Volatility Mean Reversion

Trading the Volatility Smile

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Abstract

Financial derivatives, or options, can best be described as a contract between a buyer and a seller. This contract is often with respect to an underlying financial asset and like those assets, trades on the secondary market. Most of these options can be classified as either a call or put. Call options provide the buyer with a right to purchase the underlying asset at a predetermined price within a certain time frame. Put options are identical to call options except the buyer has the option of selling the asset instead. Options are said to have expired once the buyer's time frame to exercise their options contract has passed. Such contracts result in asymmetric payoff graphs with respect to movement in the underlying, different implied volatility (IV) values at different options strikes resulting in what's called the IV smile, and higher IV values for options contracts closer to expiration. Like all other assets, every option that trades in the secondary market has a theoretical fair price. At this price, both the buyer and seller can expect to break even upon contract expiry. One such model to determine the fair price of an option is the Black Scholes Options Pricing Model (Black Scholes). This paper aims to analyse how the IV smile changes over time using Black Scholes and identify possible arbitrage opportunities as the result of any market inefficiencies.

1 Rotman International Trading Competition

The topic of this paper was inspired by the 2020 Rotman International Trading Competition (RITC) options volatility case by Matlab. The premise of the case was identifying and trading incorrectly priced options according to the Black Scholes equation. Results were scored on the basis of greatest profit generated through delta neutral positions. The RITC options case consisted of one underlying security with a total of 10 different option strikes. This resulted in a total of 40 unique option contracts due to there being 2 different expiry months in addition to the the ability to trade both calls and puts.

Almost all opportunities in the case presented themselves in one of two different methods: single option miss pricing and chain miss pricing. Single option miss pricing opportunities were far more apparent as one option would appear drastically over priced or under priced compared to the others. However, this scenario is not realistic as markets are quick to realize when individual options are not trading at fair value. However, option chain miss pricing has been observed in the market, with one example being the volatility smile. In the RITC case, aggregate IV of options were altered and deviations from the mean resulted in potential arbitrage opportunities. This is different from the skew observed in the IV smile, which can be observed when plotted against option moneyness. However, the concept is similar - as all options of the same term should theoretically have the same IV. Options that are too far out of the money (OTM) or in the money (ITM) may not follow due to market inefficiencies such as fees and minimal portfolio return requirements, but those closer to at the money (ATM) should display this trend none the less.

2 Bloomberg Volatility Surfaces

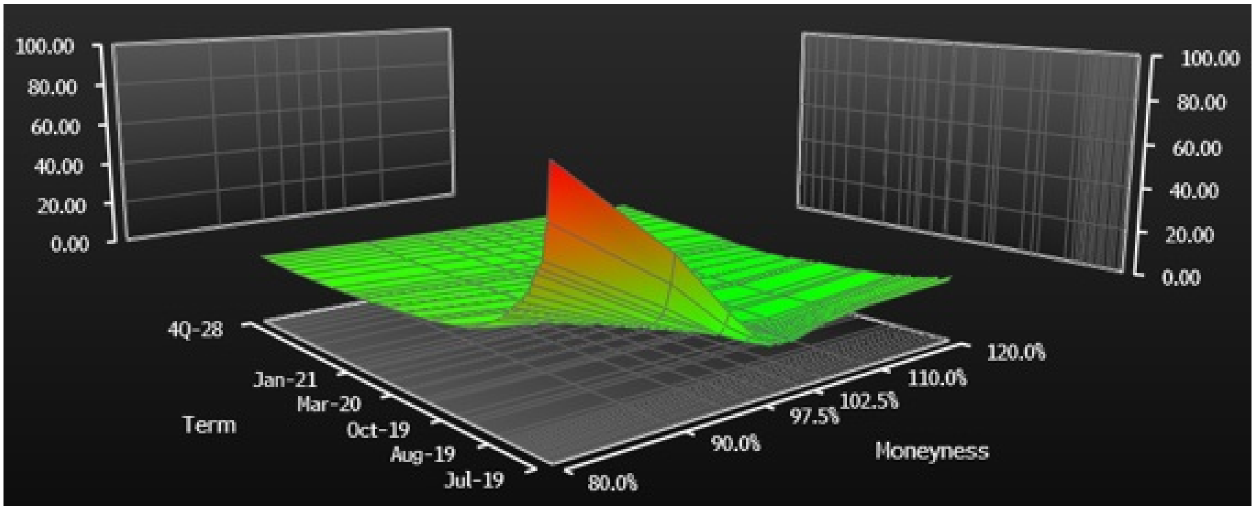

Figure 1: Bloomberg SPY Volatility Surface for July 1, 2019

Figure 1 is SPY volatility surface for July 1st, 2019. Notable features of the surface include higher IV for closer term options and the highest IV for options at an 80% moneyness. However, there is very low IV for 120% moneyness options. Additionally, the overall volatility is lower when compared to August 1st, 2019, which is shown in figure 2. The volatility surfaces for September and October can be found in the appendix.

