Project Presentation

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Table of Contents

Cellular-level Neuronal Dynamic Modeling

- Second-order FitzHugh-Nagumo (FHN) oscillator model.

$$\dot{x} = \alpha \left[y + x - \frac{x^3}{3} + z \right]$$

$$\dot{y} = \frac{1}{\alpha} \left[\omega^2 x - a + by \right]$$

where α, ω^2, a, b are model parameters and z is input stimulus.

Cellular-level Neuronal Dynamic Modeling

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where α, ω^2, a, b are model parameters and z is input stimulus.

- For this project, the neuronal model is modeled with the following:

$$\alpha = 3 \quad a = 0.7 \quad b = 0.8 \quad \omega^2 = 1$$

Investigating Intrinsic Frequency of the Oscillator Output

Objectives

- Identifying the range of $z = k$ that provides an oscillatory behavior in system output.
- Identifying the range of corresponding intrinsic frequency of oscillations.

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Investigating Intrinsic Frequency of the Oscillator Output

Objectives

- Identifying the range of $z = k$ that provides an oscillatory behavior in system output.
- Identifying the range of corresponding intrinsic frequency of oscillations.

- Method:
  - 2-D differential equation model is solved using ODE15s stiff solver.
  - Preliminary investigation: Coarse search within an input stimuli $z_{pre} = [-4, +4]$ with step size 0.5.
  - Secondary investigation: Finer search within the identified range for $z = k$.

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Investigating Intrinsic Frequency of the Oscillator Output

- Results:
  - The neural model outputs brain rhythms within $0.0740 - 0.1409Hz$.

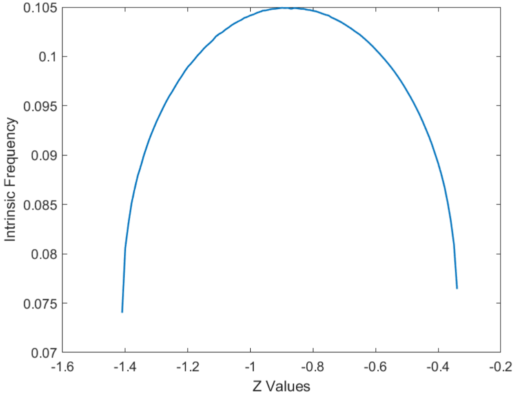

Figure: Intrinsic frequency of the oscillator output

Bidirectional coupling of FHN oscillators

- Let O_1 and O_2 be two FHN oscillators with a bandwidth (i.e., an intrinsic frequency range) of $0.0740 - 0.1409 Hz$.

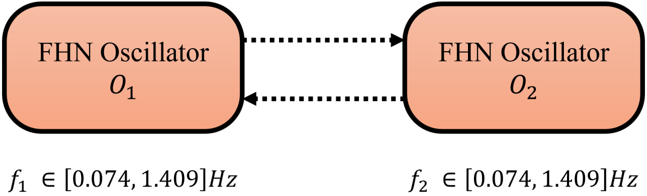

Figure: Bi-directional Coupling

- Bi-directional Coupling: O_1 and O_2 are connected to each other with equal strength, and both oscillators are influenced by each other in turn.

Bidirectional coupling of FHN oscillators

$$O_1 = \begin{cases} \dot{x}_1 = \alpha \left[y_1 + x_1 - \frac{x_1^3}{3} + (k_1 + c x_2) \right] \\ \dot{y}_1 = -\frac{1}{\alpha} \left[\omega^2 y_1 - a + b x_1 \right] \end{cases}$$

$$O_2 = \begin{cases} \dot{x}_2 = \alpha \left[y_2 + x_2 - \frac{x_2^3}{3} + (k_2 + c x_1) \right] \\ \dot{y}_2 = -\frac{1}{\alpha} \left[\omega^2 y_2 - a + b x_2 \right] \end{cases}$$

where $\alpha = 3,\ a = 0.7,\ b = 0.8,\ \omega = 1.$

- k_1 and k_2: Parameters related to intrinsic properties.
- c: Symmetric coupling coefficient (Amount of influence from each oscillator).

Phase Coherence Index (R) of coupled FHN oscillators

Phase Coherence Index (R)

Let $\phi_{x_1}[n]$ and $\phi_{x_2}[n]$ be two-phase time series obtained from complex Wavelet Transform.

$$R = \left| \frac{1}{N} \sum_{k=1}^{N} e^{i\left[\phi_{x_1}[k] - \phi_{x_2}[k]\right]} \right|$$

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- Objective: Identify different regions of system behavior.
    - Entrainment Region $\Rightarrow$ Strong correlation between the two oscillators i.e., $R \approx 1$.
    - Transition Region $\Rightarrow$ Region where the R-index changes from a higher value to a pre-defined lower value.

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Phase Coherence Index (R) of coupled FHN oscillators

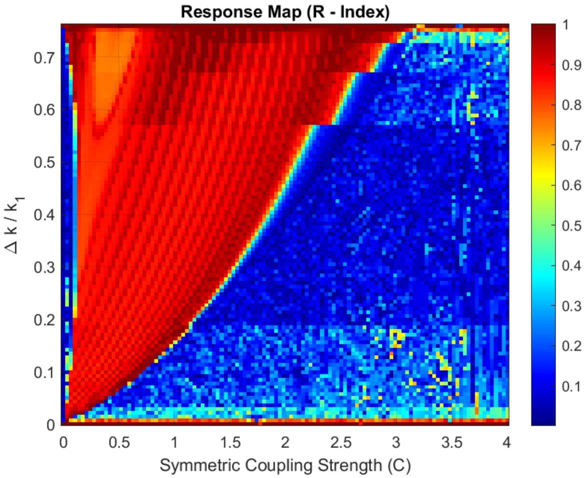

Figure: Phase coherence index (R) response map

Phase Coherence Index (R) of coupled FHN oscillators

Observations

- The degree of correlation changes based on the C which is further characterized by k_i's that depend on their intrinsic frequencies.
- Red Zone - Entrainment Region where $R \geq 0.8$ i.e., high synchronization between the two oscillators is present.
  - Boundary of the region - An exponential function of C.
  - The chaotic System Behaviour - distributed along the boundary of the entrainment region.
- For $\mathrm{C} > 1$ - the possibility of **active** coupling i.e., synchronization is achieved more rapidly than the case in passive coupling.

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Maximum Lyapunov Exponent

- Measures the complexity of the system.

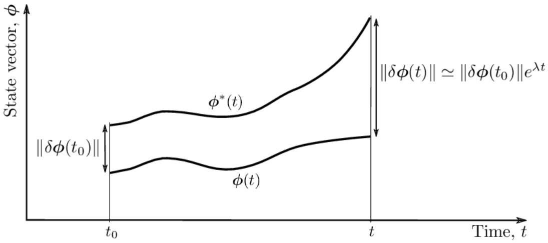

Figure: Phase trajectories and Lyapunov Exponent

- $\lambda > 0 \Rightarrow$ the trajectories diverge (*chaotic*).
- $\lambda < 0 \Rightarrow$ the trajectories converge (*hour-glass* i.e., non-oscillatory).
- $\lambda = 0 \Rightarrow$ Periodic (oscillatory) system

Maximum Lyapunov Exponent

Implementation

- Algorithm - Rosenstein's algorithm (from MATLAB)
- Original Max. Lyapunov Maps are NOISY!!

