

From S4 to Mamba: A Comprehensive Survey on Structured State Space Models

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Recent advancements in sequence modeling have led to the emergence of Structured State Space Models (SSMs) as an efficient alternative to Recurrent Neural Networks (RNNs) and Transformers, addressing challenges in long-range dependency modeling and computational efficiency. While RNNs suffer from vanishing gradients and sequential inefficiencies, and Transformers face quadratic complexity, SSMs leverage structured recurrence and state-space representations to achieve superior long-sequence processing with linear or near-linear complexity. This survey provides a comprehensive review of SSMs, tracing their evolution from the foundational S4 model to its successors like Mamba, Simplified Structured State Space Sequence Model (S5), and Jamba, highlighting their improvements in computational efficiency, memory optimization, and inference speed. By comparing SSMs with traditional sequence models across domains such as natural language processing (NLP), speech recognition, vision, and time-series forecasting, we demonstrate their advantages in handling long-range dependencies while reducing computational overhead. Despite their potential, challenges remain in areas such as training optimization, hybrid modeling, and interpretability. This survey serves as a structured guide for researchers and practitioners, detailing the advancements, trade-offs, and future directions of SSM-based architectures in AI and deep learning.

CCS Concepts: • Computing methodologies → Machine learning; Deep learning theory; Structured State Space Models (SSMs); Sequence modeling; Computational efficiency; Mamba and structured sequence models; Efficient transformer alternatives.

Additional Key Words and Phrases: Structured State Space Sequence Models, Mamba, Jamba, SSM

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1 Introduction

Traditional sequence modeling architectures, such as Recurrent Neural Networks (RNNs) and Transformers, have demonstrated significant limitations in handling long-range dependencies,

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particularly in domains such as natural language processing (NLP), speech processing, vision, and time-series forecasting. RNNs suffer from the vanishing and exploding gradient problem, limiting their ability to retain information over extended sequences, and their inherently sequential nature restricts parallelization, making them inefficient for long-range tasks [1]. While Long Short-Term Memory (LSTM) networks alleviate some of these issues through gating mechanisms, they still introduce additional complexity and computational overhead. Transformers, on the other hand, rely on self-attention mechanisms that capture global dependencies but scale poorly due to their quadratic time and space complexity ($O(N^2)$) [1]. Moreover, Transformers face challenges with fixed-length positional embeddings and excessive memory consumption, making them inefficient for processing extremely long sequences.

Beyond these challenges, RNNs struggle with slow training times due to their sequential processing, while Transformers become impractical for extremely long inputs, such as document-level NLP tasks or long-duration speech modeling, due to their memory-intensive pairwise attention computations [2]. These issues have motivated the development of Structured State Space Models (SSMs), which offer a fundamentally different approach to sequence modeling. Unlike RNNs, SSMs avoid vanishing gradients and sequential processing constraints by employing latent state-space representations that can maintain long-term dependencies more efficiently [2]. Compared to Transformers, SSMs mitigate the quadratic scaling problem by replacing attention mechanisms with state-space formulations that enable linear or near-linear complexity ($O(N + L)$) in sequence length, making them more efficient for long-sequence processing.

SSMs have recently emerged as an efficient alternative, overcoming many of these limitations by leveraging structured recurrence and continuous-time representations to capture long-range dependencies without suffering from memory decay [3]. Unlike RNNs, SSMs employ mathematical reparameterization techniques that stabilize long-term memory retention, avoiding exponential memory decay. Compared to Transformers, SSMs significantly reduce computational complexity, achieving near $O(N \log N)$ scaling by utilizing parallel scan algorithms and Fast Fourier Transform methods, making them well-suited for long-sequence modeling [3]. Notably, the Structured State Space Sequence Model (S4) introduces an optimized parameterization that substantially reduces memory overhead while preserving the advantages of state-space modeling [2]. Additionally, hybrid architectures such as State sPace AugmenteD TransformEr (SPADE) incorporate SSMs into Transformer layers to enhance global information capture while maintaining local contextual refinement through efficient attention mechanisms [4].

Empirical studies demonstrate that SSM-based models achieve state-of-the-art results on long-range dependency benchmarks, such as the Long Range Arena (LRA), outperforming both RNNs and Transformers while providing up to $60\times$ faster inference for sequence modeling tasks [2]. Beyond these established domains, recent research has shown that SSM-based architectures, such as MambaNet, can also be successfully applied to structured tabular data, particularly in transportation safety. Studies on crash severity prediction for high-risk road users, including young motorcyclists [5] and child bicyclists [6], have demonstrated that MambaNet outperforms traditional models in classifying injury severity while maintaining computational efficiency. These findings suggest that SSMs hold promise for structured data modeling applications that require real-time decision-making and predictive analytics, further expanding their impact beyond sequence learning.

Selective SSMs further optimize memory usage through hierarchical gating mechanisms, ensuring that only relevant information is retained while minimizing storage requirements [7]. These advancements have demonstrated superior performance in long-sequence modeling across multiple domains, including NLP, speech recognition, vision, and time-series forecasting, where efficient handling of long-range dependencies is critical.

1.1 Objective of the Survey

SSMs have emerged as a promising alternative to Transformer-based architectures, offering efficient sequence modeling with linear computational complexity. The evolution of SSMs, beginning with the foundational S4 model, has led to significant advancements, including Mamba, S5, and Jamba, which incorporate selective mechanisms to improve performance across various domains. Early SSMs, such as S4, were primarily designed for long-range sequence modeling but had limitations in expressive power, particularly in content-based reasoning tasks [8]. The introduction of Mamba refined these models by integrating a selective state-space mechanism that allows input-dependent processing, significantly enhancing expressivity and accuracy [8]. Further developments, such as Mamba-2, have optimized efficiency, making SSMs competitive with Transformers while being 2-8 \times faster than their predecessors [9].

SSMs offer a computational advantage over Transformers, which scale quadratically in sequence length, making them expensive for long-form processing. Instead, models like Mamba and S5 achieve linear scaling, improving inference speed and reducing memory requirements while maintaining comparable accuracy [8, 10]. S5 introduces a multi-input, multi-output (MIMO) SSM, enhancing parallelism and efficiency over S4, allowing it to match Transformer performance while using fewer computational resources [10]. This advantage has led to SSMs being increasingly adopted in various real-world applications, such as speech processing [10, 11], medical imaging [12], and NLP [9]. While Transformers still dominate NLP tasks, hybrid models like Jamba, which combine Transformer layers with Mamba components, have demonstrated superior efficiency and scalability, particularly in long-context applications [13].

Despite these advancements, challenges remain. Training SSMs efficiently at scale is still an open problem compared to Transformer-based architectures, which benefit from a well-established optimization ecosystem [9]. Moreover, hybrid architectures like Jamba have shown that a combination of Transformers and SSMs can lead to improvements in throughput and memory efficiency, reducing Key-Value (KV) cache memory by an order of magnitude compared to pure Transformer models [13]. SSMs are also being explored for multimodal learning, where their efficiency could complement Transformer-based representations [12]. Future research directions include enhancing hardware optimization, improving selective state-space mechanisms, and expanding SSMs' applicability in multimodal and real-time processing tasks [14].

While these advancements highlight the growing potential of SSMs as a scalable and efficient alternative to Transformers, a systematic analysis of their evolution, capabilities, and limitations is essential to fully understand their impact on modern AI architectures. To this end, this survey aims to provide a structured review of SSMs, their advancements, and their comparative strengths and weaknesses in sequence modeling.

Recent advancements in sequence modeling have shifted the focus from RNNs and Transformers to SSMs, which offer improved computational efficiency, scalability, and superior handling of long-range dependencies. As SSMs continue to evolve, models such as S4, Mamba, S5, and Jamba have demonstrated their potential to rival or surpass Transformers in various domains, from NLP and speech recognition to vision and time-series forecasting. However, while these models offer notable advantages in efficiency and scalability, challenges remain in optimizing their training dynamics, enhancing interpretability, and exploring hybrid architectures that integrate SSM and Transformer components.

This survey aims to provide a comprehensive overview of SSMs, tracing their evolution from S4 to Mamba, S5, and Jamba, and evaluating their comparative strengths and limitations relative to Transformer-based architectures. The survey is structured around three key objectives:

- (1) **Comprehensive Review of SSMs** – Analyzing key architectures (S4, Mamba, S5, Jamba), their design principles, and how they improve upon previous models.
- (2) **Comparison with Transformers** – Evaluating trade-offs in memory efficiency, computational cost, and performance on long-sequence tasks, highlighting scenarios where SSMs provide advantages over Transformers.
- (3) **Applications & Future Directions** – Exploring SSMs' impact on NLP, vision, speech processing, and time-series forecasting, while identifying open research challenges in scalability, interpretability, and the development of hybrid SSM-Transformer models.

This survey provides a structured perspective on the evolution, capabilities, and future potential of SSMs, bridging the gap between theoretical advancements and real-world applications in AI research and industry.

2 Fundamentals of State Space Models (SSMs)

2.1 Mathematical Foundation of SSMs

2.1.1 Introduction: SSMs constitute a powerful mathematical approach broadly employed in diverse fields, including control theory, systems engineering, economics, and artificial intelligence, to describe complex dynamical systems. At their core, SSMs characterize the evolution of system states over time through first-order differential or difference equations, explicitly capturing the influence of external inputs and their relationship with observable outputs [15]. Distinguished from classical autoregressive models, SSMs offer substantial advantages, such as the intuitive handling of multivariate systems, straightforward interpretation of internal dynamics, and seamless transitions between continuous and discrete-time domains [2]. Recent advancements have expanded the computational efficiency and interpretability of SSMs by demonstrating equivalence among their continuous, recurrent, and convolutional representations, further solidifying their relevance in modern sequence modeling frameworks [2, 14].

2.1.2 General Formulation: Classical SSMs are mathematically defined by equations that describe the evolution of hidden states over time and the generation of observable outputs, as introduced by Kalman [16]. These models serve as a foundational framework for modeling dynamic systems across various scientific and engineering domains, such as control theory, signal processing, and economics. Classical SSMs encapsulate how a system's internal states evolve in response to external inputs and how these states influence observable outputs [15]. A notable strength of classical SSMs lies in their ability to explicitly represent both internal dynamics and external interactions using first-order differential equations, making them indispensable for capturing complex behaviors in dynamic systems. Figure 1 illustrates the fundamental structure of SSMs, showcasing three key perspectives: continuous-time state-space dynamics, long-range dependencies, and discrete-time representations.

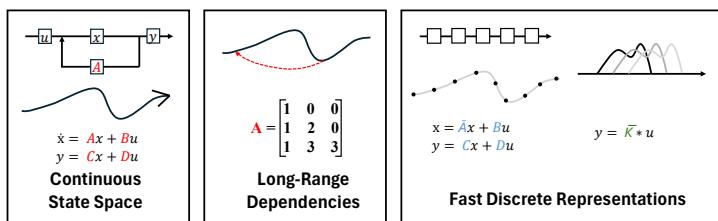


Fig. 1. Conceptual Representation of State Space Models, adapted from [2]

In the **left panel**, a continuous-time SSM is depicted, where an input $u(t)$ influences the latent state $x(t)$ via the system matrix A , with matrices A, B, C , and D defining state evolution, input interaction, and output mapping. The **middle panel** illustrates how specialized A matrices enable deep SSMs to capture long-range dependencies, ensuring stable memory retention over long sequences—an improvement over traditional recurrent models. The **right panel** demonstrates how structured parameterization allows SSMs to transition into fast discrete representations, efficiently leveraging either a recurrent or convolutional framework. This enables scalability in deep learning applications like speech recognition, time-series forecasting, and NLP. .