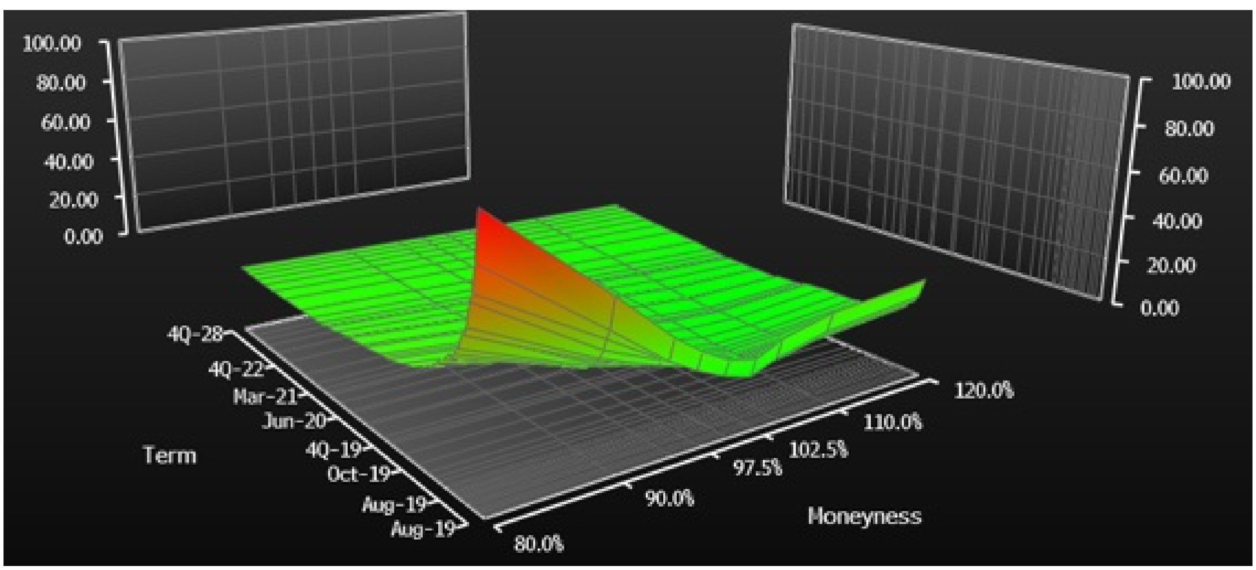

Figure 2: Bloomberg SPY Volatility Surface for August 1, 2019

Each volatility surface can be decomposed into three dimensions: term, moneyness, and volatility. As volatility is the dependent variable, the relevant dimensions that can be set are term and moneyness. Figures 3 and 4 decompose Figure 1 into term volatility and moneyness volatility respectively. Similar figures for the volatility surfaces of August, September, and October can be found in the appendix. It is also important to note that the term volatility is an average of moneyness volatility for a given expiry.

