Name ______________________
Physics 51M Section ___ Box # _____
Problem Set 5
14 October 2019

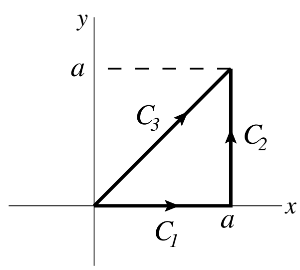
**Collaborators:**

**Problem 1**

(a) Calculate a line integral of a vector function $\vec{G}(x, y, z) = z^2\hat{x} + x^2\hat{y} - y^2\hat{z}$ over a path $C_3$ in the $(x - y)$ plane, as shown in the figure.

(b) Now repeat the calculation over the piecewise path $C_1 + C_2$.

(c) A vector field is called conservative if its line integral is path independent between any two endpoints. Based on your results above, is $\vec{G}$ a conservative vector field?

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**Problem 2** Calculate the work done by the force $\vec{F} = -y\hat{x} + x\hat{y}$ along the closed path defined by the parametric trajectory

$$\vec{r}(t) = a\cos\left(\frac{t}{t_0}\right)\hat{x} + a\sin\left(\frac{t}{t_0}\right)\hat{y} + a\cos\left(\frac{2t}{t_0}\right)\hat{z}$$

where $a$ and $t_0$ are constants with appropriate units. Is $\vec{F}$ a conservative vector field?

*Hint:* Start by justifying that $t = 0$ and $t = 2\pi t_0$ give valid starting and ending integration points.

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**Problem 3*** Suppose an electric field is given by $\vec{E} = k\frac{\hat{r}}{r}$ in spherical coordinates, where $k$ is a constant.

(a) Find the volume charge density $\rho$ everywhere in space.

(b) Using Gausss Law, find the total charge contained in a sphere of radius $R$, centered at the origin (at $r = 0$).

(c) Repeat part (b), but this time find the total charge by direct integration of $\rho$ over the volume of the sphere. Do you get the same result?

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**HRK E28.2** Derive an expression for the work required by an external agent to put the four charges together as indicated in Fig 28-28. Each side of the square has length $a$. Comment on whether the external agent had to do positive work to construct this charge configuration or whether positive work was done on the agent by the electric field.

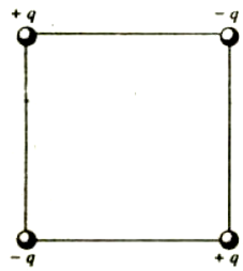


**FIGURE 28-28.** Exercise 2.

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**HRK P28.6** A particle of mass $m$, charge $q > 0$, and initial kinetic energy $K$ is projected (from an infinite separation) toward a heavy nucleus of charge $Q$, assumed to have a fixed position in our reference frame.

(a) If the aim is "perfect", how close to the center of the nucleus is the particle when it comes instantaneously to rest?

(b) With a particular imperfect aim, the particle's closest approach to the nucleus is twice the distance determined in part $(a)$. Determine the speed of the particle at this closest distance of approach. Assume that the particle does not reach the surface of the nucleus.

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**HRK P28.10** A total amount of positive charge $Q$ is spread onto a nonconducting, flat, circular annulus of inner radius $a$ and outer radius $b$. This charge is distributed so that the charge density (charge per unit area) is given by $\sigma = k/r^3$, where $r$ is the distance from the center of the annulus to any point on it. Show that (with $V = 0$ at infinity) the potential at the center of the annulus is given by

$$V = \frac{Q}{8\pi\epsilon_0}\left(\frac{a+b}{ab}\right)$$

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