Please enter your name and uID below.

Name: Devin White

uID: u0775759

Collaborators, if any, and how you collaborated:

Submission notes

- Due at 11:59 pm (midnight) on Thursday, Dec 8th.
- Solutions must be typeset using one of the template files. For each problem, your answer must fit in the space provided (e.g. not spill onto the next page) *without* space-saving tricks like font/margin/line spacing changes.
- Upload a PDF version of your completed problem set to Gradescope.
- Teaching staff reserve the right to request original source/tex files during the grading process, so please retain these until an assignment has been returned.
- Please remember that for this problem set, you are allowed to collaborate in detail with your peers, as long as you cite them. However, you must write up your own solution, alone, from memory. If you do collaborate with other students in this way, you must identify the students and describe the nature of the collaboration. You are not allowed to create a group solution, and all work that you hand in must be written in your own words. Do not base your solution on any other written solution, regardless of the source.

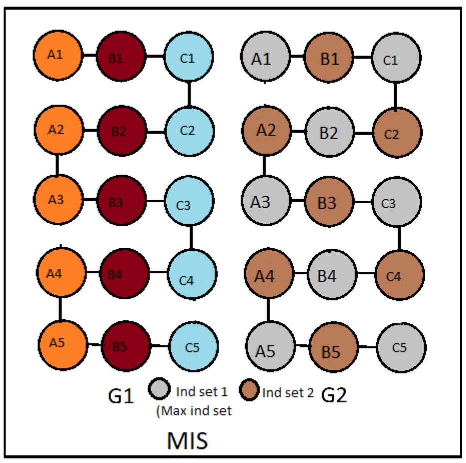
2. (Course Policies)
Here I will argue that "Course Policies" cannot possibly be solved with an algorithm in polynomial time, and is in fact, an NP hard problem. In order to show that this is in fact the case, I will use a reduction from a known NP hard problem mentioned in the textbook, Max-independent-set, or MIS, which is described as *"an independent set that is not a subset of any other independent set. In other words, there is no vertex outside the independent set that may join it because it is maximal with respect to the independent set property"*. Similarly, a solution to to "course policies" is one in which each student has at least one of their "strongly favor" policies implemented, or "strongly opposed" policies not implemented. In order to prove Course policies is NP hard, the following steps will be performed as is typical for an NP reduction problem:

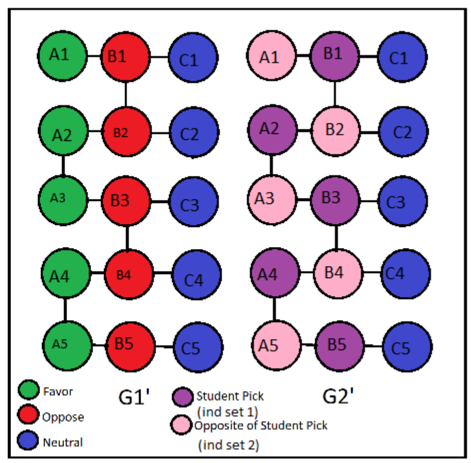
(a) Show a certificate of each problem

(b) transform MIS into course policies

(c) → Show that if Max-independent-set is a YES-instance then an instance of "Course policies" is a YES instance

(d) ← Show that a YES-instance of "Course policies" is a YES instance of Max-independent-set