Maximum Lyapunov Exponent

Implementation

- Algorithm - Rosenstein's algorithm (from MATLAB)
- Original Max. Lyapunov Maps are NOISY!!
- WHY?
  - Due to numerical inaccuracies of software packages, $\lambda = 0$ is not correctly calibrated.

Maximum Lyapunov Exponent

Implementation

- Algorithm - Rosenstein's algorithm (from MATLAB)
- Original Max. Lyapunov Maps are noisy.
- Why?
  - Due to numerical inaccuracies of software packages, $\lambda = 0$ is not correctly calibrated.
- Solution: **Zero-Lyapunov Exponent Calibration**
  - When $c = 0$, oscillators will have their intrinsic behavior.
  - We know the intrinsic behaviors of the two oscillators for the selected k_1, k_2 are periodic and oscillatory.
  - Let λ_{0,k_1,k_2} be the max. Lyapunov exponent for $c = 0$.
  - Define

$$\tilde{\lambda}_{c,k_1.k_2} = \lambda_{c,k_1.k_2} - \lambda_{0,k_1,k_2}$$

$$\forall c \in [0,4], k_1, k_2 \in [-1.41, -0.38]$$

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Example: Zero-Lyapunov Exponent Calibration

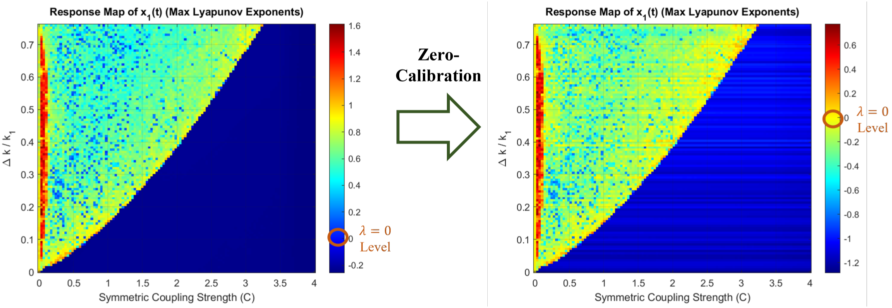
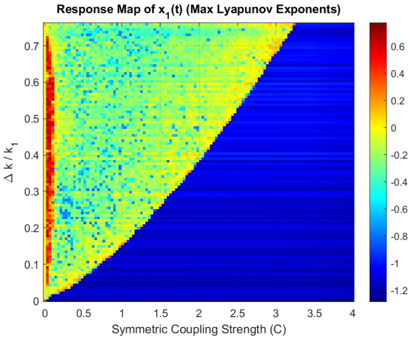


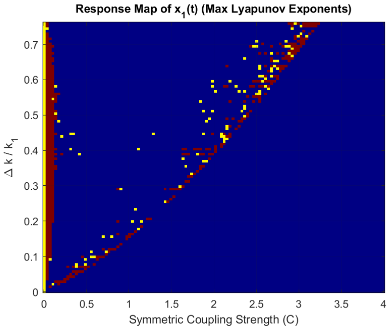
Figure: Zero-Lyapunov Exponent Calibration for x_1

Maximum Lyapunov Exponent

Response Maps - x_1



(a) Response Map

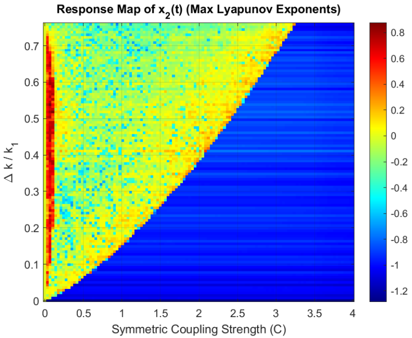


(b) Masked Response Map

Figure: Max. Lyapunov Exponent Response Map for x_1

Maximum Lyapunov Exponent

Response Maps - x_2



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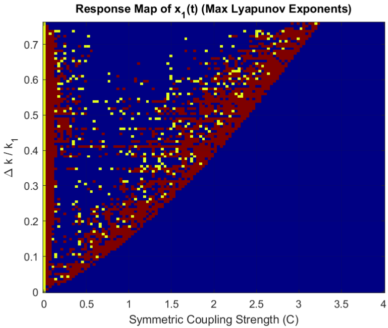


(b) Masked Response Map

Figure: Max. Lyapunov Exponent Response Map for x_2

Maximum Lyapunov Exponent

Observations

- Chaotic system nature is strong at the transition region.
  - Also, this is distributed along the exponential boundary of the entrainment region as well.
  - However, the divergence rate (i.e., measured by the Max. Lyapunov Exponent) is lesser compared to the transition region.
- Periodic clock system behavior is very limited once coupled.

Correlation Dimension

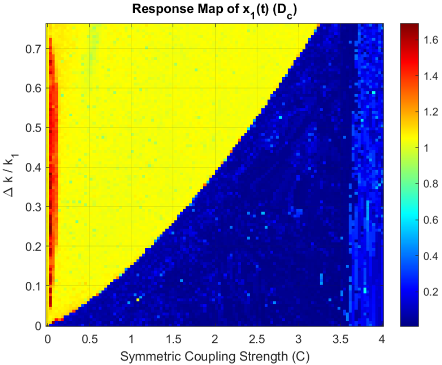
- Another measure of the complexity of the system.

Implementation

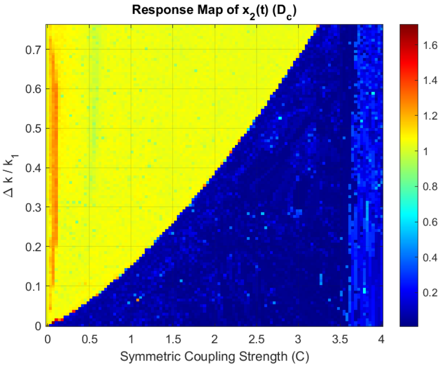
- Algorithm - Grassberger-Procaccia algorithm (from MATLAB)
  - Optimal Lag (τ) and Embedding Dimension (d) estimated by Phase Space Reconstruction (using MATLAB).
  - Then estimate the correlation dimension D_c for each system configuration.
- High $D_c (\geq 1) \Rightarrow$ Chaotic System Behaviour

Correlation Dimension

Response Maps - x_1 & x_2



(a) x_1 Signal



(b) x_2 Signal

Figure: Correlation Dimension Response Maps

Table of Contents

Discrete-Time Wiener-Bose Model

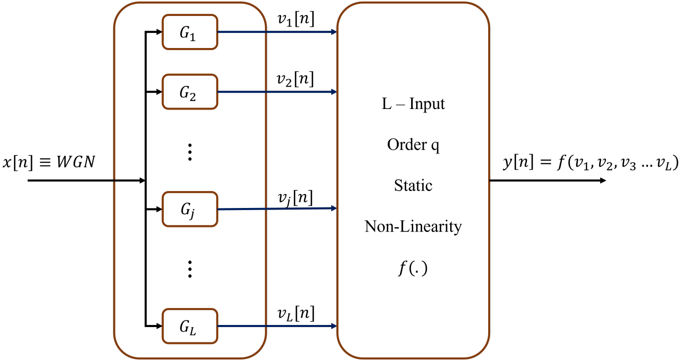


Figure: Discrete-Time Wiener Bose Model

$\mathrm{G_j} \triangleq$ Linear Filter with ℓ_{j-1} as impulse response.

