University of Colombo School of Computing

SCS 1308 - Foundations of Algorithms

Tutorial - 03

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1. Disprove the claim: "For all integers x, $x2 - 3x + 2 \geq 0$".

   Counterexample: Let x=2:

   $x^2 - 3x + 2$ **=** $2^2 - 3(2) + 2$ **= 4-6+2 = 0**

   Can't disprove this statement.

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2. Find out the loop invariant conditions in the following selection sort algorithm.

   In the above pseudo code there are two loop invariant condition:

   - In the outer loop, array is sorted for **first i elements**.
   - In the inner loop, **min** is always the minimum value in **A[i to j]**.

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3. Prove that the sum of the first n powers of 2 is 2^n-1.

   The sum of the first n powers of 2 is given by:

   $S(n) = 2^0 + 2^1 + 2^2 + ... + 2^{n-1} = 2^n - 1$

   Step 1: Base Case (n=1)

   For n=1, the sum is:

   $S(n) = 2^1 - 1 = 1$

   Step 2: Inductive Hypothesis

   Assume the formula holds for n=k:

   $S(k) = 2^0 + 2^1 + 2^2 + ... + 2^{k-1} = 2^k - 1$

   Step 3: Inductive Step

   Prove the formula holds for n=k+1:

   $S(k + 1) = 2^0 + 2^1 + 2^2 + ... + 2^{k-1} + 2^k$

   Substitute the inductive hypothesis $S(k) = 2^k - 1$ into the equation:

   $S(k + 1) = (2^k - 1) + 2^k$

   Simplify:

   $S(k + 1) = 2^k + 2^k - 1 = 2.\ 2^k - 1 = 2^{k+1} - 1$

   Thus, the formula holds for n=k+1

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4. Find out the loop invariant conditions in the following insertion sort algorithm.

   In insertion sort, loop invariant condition is that the **subarray A[0 to i-1]** is always sorted.

5. Prove the correctness of the insertion sort algorithm using loop invariant.

6. Prove the correctness of the following algorithm to compute the sum of elements in an array:

```c
#include <stdio.h>

int main() {
    int n;
    printf("Enter the number of elements in the array: ");
    scanf("%d", &n);

    int A[n], sum = 0; // Initialize the array and the sum
    printf("Enter the elements of the array:\n");
    for (int i = 0; i < n; i++) {
        scanf("%d", &A[i]); // Input each element
    }
    // Compute the sum directly in the main function
    for (int i = 0; i < n; i++) {
        sum += A[i]; // Add each element to the sum
    }

    printf("The sum of the array elements is: %d\n", sum);
    return 0;
}
```

Invariant: At the start of each iteration, **sum** equals the sum of the first i elements of the array.