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Project 01 - Results and Discussion

JEB 1444 - Neural Engineering

Part 1: Investigating intrinsic frequency of the oscillator output

In this part of the project, FitzHugh-Nagumo oscillator model is considered to identify stimulus intensities that provide an oscillatory behavior (i.e., a stimulus that is sufficient to drive the system towards the excited state) in the output and the corresponding intrinsic frequency of oscillations. The FitzHugh-Nagumo oscillator model is characterized by two differential equations:

$$\dot{x} = \alpha \left[y + x - \frac{x^3}{3} + z \right]$$

$$\dot{y} = -\frac{1}{\alpha} \left[\omega^2 x - a + by \right]$$

The investigation was carried out with the following model parameters:

$$a = 0.7 \quad b = 0.8 \quad \omega^2 = 1 \quad \alpha = 3$$

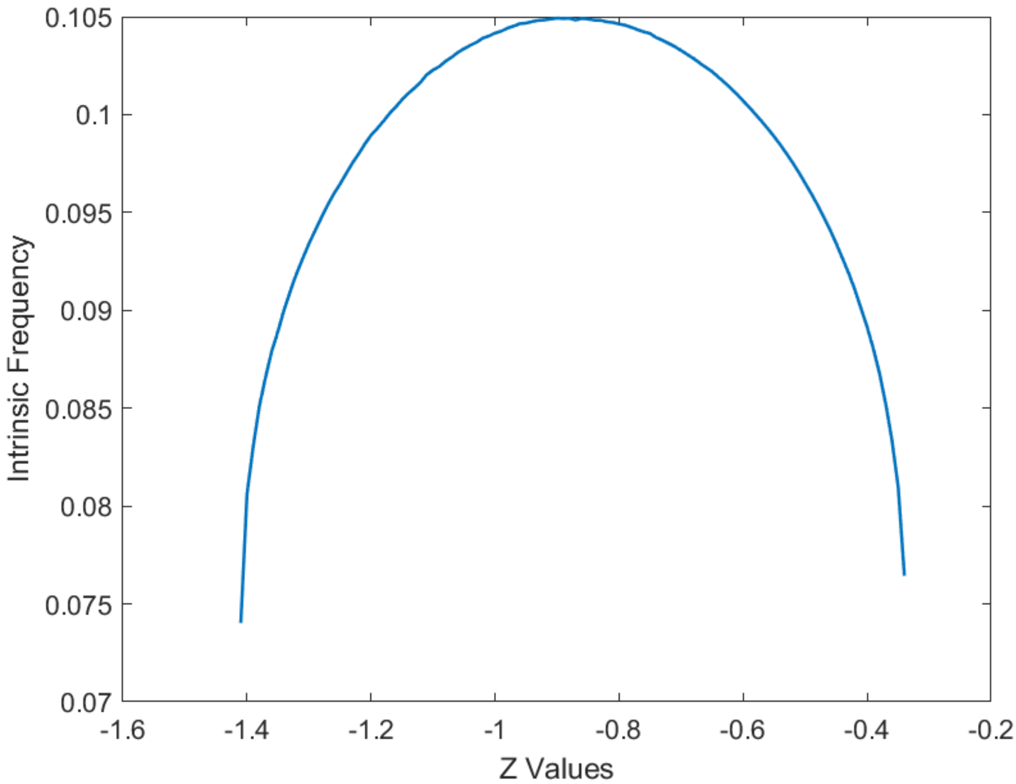

Figure 1: Intrinsic frequency of the oscillator output against input stimulus intensity

Discussion

The 2-D differential equation model was solved using the *ODE15s* stiff solver for different input stimulus intensity levels. At first, this was performed coarsely to identify the range of z that causes oscillatory outputs. Once the range was identified, the intrinsic frequencies that correspond to oscillatory outputs were investigated more closely and the Fig. 1 illustrated how the intrinsic frequency changes with the stimulus strength. Based on this, we can conclude that approximately $z \in [-1.4, -0.3]$ provides oscillatory outputs.