$\mathrm{h_r[n_1, n_2, \dots, n_r]} \triangleq$ q$^{\text{th}}$-order Wiener Kernel

Discrete-Time Wiener-Bose Model

- Using $q = 2$ system:

$$y[n] = c_0 + \sum_{j=1}^{L} c_1(j) v_j[n] + \sum_{j_1=1}^{L} \sum_{j_2=1}^{L} c_2(j_1, j_2) v_{j_1}[n] v_{j_2}[n]$$
$$= f(v_1, v_2, \ldots, v_j, \ldots, v_L)$$

and

$$v_j[n] = \sum_{k} \ell_{j-1}[k] x[n-k]$$

- The r^{th} Wiener kernel can be estimated as:

$$\mathrm{h_r}[\mathrm{n_1}, \mathrm{n_2}, \ldots, \mathrm{n_r}] = \sum_{j_1} \cdots \sum_{j_r} c_r(j_1, j_2, \ldots, j_r) l_0[n] \ldots \ell_{r-1}[n]$$

where $0 \leq r \leq 2$

Estimating Wiener-Bose Model for Coupled Oscillators

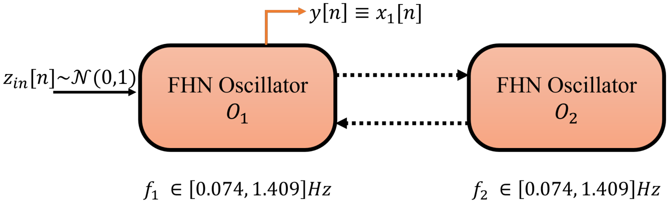


Figure: Project setup to estimate the Wiener-Bose Model

- Inject Bandlimited WGN to Oscillator No. 1 ($x[n] \equiv z_{in}[n]$)
- Measure the output $x_1[n]$ from Oscillator No. 1 ($y[n] \equiv x_1[n]$)
- Use *Laguerre-Expansion Technique* to find Wiener-Bose model parameters for $q = 2$ i.e., $\{c_r(1, 2, \ldots, r), 0 \le r \le 2\}$

Estimating Wiener-Bose Model for Coupled Oscillators

System Input

- Why **White Gaussian Noise**?
  - Wiener kernals become **orthogonal** for WGN input.
  - Can estimate h_r for $0 \leq r \leq 2$ kernels independently i.e., no sequential estimation of kernels is needed.
  - **EASY!!**
- Why **Bandlimited**?
  - Should ensure that the noise bandwidth is sufficiently larger than the system bandwidth.
  - Should ensure that it follows the sampling theory to avoid any aliasing effects on the system output.

System Output

- Solution for the Oscillator No. 1 output i.e., x_1, by solving . . .

$$\dot{x}_1 = \alpha \left[y_1 + x_1 - \frac{x_1^3}{3} + (k_1 + c x_2) + z_{in}^1 \right]$$

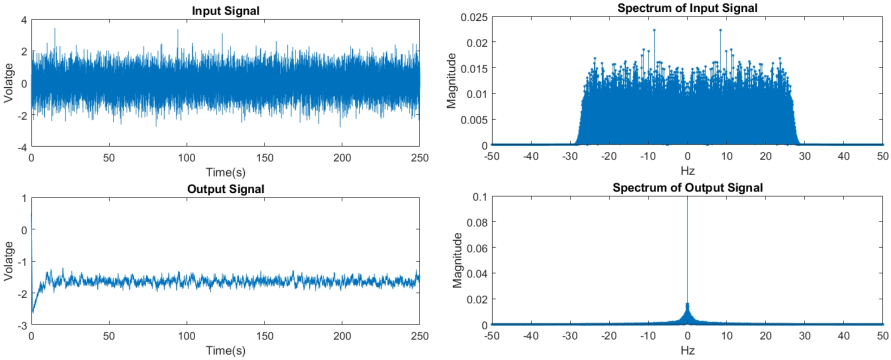
$$\dot{y}_1 = -\frac{1}{\alpha} \left[\omega^2 x_1 - a + b y_1 \right]$$

$$\dot{x}_2 = \alpha \left[y_2 + x_2 - \frac{x_2^3}{3} + (k_2 + c x_1) \right]$$

$$\dot{y}_2 = -\frac{1}{\alpha} \left[\omega^2 x_2 - a + b y_2 \right]$$

Estimating Wiener-Bose Model for Coupled Oscillators

Example Input-Output Signals: (c = 0.95, k_1 = -1.41, k_2 = -0.68)

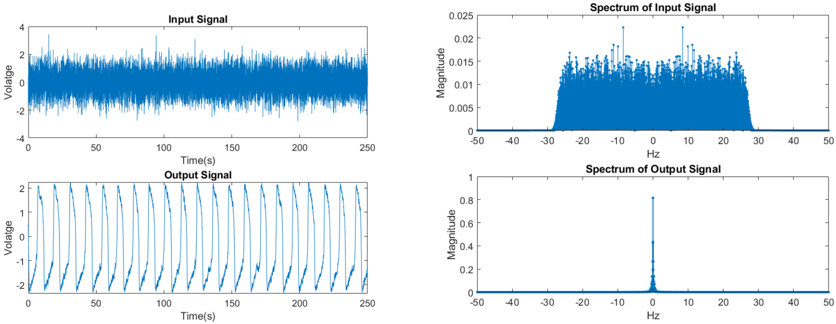


(a) Temporal characteristics

(b) Spectral characteristics

Figure: Filtered WGN Input with F_p = 25Hz and System Output

Example Input-Output Signals: (c = 0.32, k_1 = -1.41, k_2 = -0.84)



(a) Temporal characteristics (b) Spectral characteristics

Figure: Filtered WGN Input with F_p = 25Hz and System Output

Estimating Wiener-Bose Model for Coupled Oscillators

- Before finding the Wiener-Bose model parameters...

Estimating Wiener-Bose Model for Coupled Oscillators

1. Estimate the bandwidth of the coupled FHN Oscillator system.

$$Y(e^{j\omega}) = \mathcal{FFT}(y[n]) \Rightarrow B_{sys} \text{ (per sample)}$$

Estimating Wiener-Bose Model for Coupled Oscillators

1. Estimate the bandwidth of the coupled FHN Oscillator system.
2. Estimating first-order kernel $k_1(\tau)$ and system memory τ_{sys}.
   - Using cross-correlation between the input-output of the oscillator system, estimate $k_1[\tau]$ kernel.

$$k_1[\tau] = \frac{1}{P_x}\left|r_{xy}[\tau]\right| = \frac{1}{P_x}\left|\sum_m x[m]\tilde{y}[m+\tau]\right|$$

where P_x is the power of input WGN and

$$\tilde{y}[n] = y[n] - \bar{y}, \qquad \bar{y} = \frac{1}{N}\sum_{n=0}^{N-1} y[n]$$

Estimating Wiener-Bose Model for Coupled Oscillators

1. Estimate the bandwidth of the coupled FHN Oscillator system.
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$$k_1[\tau] = \frac{1}{P_x}\left|r_{xy}[\tau]\right| = \frac{1}{P_x}\left|\sum_m x[m]\tilde{y}[m+\tau]\right|$$

   - Estimate the sample at which $k_1[\tau]$ drops below $|k_1(\tau)|_{max}\exp(-5)$

$$\tau_{sys} \approx 4.5RC$$

Estimating Wiener-Bose Model for Coupled Oscillators

1. Estimate the bandwidth of the coupled FHN Oscillator system.
2. Estimating system memory of the coupled FHN Oscillator system.
3. Estimate the no. of Laguerre Functions (L) and α.