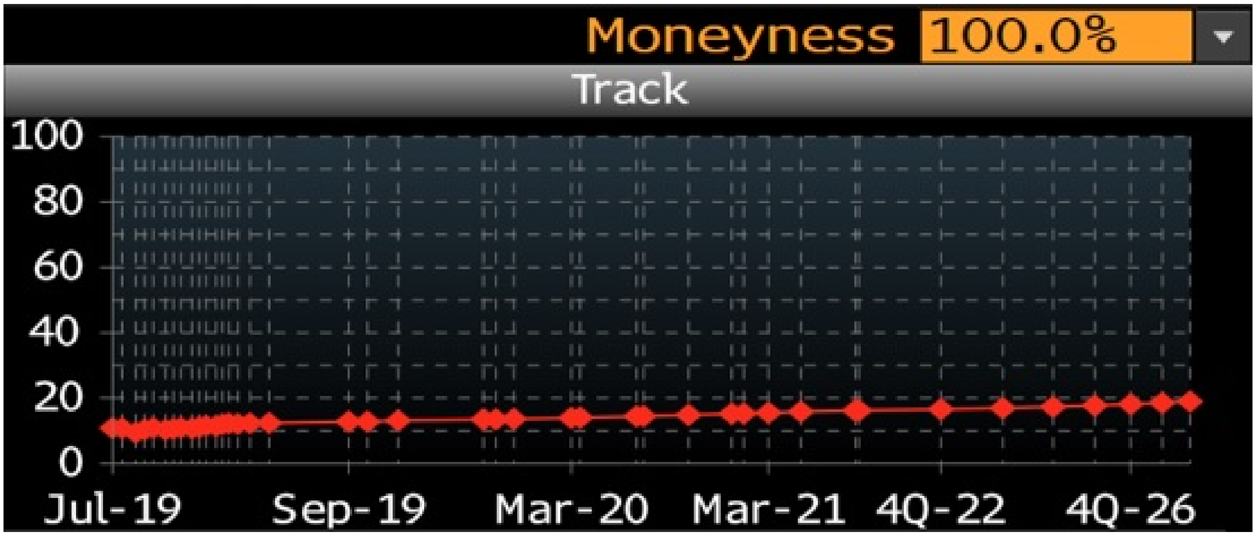

Figure 3: Bloomberg SPY Term Volatility for July 1, 2019

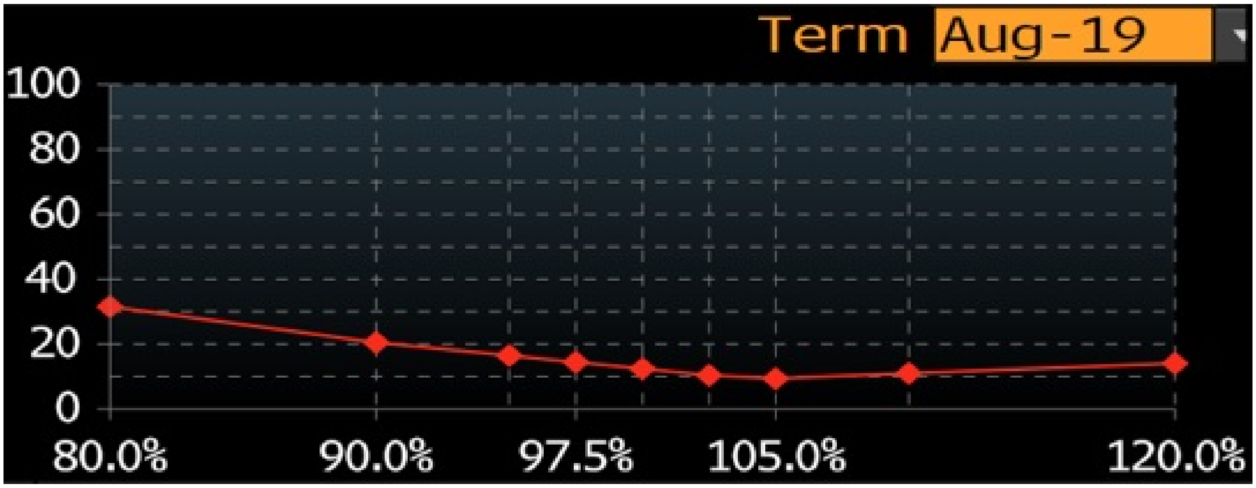

Figure 4: Bloomberg SPY Volatility Smile for July 1, 2019, August Contract

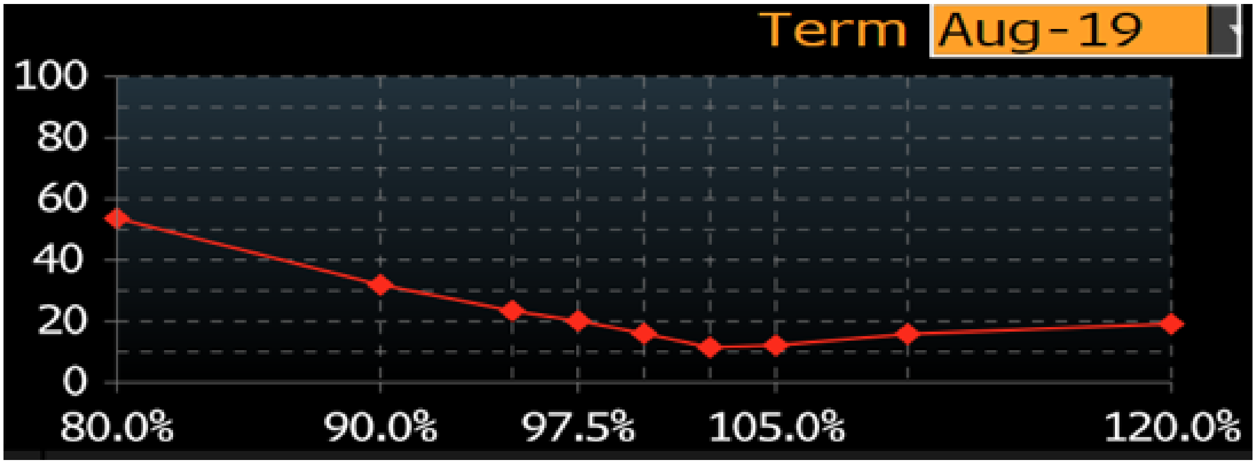

Figure 4: Bloomberg SPY Volatility Smile for August 1, 2019, August Contract

3 Black Scholes Options Pricing Model

The Black Scholes equation is one method of determining option prices. Other methods include the Binomial models and Monte Carlo methods. However, the strength of Black Scholes lies not within its ability to determine the price of any call or put, but rather the ability to determine the IV of an options contract. To do this, Black Scholes requires six different inputs: option price (W), current underlying price (P), annualized trading time until expiry (t), strike price (K), interest rate (r), and dividend yield (D). Call and put options are priced differently but use the same inputs. To calculate the price of a call option, equation 1 must be used:

