Homework 3

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Let $V = \mathbb{R}^2$, let β be the standard basis of V, and let L be the line $y = mx$ for m a non-zero real number.
$\beta = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

1 Question 1

Let T be the reflection of V across the line L. Find a basis γ for V in which $[T]_\gamma$ is diagonal. Then find $[T]_\beta$
for T_γ to be diagonal, the entries across the diagonal must be equal to 0. Let that basis be $\left\{ \begin{pmatrix} 1 \\ m \end{pmatrix}, \begin{pmatrix} -m \\ 1 \end{pmatrix} \right\}$ Then $T(1, m)$ is equivalent to itself, as it is on the line y = mx and $T(-m, 1)$ is flipped in reverse, so is equal $-1 \begin{pmatrix} -m \\ 1 \end{pmatrix}$ Therefore, the coordinates for the matrix are $\begin{pmatrix} 1 * V_1 & 0 * V_1 \\ 0 * V_2 & -1 * V_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Which is a diagonal matrix as desired. To find $[T]_\beta$, we note that $[T]_\beta$ is similar to $[T]_\gamma$ by way of $[1]_\gamma^\beta$ such that $[T]_\beta = [1]_\gamma^\beta [T]_\gamma [1]_\gamma^{\beta -1}$ where $[1]_\gamma^\beta = \begin{pmatrix} 1 & -m \\ m & 1 \end{pmatrix}$ and whose inverse is $\begin{pmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{-m}{1+m^2} & \frac{1}{1+m^2} \end{pmatrix}$ so that $[T]_\beta = \begin{pmatrix} 1 & -m \\ m & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{-m}{1+m^2} & \frac{1}{1+m^2} \end{pmatrix} = \frac{1}{1+m^2} \begin{pmatrix} 1 - m^2 & 2m \\ 2m & m^2 - 1 \end{pmatrix}$

2 Question 2

Let T be the projection of V onto the line L, do as above. There is no basis that will provide with a diagonal transformation matrix, as all transformation matrices for projections are of the form: $\begin{pmatrix} 1 & m \\ m & m^2 \end{pmatrix}$ since every projection will be a multiple of any vector on the line $y = mx$, therefore, one cannot find a linearly independent set of vectors to form a diagonal matrix with, therefore there does not exist a basis γ that provides a diagonal transformation matrix. The projection applied to β equals $\begin{pmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{m}{1+m^2} & \frac{m^2}{1+m^2} \end{pmatrix}$ so $[T]_\beta = \begin{pmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{m}{1+m^2} & \frac{m^2}{1+m^2} \end{pmatrix}$ As those are its coordinates with respect to β