$$M = \tau_{sys} \times B_{sys}$$

$$\mathrm{L} = \max\left(1, \left\lceil \frac{\mathrm{M}}{\mathrm{L}_{\mathrm{Thresh}}} \right\rceil\right) \text{ where } \mathrm{L}_{\mathrm{Thresh}} = 25,$$

and

$$\alpha = \exp\left(-\frac{\alpha_{Thresh}}{\tau_{Sys} \times T_s - 1}\right) \text{ where } T_s = 0.01s \text{ and } \alpha_{Thresh} = 45.$$

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Estimating Wiener-Bose Model for Coupled Oscillators

- **Non-Oscillatory Region ($\lambda < 0$)** $c = 0.95, k_1 = -1.41, k_2 = -0.68$

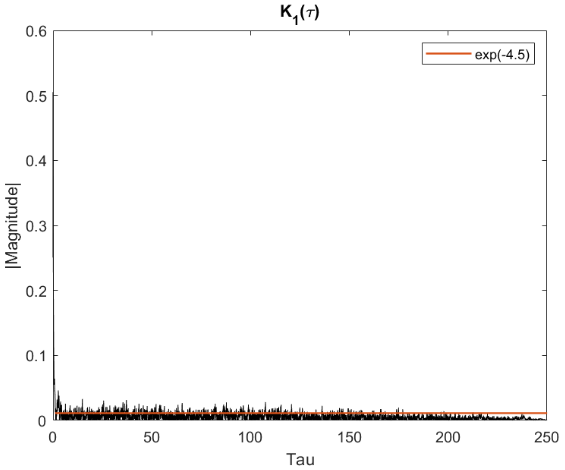

Figure: $k_1(\tau)$ estimation

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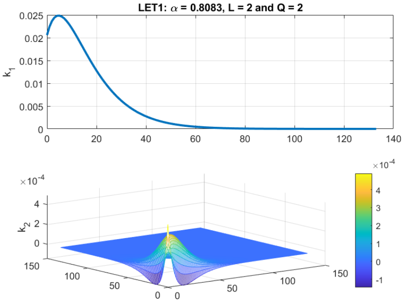
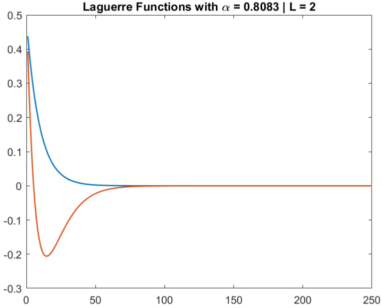


(a) Estimated Wiener Kernels



(b) Laguerre Functions in Linear Part

Figure: Estimated Wiener-Bose Kernels and Laguerre Functions

Estimating Wiener-Bose Model for Coupled Oscillators

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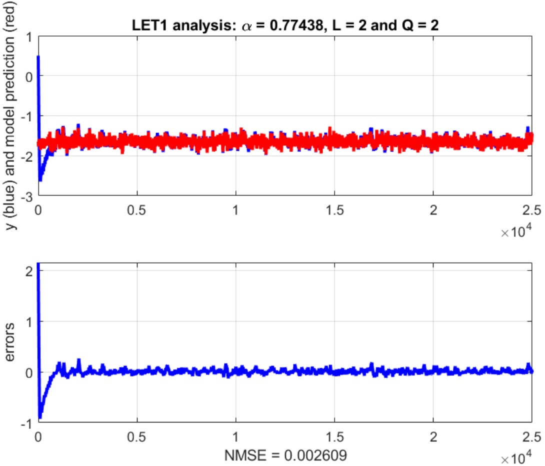

Figure: Waveform comparison between the FHN Model & Wiener-Bose Model

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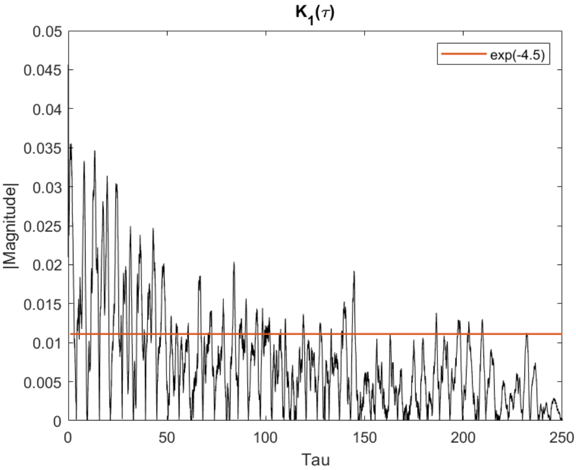
Estimating Wiener-Bose Model for Coupled Oscillators

- **Oscillatory Region ($\lambda \approx 0$)** $c = 0.32, k_1 = -1.41, k_2 = -0.84$



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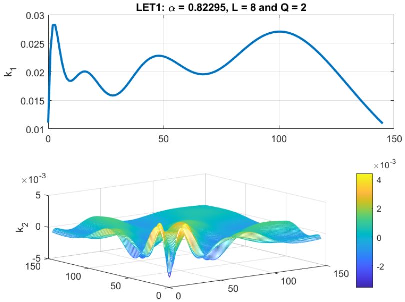

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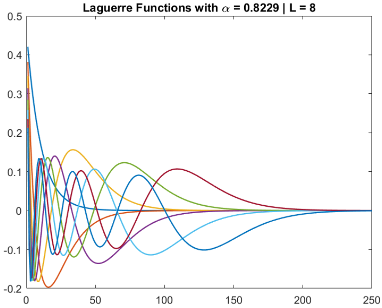

(b) Laguerre Functions in Linear Part

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Estimating Wiener-Bose Model for Coupled Oscillators

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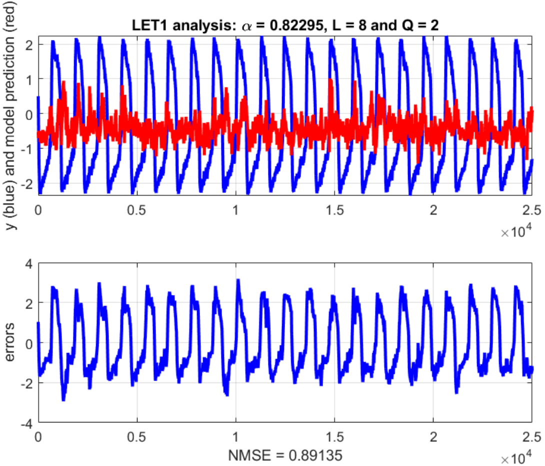

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- **Chaotic Region ($\lambda > 0$)** $c = 0.01, k_1 = -1.41, k_2 = -0.61$

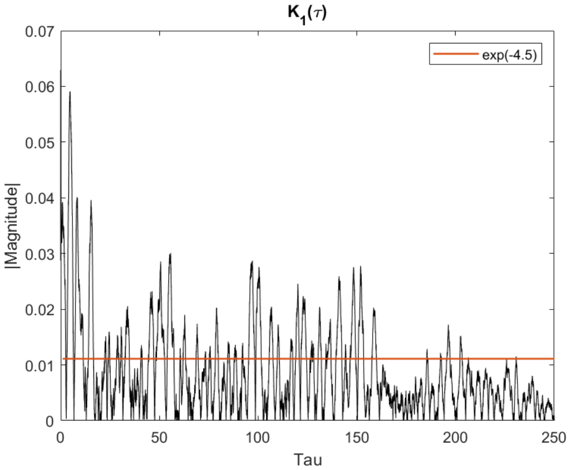

Figure: $k_1(\tau)$ estimation

Estimating Wiener-Bose Model for Coupled Oscillators

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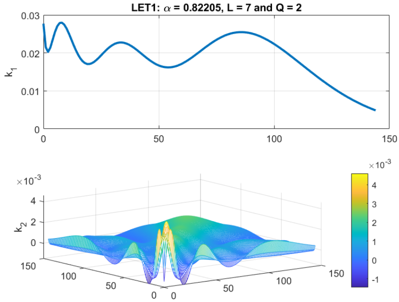