Part 2: Investigating the phase coherence index (R) of the coupled oscillators

In this part of the project, we explore the system dynamics of bidirectionally coupled FitzHugh-Nagumo oscillators that have intrinsic frequency responses derived in Part 1. The coupled system is characterized as:

$$\dot{x}_1 = \alpha \left[y_1 + x_1 - \frac{x_1^3}{3} + (k_1 + c x_2) \right] \qquad \dot{y}_1 = -\frac{1}{\alpha} \left[\omega^2 x_1 - a + b y_1 \right]$$

$$\dot{x}_2 = \alpha \left[y_2 + x_2 - \frac{x_2^3}{3} + (k_2 + c x_1) \right] \qquad \dot{y}_2 = -\frac{1}{\alpha} \left[\omega^2 x_2 - a + b y_2 \right]$$

where k_1 and k_2 are related to the intrinsic frequencies of the oscillators (i.e., when $c = 0$) and we use a symmetric coupling coefficient c between the two oscillators.
In this part of the project, the system dynamics for different coupling strengths and intrinsic frequency choices are characterized by the phase coherence index (R) which measures the degree of correlation between the two oscillators.

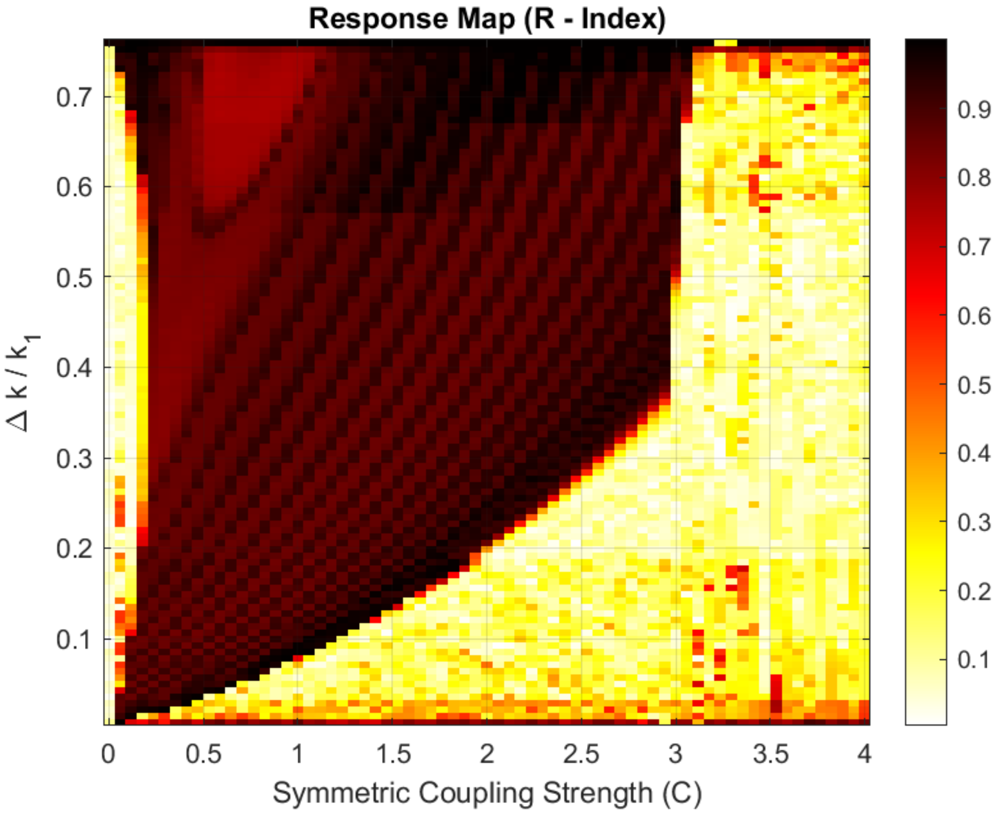

Figure 2: Phase coherence index (R) response map

Discussion

From the previous part, I identified the stimulus range $z = k$ that will drive the system into an excited state which results in an oscillatory output. For computational easiness, I fixed k_1 to the least value of the z's identified and varied k_2 within the range of identified z range in Part 1. Thus the percentage frequency difference $\frac{k_2 - k_1}{k_1}$ is always positive. After I solve for the x_1 and x_2 using *ODE15s* stiff differential solver, I obtained their complex Wavelet transformations separately.
During the coupling, the intrinsic frequencies associated with k_1 and k_2 may change. To avoid the issues in selecting the corresponding phase time series from the Wavelet transformations, I selected the Wavelet response that corresponded to the closest frequency (i.e., scale) to the intrinsic frequency of the chosen k_i value for $i = 1, 2$.