Recent advancements in deep learning have led to the development of SSMs, a specialized class of SSMs designed for efficient sequence modeling. Unlike classical SSMs, which focus on modeling physical and control systems, Structured SSMs such as S4, Mamba, and S5 leverage structured recurrence and diagonalized state-space representations to capture long-range dependencies in sequential data. These models have demonstrated superior scalability in applications such as natural language processing, speech recognition, and time-series forecasting. Throughout this paper, we refer to "Structured SSMs" when discussing their role in deep learning, differentiating them from classical state-space models."

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t) \quad (1)$$

Equation (1) represents the State-Space Representation of a dynamical system.

Here, $x(t) \in \mathbb{R}^n$ denotes the state vector encapsulating the system's internal states at time t , while $u(t) \in \mathbb{R}^m$ represents the input or control vector externally influencing the system. The output vector $y(t) \in \mathbb{R}^p$ describes observable quantities [14]. The matrices A, B, C, D are constant and define system dynamics, input influence, state-to-output mapping, and direct input-to-output relationships, respectively. The structure and dimensions of these matrices depend on the complexity and nature of the system being modeled.

where:

- $x(t) \in \mathbb{R}^n$ is the state vector representing the internal state of the system=.
- $u(t) \in \mathbb{R}^m$ is the input (control) vector influencing the state evolution=.
- $y(t) \in \mathbb{R}^p$ is the output (observation) vector.
- $A \in \mathbb{R}^{n \times n}$ is the state transition matrix, governing the dynamics of the system.
- $B \in \mathbb{R}^{n \times m}$ is the control matrix, mapping input effects to the state.
- $C \in \mathbb{R}^{p \times n}$ is the observation matrix, defining how the internal state maps to the output.
- $D \in \mathbb{R}^{p \times m}$ is the feedthrough matrix, which directly relates the input to the output.

2.1.3 Discrete-Time Representation of SSMs: SSMs originate from control theory, where they are traditionally formulated in continuous time using differential equations. However, for practical digital computation, deep learning applications, and real-time simulations, these continuous representations must be discretized. The process of converting continuous-time equations into discrete-time involves numerical discretization techniques such as the bilinear transform, Euler method, and Z-transform. Among these, the bilinear transform is often preferred due to its ability to preserve system stability while maintaining computational efficiency [17, 18].

A discrete-time SSM is mathematically defined as:

$$x_{t+1} = A_d x_t + B_d u_t, \quad y_t = C_d x_t + D_d u_t \quad (2)$$

where A_d, B_d, C_d, D_d are the discretized counterparts of their continuous-time parameters. These parameters govern the evolution of the system's state over discrete time steps, enabling efficient

sequential modeling of time-series data [2, 19]. The transformation from continuous to discrete form is commonly performed using matrix exponentials, formulated as:

$$A_d = e^{A\Delta t}, \quad B_d = \int_0^{\Delta t} e^{A\tau} B d\tau \quad (3)$$

where Δt represents the sampling interval used for discretization. This method ensures that the system's temporal dynamics are accurately captured, allowing for efficient computation and numerical stability compared to simpler approximation methods [10].

An alternative approach for discretization is using the bilinear transformation, which improves system stability while maintaining accuracy. The discrete transition matrix \bar{A} is computed as:

$$\bar{A} = (I - \alpha\Delta t A)^{-1}(I + (1 - \alpha)\Delta t A) \quad (4)$$

where:

- I is the identity matrix,
- A is the continuous-time state transition matrix,
- α is a weighting parameter that determines the approximation method,
- Δt is the discretization step size.

This transformation ensures that the discrete-time system remains numerically stable while effectively approximating the continuous dynamics of the model. The bilinear transformation is particularly useful in machine learning applications, where preserving long-range dependencies and computational efficiency is crucial for model scalability.

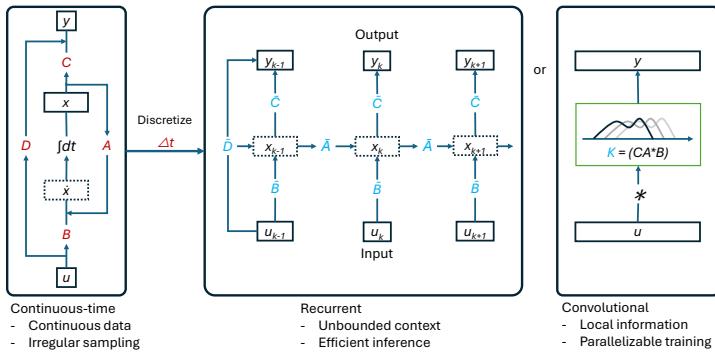


Fig. 2. Three Views of Linear State Space Layer, adapted from [19].

As illustrated in Figure 2, the discrete-time SSM can be implemented in either a recurrent or convolutional framework. In the recurrent formulation, the model updates its hidden state sequentially at each time step, following a structured recurrence similar to RNNs. This allows for effective memory retention and autoregressive sequence modeling, making it well-suited for applications like speech recognition, reinforcement learning, and finance [2, 19]. The structured recurrence also improves gradient propagation, addressing the vanishing gradient issue found in traditional RNNs.

In contrast, the convolutional representation of discrete-time SSMs offers parallelized training and inference by applying a discretized convolutional kernel across the entire input sequence. This representation significantly reduces computational bottlenecks, making it ideal for large-scale sequence modeling tasks such as NLP, genomics, and time-series forecasting [9]. Unlike standard

convolutional neural networks (CNNs), which operate with fixed kernel sizes, convolutional SSMs use learned state-space parameterizations, allowing for more flexible and efficient long-sequence modeling.

The dual formulation of discrete-time SSMs—recurrent and convolutional—demonstrates their versatility and adaptability to different computational constraints. Recurrent SSMs are more suitable for streaming and autoregressive tasks, where maintaining sequential order is essential. Convolutional SSMs, on the other hand, excel in batch processing and large-scale training, benefiting from depthwise computation and GPU acceleration. Recent advancements, such as Mamba and S4, have introduced structured state-space optimizations, enabling SSMs to achieve state-of-the-art performance while maintaining linear computational complexity.

As research continues to evolve, the discrete-time representation of SSMs is proving to be a powerful alternative to Transformers for many real-world applications. By leveraging structured state-space dynamics, these models can efficiently handle long sequences, bridging the gap between differential equation-based models, RNNs, and convolutional architectures [19].

2.1.4 Convolutional Formulation of SSMs: The convolutional perspective of SSMs transforms the input sequence through a structured, linear state transition, allowing for parallel computation over long sequences. As depicted in the provided image, SSMs can be rewritten in a discretized convolutional form, where the latent state evolution is governed by a transition matrix A and input transformation matrix B [19]. This results in a system response that can be computed efficiently using the following discrete-time formulation:

$$h_t = Ah_{t-1} + Bu_t, \quad y_t = Ch_t + Du_t \quad (5)$$

where:

- h_t represents the hidden state,
- u_t is the input at time step t ,
- A, B, C, D are learned parameters that define state transitions.

In the convolutional interpretation, the state-space model behaves as an implicit filter, where the system response is equivalent to convolving the input sequence with a learned kernel:

$$K = (CA^*B) \quad (6)$$

This formulation enables efficient parallelization, similar to CNNs, making SSMs highly scalable for long-sequence processing [9, 19]. Unlike standard CNNs, where the kernel size is fixed, SSMs implicitly parameterize kernels through structured matrices, offering a more flexible and learnable representation.

Convolutional SSMs offer several advantages over traditional recurrent architectures. One key benefit is parallel computation: unlike RNNs, which require sequential updates, convolutional SSMs can process entire sequences in parallel, significantly reducing training time, making them efficient for handling large-scale datasets. Moreover, another advantage is their ability to model long-range dependencies effectively. The implicit kernel representation enables better memory retention over long sequences, making these models well-suited for applications such as genomics, video processing, and speech recognition. By leveraging structured matrices, SSMs can learn more flexible representations, surpassing the limitations of traditional CNNs with fixed kernel sizes. Finally, hardware efficiency is a crucial factor. Convolutional SSMs utilize structured matrix multiplications, allowing them to efficiently exploit modern hardware accelerators such as GPUs and TPUs. This computational advantage makes them ideal for high-performance machine learning applications where both scalability and efficiency are critical.

2.1.5 Recurrent Formulation of SSMs: In contrast to the convolutional approach, the recurrent perspective of SSMs treats the hidden state as an evolving memory, similar to RNNs but with structured linear recurrence. This formulation explicitly updates the state at each time step, preserving sequential information flow without the need for explicit convolutional filtering.

The recurrent form of an SSM follows:

$$h_t = Ah_{t-1} + Bu_t \quad y_t = Ch_t \quad (7)$$

where the state h_t is updated at each time step rather than applying a global convolution over the input. This design shares similarities with traditional RNNs but introduces structured state transitions that mitigate issues such as vanishing gradients [1, 19].

A key difference between SSM recurrence and classical RNNs is that SSMs employ structured matrices for the transition dynamics. Unlike vanilla RNNs, where the hidden state evolution is unconstrained, SSMs impose a structured transformation on A , allowing for more stable long-range dependency modeling [9, 19].

Recurrent SSMs offer several advantages over conventional sequence models. First, they reduce computational complexity compared to Transformers, which scale quadratically with sequence length. SSMs maintain a linear complexity of $O(T)$, making them efficient for long sequences. Second, their use of structured state-space matrices ensures stable training dynamics, mitigating issues such as exploding and vanishing gradients that commonly affect standard RNNs. Finally, SSM recurrence enables efficient sequential processing while preserving long-range dependencies, leading to superior performance in tasks such as time-series prediction and reinforcement learning.

2.1.6 RNNs: RNNs have a rich history dating back to the early work of Elman [20], who introduced the foundational *Elman network* as a simple recurrent model designed for sequence processing. RNNs were developed to handle sequential data by employing recurrent connections that allow the hidden state to retain information from previous time steps, effectively capturing temporal dependencies [1]. Early RNN models, including *Jordan networks* [21] and *Elman networks* [20], established the core principle of using hidden states to model sequential dependencies [19]. However, traditional RNNs struggled with vanishing gradients when learning long-range dependencies, leading to the development of architectures such as *LSTM* by Hochreiter & Schmidhuber [22]. Mathematically, the hidden state h_t at time step t is updated based on the current input x_t and the previous hidden state h_{t-1} , following the equation:

$$h_t = \sigma(W_h h_{t-1} + W_x x_t + b) \quad (8)$$

where W_h , W_x , and b are trainable parameters, and σ is typically a nonlinear activation function such as the hyperbolic tangent (*tanh*). Despite their theoretical strength in capturing temporal dependencies, RNNs suffered from practical issues like the *vanishing* and *exploding gradient* problems, which severely restricted their effectiveness in modeling long sequences [22]. These limitations necessitated improvements, eventually leading to the introduction of gated architectures such as LSTMs and Gated Recurrent Units (GRUs) [23].

2.1.7 CNNs: CNNs, first introduced by LeCun et al. [24], were initially designed for computer vision tasks but later adapted for sequential data processing. CNNs leverage convolutional layers to efficiently capture local patterns by applying *kernels* (*filters*) across input data. In sequence modeling, a *one-dimensional convolution operation* is commonly defined as:

$$y_i = \sum_{j=0}^{k-1} x_{i+j} w_j + b \quad (9)$$

where x represents the input sequence, w denotes the kernel weights, b is a bias term, and k is the kernel size. CNNs exhibit *high computational efficiency* due to their parallelization capabilities, making them more scalable than RNNs. However, their *limited receptive fields* inherently restrict their ability to model long-range dependencies, requiring deeper architectures or advanced techniques such as *dilated convolutions* to extend their contextual reach [25].