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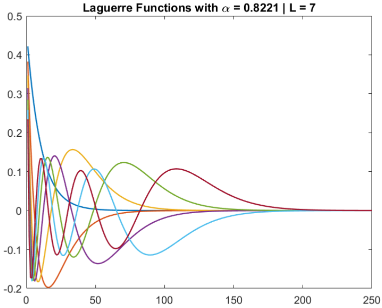

(b) Laguerre Functions in Linear Part

Figure: Estimated Wiener-Bose Kernels and Laguerre Functions

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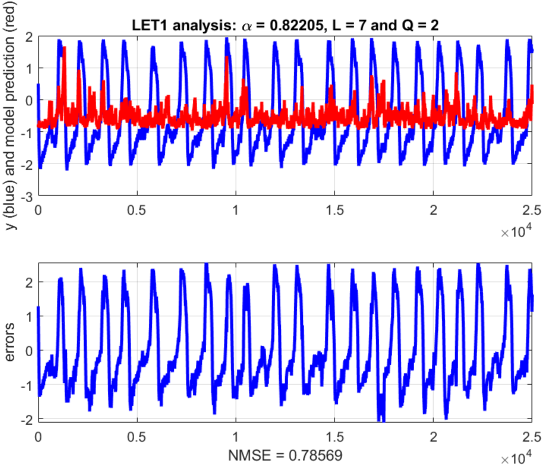


Figure: Waveform comparison between the FHN Model & Wiener-Bose Model

Principal Dynamic Modes for Modular Volterra Model

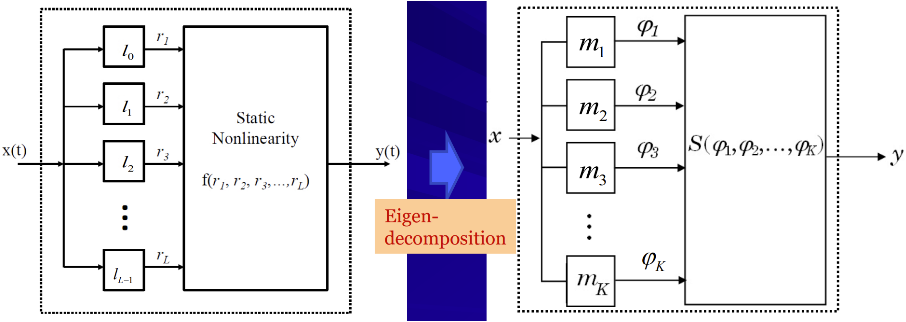


Figure: From Wiener-Bose Model to Modular Volterra Model

- Need to estimate the number of Principal Dynamic Modes (PDMs) K, each PDM m_j and their Associated Non-linear Functions (ANFs).

Principal Dynamic Modes for Modular Volterra Model

Estimating Number of PDMs (K)

1. Construct the $\mathbf{Q}$ matrix

$$\mathbf{Q} = \begin{bmatrix} h_0 & \frac{1}{2}\mathbf{h_1}^T \\ \frac{1}{2}\mathbf{h_1} & \mathbf{h_2} \end{bmatrix} \in \mathbb{R}^{(d+1)\times(d+1)}$$

   where

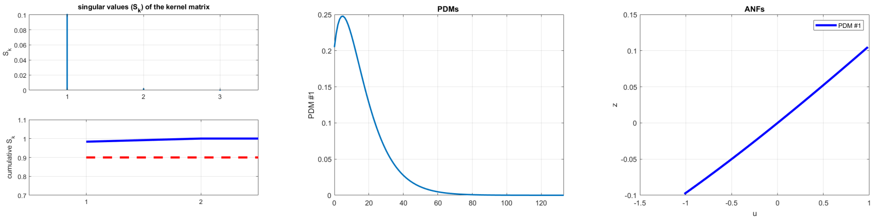
$$\mathbf{h_1} \in \mathbb{R}^{d\times 1}, \mathbf{h_2} \in \mathbb{R}^{d\times d}$$

2. Eigen Decomposition $\mathbf{Q} = \mathbf{V\Lambda V}^T$
3. Estimate K

$$K = \left| \left\{ \lambda_i \;\middle|\; \frac{|\lambda_i|}{\sum |\lambda_j|} \geq \epsilon \right\} \right| \text{ where } \epsilon = 0.01$$

Principal Dynamic Modes for Modular Volterra Model

- **Non-Oscillatory Region ($\lambda < 0$)** $c = 0.95, k_1 = -1.41, k_2 = -0.68$



(a) Eigen Values of **Q** matrix (b) Estimated PDMs (c) Estimated ANFs

Figure: PDM Estimation

- **Non-Oscillatory Region ($\lambda < 0$)** $c = 0.95, k_1 = -1.41, k_2 = -0.68$

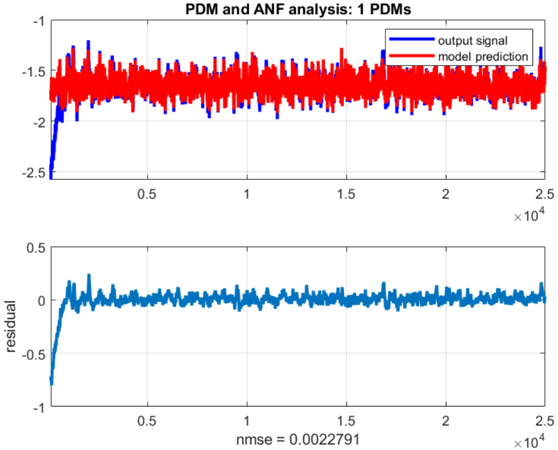
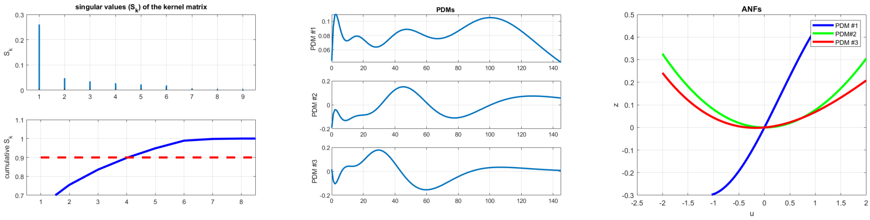


Figure: Waveform comparison between the FHN Model & Modular Volterra Model

Principal Dynamic Modes for Modular Volterra Model

- **Oscillatory Region ($\lambda \approx 0$)** $c = 0.32, k_1 = -1.41, k_2 = -0.84$



(a) Eigen Values of **Q** matrix (b) Estimated PDMs (c) Estimated ANFs

Figure: PDM Estimation

Principal Dynamic Modes for Modular Volterra Model

- **Oscillatory Region ($\lambda \approx 0$)** $c = 0.32, k_1 = -1.41, k_2 = -0.84$