$$W(P,t,K,r,D) = e^{\text{-Dt}} P N(d_1) - e^{\text{-rt}} K N(d_2) \tag{1}$$

where $N(x)$ = Probability (Standard Normally Distributed Random Variable $\leq x$). The formulas for d_1 and d_2 are shown below in equations 2 and 3 respectively:

$$d_1 = \frac{ln(P/K) + (r - D - IV^2/2)t}{IV\sqrt{t}} \tag{2}$$

$$d_2 = d_1 - IV\sqrt{t} \tag{3}$$

The price of a put option is calculated using equation 4 shown below:

$$W(P,t,K,r,D) = e^{\text{-rt}} K N(-d_2) - e^{\text{-Dt}} P N(-d_1) \tag{4}$$

3.1 IV and Option Greeks

From equation 2, it is apparent that there is no way to rearrange the formula to solve to IV. However, since the output of equations 1 and 4 are determined by the market, IV can be determined through a series of iterative calculations. The price for which the input IV results in an option price equivalent to that of the market will be the IV of that option.

Just being able to predict IV is not sufficient for trading purposes as IV is only one of the 5 inputs required to determine the fair price of an option. In order to trade IV, exposure to the other variables must be limited. For IV and each of the inputs in equation 1, with the exception of K and D, there is a corresponding Greek which measures the option's absolute dollar sensitivity, to each of the respective inputs. $P, t, IV\ and\ r$ correspond to option Greeks delta, theta, vega, and rho respectively. Each is a derivative with respect to the corresponding variable of call/put options pricing equations. There is also an additional Greek, gamma, which measures delta's sensitivity to price. The equations for each are shown below.

$$delta_{call} = e^{\text{-Dt}} N(d_1) \tag{5}$$

$$delta_{put} = e^{\text{-Dt}} (N(d_1) - 1) \tag{6}$$

$$gamma = \frac{e^{\text{-Dt}}}{P\sqrt{t}} \frac{1}{IV\sqrt{2\pi}} e^{-d_1^2/2} \tag{7}$$

$$theta_{call} = \frac{1}{252}(-(\frac{Pe^{-Dt}}{2\sqrt{t}} \frac{IV}{\sqrt{2\pi}} e^{-d_1^2/2}) - rKe^{-rt}N(d_2) + DPe^{-Dt}N(d_1)) \tag{8}$$

$$theta_{put} = \frac{1}{252}(-(\frac{Pe^{-Dt}}{2\sqrt{t}} \frac{IV}{\sqrt{2\pi}} e^{-d_1^2/2}) + rKe^{-rt}N(-d_2) - DPe^{-Dt}N(-d_1)) \tag{9}$$

With respect to equations 8 and 9, the scaling factor of 252 is sometimes seen as 365. This number represents how many days there are in a year. 252 is used with preference over 365 as there is an average of 252 trading days in a year. The exception would be a leap year with the possibility of having 253 trading days. The difference is trivial for this analysis.

$$vega = \frac{Pe^{-Dt}}{100}\sqrt{\frac{t}{2\pi}}e^{-d_1^2/2} \tag{10}$$

$$rho_{call} = \frac{Kte^{-rt}N(d_2)}{100} \tag{11}$$

$$rho_{put} = \frac{-Kte^{-rt}N(-d_2)}{100} \tag{12}$$

The risk free rate used for Black Scholes inputs are calculated using the floating LIBOR rate based on 3 month Eurodollar loans. The equation for r is shown below, which is adjusted for continous time.

$$r = 4\ ln(1 + \frac{(100 - Eurodollar)/100}{4}) \tag{13}$$

3.2 Isolating IV Exposure

Immediately it is apparent that when purchasing a option, the buyer is not exclusively long vega. the prices of each option are also sensitive to delta, theta, and rho. Of all the Greeks, only delta, theta, and vega have major impacts on option pricing. The other two Greeks, gamma and rho, can be ignored. As gamma is a derivative on delta, successfully hedging delta exposure will also minimize gamma exposure. However, there is no easy way to hedge rho without directly minimizing interest rate exposure. The method used to hedge rho exposure will be explored the following section.

3.3 Flaws of Black Scholes, Assumptions, and Controls

One of the results of Black Scholes is that IV is the same across option contracts regardless of strike price. This is made obvious through equations 1 and 2 where there is no relationship between input variables V and K. Theoretically, the IV of any underlying asset can be determined using any option contract. However, the IV smile shows the error of this assumption.

Given such a limitation, Black Scholes is still useful when trying to determine the IV of option contracts rather than the underlying asset. However, the underlying asset can still only have one IV value. Naturally, the problem becomes identifying the correct IV of the underlying based on what's seen within the option chain. The mean reversion of volatility can be seen through the term volatility of an asset's option chain, with higher IV in the near and far terms. However, as term IV is the aggregate of all strike IV, it can follow that the highest IV strikes should see a decrease in IV with the lowest IV strikes seeing an increase.