2.1.8 Transformers: The Transformer architecture, introduced by Vaswani et al.[26], revolutionized sequence modeling, particularly in natural language processing, by introducing *self-attention* mechanisms. Self-attention allows each element in a sequence to directly attend to every other element, effectively capturing *global dependencies*. The core mathematical operation of self-attention is defined as:

$$\text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V \quad (10)$$

$$Q = W_q X, \quad K = W_k X, \quad V = W_v X \quad (11)$$

where Q , K , and V are linear transformations of the input, representing *queries*, *keys*, and *values*, respectively, and d_k is the dimensionality of the keys. Transformers mitigate the long-range dependency issues observed in RNNs and CNNs, enabling efficient parallel computation. However, they incur a quadratic computational complexity $O(T^2)$ with respect to sequence length, posing scalability challenges for extremely long sequences [26].

2.1.9 SSMs: SSMs originate from control theory and signal processing [16], but recent advancements have positioned them as highly efficient alternatives to Transformers for sequence modeling, particularly for very long sequences. Unlike Transformers, which have quadratic complexity due to self-attention operations, SSMs provide a structured linear representation of latent state dynamics, significantly reducing computational complexity and enabling efficient parallel computation [2, 14, 19]. A comprehensive mathematical formulation, including continuous and discrete-time representations and discretization techniques, has been detailed previously in Section 2.

2.2 Comparison with Other Sequence Models

A concise comparison of RNNs, Transformers, CNNs, and SSMs in terms of computational complexity, strengths, and weaknesses is provided in Table 1.

RNNs process sequences efficiently with linear complexity $O(T)$, updating hidden states at each step based on prior states and inputs. However, they accumulate unnecessary information over long sequences, leading to inefficiencies and vanishing gradient issues, which hinder their ability to model long-range dependencies effectively [22]. While LSTMs and GRUs introduce gating mechanisms to mitigate these limitations, they still scale poorly with sequence length due to their reliance on storing past states [27].

Transformers [26] address this limitation by using self-attention to model global interactions across sequence elements. However, this flexibility comes at a cost: quadratic complexity $O(T^2)$, making Transformers computationally expensive, particularly for long sequences.

CNNs, originally designed for vision tasks, have been adapted for sequence modeling using 1D convolutions over temporal data [7]. CNNs efficiently capture local patterns in sequences, making them computationally efficient. However, their fixed kernel sizes limit their ability to model long-range dependencies unless large receptive fields or stacked layers are used, which increases computational cost [28].

In contrast, SSMs, including architectures such as S4 and Mamba, offer an efficient alternative with linear complexity $O(T)$ while maintaining strong long-range dependency modeling. Unlike

Transformers, which rely on self-attention, SSMs utilize structured state transitions to model dependencies more efficiently [2, 19]. Moreover, Selective SSMs dynamically update their hidden states, retaining only relevant information rather than storing full past states like RNNs. This enables efficient memory compression, reducing computational overhead while maintaining scalability [29].

Unlike CNNs, which rely on fixed receptive fields and struggle with long-range dependencies, SSMs do not require predefined kernel sizes and can adaptively model dependencies across varying timescales [30]. Their structured approach ensures scalability and efficiency, making them a strong contender for sequence modeling tasks that demand both long-term memory and computational efficiency.

By balancing memory retention, efficiency, and scalability, Selective SSMs provide a more flexible and computationally efficient framework than RNNs, CNNs, and Transformers. Their ability to selectively retain critical information while discarding irrelevant data makes them particularly effective for long-sequence tasks such as NLP, time-series forecasting, and signal processing.

Table 1. Comparison of Sequence Models

Model Source	Complexity	Strengths	Weaknesses
RNNs [22, 27]	$O(T)$	Sequential processing, low memory usage.	Vanishing gradients, limited long-range memory.
CNNs [7, 28]	$O(T)$	Local feature extraction, parallelizable.	Fixed receptive fields, limited long-range dependencies.
Transformers [26]	$O(T^2)$	Strong global attention, effective in NLP and vision tasks.	Expensive for long sequences, high memory and computational costs.
SSMs [2, 19, 29, 30]	$O(T)$	Long-range dependencies, efficient memory compression.	Limited in-context learning, requires specific initialization for stability.

3 The Foundation of Modern SSMs

3.1 Introduction to S4

The advancement of deep learning has led to remarkable progress in sequence modeling tasks such as NLP, time series forecasting, and speech recognition. However, conventional models like RNNs and transformers face significant challenges when handling extremely long sequences. RNNs struggle with vanishing and exploding gradients, making them inefficient for long-range dependencies [22, 31]. Meanwhile, transformers, which rely on self-attention mechanisms, suffer from quadratic complexity $O(N^2)$ for sequence length, limiting their scalability for long-sequence modeling [26]. To address these limitations, structured SSMs have gained attention as an alternative, particularly the S4 model, which provides a novel approach for handling long-range dependencies efficiently [2].

S4 is a structured state-space model that significantly improves upon previous RNN and attention-based methods by leveraging continuous-time state-space formulations [32]. The foundation of S4 lies in classical state-space models, which have been extensively used in control theory and signal processing [16, 33]. These models define system dynamics through differential equations, which are then discretized for computation [34]. While traditional SSMs were impractical for deep learning due to their computational overhead, S4 introduces a structured formulation that allows for fast and scalable computation [35]. This innovation enables S4 to efficiently capture long-range dependencies without suffering from gradient degradation, making it a promising solution for long-sequence modeling tasks.

3.1.1 First Structured SSM Designed for Long-Sequence Modeling: S4 represents a paradigm shift in sequence modeling by introducing a mathematically grounded and computationally efficient alternative to traditional architectures. A key innovation in S4 is the use of the High-Order Polynomial Projection Operator (HiPPO) framework, which provides a mathematically rigorous approach to preserving long-range dependencies in sequential data [35]. HiPPO enables continuous memory retention through an optimal polynomial projection mechanism, addressing the limitations faced by earlier sequence models such as LSTMs [22], GRUs [36], and vanilla RNNs [31].

Figure 3 illustrates the HiPPO framework, which optimally projects past information onto a polynomial basis to maintain a compressed yet expressive memory representation. In this process, a function $f(t)$ is approximated by projecting it onto polynomial bases using a weighting measure $\mu(t)$. The function's history is then represented as a set of evolving coefficients governed by a structured ODE:

$$\frac{d}{dt}c(t) = A(t)c(t) + B(t)f(t).$$

To ensure efficient computation, this continuous ODE is discretized into a recurrence relation:

$$c_{k+1} = A_k c_k + B_k f_k.$$

This approach allows sequence models to retain information across long time horizons efficiently, a key principle that later influenced S4's structured state-space design.

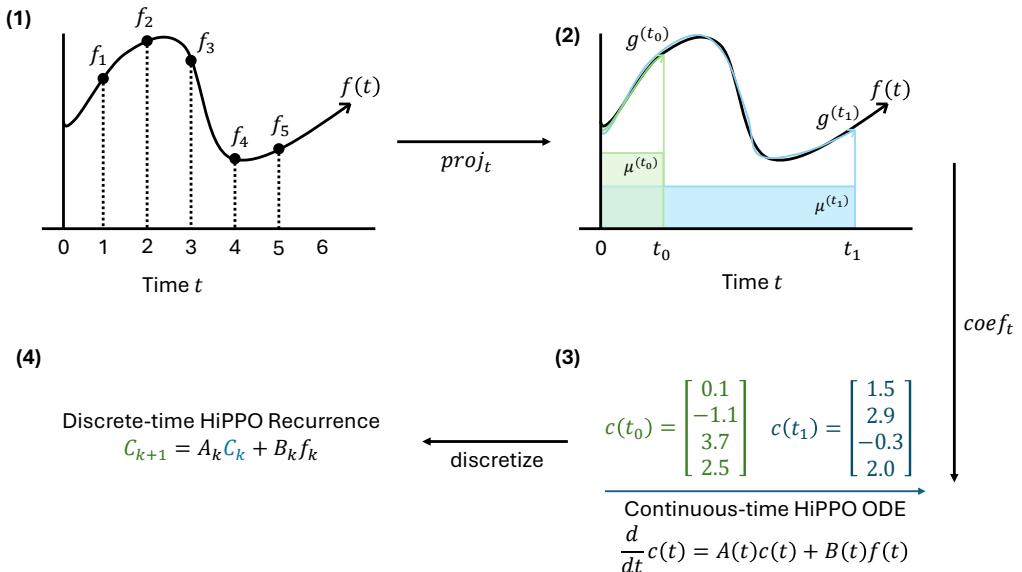


Fig. 3. HiPPO Framework for Online Function Approximation, adapted from [35]

Unlike previous architectures that relied on explicit gating mechanisms or self-attention, S4 leverages structured matrices for efficient long-term information retention [37, 38]. Another breakthrough of S4 is its ability to achieve subquadratic computational complexity while maintaining high expressivity. Transformers, while powerful, require $O(N^2)$ complexity due to self-attention computations, making them infeasible for very long sequences [26, 39]. In contrast, S4 achieves $O(N \log N)$ complexity by leveraging structured state-space computations, significantly reducing

computational overhead [32]. This advantage has positioned S4 as a more scalable alternative to transformers, particularly in time series forecasting, speech recognition, and text processing [40, 41].

S4’s efficiency is also attributed to its use of diagonal plus low-rank parameterization, which allows it to approximate long-sequence dependencies while remaining computationally stable [32]. This contrasts with previous state-space representations that required expensive matrix multiplications, making them impractical for deep learning applications. By introducing structured recurrence, S4 can effectively learn from long sequential data without explicit recurrence mechanisms like those in standard RNNs [42]. One of the most important contributions of S4 is its compatibility with modern hardware accelerators, such as GPUs and TPUs, which enable efficient parallelization [43, 44]. Many earlier sequence models, including attention-based transformers, require significant memory overhead, limiting their feasibility for long-sequence processing [43]. S4, however, benefits from its structured state-space computations, allowing it to handle sequences of tens of thousands of tokens without excessive memory consumption [45, 46].

Beyond efficiency, S4 has demonstrated state-of-the-art performance across various domains. In NLP, it has been applied to long-form text generation, outperforming traditional transformers on tasks requiring extended memory retention [32]. In time series analysis, S4 has shown strong predictive accuracy in forecasting complex temporal patterns, outperforming previous state-of-the-art models such as Temporal Fusion Transformers [41]. Additionally, in biomedical signal processing, S4 has been leveraged for EEG and ECG analysis, demonstrating superior performance in capturing long-range dependencies in physiological signals [47]. Despite its advantages, implementing S4 is not without challenges. One of the primary difficulties is the optimization of structured state-space matrices, which requires specialized initialization techniques to maintain numerical stability [32]. Unlike standard architectures such as transformers, where training dynamics are well-understood, S4 introduces complexities in parameter tuning that necessitate novel training techniques such as spectral normalization [48].

S4’s success has inspired the development of several extensions. Structured State-Space Sequence Model with Simplified Representations (S5) introduces a streamlined version of S4 with fewer hyperparameters, improving ease of implementation [10]. Additionally, hybrid models that combine S4 with self-attention mechanisms have been explored to further enhance their modeling capabilities [49]. These developments indicate a growing interest in structured state-space approaches as a replacement for traditional sequence models.

3.2 Key Innovations in S4

The S4 model introduces several groundbreaking innovations that distinguish it from traditional sequence models such as RNNs, LSTMs, and Transformers. These innovations primarily revolve around structured recurrence, efficient convolutional formulations, and FFT acceleration, all of which contribute to making S4 scalable, efficient, and well-suited for long-sequence modeling tasks [2].