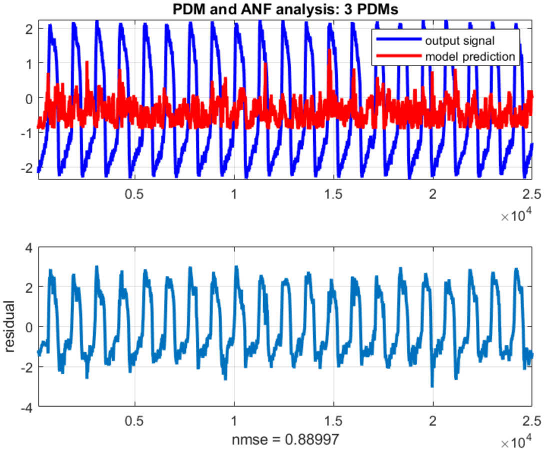
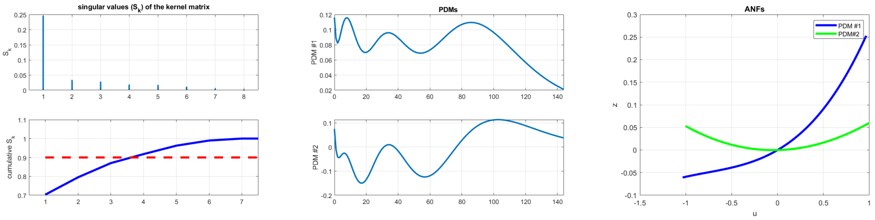


Figure: Waveform comparison between the FHN Model & Modular Volterra Model

Principal Dynamic Modes for Modular Volterra Model

- **Chaotic Region ($\lambda > 0$)** $c = 0.01, k_1 = -1.41, k_2 = -0.61$



(a) Eigen Values of **Q** matrix (b) Estimated PDMs (c) Estimated ANFs

Figure: PDM Estimation

- **Chaotic Region ($\lambda > 0$)** $c = 0.01, k_1 = -1.41, k_2 = -0.61$

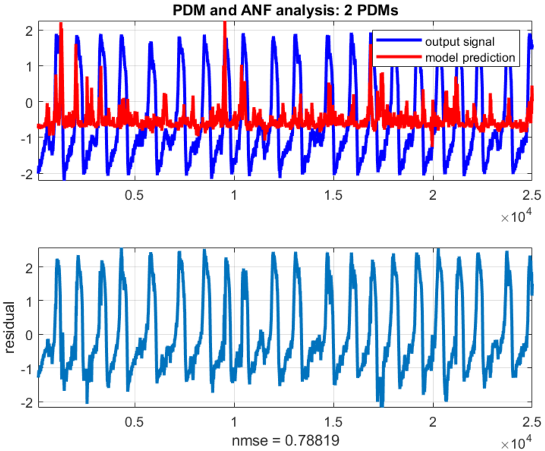


Figure: Waveform comparison between the FHN Model & Modular Volterra Model

Parametric vs. Non-Parametric Model Output Comparison

- FHN model can generate action potentials using the **Threshold Phenomena**.

Parametric vs. Non-Parametric Model Output Comparison

- FHN model can generate action potentials using the **Threshold Phenomena**.
- Can Non-Parametric Models (i.e., Wiener-Bose/Modular Volterra Models) do the same?

NO!!

Parametric vs. Non-Parametric Model Output Comparison

- FHN model can generate action potentials using the **Threshold Phenomena**.
- Can Non-Parametric Models (i.e., Wiener-Bose/Modular Volterra Models) do the same?

NO!!

- Solution: **Threshold Triggering**

$$y_{trig}[n] = 0.5 + 0.5 \cdot \mathrm{sgn}\left(y[n] - \beta\right)$$

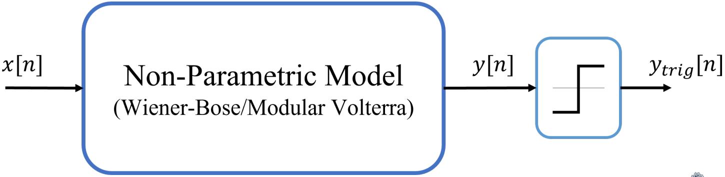
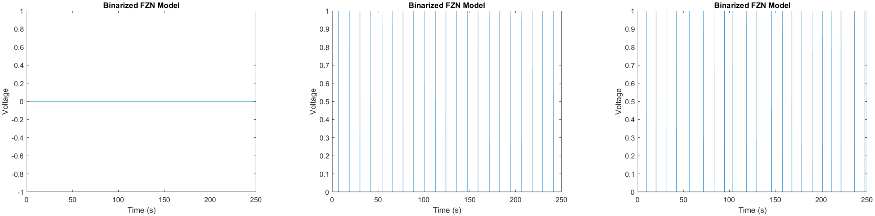


Figure: Spiking Model Structure

Parametric vs. Non-Parametric Model Output Comparison

Before comparing output waveforms from the Parametric (FHN) Model and Non-Parametric Models...

1. Binarize the FHN model output to $\{0, 1\}$



(a) Non-Oscillatory System (b) Oscillatory System (c) Chaotic System

Figure: Binarized Waveforms from FHN Model

Parametric vs. Non-Parametric Model Output Comparison

Before comparing output waveforms from the Parametric (FHN) Model and Non-Parametric Models...

1. Binarize the FHN model output to $\{0, 1\}$
2. Find the optimal threshold β^* to threshold trigger non-parametric model outputs using training noise sequence.

$$\beta^* = \arg\max_{\beta} \mathrm{TPR}(\beta) \text{ s.t. } \mathrm{FPR}(\beta) \leq \gamma_{allowed}$$

where

$$\mathrm{TPR} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN}} \text{ True Positive Rate,}$$

$$\mathrm{FPR} = \frac{\mathrm{FP}}{\mathrm{FP} + \mathrm{TN}} \text{ False Positive Rate.}$$

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- How to define True Positives (TP), False Positives (FP), True Negatives (TN), and False Negatives (FN)?

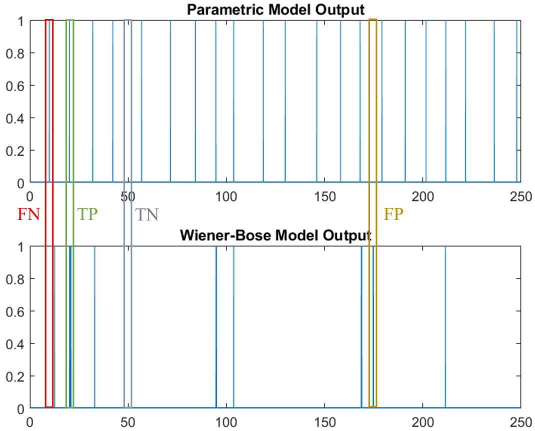
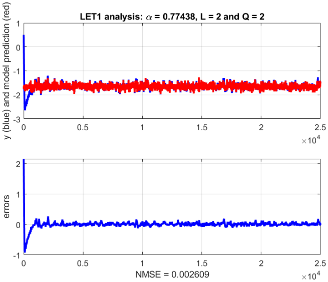


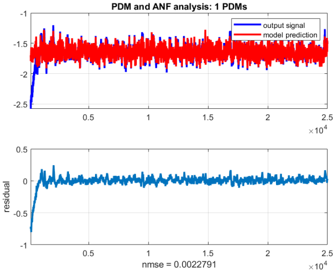
Figure: Time point-wise definitions for TP, FP, TN, and FN

Model Evaluation (Training)

1. Non-Oscillatory Systems
   - There are no action potentials to compare in the FHN model.
   - ∴ waveform comparison between the outputs from the FHN model and Non-Parametric models is enough.