Prior to testing this assumption, several controls had to be placed in order to minimize market noise. Options that are close to expiry have a tendency to over react to market volatility. However, options that are further from expiry tend to be agnostic to market volatility. Thus, the option contracts will be analysed from the beginning of the month prior to the expiry month until the end of that same month. As options typically expire on the third Friday of every month, this time frame should avoid most of the extreme volatility in the last two weeks of an option's life.

Liquidity is also an issue when if comes to option trading. Often, derivative markets tend to offer less liquidity than the secondary market for the corresponding underlying asset. To provide the greatest liquidity, the most liquid ETF - SPY - was chosen as the underlying asset, with average daily trading volume exceeding 100 million shares as of March, 2019.

Finally, with respect to option Greeks, two separate controls were placed: one with respect to historical volatility and the other with respect to interest rate. The assumption that gamma would be naturally hedged through hedging delta only holds true if delta remains near zero for the entire duration of a trade. However, this is not realistically feasible as commissions would be extremely high. Thus, three months were chosen on the basis that stock movement between the first and last days of the month are minimal such that the ATM strike remains the ATM strike for the majority of the time. Any delta exposure under these controls will also be hedged with the underlying, rebalanced on a daily basis. A similar approach is taken with respect to interest rate. It is ensured that the LIBOR rate does not vary by more than 50 basis points between the beginning and end of each month. This will result in minimal price variation for options with respect to interest rate change.

With two Greeks remaining, theta and vega, it is still difficult to conclusively determine whether or not IV demonstrates mean reversion at a level which can be exploited through option trading. Thus, theta must also be hedged. The most straightforward method of hedging theta is through taking on a combination of long and short positions such that the theta exposure is zero. This is also the preferred method as there is no need to place controls on vega other than deciding which options to buy versus sell.

3.4 Moneyness Composition of the IV Smile

All SPY options can also be broken down into two different value components: intrinsic and extrinsic value. An option only has intrinsic value when moneyness is greater than 100. Alternatively, this can be decomposed into the sum of the exercise value and the option premium. The option premium can then be further broken down into two components: time value and volatility, which correspond to theta and vega respectively. As a result, OTM options have zero exercise value so everything must be priced into either time or volatility. Thus, it follows that OTM options will provide a clearer picture of what IV is. Similarly, figure 4 in section 2 is comprised of almost entirely from the IV of OTM options. At a moneyness less than 100%, the IV is calculated using puts while at a moneyness greater than 100%, the IV is calculated using calls.

4 Analysis

Given all the controls described, July, August and September of 2019 were chosen as the appropriate time periods, with the next month options as the traded asset. Trades are placed daily starting from the first day of the month until the second last day of the month. The net total position is expected to re-balance both theta and delta exposure to $\leq \$0.01$ and $\leq \$1.00$ on a daily interval with a maximum error of $\pm\$0.005$ and $\pm\$0.5$ respectively. Theta exposure will be hedged using option contracts where as delta exposure will be hedged using the underlying. Much of the analysis was computed using VBA, for greater clarity and optimization, which can be viewed through the developer tab. The sheet requires that macros are enabled for calculation with respect to IV and all option Greeks.

4.1 Results

Time Frame	July	August	September
Option Profit	-$20,752.00	$22,998.00	-$19,686.00
Stock Profit	$27,589.53	-$28,456.74	$21,135.59
Non-Delta Profit	$6,837.53	-$5,458.74	$1,449.59
Average Vega	1574.94	3866.19	718.34
Change in IV	3.69%	2.83%	1.12%
Total Contract Volume	1159.00	2985.00	722.00

Figure 1: Trading results for 2019

Figure 1 in section 4.1 shows the trading results. Contrary to the theory behind the approach, it would appear that there is no correlation between an option's IV versus the average of all option IVs along the same chain for a given asset. However, further analysis with respect to changes in the term volatility - as the trading strategy that was chosen increases leverage over time - is able to identify potential causes for the apparent lack of a relationship between vega exposure and IV.

