PHYS 506 Homework 1

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Problem 1

Propagate uncertainty in each of the following. Explain what method you are using and why. Give both symbolic and numeric answers.

(a) $y = d\sin\theta$ for $d = (2.00 \pm 0.05)$ m, $\theta = (15 \pm 1)\,\text{deg}$.

(b) $P = (V - IR)I$ for $V = 5.0 \pm 0.2$ V, $I = 1.24 \pm 0.03$ A, $R = 0.13\Omega \pm 5\%$

(c) $y = \sin\theta - \theta$ for $\theta = 0.000 \pm 0.087$ radians.

Solution

(a) This is a multiplication of 2 uncertain values, so we see

$$\frac{\delta y}{y} = \sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta(\sin\theta)}{\sin\theta}\right)^2}$$

To calculate $\delta(\sin\theta)$ we must use the fact that this is a function of a single variable, so we have

$$\delta(\sin\theta) = \delta\theta\frac{\partial}{\partial\theta}(\sin\theta) = \delta\theta\cos\theta$$

So then we have

$$\begin{aligned}\frac{\delta y}{y} &= \sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta(\sin\theta)}{\sin\theta}\right)^2}\\ &= \sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta\theta\cos\theta}{\sin\theta}\right)^2}\end{aligned}$$

$$\boxed{\delta y = y\sqrt{\left(\frac{\delta d}{d}\right)^2 + (\delta\theta\cot\theta)^2}}$$

So then numerically we have

$$y = (2.00\,\text{m})\sin(15\,\text{deg}) = 0.518\,\text{m}$$

$$\delta y = (0.518\,\text{m})\sqrt{\left(\frac{0.05\,\text{m}}{2.00\,\text{m}}\right)^2 + ((1\,\text{deg}\cdot 2\pi/360)\cot(15\,\text{deg}))^2} = 0.0361\,\text{m}$$

So our final value is

$$\boxed{y \pm \delta y = 0.518 \pm 0.0361\,\text{m}}$$

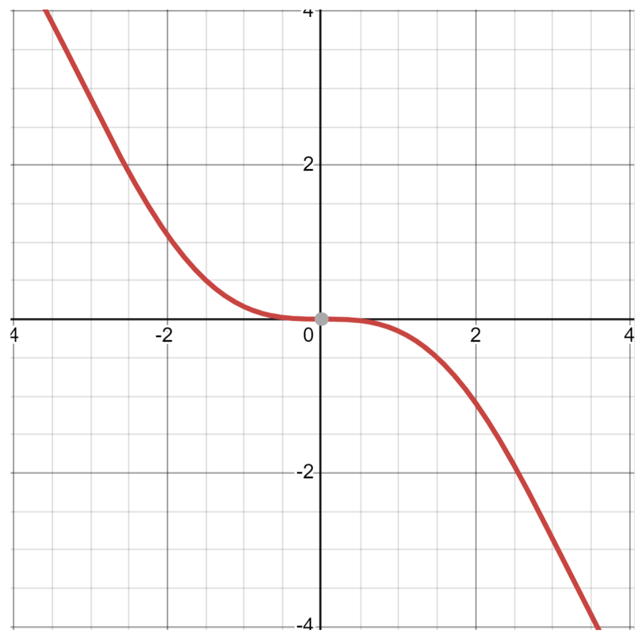
(b) This is simply a series of linear arithmetic operations, so we can simply add the proportional uncertainties in quadrature. We see

$$\frac{\delta P}{P} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2 + \left(\frac{\delta I}{I}\right)^2} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + 2\left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2}$$

So then numerically we have

$$P = (5.0 - (1.24)(0.13))(1.24) = \boxed{6.00}$$

$$\delta P = (6.00)\sqrt{\left(\frac{0.2}{5.0}\right)^2 + 2\left(\frac{0.03}{1.24}\right)^2 + (.05)^2} = \boxed{.4107}$$



(c) We see that the graph is given by This is approximately linear in a neighborhood of zero, so we can simply use the first order Taylor approximation and use the fact that $\sin x \approx x$ about zero. So then the uncertainty in y is given by the same uncertainty in x. The use of any other method should result in the uncertainty being zero, which cannot be reasonable.

Problem 2

n experiment measures the rate of a certain type of cosmic ray over 16 years, exactly 5844 days. They divide the data by "season" into 4 equal-sized data sets, each 1461 days long. They observe 771, 762, 686, and 802 events in the spring, summer, fall, and winter data sets respectively. The expected "background" of false events in each data set is $\mu_B = 51.4$. The number of events observed by the experiment in season i is expected to be a Poisson random variable with mean $\mu_i = \mu_{Ci} + \mu_B$, where μ_{Ci} is the actual mean rate expected number of cosmic rays in that season. Find:

(a) Best estimate and standard (1-sigma) uncertainty for μ_{Ci} for each season.

(b) Best estimate for overall mean rate number μ_C of the cosmic rays averaged over all seasons.

(c) Deviation $\Delta\mu_{Ci}$ of the mean rate count in each 16-season bin from the yearly mean, with uncertainty.