Once the phase series (ϕ_{x_1}, ϕ_{x_2}) for each (k_1, k_2, c) are extracted, R-index for each grid point is calculated as:

$$R = \left| \frac{1}{N} \sum_{n=1}^{N} e^{i\left[\phi_{x_1}[n] - \phi_{x_2}[n]\right]} \right|$$

The obtained system response map is shown in Fig. 2. Based on it, we can observe how the degree of correlation changes based on the coupling strength between the oscillators which are further characterized by k_i's that depend on their intrinsic frequencies. For coupling strengths less than 3, we can observe an exponential envelope that binds the region where the phases are locked between the two coupled oscillators. When the coupling strength increased beyond 3, the system seems to be uncorrelated and the oscillators are out of phase.

It will be interesting to try out all the possible perturbations of k_1 and k_2. Further exploring the responses with negative coupling coefficients and/or non-symmetric coupling coefficients between the two oscillators should be interesting.

Part 3: Investigating the two complexity measures for the coupled oscillators

Lyapunov Exponent

In this section, the system behavior is explored using the maximum Lyapunov exponent which is one of the measures that characterizes the system's complexity. It provides a measure of the exponential rate of divergence (or convergence) of a trajectory in the state space from another nearby trajectory differing only by its initial start conditions.

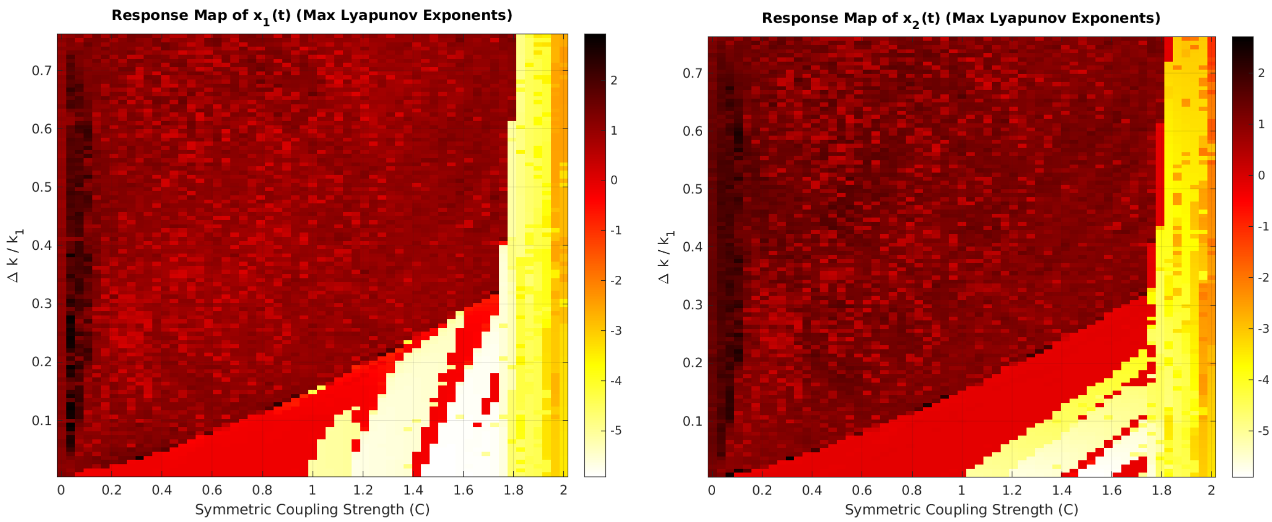

Figure 3: Maximum Lyapunov exponent response maps

Discussion

The coupled system is the same as in Part 2 and the maximum Lyapunov exponents are calculated for each time-series x_1 and x_2 using Rosenstein's algorithm. The results obtained are shown in Fig. 3.

