Traveling Salesman Problem

Khalid Hourani

April 23, 2017

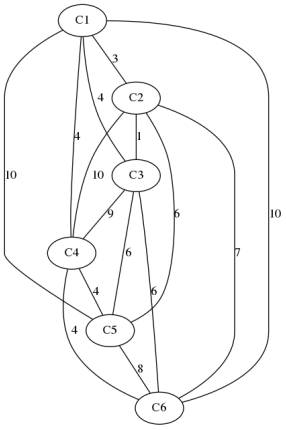
The Traveling Salesman Problem:

Given a list of cities and a the distance between any two cities, what is the shortest route that visits each city exactly once and returns to the original city?

This can be visualized as a Graph Problem by creating an undirected, weighted graph in which the cities are represented by vertices and the distances are represented by weighted edges. For example, consider the following cities and distances:

City	C1	C2	C3	C4	C5	C6
C1	0	3	4	4	10	10
C2	3	0	1	10	6	7
C3	4	1	0	9	6	6
C4	4	10	9	0	4	4
C5	10	6	6	4	0	8
C6	10	7	6	4	8	0

This can be represented with the following undirected, weighted graph:



A brute-force search of all possible paths shows that the shortest distance is given by traveling in the following order:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 4$$

For a graph with n vertices and edges, there are $n!$ possible routes. This means that, as the number of cities increases, a brute-force search of all possible routes becomes unfeasible. In fact, with $n = 20$, there are $20! = 2432902008176640000 \approx 2.4 \times 10^{18}$ possible routes to search.