Above are two certificates, one for MIS on the left "Course policies" on the right,where. For each, the certificate consists of a graph, G, and a series of vertices and edges, and a YES instance consists of any case where there is a MIS.
we can see that there are two graphs, G1 and G2 for MIS, and G1' and G2' for "Course policies". Where G1 and G1' represent a more "generalized graph" and G2 and G2' show graphs with independent sets highlighted(G2' will be used later...) A solution to the Max independent set problem consists of a set of vertices such that no two vertices in G are adjacent.
Taking vertices of the Max independent set of G1, $G1 = \{A_1, B_1, C_1......A_n, B_n, C_n\}$ we can get our max independent set, shown in G2, which consists of the vertices shown in Grey. we can transform this into "course policies", on the right, with similar vertices $G'1 = \{A_1, B_1, C_1......A_n, B_n, C_n\}$, and similarly, a "possible" MIS for an arbitrary selection of student choices represented in a graph in G2'.
In order to truly prove that the transformation from MIS to "Course policies" is valid, there must first be a few additional verifications. Firstly, whether there is a policy on the right in G1' or G2' which satisfies the requirement in which a policy the student favors is implemented, or disfavors is not implemented, as well as G1 and G1' having the same Max independent set. Or More specifically, some arbitrary G and its transformation G' have Max independent sets. If those 2 requirements are satisfied, it can be said that MIS can be reduced to course policies, and vice versa, and therefore Course Policies is NP-hard
In order to perform the transformation from Max Independent set to course policies, we must

follow the steps below. This will be done using the same graphs as above(G1' and G2') and the choices it has made for ease and additional clarity.

- A = Green = Strongly favor policy
- B = Red = Strongly oppose Policy
- C = Blue = Neutral on policy

$\rightarrow$
Since a solution to MIS is a set of vertices in which no two are adjacent, we can transform this into "Course policies" by creating a graph of students where the given choices of "strongly favor", "strongly oppose" and "neutral" are the set of vertices. We can think of each student's responses as a subgraph part of a larger whole in G1', if the ith student chooses "strongly oppose" or "strongly favor" we color in the ith students choice for the ith policy purple, while coloring the choice which is the opposite of their choice pink. Since "Neutral" has no effect on whether or not a given student is happy(only affecting what can be picked for other students), we can leave blue alone, essentially ignoring the Blue vertices/"Neutral on policy"/C, which will be a general trend in the graph. For example, if the Student Strongly oppose for policy i, then for row i Ai = pink, Bi = purple, and Ci remains the same. if the student chose Strongly favor for policy i, then Ai=purple, Bi = pink, Ci remains the same, and so on.

(a) The next step is to change the each edge of the graph connecting C's, E.G $C_i \rightarrow C_i + 1$ to the edges $B_i \rightarrow B_i + 1$

(b) Next, use some method to find the Maximum independent set of the graph

(c) Repeat the above for each subgraph in G until all subgraphs are completed. Essentially, it is possible to choose a subset S of k vertices corresponding to "Favor" and "Not favor" in course policies, such that no two vertices are connected directly, as one cannot favor, not favor, and be neutral on the same policy. Thus "Course policies can make an independent set"
$\leftarrow$

(d) It is assumed that it is possible to make all students happy, since each student can strongly favor, or strongly oppose any of the i course policies, with neutral not affecting the student's own happiness in general, The combination of students responses should form some series of independent sets in G'.
After Step C, all the created sub-graphs in our student response graph can be compared, and we check whether or not the intersection of all the Maximum independent sets has at minimum one vertex, or a size of 1. This is our indicator of whether a policy that the student strongly favors is implemented or not, or whether policies they disfavor are not implemented, and therefore will indicate whether or not a given student is "happy"
If it is possible to "make all students happy" by implementing "Strongly favor"(A) or not implementing "Strongly oppose"(B) on any policy such that all students match on at least one of these same policies(either every student opposes or favors a given policy), then it can be concluded that the graph G' in course policies has a Maximum independent set consisting of at minimum one vertex in all $s_i \epsilon G$ intersections with this policy.

(e) If there is not at minimum one vertex at all $s_i \epsilon G$ intersections, then there is not a maximum independent set and all students for this particular course can not be made happy, and there is no way to implement a policy or not implement a policy which will make all students happy.

Since Course policies and MIS can be transformed into one another, and a solution to Course policies matches a solution to MIS, then we can conclude that "Course policies" is at minimum NP-Hard