3.2.1 State-space Formulation (Structured Recurrence). One of the most fundamental innovations in S4 is its state-space formulation, which provides a structured alternative to traditional recurrence-based models. Unlike standard RNNs, which rely on hidden states that evolve sequentially over time, S4 models sequences using continuous-time state-space equations, allowing for a more mathematically rigorous representation of long-range dependencies [32]. Traditional SSMs have long been used in control theory, signal processing, and dynamical systems, but their direct application to deep learning has been limited due to their computational inefficiency [16, 33]. S4 overcomes these challenges by introducing a structured representation that enables efficient

parallelization while retaining the expressiveness of SSMs [34, 41]. The core equation governing S4’s structured recurrence is:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (12)$$

$$y(t) = Cx(t) + Du(t) \quad (13)$$

where A , B , C , and D are learnable structured matrices that define the evolution of the internal state $x(t)$ over time [2]. Unlike conventional RNNs that require sequential updates of hidden states, S4 converts these state-space operations into an efficient convolutional representation, allowing for fast parallel computations [19, 32]. The advantage of this structured formulation is that it implicitly maintains long-range dependencies, addressing one of the fundamental weaknesses of both recurrent architectures and self-attention mechanisms [37, 46].

This structured recurrence mechanism is illustrated in Figure 4, which provides an overview of the S4 layer architecture. The model is formulated as a large state-space system (SSM) with state size HN , where each independent SSM operates in parallel, leveraging block-diagonal state, input, and output matrices for efficient sequence modeling. The architecture integrates HiPPO-based initialization to approximate long-range dependencies and employs a Diagonal Plus Low-Rank parameterization to derive an optimized convolutional kernel. This parameterization enables efficient memory retention and fast sequence processing while reducing computational complexity. By structuring the recurrence in this manner, S4 efficiently models long-range dependencies with reduced memory overhead, making it a scalable and expressive alternative to traditional sequence modeling architectures.

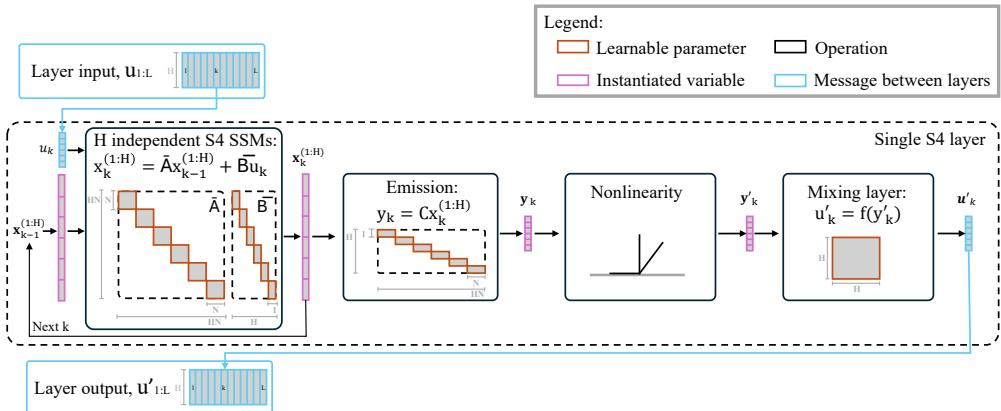


Fig. 4. S4 Layer Structure adapted from [10]

3.2.2 Efficient Convolutions Replacing RNN-style Recurrence. S4 replaces traditional RNN-style recurrence with convolutional operations, allowing it to handle long sequences in a much more efficient manner [2]. While classical recurrent architectures update states sequentially, S4 instead applies a convolutional filter over the input sequence, which effectively replaces recurrence with a learned convolutional response. This shift is significant because convolutions are inherently parallelizable, unlike recurrence, which requires sequential dependencies. In contrast, RNNs update hidden states one step at a time, preventing parallel execution and leading to inefficiencies when

processing long sequences [22]. The convolutional nature of S4 also enables smoother long-range dependencies, allowing the model to capture patterns spanning thousands of tokens without requiring explicit memory gating mechanisms like those used in LSTMs or GRUs [36]. This makes S4 particularly advantageous for tasks such as video understanding [40], protein sequence modeling [50], and large-scale language modeling [32].

3.2.3 FFT Acceleration for Parallelized Computations. Another major innovation in S4 is its use of FFT acceleration, which enables highly efficient sequence processing [2]. By converting state-space operations into the frequency domain using FFT, S4 significantly reduces computational overhead and improves scalability. FFT acceleration allows S4 to compute convolutions in $O(N \log N)$ time, making it dramatically faster than traditional sequence models. This optimization is particularly important in real-time applications, such as speech recognition and time series forecasting, where rapid inference is critical [41]. Furthermore, FFT-based parallelization enables S4 to be highly optimized for GPU and TPU architectures, making it practical for large-scale deployments [43, 44]. This sets S4 apart from traditional RNNs, which suffer from poor hardware utilization due to their inherently sequential nature.

3.3 Strengths & Limitations

While S4 introduces a range of powerful innovations, it also comes with certain challenges and trade-offs. This section discusses both the strengths and limitations of the model.

3.3.1 Strengths of S4. One of the primary advantages of S4 is its ability to process extremely long sequences efficiently, outperforming standard transformers and RNNs in terms of scalability [2]. Its structured recurrence and FFT acceleration enable it to handle sequences of tens of thousands of tokens, making it ideal for applications like DNA sequence modeling [49], weather forecasting [41], and long-context NLP tasks [51]. Unlike RNNs, which suffer from sequential computation bottlenecks, S4 is highly parallelizable, making it suitable for GPU and TPU acceleration [44, 52]. This makes S4 a practical choice for large-scale machine learning applications. S4's structured recurrence provides an implicit memory mechanism that outperforms LSTMs and Transformers in retaining long-range dependencies [19]. This is particularly useful for tasks requiring long-term context understanding, such as speech-to-text processing [53] and financial time series analysis [54].

3.3.2 Limitations of S4. One of the major drawbacks of S4 is its complex initialization process. Training S4 requires specialized parameter tuning to maintain numerical stability, as the structured state-space formulation can be sensitive to improper initialization [32]. Spectral normalization techniques have been explored to mitigate this issue, but further research is needed to simplify S4's optimization dynamics [55]. Unlike attention-based models, where attention weights provide interpretability, S4's state-space formulation lacks an easily interpretable mechanism for analyzing how it processes sequential information [51]. This makes it challenging to understand how the model prioritizes different parts of an input sequence, particularly in applications such as medical decision-making. While S4 is designed for efficiency, it requires carefully optimized implementations to utilize its computational benefits fully [32]. This can be a limitation in environments where high-performance GPUs/TPUs are unavailable.

4 Evolution of S4-Based Models

4.1 Mamba: Optimized Successor to S4

The rapid advancement of sequence modeling has led to the widespread adoption of Transformers, yet their quadratic computational complexity poses significant scalability challenges, particularly for long-sequence tasks. SSMs, such as S4, have emerged as promising alternatives by leveraging

structured state-space dynamics to enhance efficiency in sequence modeling. However, S4's reliance on static parameters and its computational overhead in handling long-range dependencies limit its scalability. Mamba, a novel Selective State Space Model (SSSM), builds upon S4 by introducing dynamic parameterization and a hardware-aware parallel algorithm, enabling linear-time complexity and substantial improvements in computational efficiency and memory utilization [9]. Unlike S4, which depends on fixed state-space updates, Mamba introduces a data-dependent selection mechanism that dynamically filters inputs, improving long-range dependency modeling [56]. It also employs an efficient hardware-aware algorithm that optimizes memory hierarchy and reduces IO operations, achieving linear-time complexity ($O(L)$), whereas Transformers suffer from quadratic complexity ($O(L^2)$) [56]. Additionally, parallel scanning, kernel fusion, and strategic recalculation further enhance computational speed while maintaining accuracy [57].

Unlike Transformers, which rely on full self-attention mechanisms, Mamba employs a Gated State Space Model (GSSM) that selectively propagates and forgets information, improving sequence retention and long-range dependency modeling. This selective propagation is further enhanced by input-dependent matrices, which replace costly self-attention mechanisms while retaining the computational efficiency of state-space models [58]. Additionally, Mamba's Selective Scan Algorithm optimizes training efficiency by improving GPU memory usage and reducing IO overhead, allowing for faster computation compared to traditional SSMs [58]. This innovation results in a fivefold increase in throughput for autoregressive sequence generation while effectively modeling long-range dependencies [59]. By eliminating the need for computationally expensive attention operations, Mamba provides a hardware-efficient alternative for sequence modeling, making it highly suitable for long-sequence tasks and high-resolution data processing [58]. These advantages extend to graph-based modeling, as seen in Graph-Mamba, which reduces GPU memory consumption by up to 74%, highlighting Mamba's effectiveness in computationally intensive tasks [56]. Specifically, Graph-Mamba replaces the attention module in the GraphGPS framework with the Graph-Mamba Block (GMB), which integrates an edge-based Message Passing Neural Network (MPNN) and a node-centric Mamba model for efficient graph learning. By incorporating node prioritization, permutation-based sparsification, and input-dependent context filtering, Graph-Mamba enhances computational scalability and long-range dependency modeling (as illustrated in Figure. 5).

Additionally, in time-series forecasting, Mamba outperforms Transformer-based architectures by mitigating permutation-invariant biases and enhancing variable selection through an optimized scan mechanism [60]. This advantage is exemplified in MambaTS, a specialized adaptation designed for time-series modeling. As illustrated in Figure 6, MambaTS consists of an embedding layer, instance normalization, multiple stacked Temporal Mamba Blocks (TMBs), and a prediction head. It employs patching and tokenization to reduce redundancy, while Variable Scan along Time (VST) arranges tokens in a structured manner to improve long-range dependency modeling. The encoder integrates SSM-based sequence modeling and gated non-linearity, enhancing efficiency and accuracy in capturing temporal patterns. Finally, a channel-independent linear decoder generates predictions, ensuring scalability and computational efficiency.

4.2 Multi-Input, Multi-Output SSM

SSMs have emerged as a powerful alternative to Transformers for sequence modeling, offering linear computational complexity and improved efficiency in handling long-range dependencies. Among these, the S5 introduces key architectural advancements that enable MIMO processing, distinguishing it from its predecessor, S4 [10]. Unlike S4, which relies on a bank of independent single-input, single-output (SISO) SSMs, S5 integrates a single MIMO SSM, streamlining computation and enhancing information exchange across multiple channels [10]. Additionally, S5 replaces S4's convolutional and frequency-domain approach with a parallel scan operation, optimizing its

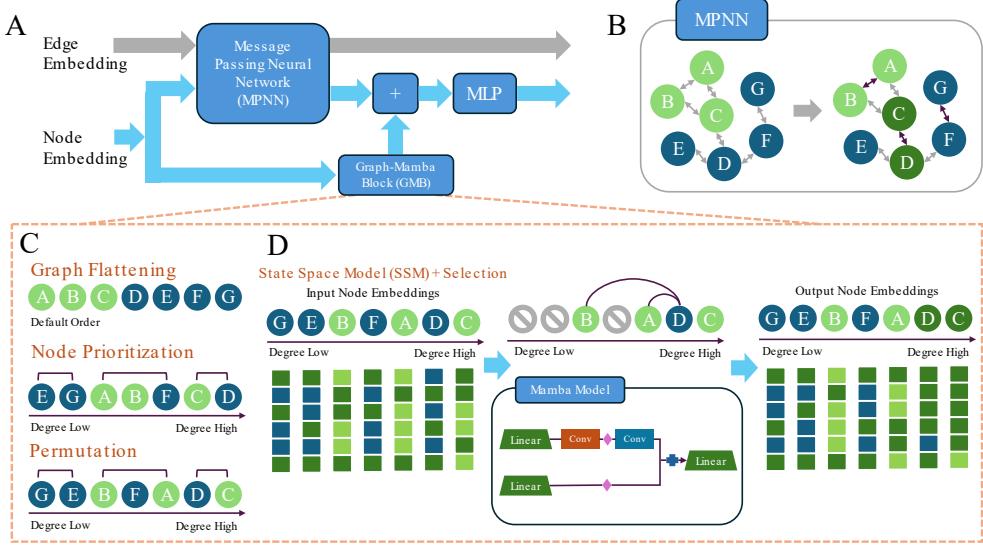


Fig. 5. Architecture of Graph-Mamba adapted from [56]