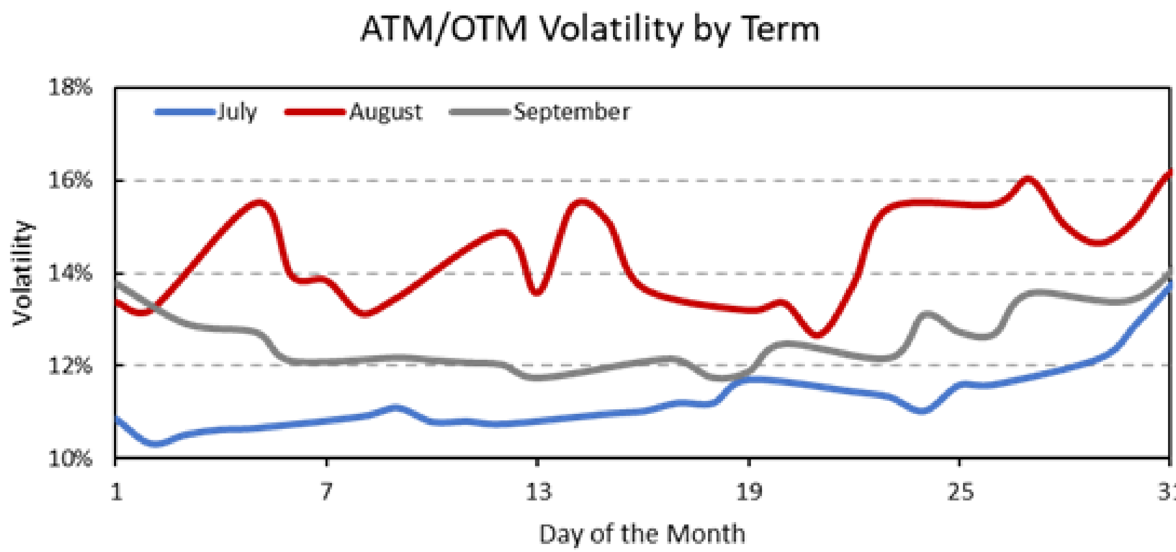

Figure 2: Term Volatility over July, August, and September

From Figure 2 in section 4.1, it can be seen that months July and September exhibited normal IV behavior whereas August varied greatly from day to day. If the results are considered with August as an outlier, there is a trend that can be established between the average vega exposure, non-delta profit, and net change in IV over a one month period. Bigger changes in IV result in greater non-delta profit as a function of average vega exposure.

5 Conclusion

The theory of IV mean reversion across different strikes for OTM option appears empirically true although not statistically significant. The returns appear to be fairly significant with respect to the overall delta exposure. However, the required capital to enter such a high number of option contracts is substantial even when spread entry is considered. Additionally, the fees required to hedge daily could potentially far exceed what the expected profits is. Although the chosen trading strategy has failed to generate an arbitrage profit, that is not to say there is no opportunity available. As long as vega exposure can be successfully isolated, with other greeks net neutral, abnormally large shifts in IV, during time periods such as the 2008 financial crisis, may lend itself well to a trading strategy that exploits this market inefficiency.

