Biharmonic Scattering

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Theorem 1. *Let*

$$T_{ik}w = \sum w_n \gamma_n e^{in\theta} \tag{0.1}$$

Where

$$\gamma_n = \frac{ik\left(H_n^{(1)}(ikR)\right)'}{H_n^{(1)}(ikR)} \tag{0.2}$$

Then we have

$$\lim_{k\to 0} \gamma_n = \frac{-|n|}{R} \tag{0.3}$$

Proof. We first use identities to calculate $\left(H_n^{(1)}(ikR)\right)'$. We see

$$\left(H_n^{(1)}(ikR)\right)' = H_{n-1}^{(1)}(ikR) - \frac{n}{ikR}H_n^{(1)}(ikR) \tag{0.4}$$

So then

$$\gamma_n = \frac{ik\left(H_{n-1}^{(1)}(ikR) - \frac{n}{ikR}H_n^{(1)}(ikR)\right)}{H_n^{(1)}(ikR)} \tag{0.5}$$

Taking the limit as $k \to 0$ we see

$$\lim_{k\to 0} \gamma_n = \lim_{k\to 0} ik\frac{\left(H_{n-1}^{(1)}(ikR) - \frac{n}{ikR}H_n^{(1)}(ikR)\right)}{H_n^{(1)}(ikR)} \tag{0.6}$$

$$= \lim_{k\to 0} \frac{ikH_{n-1}^{(1)}(ikR)}{H_n^{(1)}(ikR)} - \frac{ik}{ik}\frac{-nH_n^{(1)}(ikR)}{RH_n^{(1)}(ikR)} \tag{0.7}$$

Now, the first term should go to zero, so if we can show that $\frac{H_n^{(1)}(ikR)}{H_n^{(1)}(ikR)} \to 1$, then we have our proof. I think we can take the limit and expand into $\Gamma(ikR)$, but I need to look into it more. □