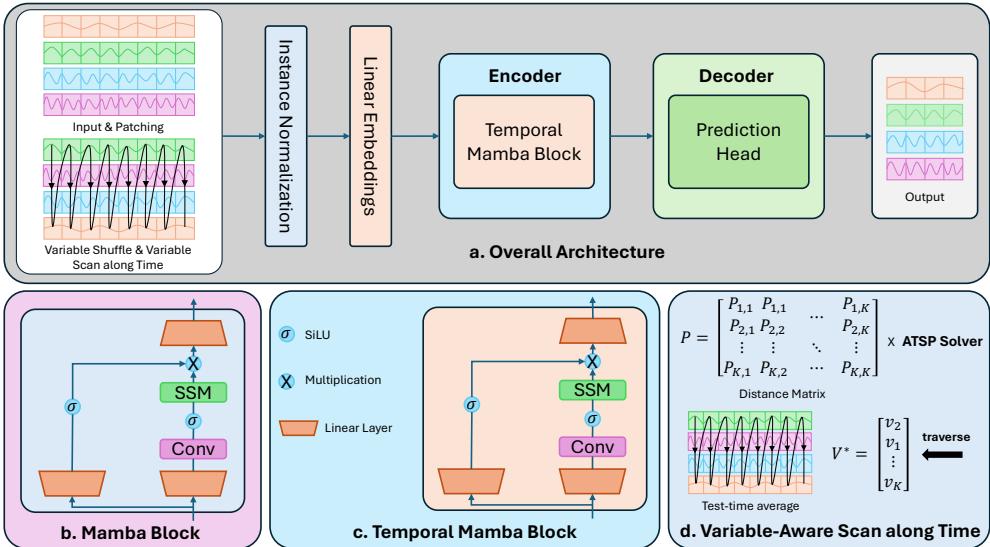


Fig. 6. Architecture of MambaTS adapted from [60]

efficiency for multi-modal data processing. This parallel scan mechanism eliminates the need for computationally expensive FFT operations, enabling S5 to scale efficiently while maintaining state transitions across multiple input-output channels [10].

The computational structure of S5 is illustrated in Figure 7. As shown, S5 enhances efficiency by utilizing a parallel scan operation on a diagonalized linear SSM, allowing it to compute outputs

recurrently in the time domain rather than relying on frequency-domain convolutions [10]. The transition from independent SISO state models in S4 to a unified MIMO framework in S5 enables more efficient computation of long sequences while preserving computational complexity. Additionally, S5 leverages HiPPO initialization schemes, a key feature inherited from S4, to optimize its internal state representations for improved sequence modeling. After computing the SSM outputs, a nonlinear activation function is applied to produce the final layer outputs, ensuring efficient transformation of sequential dependencies.

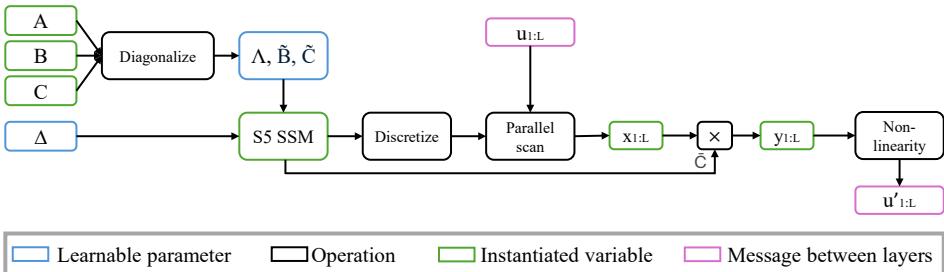


Fig. 7. Computational Components of the S5 Layer With Parallel Scan on a Diagonalized Linear SSM for Sequence Modeling, Followed by a Nonlinear Activation adapted from [10]

The Mamba architecture further refines these structured state space models by introducing Selective State Space Mechanisms (S6), which dynamically adjust hidden states based on input context, improving scalability and enabling efficient parallelization [9]. Mamba also leverages hardware-aware optimizations, achieving up to $40\times$ faster throughput compared to conventional SSM implementations [9]. Compared to S5, Mamba's associative parallel scans extend beyond structured sequence modeling, improving scalability in long-sequence tasks [10]. This makes Mamba particularly effective for multi-modal processing, as seen in architectures like VL-Mamba and Coupled Mamba, which introduce state-coupling mechanisms to enhance cross-modal information fusion [61, 62].

Additionally, Coupled Mamba improves upon SSM-based architectures by enabling inter-modal state interactions while preserving intra-modal independence, leading to superior parallelism and computational efficiency in multi-modal learning [61]. KalMamba further extends Mamba by integrating probabilistic inference with Kalman filtering, allowing efficient time-parallel belief state estimation for reinforcement learning applications, although it is not explicitly focused on multi-modal fusion [63]. Furthermore, SiMBA, a Mamba-based variant, introduces Einstein FFT (EinFFT) for spectral channel mixing, specifically optimizing Mamba for vision and multivariate time-series tasks, thereby addressing scalability concerns in high-dimensional data processing [64].

4.3 Jamba: MoE-Based Hybrid SSM-Transformer

The rapid evolution of deep learning architecture has led to the emergence of hybrid models that integrate Mixture-of-Experts (MoE), Transformer mechanisms, and SSMs to achieve superior efficiency and scalability in long-context sequence modeling. Traditional Transformer-based architectures, while excelling in expressivity and generalization, suffer from quadratic complexity in attention operations, limiting their feasibility for tasks requiring extended context lengths [65]. Meanwhile, SSMs, such as Mamba, provide linear-time sequence modeling with constant memory usage, making them particularly advantageous for handling long sequences with reduced

computational overhead [66]. Recent hybrid architectures Jamba, Hymba, Samba, and Zamba seek to combine the strengths of MoE, Transformers, and SSMs, offering new solutions for efficient language modeling and structured sequence learning. Jamba, for instance, interleaves Transformer and Mamba layers to balance expressivity and efficiency while incorporating MoE layers to scale model capacity without a proportional increase in active parameters [13, 65]. This design enables Jamba to support 256K token-long contexts with significantly lower memory usage, outperforming pure Transformers in throughput and inference efficiency. Similarly, Zamba adopts a Mamba backbone with a shared Transformer attention module, reducing inference latency and memory requirements while maintaining competitive performance in long-sequence tasks [67]. Meanwhile, Samba fuses selective SSMs with sliding window attention, optimizing context recall while retaining high computational efficiency [66]. Hymba follows a similar paradigm but leverages MoE more extensively, enhancing specialization among expert layers to maximize parameter efficiency [66].

5 Other Variants of S4

The Structured State-Space Sequence model (S4) revolutionized sequence modeling by efficiently capturing long-range dependencies. However, challenges such as complex initialization and limitations in discrete data tasks led to the development of various S4 variants. These models refine S4’s architecture to enhance efficiency, scalability, and adaptability to different domains.

This section explores key S4 variants, including S4ND, DSS, Liquid-S4, Hyena, S4D, Mega-S4, RWKV-S4, and U-Mamba. Each introduces improvements such as multi-dimensional processing, simplified state-space formulations, or hybrid architectures. We summarize their strengths, weaknesses, and applications, providing a comparative analysis for reference.

5.1 Detailed Variant Description

Below we detail each S4 variant, their key mathematical formulations, design principles, and identified strengths and weaknesses outlined in the literature.

5.1.1 Multi-Dimensional S4 (S4ND): S4ND extends S4 to multi-dimensional signals like images and videos by converting state-space equations to partial differential equations (PDEs) [68]. Unlike S4, which operates on one-dimensional sequences, S4ND characterizes multi-dimensional signals in terms of:

$$\frac{\partial \mathbf{x}}{\partial t^{(1)}} = A^{(1)} \mathbf{x} + B^{(1)} \mathbf{u}, \quad (14)$$

$$\frac{\partial \mathbf{x}}{\partial t^{(2)}} = A^{(2)} \mathbf{x} + B^{(2)} \mathbf{u}. \quad (15)$$

Discretization of the above PDEs allows S4ND to construct multi-dimensional convolution kernels that can effectively process high-dimensional data. S4ND can be used to substitute Conv2D and self-attention layers to improve performance on vision tasks. For instance, S4ND outperformed a Vision Transformer baseline by 1.5% on ImageNet-1k and improved video classification accuracy by 4% compared to a 3D ConvNeXt on HMDB-51 [68].

One of its strengths is its resolution-invariant, implicitly learned convolutional kernels that encourage scale generalization. A band-limiting extension also minimizes aliasing, enhancing zero-shot performance—trained on 8×8 images and tested on 32×32, it outperformed Conv2D by 40% [68]. While S4ND’s multiple-state matrix support increases computational cost, its ability to capture spatial dependency makes it an interesting candidate for medical imaging and video recognition tasks.

5.1.2 Diagonal State Spaces (DSS): DSS simplifies the S4 model by constraining the state matrix A to be diagonal, significantly reducing computational load while maintaining most of S4's performance [69]. The state-space method is formulated as:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t). \quad (16)$$

DSS avoids low-rank corrections in S4, resulting in a more efficient implementation. The convolution kernel is reduced to:

$$K_l = \sum_{n=1}^N C_n e^{\lambda_n \Delta t} B_n. \quad (17)$$

This design allows DSS to achieve competitive performance with S4 on long-range sequence modeling tasks while being easier to implement and optimize [69]. However, the use of diagonal matrices limits interactions between state dimensions, reducing expressiveness in complex sequence tasks. Despite this, DSS performs comparably to S4 on benchmarks such as the LRA and speech classification [69].

5.1.3 Liquid S4: Liquid-S4 augments S4 with an input-sensitive state transition scheme, drawing inspiration from Liquid Time-Constant (LTC) networks [70]. The state-space representation is modified as:

$$\dot{x}(t) = (A + B \odot f(x, u))x + B \odot f(x, u). \quad (18)$$

Here, $f(x, u)$ is a dynamically changing nonlinear function with respect to the input, enabling the model to selectively filter or emphasize information. This responsiveness allows Liquid-S4 to better handle variable-length dependencies compared to regular SSMs.

Compared with standard S4, which relies on a fixed state transition matrix, Liquid-S4's adaptive mechanism enables generalization across more diverse input sequences. Empirical evaluations show that Liquid-S4 achieves state-of-the-art results on long-range sequence modeling benchmarks like the LRA and surpasses ConvNeXt in speech recognition tasks with fewer parameters [70]. However, its non-linearity imposes additional computational costs and may require more regularization to ensure stable training.

5.1.4 Hyena: Hyena serves as a replacement for self-attention, utilizing long convolutions coupled with gating mechanisms to achieve sub-quadratic complexity without sacrificing expressiveness [71]. The model output is given by:

$$y(t) = g(t) \odot (W *_l x)(t), \quad (19)$$

where $(W *_l x)$ represents a convolutional kernel and $g(t)$ is a data-dependent gating function. This architecture enables Hyena to efficiently capture long-range dependencies with a lower computational expense compared to Transformers.

Unlike self-attention, which performs pairwise interactions between all tokens, Hyena integrates data-dependent implicit long convolutions. This allows it to preserve sequence-wide context while avoiding the quadratic complexity of attention mechanisms [71]. Hyena has demonstrated competitive performance in long-context modeling tasks such as language modeling on WikiText-103 and The Pile, achieving perplexity levels comparable to Transformers while reducing training compute by 20% [71].

Although Hyena is superior in processing efficiency, it may fall short of the flexibility of attention-based models when token-specific information retrieval is necessary. Nevertheless, its hybrid convolutional approach presents a promising solution for scalable sequence modeling.