6 References

1. http://ritc.rotman.utoronto.ca/documents/2020/RITC2020_Case_Package.pdf

2. https://vegacapital.net/explaining-the-volatility-smile/

3. https://www.macroption.com/black-scholes-formula/

4. https://www.globalcapital.com/article/k65scnxh3mcr/

5. Bloomberg Terminal

7 Appendix

7.1 Volatility Surfaces

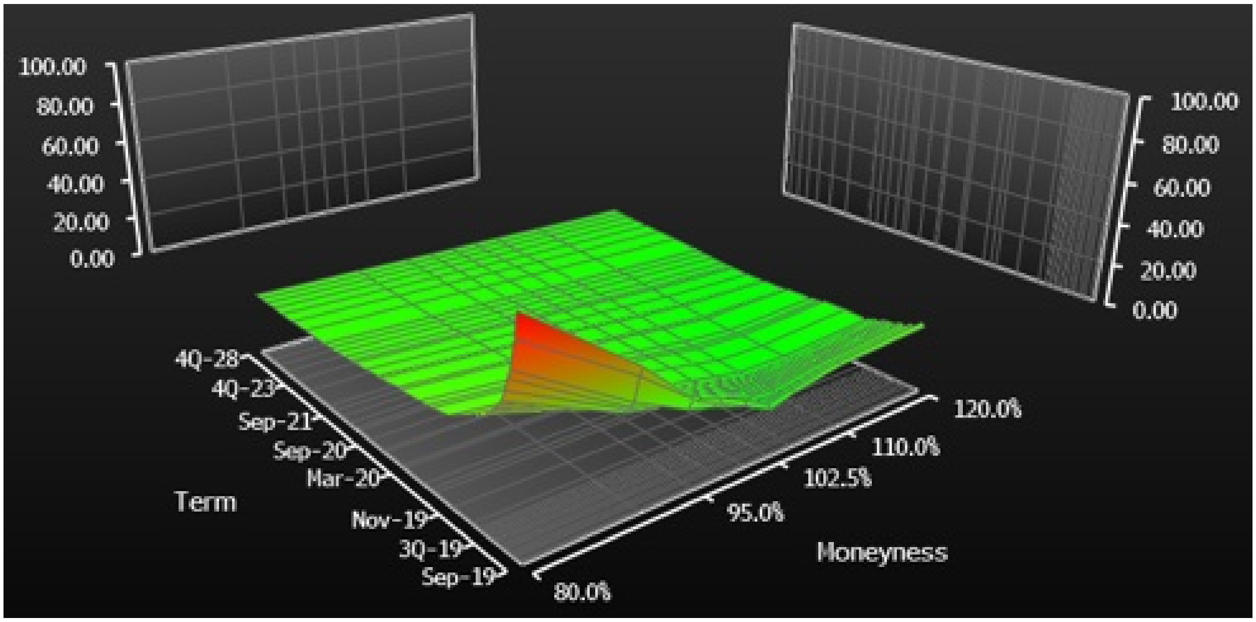

Figure A1: Bloomberg SPY Volatility Surface for September 1, 2019

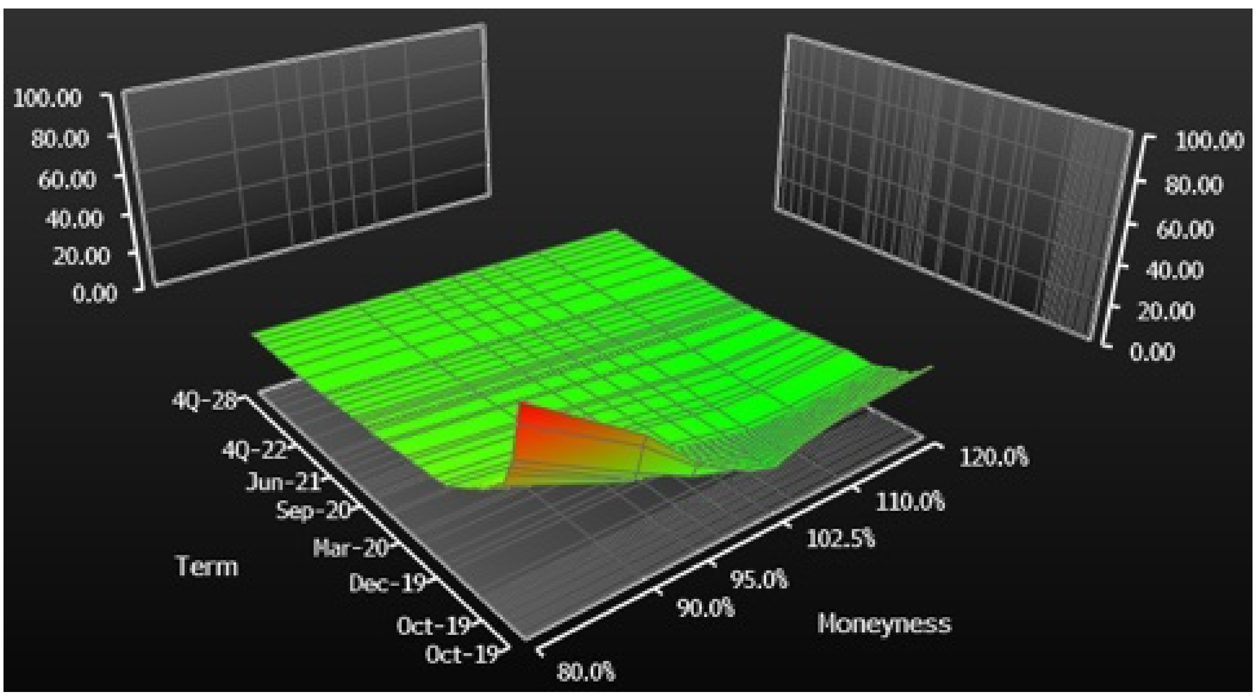

Figure A2: Bloomberg SPY Volatility Surface for October 1, 2019

7.2 Term Volatility

Figure A3: Bloomberg SPY Term Volatility for August 1, 2019

Figure A4: Bloomberg SPY Term Volatility for September 1, 2019

