

We consider each sort on a 0-indexed array A of n elements.

1. Selection Sort

On the i^{th} iteration (beginning with 0), we pass through the array from A_i to A_{n-1} to find the smallest value. Thus, the number of operations is given by

$$(n-1) + (n-2) + \dots + 2 + 1 = \frac{n(n-1)}{2}$$

A selection sort is therefore $O(n^2)$.

2. Insertion Sort

On the i^{th} iteration (beginning with 0), we place A_{i+1} in the correct position relative to the first $i+1$ elements. In the **worst case**, the array will be in reverse-order, and elements A_0 through A_i must be shifted one to the right, with A_{i+1} being placed at the beginning of the array. That is a total of

$$1 + 2 + \dots + (n-2) + (n-1) = \frac{n(n-1)}{2}$$

operations. An insertion sort is therefore $O(n^2)$ **in the worst case**.

In the **best case**, the array is already sorted and we only need to pass through the array once, in which case the sort is $O(n)$.

3. Merge Sort

The number of recursive calls to the merge sort will be $\lceil \log(n) \rceil$. At each step, the number of operations in merging the sublists will be proportional to the number of elements. Thus, a merge sort is $O(n \log(n))$.

4. Quick Sort

For each level the number of operations will be $O(n)$. In the **worst case**, the pivot will be the least or greatest value of each array, and thus there will be n levels, giving an $O(n^2)$ performance. In the **average** and **best case**, the number of levels will be $O(\log(n))$, and the performance is therefore $O(n \log(n))$.

In summary:

Name	Best	Average	Worst
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$
Merge Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick Sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$