5.1.5 Diagonal S4 (S4D): S4D strengthens DSS by incorporating S4 initialization methods while retaining a diagonal state matrix [72]. It generalizes DSS’s diagonal structure but enhances long-range dependency modeling through specialized initialization techniques. The convolution kernel is given by:

$$K_l = \sum_{n=0}^{N-1} C_n \lambda_n^l B_n. \quad (20)$$

S4D balances expressiveness and computational cost by maintaining the advantages of DSS while avoiding the computational burden of full-rank state matrices. It is compatible with Vandermonde matrix-based kernel computation, retaining the same theoretical complexity as S4 but with a simpler implementation [72].

Compared to DSS, S4D supports more flexible initialization schemes, such as HiPPO matrix-based methods, leading to improved performance on long-range sequence modeling tasks. It has been shown to perform robustly on image, audio, and time-series benchmarks, often surpassing DSS in both efficiency and accuracy [72].

5.1.6 Mega-S4: Mega combines gated attention with Exponential Moving Averages (EMA) to enhance sequence modeling efficiency [73]. The model refines its hidden state as:

$$y_t = \sigma(q_t \odot k_t) \odot v_t, \quad (21)$$

where (q_t, k_t, v_t) are learned feature representations, and σ is an element-wise gating function. By combining EMA with attention-like gating, Mega achieves a balance between computational efficiency and expressiveness, maintaining linear time and space complexity compared to standard Transformers.

Unlike traditional attention mechanisms burdened by quadratic complexity, Mega employs multi-dimensional damped EMA to capture long-term dependencies while ensuring stability [73]. This allows it to outperform state-of-the-art models like Transformers and S4 in tasks such as language modeling (WikiText-103) and neural machine translation, while also offering faster inference speeds [73].

However, tuning EMA decay parameters can be challenging, and the gating mechanism introduces additional hyperparameters requiring careful calibration. Despite these challenges, Mega presents a strong alternative to attention-based models for long-sequence modeling.

5.1.7 Receptance Weighted Key Value (RWKV)-S4: Receptance Weighted Key Value (RWKV) blends the efficiency of RNNs with the parallelizability of Transformers, offering a scalable and memory-efficient approach to long-sequence modeling [74]. The model employs a RWKV mechanism, updating its hidden state through:

$$p_t = \omega \odot p_{t-1} + e^{k_t} \odot v_t, \quad q_t = \omega \odot q_{t-1} + e^{k_t}, \quad (22)$$

$$h_t = \sigma(r_t) \odot \frac{p_t}{q_t}. \quad (23)$$

This recurrence-based architecture allows RWKV to process sequences with constant memory usage, making it highly efficient for both training and inference [74].

Unlike standard RNNs, RWKV does not suffer from the vanishing gradient problem and can be parallelized at training time, similar to Transformers. It achieves performance levels comparable to large Transformer models while significantly reducing computational costs. Empirical studies

demonstrate RWKV's effectiveness in language modeling tasks, where it outperforms traditional RNNs and performs competitively on datasets such as The Pile and WikiText-103 [74].

However, RWKV compresses long-range dependencies into a single fixed hidden state, which can lead to information loss compared to full self-attention models. Nevertheless, its efficiency and scalability make it a compelling alternative for handling long sequences.

5.1.8 U-Mamba: U-Mamba simplifies state-space modeling for biomedical image segmentation by integrating Mamba into a U-Net framework [75]. U-Mamba leverages a CNN-SSM hybrid model to capture long-range relationships while preserving spatial coherence in medical imaging. The state-space representation is defined as:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t). \quad (24)$$

In its discrete-time formulation, U-Mamba follows:

$$x_k = A_d x_{k-1} + B_d u_k, \quad y_k = C_d x_k. \quad (25)$$

where A_d, B_d, C_d are the discretized parameters.

By embedding Mamba into a U-Net-like framework, U-Mamba combines local feature extraction via convolutional layers with structured state-space models for sequence modeling across distant regions [75]. This hybrid design allows it to outperform traditional CNN-based architectures (e.g., U-Net, SegResNet) and Transformer-based models (e.g., UNETR, SwinUNETR) on multiple biomedical segmentation tasks [75].

While U-Mamba improves segmentation accuracy on modalities such as CT, MRI, and endoscopy, integrating state-space layers incurs additional computational costs. However, its self-configurability mitigates this issue by dynamically tuning network hyperparameters across datasets, enabling performance improvements without extensive manual adjustments.

5.1.9 FusionMamba: FusionMamba introduces a dynamic feature enhancement framework for multimodal image fusion by integrating Mamba into an image fusion architecture [76]. Traditional convolution-based fusion models struggle with global feature extraction, while Transformer-based methods incur high computational costs. FusionMamba addresses these issues by leveraging a selective structured state-space model (SSM) to efficiently model long-range dependencies with linear complexity.

The state-space representation for FusionMamba follows:

$$\dot{h}(t) = Ah(t) + Bx(t), \quad y(t) = Ch(t) + Dx(t). \quad (26)$$

Its discretized form is given by:

$$h_k = \bar{A}h_{k-1} + \bar{B}x_k, \quad y_k = Ch_k + Dx_k. \quad (27)$$

where \bar{A}, \bar{B}, C, D are the discretized parameters optimized for multimodal fusion.

FusionMamba introduces the Dynamic Feature Fusion Module, which enhances feature representation by dynamically adjusting the texture and disparity perception in multimodal images. It further integrates the Cross-Modal Fusion Mamba Module (CMFM) to enhance inter-modal correlation while suppressing redundant information [76].

As shown in Figure 8, the FusionMamba framework fuses two source images (I_1, I_2) using Dynamic Vision State Space (DVSS) blocks for feature extraction, Dynamic Feature Fusion Module for multimodal enhancement, and Learnable Descriptive Convolution (LDC) for texture refinement. The decoder, incorporating DVSS and patch-expanding blocks, reconstructs the fused image (F) while preserving both local and global features.

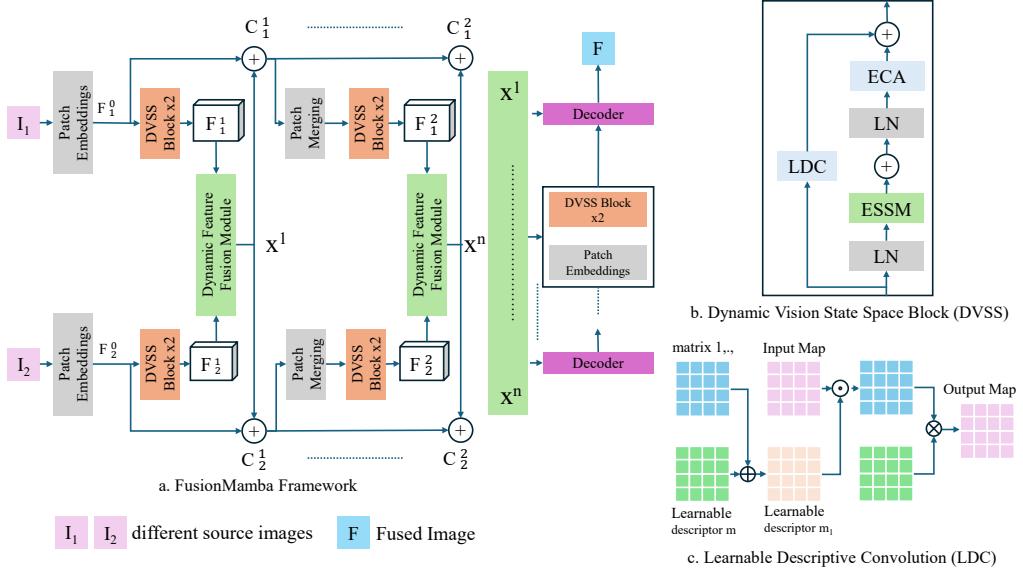


Fig. 8. Overview of the FusionMamba framework adapted from [76]

By embedding Mamba into a feature fusion pipeline, FusionMamba achieves state-of-the-art performance in multimodal medical image fusion tasks, including infrared-visible (IR-VIS), CT-MRI, PET-MRI, and SPECT-MRI fusion. Unlike purely CNN- or Transformer-based approaches, FusionMamba efficiently balances local feature refinement and global modeling, leading to enhanced image clarity, improved diagnostic accuracy, and robust feature extraction for downstream vision tasks [76].

Despite its advantages, FusionMamba's integration of dynamic convolution and state-space modeling introduces additional computational complexity. However, the model's parallel scanning algorithm optimizes efficiency on modern hardware, making it suitable for high-resolution medical imaging and real-time applications.

SSMs like FusionMamba build upon advancements in structured sequence modeling, including S4 and its variants, which have demonstrated strong performance across multiple domains. The following table (Table 2) provides a comparative analysis of S4-based architectures and related models, highlighting their key features, strengths, and limitations in tasks ranging from vision and language modeling to biomedical segmentation.

Table 2. Comparison of S4 Variants and Related Models

Model Source	Key Features	Strengths	Weaknesses
S4ND [68]	Multi-dimensional state-space (PDE) extension of S4; continuous spatial and temporal modeling.	Handles images/videos as continuous signals; Multi-resolution capability (train low-res, test high-res); Matches CNN/ViT performance on vision tasks.	Complex to implement (multi-dimensional PDE, multiple A matrices); More parameters for multi-dim. states; Less useful for inherently 1D data (text).

Model Source	Key Features	Strengths	Weaknesses
DSS [69]	Diagonal State Spaces: A matrix is diagonal; simple ZOH discretization.	Very simple and fast (no HiPPO, closed-form kernel); With proper initialization, can outperform original S4; Fewer parameters, easier to tune.	Needs careful parameter schemes (exp/softmax); Lacks coupling between state dimensions; Less effective on discrete data without further gating.
Liquid-S4 [70]	Input-dependent (liquid) SSM; A adapts as $A + B \odot f(x, u)$; combines S4 with LTCs.	Dynamic “attention-like” gating of state; Excellent long-term memory adaptivity; SoTA on LRA and strong results in speech and physiological signals.	More complex recurrence may be harder to train (nonlinear dynamics); Slight overhead per step for $f(x, u)$; Underperforms on language vs. full attention.
Hyena [71]	Implicit long convolution + gating; no explicit attention; hierarchical convolutional filters.	Sub-quadratic time; extremely fast for long sequences (e.g., 100x faster than attention at 64K seq length); Matches Transformer accuracy on language modeling.	Architectural complexity (implicit kernel generation, many hyperparameters); Requires large scale to shine; Primarily validated on text, behavior on other modalities less known.
S4D [72]	Diagonal S4 with principled initialization; unified S4’s math with DSS simplicity.	Simple (diagonal A) yet retains S4 performance; Flexible discretization (bilinear or ZOH); Faster training with similar accuracy to S4.	Fundamentally a linear model (requires stacking or additional gating for complex features); Handling complex eigenvalues adds implementation nuance.
Mega [73]	Single-head gated attention + EMA (moving average); Transformer variant with linear complexity.	Linear time and memory with chunking; Best-in-class on long-range benchmarks; Combines benefits of RNN (EMA bias) and attention (content gating).	Chunking may lead to indirect long-range interactions; New hyperparameters (chunk size, EMA decay) to tune; Less tested on non-text modalities.
RWKV [74]	Recurrent architecture mimicking Transformer attention; maintains hidden state, uses exponential decay.	Parallelizable training with RNN-style inference (constant memory); Matches Transformer quality in language tasks; Efficient hidden state compression.	Fixed hidden state may become an information bottleneck; Ecosystem and tooling are still maturing; Advantages diminish for short contexts.
U-Mamba [75]	U-Net + Mamba SSM blocks for segmentation; hybrid CNN-SSM architecture.	Captures global context in segmentation, outperforming pure CNNs or ViTs on biomedical tasks; Self-configuring to adapt to dataset scale and noise.	Specialized to segmentation; may require significant modification for other tasks; Increased model complexity due to CNN and SSM integration.

