Comparison of the effect by different Heuristic values on solving modified N-puzzle using A* algorithm

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Introduction

The N-puzzle is a puzzle that contains a square matrix with numbers assigned to each position. These positions are called tiles. The typical N-puzzle has a single blank position, allowing the other tiles to be slid and re-arranged. Given a start configuration and a goal configuration, the puzzle is solved when the goal configuration is obtained by rearranging the start configuration accordingly. In this assignment, instead of the traditional N-puzzle, a modified version with two blanks is used

A* algorithm is a traversal algorithm that is widely used in pathfinding. It uses a heuristic value to conduct an informed search to find the shortest path from one node to another. Compared to conventional traversal algorithms, A* is considered to be smart[1]. This can be used to solve the modified N-puzzle by considering each puzzle configuration as a node.

The algorithm may use any appropriate heuristic value. In the case of the N-puzzle, the following heuristic values are used and compared.

- Total misplaced tiles - number of tiles which has a different position compared to the goal configuration
- Total manhattan distance - summation of the manhattan distance of each tile from its position in the goal configuration

In the following process, the heuristic values are compared in terms of the number of intermediate states the A* algorithm considered before finding the best path to the goal[2]. All the relevant source codes are provided in the folder containing this report.

Testing using Paired t-tests

100 N-puzzle start configurations were randomly generated such that the puzzle size was varying from 5 to 20. For each start configuration, a corresponding goal configuration was also generated. Then for each of these start, goal pairs, the A* algorithm was run twice, once with the misplaced tile count as the heuristic value and once with the total manhattan distance as the heuristic value, and the number of intermediate states considered by the algorithm was recorded.

When generating the goal configurations, instead of making it entirely random, the tiles of the start configuration itself were altered 10-20 times randomly. This step was followed because it reduces the time taken to solve puzzles with larger sizes.

Result

Here, “Misplaced tile moves” contains the number of moves taken by the algorithm to reach the goal when the heuristic is calculated by the number of misplaced tiles. In the same way, “Manhattan moves” contains the number of moves taken by the algorithm to reach the goal when the heuristic is calculated by the number of misplaced tiles

“Differences” contains the difference(misplaced_tile_moves - manhattan_moves) between the number of moves for a particular start, goal pair.

No of tests: 100
Puzzle SIZE lower limit(>3): 5
Puzzle SIZE greater limit(>3 and >lower limit): 20

No. of tests with misplaced tiles as h value: 100
No. of tests with manhattan distance as h value: 100

Misplaced tile moves :
[12, 12, 7, 9, 16, 28, 8, 38, 18, 20, 152, 5, 389, 5, 20, 7, 75, 10, 14, 24, 8, 14, 40, 12, 9, 7, 13, 14, 7, 16, 29, 11, 11, 10, 18, 16, 11, 6, 8, 6, 22, 46, 24, 52, 24, 9, 13, 13, 12, 24, 121, 8, 4, 6, 12, 10, 14, 9, 6, 23, 17, 4, 11, 4, 7, 20, 13, 19, 50, 107, 24, 31, 6, 13, 11, 6, 13, 11, 7, 17, 5, 167, 7, 14, 8, 47, 24, 20, 8, 22, 159, 39, 14, 10, 6, 25, 19, 22, 24, 16]

Manhattan moves :
[24, 30, 15, 18, 30, 35, 16, 32, 42, 50, 52, 8, 270, 8, 42, 7, 32, 21, 25, 48, 14, 32, 56, 16, 10, 12, 20, 30, 12, 28, 26, 20, 28, 18, 19, 30, 28, 10, 7, 6, 27, 59, 16, 82, 56, 21, 28, 14, 28, 42, 60, 12, 6, 8, 27, 9, 30, 16, 8, 17, 28, 6, 28, 6, 12, 45, 30, 42, 55, 116, 48, 70, 10, 36, 28, 12, 24, 27, 12, 36, 4, 152, 15, 35, 15, 59, 54, 65, 15, 45, 114, 50, 14, 20, 10, 60, 48, 42, 26, 30]

Differences :
[-12, -18, -8, -9, -14, -7, -8, 6, -24, -30, 100, -3, 119, -3, -22, 0, 43, -11, -11, -24, -6, -18, -16, -4, -1, -5, -7, -16, -5, -12, 3, -9, -17, -8, -1, -14, -17, -4, 1, 0, -5, -13, 8, -30, -32, -12, -15, -1, -16, -18, 61, -4, -2, -2, -15, 1, -16, -7, -2, 6, -11, -2, -17, -2, -5, -25, -17, -23, -5, -9, -24, -39, -4, -23, -17, -6, -11, -16, -5, -19, 1, 15, -8, -21, -7, -12, -30, -45, -7, -23, 45, -11, 0, -10, -4, -35, -29, -20, -2, -14]

Mean Difference: -7.03
Standard Deviation: 22.34371310126218
Standard Error of the Mean Difference: 2.234371310126218
T-statistic: -3.146298902129602

Statistical analysis

The differences between the moves were then used to calculate the mean, standard deviation, standard error, and t-value using the following formulas[3].

Standard Error = Standard Deviation / Sqrt(n)
n = Number of elements in the list(number of test cases)

T-value = Mean Difference / Standard Error

The t-value was found as -3.146298902129602

Using a confidence level of 99%, and 99(equals to n - 1) degrees of freedom, the critical t-value can be found[4].

Critical t-value = 1.6604 > -3.146298902129602

Conclusion

The calculated t-value is much less than the critical t-value. This means that the algorithm checks fewer intermediate nodes when the number of total misplaced tiles is set as the heuristic value than when the manhattan distance is set as the heuristic value.
Therefore, the total number of misplaced tiles gives a more efficient output compared to the total manhattan distance in terms of the number of intermediate nodes checked in the process.
Hence, the total number of misplaced tiles is the better heuristic of the two.

References

[1] GeeksforGeeks. 2022. *A* Search Algorithm - GeeksforGeeks*. [online] Available at: <A* Search Algorithm - GeeksforGeeks/> [Accessed 19 October 2022].

[2] Insight into programming algorithms. 2022. *Implementing A-star(A*) to solve N-Puzzle*. [online] Available at: <Implementing A-star(A*) to solve N-Puzzle> [Accessed 19 October 2022].

[3]*Statistics: 1.1 paired T-tests - statstutor* (no date). Available at: Statistics: 1.1 Paired t-tests (Accessed: October 20, 2022).

[4]Szczepanek, A. (2022) *Critical value calculator*, *Omni Calculator*. Omni Calculator. Available at: Critical Value Calculator (Accessed: October 20, 2022).