The two oscillators demonstrate almost identical system complexities. It is evident that the two systems demonstrate chaotic nature (with $\lambda_{max} > 0$) and have similar envelopes to that of the system response map obtained in Part 2 except that the chaotic system responses are evident until the coupling coefficient is around 1.8. We can see that two oscillators will have higher chaotic behavior at the smaller coupling values around $0.1 \sim 0.2$.

Correlation Dimension

Another measure that can be used to characterize the complexity of the two oscillators is *correlation dimension*. The correlation dimension is estimated using a correlation sum that depends on m-dimensional embedding vectors that were created based on the delayed version of the original time series.

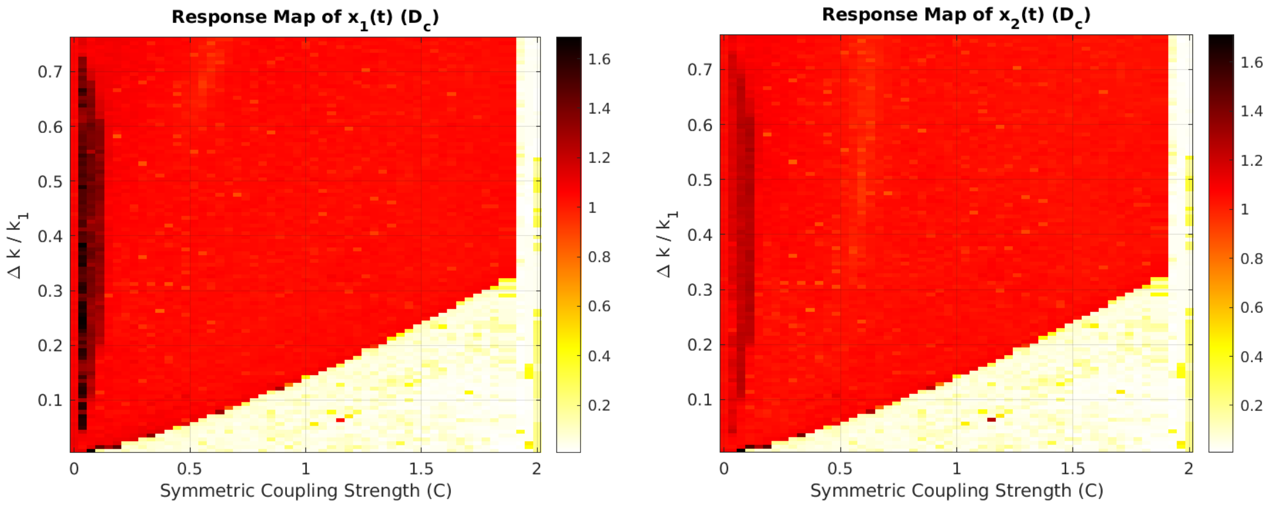

Figure 4: Correlation dimension response maps

Discussion

After solving for the x_1 and x_2, the state space is reconstructed using the delayed time series to estimate the embedding dimension (m) and optimal delay parameters (τ). Using these estimates, the correlation dimension is calculated using the Grassberger-Procaccia algorithm. The results are shown in Fig. 4.
Based on the results, the chaotic system behavior is illustrated within a similar region that was observed with maximum Lyapunov exponents. Higher the correlation dimension, the system is chaotic and we can observe that with a coupling strength around $0.1 \sim 0.2$, the system is highly chaotic with a correlation dimension exceeding 1.4.