6 Other Categorization of SSMs

6.1 Recurrent SSMs

6.1.1 RWKV. The RWKV model is a hybrid neural architecture that combines the parallelized training efficiency of Transformers with the sequential inference benefits of RNNs [74]. By employing a linear attention mechanism, RWKV efficiently scales to long sequences while significantly reducing computational complexity. This design allows it to overcome the quadratic memory and compute requirements of Transformers while retaining expressiveness [74, 77].

RWKV integrates key architectural elements from both paradigms. It employs a Receptance Vector (R) to regulate memory retention, a Weight Decay Vector (W) to modulate historical influence, and Key (K) and Value (V) vectors similar to attention mechanisms in Transformers. These components are structured within stacked residual blocks, where time-mixing and channel-mixing sub-blocks facilitate efficient long-range dependency modeling. The time-mixing mechanism employs a weighted decay function to manage token interactions, while channel-mixing enhances expressivity across feature dimensions. Unlike traditional Transformers, RWKV enables parallelized

training while maintaining memory-efficient sequential inference, making it particularly effective for long-sequence tasks [74, 77].

The RWKV model has undergone several refinements, with RWKV-4 introducing time- and channel-mixing mechanisms, RWKV-5 (Eagle) incorporating multi-head matrix-valued states to improve expressiveness, and RWKV-6 (Finch) optimizing data-driven interpolation for enhanced sequence modeling [77]. These iterations have broadened RWKV's applicability across diverse domains, including NLP, computer vision, and audio processing. In NLP, RWKV has achieved competitive performance in language modeling and machine translation, serving as a viable alternative to Transformer-based architectures [74, 77]. It has also been effectively deployed in text generation, conversational AI, and virtual assistants. Beyond NLP, RWKV has been applied to high-resolution medical image segmentation, 3D point cloud perception, Automatic Speech Recognition (ASR), and AI-assisted music composition, demonstrating its versatility across sequential learning tasks.

6.1.2 Linear Recurrent Units (LRU). LRUs provide an efficient alternative to traditional RNNs and Transformer-based architectures for sequence modeling. Unlike conventional RNNs, which suffer from vanishing gradients, and Transformers, which have quadratic complexity, LRUs maintain computational efficiency while effectively capturing long-range dependencies [1]. By employing linear recurrence, LRUs eliminate non-linearity in recurrent updates, enabling parallelized training and efficient inference. These characteristics make LRUs particularly suitable for NLP, time-series forecasting, and recommendation systems [1].

LRUs retain the recurrence advantages of RNNs while addressing their inefficiencies. They utilize diagonal parameterization for enhanced computational stability and allow parallel state updates during training, significantly improving efficiency while preserving sequential inference. Additionally, recursive parallelization reduces memory costs, making LRUs highly effective for long-sequence modeling tasks such as economic forecasting, medical data analysis, and next-item prediction in recommendation systems. Empirical studies demonstrate that LRUs outperform traditional RNN-based and self-attention-based models in these domains, offering a scalable and computationally efficient solution [1, 78].

6.1.3 Hierarchical Graph Recurrent Networks (HGRN). HGRNs integrate graph neural networks (GNNs) with RNNs to facilitate efficient communication, memory retention, and decision-making in multi-agent systems [79]. These models are particularly effective in multi-agent deep reinforcement learning (MADRL) scenarios, where decentralized agents must operate under partially observable environments while maintaining long-term dependencies. By leveraging Hierarchical Graph Attention Networks (HGAT) for inter-agent communication and GRU for long-term memory storage, HGRNs enhance coordination and adaptability in complex decision-making tasks[79, 80].

HGRNs represent multi-agent systems as graph-based structures, where each agent is treated as a node with embedded feature representations. Connections between nodes are established based on predefined criteria, such as spatial proximity or network connectivity, enabling efficient information sharing. HGAT extracts valuable contextual information from neighboring agents, while GRUs retain historical data, allowing agents to recall past states and optimize future decisions [79, 80]. Variants such as Soft-HGRN and SAC-HGRN have been introduced to enhance learning strategies, with Soft-HGRN employing maximum-entropy learning for improved stochastic policy training, while SAC-HGRN integrates an actor-critic framework to refine decision-making.

HGRNs have been successfully applied in multi-agent coordination tasks, including Unmanned Aerial Vehicle (UAV) fleet control, where they improve navigation and communication efficiency. Additionally, they have been implemented in multi-agent traffic control systems, reducing congestion and optimizing traffic flow [79, 80]. Other applications include cooperative treasure collection and

pursuit games, where agents operate with incomplete information yet must coordinate effectively to achieve shared objectives.

6.2 Gated & Hybrid SSMs

6.2.1 Gated State Space Models (GSS). GSS models enhance state-space modeling efficiency and scalability by incorporating gating mechanisms to regulate information flow selectively [81]. Building upon S4 and DSS, GSS addresses computational inefficiencies by dynamically activating state updates, improving performance in autoregressive sequence modeling.

The GSS architecture consists of a gated state-space module that controls information flow between dense layers and state-space components, allowing efficient long-range dependency modeling [81]. By utilizing FFT-based convolutions, GSS reduces complexity from $O(L^2)$ in Transformers to $O(L \log L)$, achieving 2-3 \times faster training times than DSS while maintaining similar accuracy. It also demonstrates zero-shot generalization to sequences up to 65K tokens, making it highly effective for language modeling, code generation, and mathematical reasoning.

A hybrid variant of GSS integrates self-attention layers, enhancing short-range modeling while maintaining state-space efficiency[81]. Despite its advantages, GSS faces trade-offs in expressivity and speed, as it remains less optimized than highly tuned Transformer models [23, 81]. Ongoing research focuses on refining hybridization strategies and optimizing GPU and TPU implementations for broader applicability.

6.2.2 Multiplicative Filtered Gated State Space Model (MEGA). MEGA is designed to address two major limitations of Transformers: their weak inductive bias and quadratic complexity in attention computation [73]. Unlike Transformers, which treat all token positions equally, MEGA introduces Moving Average Equipped Gated Attention, improving efficiency and long-sequence modeling by incorporating position-aware local dependencies.

A key innovation in MEGA is the integration of EMA into attention computation, which smooths input signals using a learnable decay factor, enhancing long-range dependency capture. Additionally, MEGA employs single-head gated attention, achieving comparable expressivity to multi-head self-attention while significantly reducing computational overhead [73]. The MEGA-Chunk variant further optimizes efficiency by applying chunk-wise computation, which reduces Transformer-like quadratic complexity to near-linear scaling, decreasing memory usage and training time while preserving contextual dependencies.

MEGA has demonstrated state-of-the-art performance across neural machine translation, language modeling, and speech/image classification [73]. Empirical results on benchmarks such as LRA, WMT16 translation, and ImageNet classification confirm its ability to efficiently model long-range dependencies while maintaining a favorable trade-off between computational cost and accuracy.

6.2.3 Toeplitz Neural Networks (TNN). The TNN is an efficient sequence modeling architecture that replaces traditional attention mechanisms with Toeplitz matrices, reducing computational complexity from quadratic to log-linear [82]. Unlike Transformers, which depend on pairwise token relations and position embeddings, TNN models sequences based solely on relative positional relations, enabling scalability to sequences up to 14K tokens while maintaining strong performance.

TNN's architecture is built on Gated Toeplitz Units (GTU) and Gated Linear Units (GLU), with the Toeplitz Neural Operator (TNO) handling token mixing purely based on relative positional information, eliminating the need for quadratic attention mechanisms. The Relative Position Encoder (RPE) enables dynamic position embeddings, allowing generalization across different sequence lengths without retraining [82]. Additionally, TNN integrates an exponential decay bias, similar to ALiBi, to enhance extrapolation capabilities for long-sequence tasks.

TNN has achieved state-of-the-art performance across multiple benchmarks, including LRA, autoregressive & bidirectional language modeling (perplexity improvements over Transformers and SSMs), and ImageNet-1K classification, demonstrating its applicability beyond text-based tasks [82]. However, while TNN reduces computational costs, it does not capture content-aware dependencies as effectively as Transformers. Future research aims to integrate self-attention mechanisms with TNN to balance efficiency and expressivity. Table 3 provides a brief overview of these algorithms.

Table 3. Comparison of SSMs

Model Source	Key Features	Strengths	Weaknesses
S4 [2, 10]	Structured SSM, HiPPO framework, supports recurrent & convolutional modes, efficient long-sequence processing.	SoTA on LRA, fast & memory-efficient, solves Path-X (16K length), strong long-range dependency handling.	Requires specialized initialization, numerical stability issues, not fully replacing Transformers in all tasks.
S5 [1, 10]	MIMO SSM, replaces independent SISO SSMs in S4, uses parallel scan for efficient sequence modeling.	Matches S4's computational efficiency, SoTA on LRA (87.4%), better long-sequence modeling.	HiPPO matrix not stably diagonalizable, requires approximations for initialization.
HiPPO [32, 83]	Memory-efficient SSMs using orthogonal basis projections, originally designed for long-range dependency modeling.	Enables efficient sequence processing, foundational for S4, supports long-range dependencies.	Initialization is complex, requires careful tuning, limited effectiveness in discrete tasks.
LRU [1, 78]	Linear recurrent SSM, removes non-linearity, diagonal state parameterization, supports parallel training.	Low memory usage, faster inference than self-attention models.	Lacks non-linearity, may struggle with complex patterns, requires tuning for stability.
HGRN [79, 80]	Recurrent SSM for multi-agent learning, integrates Hierarchical Graph Attention (HGAT) and GRU for spatio-temporal modeling.	Handles partial observability, scales to large multi-agent systems, improves communication & cooperation.	High computational cost, requires careful tuning for stability.
Mamba [23, 84]	Selective SSM, input-dependent parameterization, no attention or MLP layers.	5× faster inference than Transformers, strong performance on NLP, speech, and genomics.	Less effective in discrete tasks without tuning, lacks established fine-tuning ecosystem.
TNN [82]	Uses Toeplitz matrices instead of attention, reducing quadratic complexity to log-linear.	Scales up to 14K sequence length, outperforms Transformers and SSMs on LRA.	Limited empirical validation outside NLP and vision tasks, may lose global context.
GSS [81]	Gated State Space model, integrates gating mechanisms with DSS.	2-3× faster training than DSS, scales well for long sequences (up to 65K tokens).	Still lags behind Transformers in some tasks, requires careful hyperparameter tuning.
Jamba [65]	Hybrid Transformer-Mamba architecture with MoE, interleaves Transformer and Mamba layers.	3× higher throughput than Mixtral, supports 256K token context length, competitive with top Transformer models.	Less established fine-tuning ecosystem, requires careful balance of Mamba and Transformer layers.
Hungry Hungry HiPPO [85]	Stacked SSMs with multiplicative interactions, FlashConv (FFT-based) for efficient training.	2× speedup on long-range benchmarks, strong in zero-shot & few-shot learning.	Perplexity gap compared to transformers, especially at 1.3B parameters.

7 Conclusions

S4 has marked a paradigm shift in sequence modeling by introducing structured recurrence mechanisms that efficiently capture long-range dependencies while maintaining computational scalability. Unlike RNNs, which struggle with vanishing gradients, and Transformers, which suffer from quadratic complexity in self-attention, S4 utilizes diagonal state-space parameterization and FFT acceleration to achieve superior efficiency in handling long sequences.

Building upon S4, advancements such as Mamba, which introduced input-dependent state selection for better expressivity, S5, which optimized MIMO processing for scalability, and Jamba, which integrated Transformer components for hybrid modeling, have further extended the capabilities of SSMs. These models have demonstrated state-of-the-art performance across diverse applications, including NLP, speech recognition, time-series forecasting, and vision tasks, solidifying SSMs as a viable alternative to traditional architectures.

