MAKERERE UNIVERSITY

COLLEGE OF ENGINEERING, DESIGN, ART & TECHNOLOGY

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

Assignment 1

Transmission Lines

Submitted by

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Group Members

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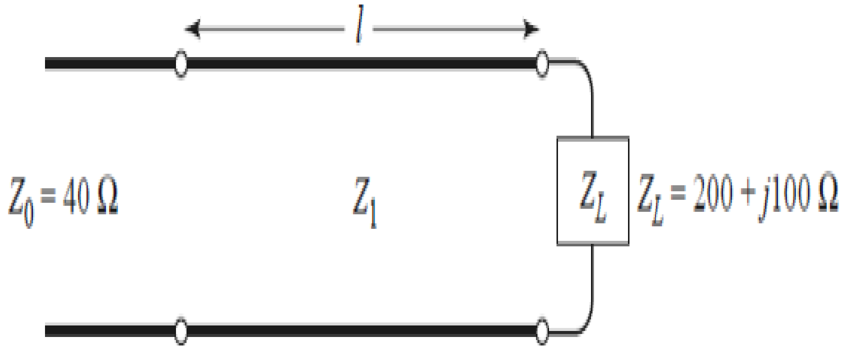
Solution to Question 1

1. Given the figure below



$$Z_{\mathbf{in}} = Z_{\mathbf{0}} = 40\Omega$$

$$Z_{\mathbf{in}} = Z_{\mathbf{1}}\left(\frac{Z_L + jZ_1\tan\beta l}{Z_1 + jZ_L\tan\beta l}\right),$$

let $t = \tan\beta l$ such that

$$Z_{\mathbf{in}} = Z_{\mathbf{1}}\left(\frac{(200 + i100) + jZ_1 t}{Z_1 + j(200 + i100)t}\right),$$

$$(40Z_1 - 4000t) + j8000t = 200Z_1 + j(100 + Z_1 t)Z_1$$

Equating real and imaginary factors
Re:

$$40Z_1 - 4000t = 200Z_1$$

$$160Z_1 = -4000t$$

$$Z_1 = -25t$$

Im:

$$5000t = Z_1(100 + Z_1 t)$$

$$8000t = -25t(100 - 25t)$$

$$000t = -2500t + 625t^3$$

$$t(625t^2 - 10500) = 0$$

Either $t = 0$
Or

$$(625t^2 - 10500) = 0$$

$$t = \pm\sqrt{16.8}$$

$$t = \pm 4.1$$

Therefore $t = -4.1$, so that $Z_1 = 102.5\Omega$
From $t = \tan\beta l$

$$\beta l = \arctan t = \arctan -4.1 = -76.3° \Leftrightarrow 104°$$

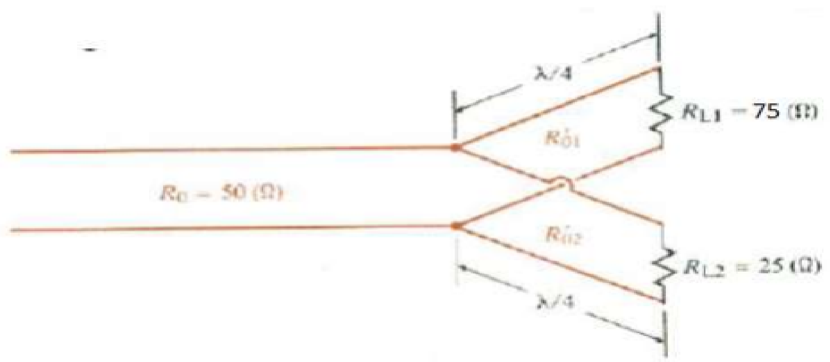
$\therefore$

$$\frac{2\pi l}{\lambda} = 104$$

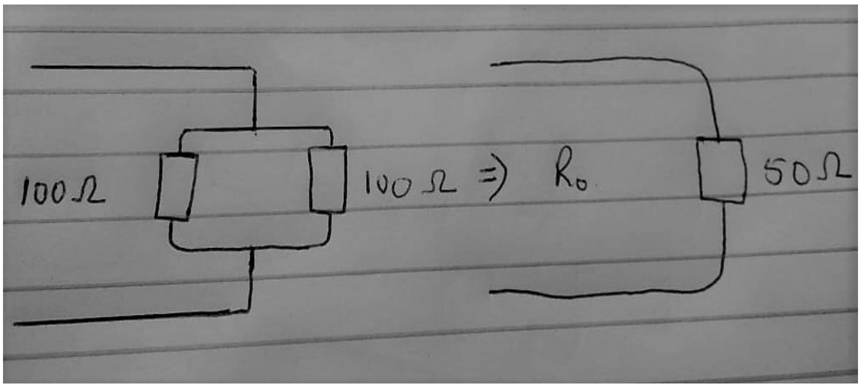
$$l = 0.289\lambda$$

Solution to Question 2

2. Given the figure below



(a) The input impedance of the two branch looking at the junction from the 50Ω line must be equal to 100Ω such that when they are in parallel the resultant impedance is 50Ω



$$Z_{\mathbf{in_1}} = R_{\mathbf{in_1}} = 100\Omega$$

The characteristics impedance

$$R_{o_1} = \sqrt{R_{\mathbf{in_1}} R_{\mathbf{L_1}}} = \sqrt{100 \times 75} = 86.6\Omega$$

Similarly

$$R_{o_2} = \sqrt{R_{\mathbf{in_2}} R_{\mathbf{L_2}}} = \sqrt{100 \times 25} = 50\Omega$$

Physical length, $l_P = 0.5c$

From

$$\frac{1}{\sqrt{\mu_o \varepsilon}} = 0.5c$$

$$\frac{1}{\sqrt{\mu_o \varepsilon_o \varepsilon_r}} = 0.5c$$

$$\begin{aligned} \varepsilon_r &= \frac{4}{\mu \varepsilon_r c^2} \\ &= \frac{4}{8.85 \times 10^{-12} \times 1.26 \times 10^{-6} \times (3 \times 10^8)^2} \end{aligned}$$

$$\varepsilon_r = 4$$

Wavelength along the transformers

$$\lambda = \frac{0.5c}{f} = \frac{0.5 \times 3 \times 10^8}{100 \times 10^6} = 1.5m$$

Physical length of the transformers

$$\frac{\lambda}{4} = \frac{1.5}{4} = 0.375m$$

(b) When matched, $SWR = 1$, on the main transmission line

$$\Gamma_{L_1} = \frac{R_{\mathbf{L_1}} - R_{\mathbf{o_1}}}{R_{\mathbf{L_1}} + R_{\mathbf{o_1}}}$$

$$= \frac{75 - 86.6}{75 + 86.6} = -0.072$$

$$SWR_1 = \frac{1 + |\Gamma_{L_1}|}{1 - |\Gamma_{L_1}|}$$

$$= \frac{1 + 0.072}{1 - 0.072} = 1.16$$

Similarly

$$\Gamma_{L_2} = \frac{R_{\mathbf{L_2}} - R_{\mathbf{o_1}}}{R_{\mathbf{L_2}} + R_{\mathbf{o_1}}}$$

$$= \frac{25 - 50}{25 + 50} = -0.33$$

$$SWR_2 = \frac{1 + |\Gamma_{L_2}|}{1 - |\Gamma_{L_2}|}$$

$$= \frac{1 + 0.33}{1 - 0.33} = 1.99$$