(a) Wiener-Bose Model

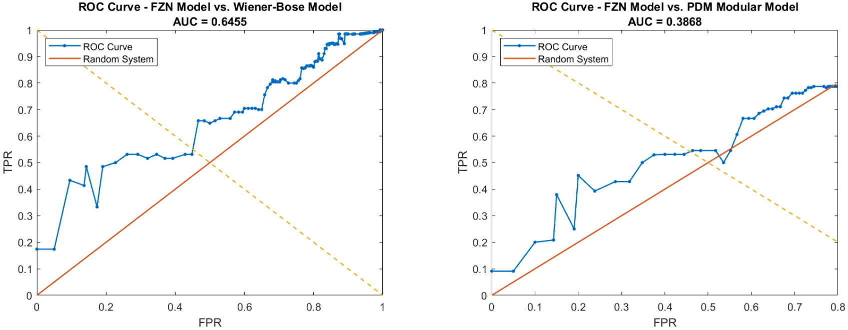


(b) Modular Volterra Model

Figure: Comparison between FHN model and Non-Parametric Models

Model Evaluation (Training)

2. Oscillatory System
   - Set $\beta = \left[\frac{\min(y[n])+\max(y[n])}{2}, \max(y[n])\right]$ and construct ROC curve.

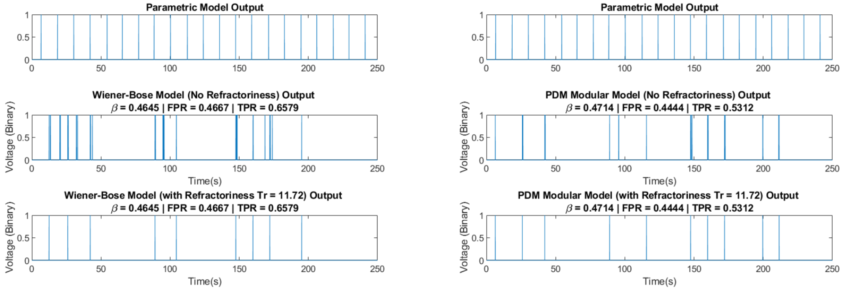


(a) Wiener-Bose (AUC $= 0.6455$) (b) Modular Volterra (AUC $= 0.3868$)

Figure: ROC Curves for Oscillatory System

Model Evaluation (Training)

2. Oscillatory System
   - Find β^* and do the threshold triggering based on it.



(a) Wiener-Bose ($\beta^* = 0.4645$) (b) Modular Volterra ($\beta^* = 0.4714$)

Figure: Action Potential Comparison between models at β^*

Parametric vs. Non-Parametric Model Output Comparison

Model Evaluation (Training)

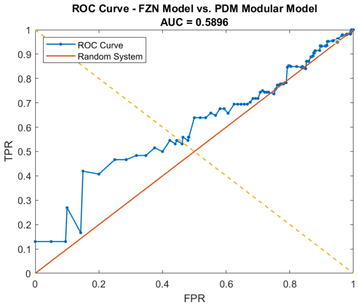
2. Oscillatory System

Observations

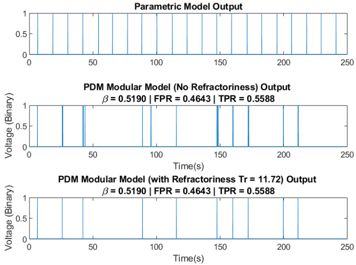
- AUC values - Better than the random coin tosses.
- BUT not a good metric to evaluate the model performance.
- For better visualization $\Rightarrow$ Threshold triggering with refractoriness
- Wiener-Bose model output is much similar to FHN model than the Modular Volterra Model output.
  - Let's add another PDM to see whether the model output improves for Modular Volterra Model!

Model Evaluation (Training)

2. Oscillatory System



(a) ROC Curve (AUC = 0.5896)



(b) Action Potentials ($\beta^* = 0.5190$)

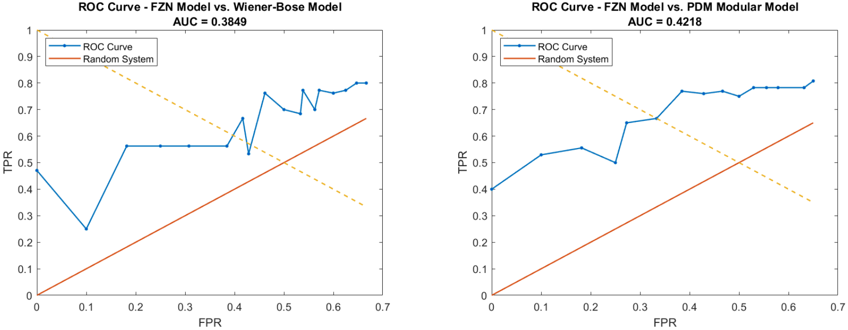
Figure: Modular Volterra Model performance with 4 PDMs

- Model performance is improved - However, the output spikes are not different!!

Parametric vs. Non-Parametric Model Output Comparison

Model Evaluation (Training)

3. Chaotic System
   - Set $\beta = \left[\frac{\min(y[n])+\max(y[n])}{2}, \max(y[n])\right]$ and construct ROC curve.



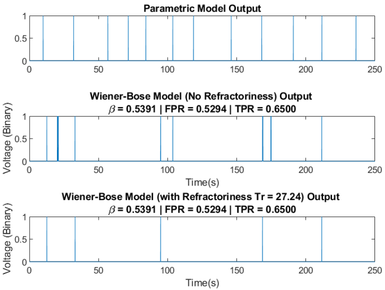
(a) Wiener-Bose (AUC = 0.3849)

(b) Modular Volterra (AUC = 0.4218)

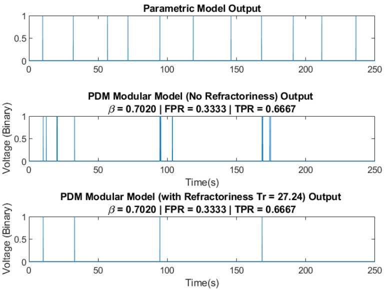
Figure: ROC Curves for Oscillatory System

Model Evaluation (Training)

3. Chaotic System



(a) Wiener-Bose ($\beta^* = 0.5373$)



(b) Modular Volterra ($\beta^* = 0.5986$)

Figure: Action Potential Comparison between models at β^*

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Parametric vs. Non-Parametric Model Output Comparison

Model Evaluation (Training)

3. Chaotic System

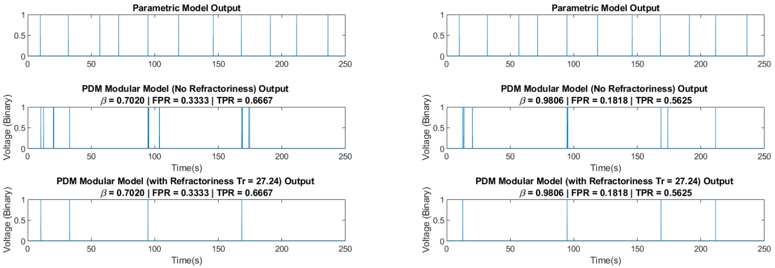
Observations

- AUC values - Better than the random coin tosses.
- BUT, not a good metric to evaluate the performance (especially for chaotic systems)
- For better visualization $\Rightarrow$ Threshold triggering with refractoriness
  - However, for a Chaotic system, an accurate refractory period cannot be estimated.
- Wiener-Bose model output is closer to the parametric model output than the Modular Volterra Model output.