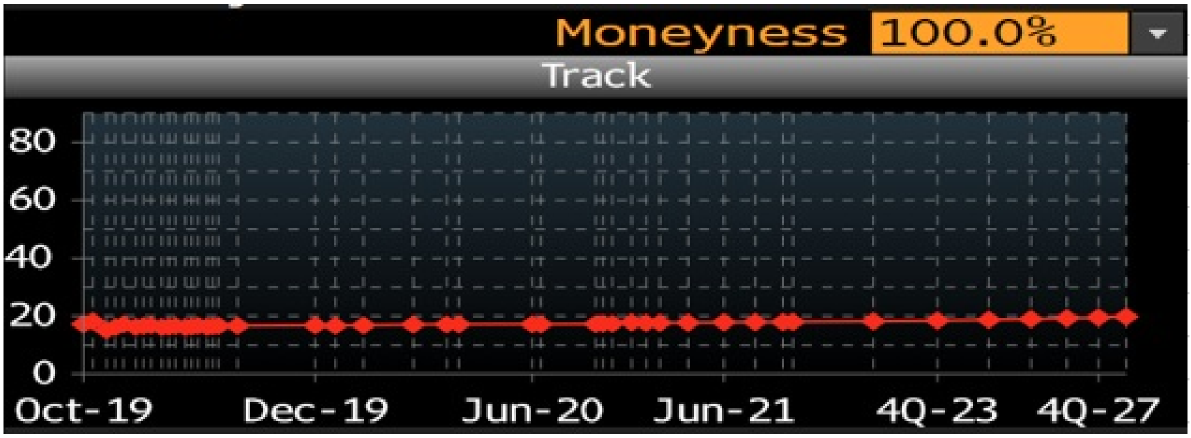

Figure A5: Bloomberg SPY Term Volatility for October 1, 2019

7.3 Volatility Smiles

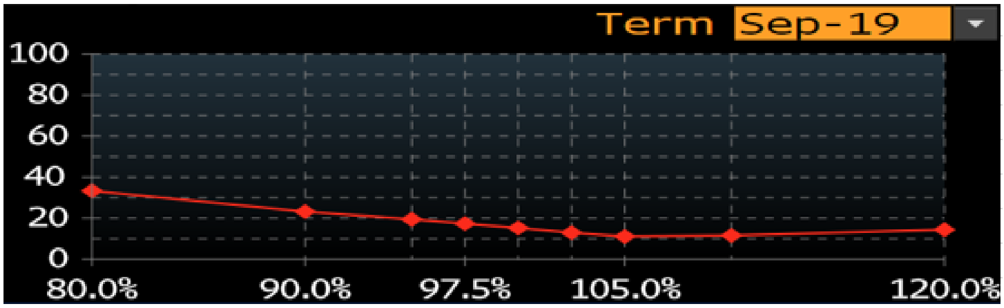

Figure A6: Bloomberg SPY Volatility Smile for August 1, 2019, September Contract

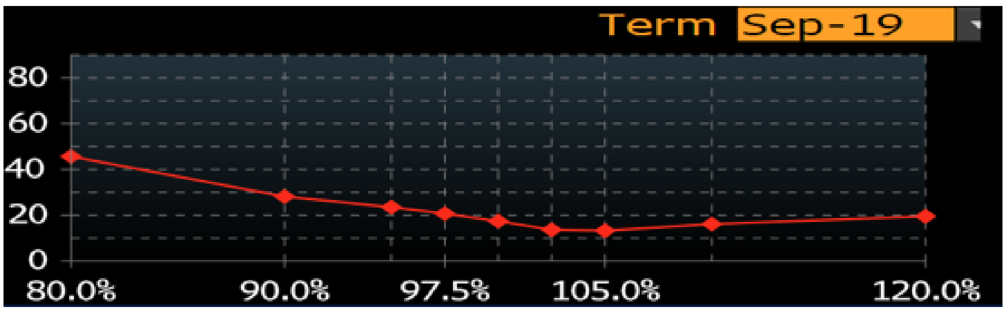

Figure A7: Bloomberg SPY Volatility Smile for September 1, 2019, September Contract

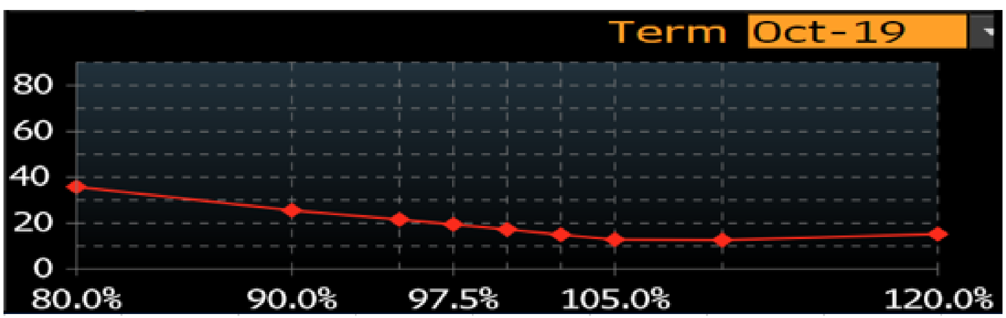

Figure A8: Bloomberg SPY Volatility Smile for September 1, 2019, October Contract

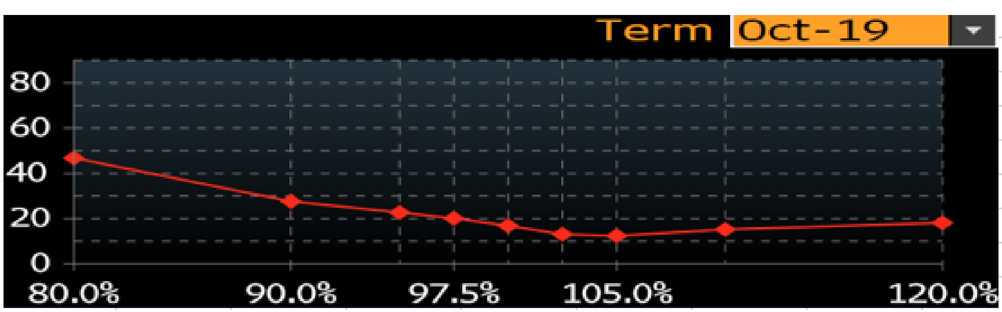

Figure A9: Bloomberg SPY Volatility Smile for October 1, 2019, October Contract