\Gamma \vdash S : \text{Void}}{\Gamma \vdash \text{do } S \text{ end} : \text{Void}} \quad (\text{T-SCOPE})$$

If

$$\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma \vdash S_1 : \text{Void} \quad \Gamma \vdash S_2 : \text{Void}}{\Gamma \vdash \text{if } e \text{ do } S_1 \text{ end else do } S_2 \text{ end} : \text{Void}} \quad (\text{T-IFELSE})$$

$$\frac{\Gamma \vdash e_1 : \text{Bool} \quad \Gamma \vdash e_2 : T_1 \quad \Gamma \vdash e_3 : T_2}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \text{lub}(T_1, T_2)} \quad (\text{T-IFTTHEN})$$

Match

$$\frac{\Gamma, p \vdash g : \text{Bool} \quad \Gamma, p \vdash e : T_r}{\Gamma \vdash \text{case } p \text{ if } g \Rightarrow e : T_r} \quad (\text{T-CASE})$$

$$\frac{\Gamma \vdash e : T_s \quad \forall i. \Gamma, x : T_s \vdash \text{case } p_i \text{ if } g_i \Rightarrow e_i : T_i}{\Gamma \vdash \text{match } e \text{ as } x \text{ case } p_i \text{ if } g_i \Rightarrow e_i \dots \text{end} : \text{lub}(T_1, \dots, T_n)} \quad (\text{T-MATCHEXPR})$$

$$\frac{\Gamma, p \vdash g : \text{Bool} \quad \Gamma, p \vdash S : \text{Void}}{\Gamma \vdash \text{case } p \text{ if } g \text{ do } S \text{ end} : \text{Void}} \quad (\text{T-CASESTMT})$$

$$\frac{\Gamma \vdash e : T_s \quad \forall i. \Gamma, x : T_s \vdash \text{case } p_i \text{ if } g_i \text{ do } S_i \text{ end} : \text{Void}}{\Gamma \vdash \text{match } e \text{ as } x \text{ case } p_i \text{ if } g_i \text{ do } S_i \text{ end} \dots \text{end} : \text{Void}} \quad (\text{T-MATCHSTMT})$$

While

$$\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma; L, \ell \vdash S : \text{Void}}{\Gamma; L \vdash \text{while } e \text{ do } S \text{ end} : \text{Void}} \quad (\text{T-WHILE})$$

Break/Continue

$$\frac{L \neq \emptyset}{\Gamma; L \vdash \text{break} : \text{Void}} \quad (\text{T-BREAK})$$

$$\frac{L \neq \emptyset}{\Gamma; L \vdash \text{continue} : \text{Void}} \quad (\text{T-CONTINUE})$$

Throw

$$\frac{\Gamma \vdash e : T_{err} \quad T_{err} \in \mathbf{E}}{\Gamma \vdash \text{throw } e : \text{Void}} \quad (\text{T-THROW})$$

Try

$$\frac{\Gamma \vdash e : T_1 \quad \Gamma \vdash e_{\text{def}} : T_2}{\Gamma \vdash \text{try } e \text{ else } e_{\text{def}} : \text{lub}(T_1, T_2)} \quad (\text{T-TRYELSE})$$

$$\frac{\Gamma \vdash S : \text{Void} \quad \forall i. \Gamma, x_i : T_i \vdash S_i : \text{Void}}{\Gamma \vdash \text{try } S \text{ catch } T_i \text{ as } x_i \text{ do } S_i \text{ end} \dots \text{end} : \text{Void}} \quad (\text{T-TRYCATCH})$$

Error

$$\overline{\Gamma \vdash \text{error}(\dots) : T} \quad (\text{T-ERROR})$$