Parametric vs. Non-Parametric Model Output Comparison

Model Evaluation (Training)

2. Chaotic System



(a) No. of PDMs = 2 (b) No. of PDMs = 3

Figure: Modular Volterra Model outputs with additional PDMs

- By adding an additional PDM i.e., (b), the model output does not improve significantly - Insignificant PDM!!

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Model Validation (Testing)

1. Oscillatory System

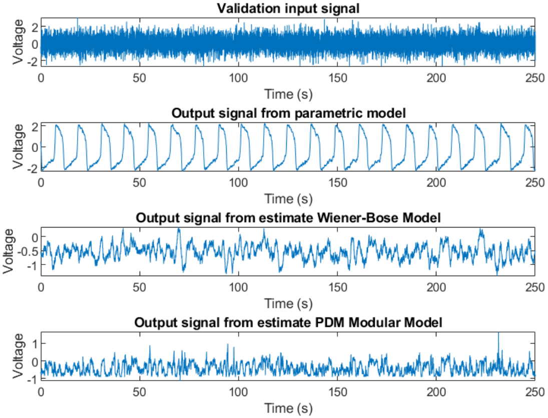
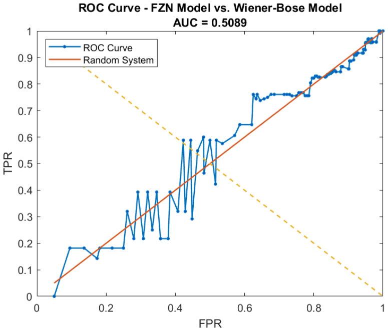


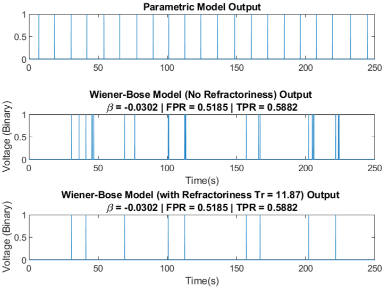
Figure: Model Validation: System Input and Outputs

Model Validation (Testing)

1. Oscillatory System



(a) ROC Curve (AUC = 0.5089)

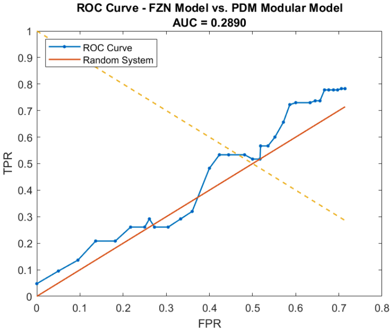


(b) Action Potentials (β = -0.0302)

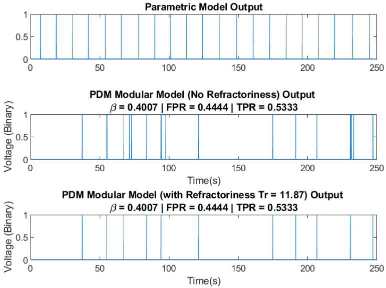
Figure: Wiener-Bose Model validation

Model Validation (Testing)

1. Oscillatory System



(a) ROC Curve (AUC = 0.2890)



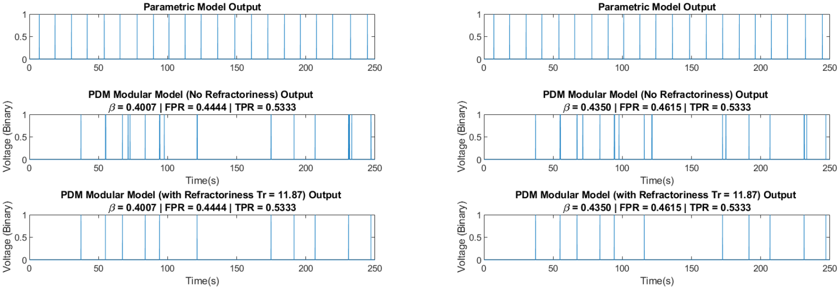
(b) Action Potentials ($\beta = 0.4007$)

Figure: Modular Volterra Model validation

Parametric vs. Non-Parametric Model Output Comparison

Model Validation (Testing)

1. Oscillatory System



(a) Using 3 significant PDMs

(b) Using 4 significant PDMs

Figure: Modular Volterra Model validation - with additional significant PDM

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Model Validation (Testing)

2. Chaotic System

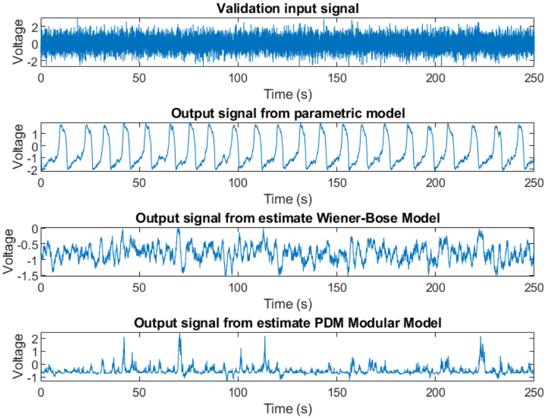
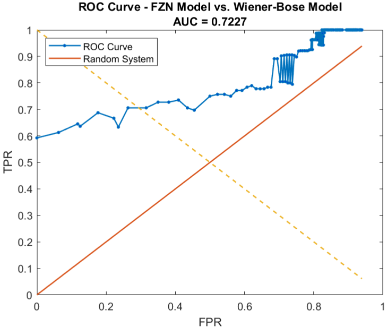


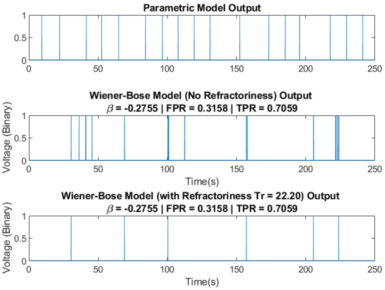
Figure: Model Validation: System Input and Outputs

Model Validation (Testing)

2. Chaotic System



(a) ROC Curve (AUC = 0.7227)

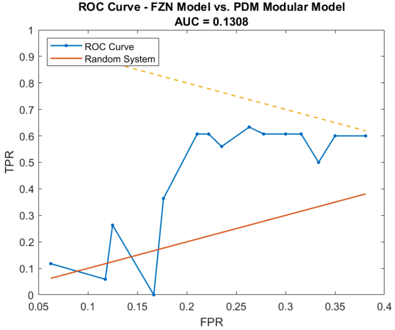


(b) Action Potentials (β = -0.2755)

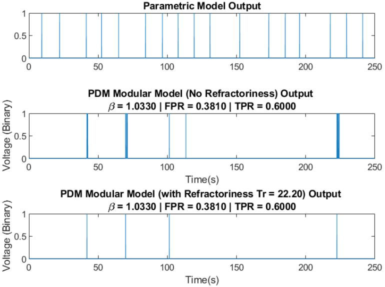
Figure: Wiener-Bose Model validation

Model Validation (Testing)

2. Chaotic System



(a) ROC Curve (AUC $= 0.1308$)



(b) Action Potentials ($\beta = 1.0330$)

Figure: Modular Volterra Model Validation

Thank You!

Q & A