Solution

(a) Since these counts are relatively large, the uncertainty in the Poisson counts can simply be given by the square root of the counts. So taking into account the fact that $\mu_i = \mu_{Ci} + \mu_B$, we see the following:

- **Spring:** $\mu_{C1} = 771 - 51.4 =$, so then $\Delta\mu_{C1} = 719.6\sqrt{\mu_{C1}} = 26.8$
- **Summer:** $\mu_{C2} = 762 - 51.4 =$, so then $\Delta\mu_{C2} = 710.4\sqrt{\mu_{C2}} = 26.66$
- **Fall:** $\mu_{C3} = 686 - 51.4 =$, so then $\Delta\mu_{C3} = 634.6\sqrt{\mu_{C3}} = 25.19$
- **Winter:** $\mu_{C4} = 802 - 51.4 =$, so then $\Delta\mu_{C4} = 750.6\sqrt{\mu_{C4}} = 27.4$

(b) For to find the value, we simply take the arithmetic mean over the four seasons. We see

$$\mu_C = \frac{771 + 762 + 686 + 802}{4} = 755$$

and the uncertainty in this mean is given by

$$\sigma_{\mu_C} = \sqrt{\frac{(771-755)^2 + (762-755)^2 + (686-755)^2 + (802-755)^2}{3}} = 50.2$$

So then we estimate our mean count to be $\mu_C = 755 \pm 50.2$ counts.

(c) There is only one measurement for each season, so our deviation is merely the difference of the two values and the uncertainty would be given by adding the uncertainties in quadrature.

Problem 3

The formula for double weighing on a balance is $W = p + \frac{r_1 - r_2}{2s}$, in which p is the sum of the weights used, $r1$ and $r2$ are the pointer readings when object is on left and right pans, respectively, and s is the sensibility of the balance [in pointer units per weight unit]. For ten weighings of the same object, the values of $r_1 - r_2$ were as follows: 0.96, 0.93, 1.08, 0.95, 0.99, 1.12, 1.02, 1.05, 0.92, 1.10. The factor $\frac{1}{2s}$ for the load used was 0.0002753 [grams per pointer unit]. Find the probable error of one weighing, and of the mean of the ten weighings. (Does p need to be given for this purpose?)

Solution

Since these are random samples, we will assume that the distribution is approximately Gaussian. So then the probable uncertainty of a single measurement is given as the standard deviation. We have symbolically

$$\sigma_W = \sqrt{\frac{1}{9}\sum_{i=1}^{10}\left(\left(p + \frac{1}{2s}x_i\right) - \mu_W\right)^2}$$

Where μ is given as the mean or

$$\mu_W = \frac{1}{10}\sum_{i=1}^{10}\left(p + \frac{1}{2s}x_i\right) = p + \frac{1}{20s}\sum_i x_i$$

So then

$$\sigma_W = \sqrt{\frac{1}{9}\sum_{i=1}^{10}\left(\left(p + \frac{1}{2s}x_i\right) - \left(p + \frac{1}{20s}x_i\right)\right)^2} = \sqrt{\frac{1}{9}\sum_{i=1}^{10}\left(\frac{9}{20s}x_i\right)^2}$$

So then numerically we have

$$\begin{aligned}\sigma_W &= \frac{1}{2s}\frac{3}{10}\sqrt{\sum_{i=1}^{10} x_i^2} \\ &= \frac{3}{10}(.002753)\sqrt{0.96^2 + 0.93^2 + 1.08^2 + 0.95^2 + 0.99^2 + 1.12^2 + 1.02^2 + 1.05^2 + 0.92^2 + 1.10^2} = 0.00265\end{aligned}$$

And the expected error of the average will be this number divided by the square root of the number of measurements, so we have

$$\sigma_{\mu_W} = \frac{\sigma_W}{\sqrt{10}} = 0.00008377$$

We do not need p for either of these calculations as it always cancels out.

Problem 4

Problem 4.28 from [Taylor]

	1	2	3	4	5
l	51.2	59.7	68.2	79.7	88.3
T	1.448	1.566	1.669	1.804	1.896
g	964.03	961.06	966.57	966.82	969.71

Solution

(a) For our values, we see The mean of the 5 measurements of g is $\mu_g = 965.638$. So then the standard deviation is given by

$$\sigma = \sqrt{\frac{(965.638-964.03)^2+(965.638-961.06)^2+(965.638-966.57)^2+(965.638-966.82)^2+(965.638-969.71)^2}{4}} =$$

So then the SDOM is given by

$$\sigma_\mu = \frac{\sigma}{\sqrt{5}} = \boxed{1.456}$$

(b) If the accepted value is 979.6, then the difference is 14.038 which is indeed almost 10 times more than our error estimate.

(c) For the error value to just include the accepted value, we first note that the function used has only one systematic uncertainty, so the uncertainty in l will be the uncertainty in g. So then our absolute uncertainty must be 14.038 and our fractional uncertainty must be $14.038/965.638 = .0145$ which is indeed around 1.5%.

(d) For our new values, we see The mean of the 5 measurements of g is $\mu_g = 970.26$. So then the standard

	1	2	3	4	5
l	53.2	61.7	70.2	71.7	90.3
T	1.448	1.566	1.669	1.804	1.896
g	1001.69	993.256	994.91	869.77	991.69

deviation is given by

$$\sigma = \sqrt{\frac{(970.26-964.03)^2+(970.26-961.06)^2+(970.26-966.57)^2+(970.26-966.82)^2+(970.26-969.71)^2}{4}} = 2.64$$

So then the SDOM is given by

$$\sigma_\mu = \frac{\sigma}{\sqrt{5}} = \boxed{1.18}$$