Despite their advantages, challenges remain in training large-scale SSMs efficiently, improving interpretability, and integrating them seamlessly into existing deep learning ecosystems. Future research should explore hardware-aware optimizations, hybrid architectures that balance expressivity and efficiency, and techniques for real-time multimodal learning to unlock the full potential of SSMs.

As deep learning evolves, SSMs—originating from S4—are expected to reshape sequence modeling by offering a scalable, memory-efficient, and computationally effective alternative to attention-based systems, paving the way for the next generation of AI architectures.

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References

- [1] Badri Narayana Patro and Vijay Srinivas Agneeswaran. Mamba-360: Survey of state space models as transformer alternative for long sequence modelling: Methods, applications, and challenges. *arXiv preprint arXiv:2404.16112*, 2024.
- [2] Albert Gu, Karan Goel, and Christopher Ré. Efficiently modeling long sequences with structured state spaces. *arXiv preprint arXiv:2111.00396*, 2021.
- [3] Shida Wang and Qianxiao Li. Stablessm: Alleviating the curse of memory in state-space models through stable reparameterization. *arXiv preprint arXiv:2311.14495*, 2023.
- [4] Simiao Zuo, Xiaodong Liu, Jian Jiao, Denis Charles, Eren Manavoglu, Tuo Zhao, and Jianfeng Gao. Efficient long sequence modeling via state space augmented transformer. *arXiv preprint arXiv:2212.08136*, 2022.
- [5] Shriyank Somvanshi, Anannya Ghosh Tusti, Subasish Das, and Rohit Chakraborty. Applying tabular deep learning models to estimate crash injury types of young motorcyclists. In *IEEE CAI*, Santa Clara, California, USA, May 5-7 2025.
- [6] Shriyank Somvanshi, Rohit Chakraborty, Anandi K Dutta, and Subasish Das. Crash severity analysis of child bicyclists using arm-net and mambanet. In *IEEE CAI*, Santa Clara, California, USA, May 5-7 2025.
- [7] Siddhanth Bhat. Mathematical formalism for memory compression in selective state space models. *arXiv preprint arXiv:2410.03158*, 2024.
- [8] Nicola Muca Cirone, Antonio Orvieto, Benjamin Walker, Christopher Salvi, and Terry Lyons. Theoretical foundations of deep selective state-space models. *Advances in Neural Information Processing Systems*, 37:127226–127272, 2024.
- [9] Tri Dao and Albert Gu. Transformers are ssms: Generalized models and efficient algorithms through structured state space duality. *arXiv preprint arXiv:2405.21060*, 2024.
- [10] Jimmy TH Smith, Andrew Warrington, and Scott W Linderman. Simplified state space layers for sequence modeling. *arXiv preprint arXiv:2208.04933*, 2022.
- [11] Koichi Miyazaki, Yoshiki Masuyama, and Masato Murata. Exploring the capability of mamba in speech applications. *arXiv preprint arXiv:2406.16808*, 2024.
- [12] Shubhi Bansal, Sreekanth Madisetty, Mohammad Zia Ur Rehman, Chandravardhan Singh Raghaw, Gaurav Duggal, Nagendra Kumar, et al. A comprehensive survey of mamba architectures for medical image analysis: Classification, segmentation, restoration and beyond. *arXiv preprint arXiv:2410.02362*, 2024.
- [13] Jamba Team, Barak Lenz, Alan Arazi, Amir Bergman, Avshalom Manevich, Barak Peleg, Ben Aviram, Chen Almagor, Clara Fridman, Dan Padnos, et al. Jamba-1.5: Hybrid transformer-mamba models at scale. *arXiv preprint arXiv:2408.12570*, 2024.
- [14] Xiao Wang, Shiao Wang, Yuhe Ding, Yuehang Li, Wentao Wu, Yao Rong, Weizhe Kong, Ju Huang, Shihao Li, Haoxiang Yang, et al. State space model for new-generation network alternative to transformers: A survey. *arXiv preprint arXiv:2404.09516*, 2024.

- [15] Katalin M Hangos, József Bokor, and Gábor Szederkényi. *Analysis and control of nonlinear process systems*. Springer Science & Business Media, 2006.
- [16] Rudolph Emil Kalman. A new approach to linear filtering and prediction problems. *Transactions of the ASME-Journal of Basic Engineering*, 82(1):35–45, 1960.
- [17] James D Hamilton. *Time series analysis*. Princeton university press, 2020.
- [18] Thomas Kailath. *Linear systems*, volume 156. Prentice-Hall Englewood Cliffs, NJ, 1980.
- [19] Albert Gu, Isys Johnson, Karan Goel, Khaled Saab, Tri Dao, Atri Rudra, and Christopher Ré. Combining recurrent, convolutional, and continuous-time models with linear state space layers. *Advances in neural information processing systems*, 34:572–585, 2021.
- [20] Jeffrey L Elman. Finding structure in time. *Cognitive science*, 14(2):179–211, 1990.
- [21] Michael I Jordan. Serial order: A parallel distributed processing approach. In *Advances in psychology*, volume 121, pages 471–495. Elsevier, 1997.
- [22] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. *Neural computation*, 9(8):1735–1780, 1997.
- [23] Albert Gu and Tri Dao. Mamba: Linear-time sequence modeling with selective state spaces. *arXiv preprint arXiv:2312.00752*, 2023.
- [24] Yann LeCun, Bernhard Boser, John S Denker, Donnie Henderson, Richard E Howard, Wayne Hubbard, and Lawrence D Jackel. Backpropagation applied to handwritten zip code recognition. *Neural computation*, 1(4):541–551, 1989.
- [25] Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, 1998.
- [26] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural information processing systems*, 30, 2017.
- [27] Roberto Cahuantzi, Xinye Chen, and Stefan Güttel. A comparison of lstm and gru networks for learning symbolic sequences. In *Science and Information Conference*, pages 771–785. Springer, 2023.
- [28] Abhinav Agrawal and Namita Mittal. Using cnn for facial expression recognition: a study of the effects of kernel size and number of filters on accuracy. *The Visual Computer*, 36(2):405–412, 2020.
- [29] Nicola Muca Cirone, Antonio Orvieto, Benjamin Walker, Cristopher Salvi, and Terry Lyons. Theoretical foundations of deep selective state-space models. *arXiv preprint arXiv:2402.19047*, 2024.
- [30] Zifeng Ding, Yifeng Li, Yuan He, Antonio Norelli, Jingcheng Wu, Volker Tresp, Yunpu Ma, and Michael Bronstein. Dygmma: Efficiently modeling long-term temporal dependency on continuous-time dynamic graphs with state space models. *arXiv preprint arXiv:2408.04713*, 2024.
- [31] Yoshua Bengio, Patrice Simard, and Paolo Frasconi. Learning long-term dependencies with gradient descent is difficult. *IEEE transactions on neural networks*, 5(2):157–166, 1994.
- [32] Albert Gu, Isys Johnson, Aman Timalsina, Atri Rudra, and Christopher Ré. How to train your hippo: State space models with generalized orthogonal basis projections. *arXiv preprint arXiv:2206.12037*, 2022.
- [33] Lennart Ljung et al. Theory for the user. *System identification*, 1987.
- [34] Ricky TQ Chen, Yulia Rubanova, Jesse Bettencourt, and David K Duvenaud. Neural ordinary differential equations. *Advances in neural information processing systems*, 31, 2018.
- [35] Albert Gu, Tri Dao, Stefano Ermon, Atri Rudra, and Christopher Ré. Hippo: Recurrent memory with optimal polynomial projections. *Advances in neural information processing systems*, 33:1474–1487, 2020.
- [36] Junyoung Chung, Caglar Gulcehre, KyungHyun Cho, and Yoshua Bengio. Empirical evaluation of gated recurrent neural networks on sequence modeling. *arXiv preprint arXiv:1412.3555*, 2014.
- [37] Angelos Katharopoulos, Apoorv Vyas, Nikolaos Pappas, and François Fleuret. Transformers are rnns: Fast autoregressive transformers with linear attention. In *International conference on machine learning*, pages 5156–5165. PMLR, 2020.
- [38] Yi Tay, Mostafa Dehghani, Dara Bahri, and Donald Metzler. Efficient transformers: A survey. *ACM Computing Surveys*, 55(6):1–28, 2022.
- [39] Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.
- [40] Max Bain, Arsha Nagrani, Gùl Varol, and Andrew Zisserman. Frozen in time: A joint video and image encoder for end-to-end retrieval. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 1728–1738, 2021.
- [41] Bryan Lim, Sercan Ö Arik, Nicolas Loeff, and Tomas Pfister. Temporal fusion transformers for interpretable multi-horizon time series forecasting. *International Journal of Forecasting*, 37(4):1748–1764, 2021.
- [42] Alex Graves, Abdel-rahman Mohamed, and Geoffrey Hinton. Speech recognition with deep recurrent neural networks. In *2013 IEEE international conference on acoustics, speech and signal processing*, pages 6645–6649. Ieee, 2013.
- [43] Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J Liu. Exploring the limits of transfer learning with a unified text-to-text transformer. *Journal of machine learning research*, 21(140):1–67, 2020.

- [44] Mohammad Shoeybi, Mostafa Patwary, Raul Puri, Patrick LeGresley, Jared Casper, and Bryan Catanzaro. Megatron-lm: Training multi-billion parameter language models using model parallelism. *arXiv preprint arXiv:1909.08053*, 2019.
- [45] Zihang Dai, Zhilin Yang, Yiming Yang, Jaime Carbonell, Quoc V Le, and Ruslan Salakhutdinov. Transformer-xl: Attentive language models beyond a fixed-length context. *arXiv preprint arXiv:1901.02860*, 2019.
- [46] Sinong Wang, Belinda Z Li, Madian Khabsa, Han Fang, and Hao Ma. Linformer: Self-attention with linear complexity. *arXiv preprint arXiv:2006.04768*, 2020.
- [47] Siyi Tang, Jared A Dunnmon, Qu Liangqiong, Khaled K Saab, Tina Baykaner, Christopher Lee-Messer, and Daniel L Rubin. Modeling multivariate biosignals with graph neural networks and structured state space models. In *Conference on health, inference, and learning*, pages 50–71. PMLR, 2023.
- [48] Takeru Miyato, Toshiki Kataoka, Masanori Koyama, and Yuichi Yoshida. Spectral normalization for generative adversarial networks. *arXiv preprint arXiv:1802.05957*, 2018.
- [49] Jimmy Smith, Shalini De Mello, Jan Kautz, Scott Linderman, and Wonmin Byeon. Convolutional state space models for long-range spatiotemporal modeling. *Advances in Neural Information Processing Systems*, 36:80690–80729, 2023.
- [50] John Jumper, Richard Evans, Alexander Pritzel, Tim Green, Michael Figurnov, Olaf Ronneberger, Kathryn Tunyasuvunakool, Russ Bates, Augustin Žídek, Anna Potapenko, et al. Highly accurate protein structure prediction with alphafold. *nature*, 596(7873):583–589, 2021.
- [51] Yusen Zhang, Ruoxi Sun, Yanfei Chen, Tomas Pfister, Rui Zhang, and Sercan Arik. Chain of agents: Large language models collaborating on long-context tasks. *Advances in Neural Information Processing Systems*, 37:132208–132237, 2024.
- [52] Yanping Huang, Youlong Cheng, Ankur Bapna, Orhan Firat, Dehao Chen, Mia Chen, HyoukJoong Lee, Jiquan Ngiam, Quoc V Le, Yonghui Wu, et al. Gpipe: Efficient training of giant neural networks using pipeline parallelism. *Advances in neural information processing systems*, 32, 2019.
- [53] Alec Radford, Jong Wook Kim, Tao Xu, Greg Brockman, Christine McLeavy, and Ilya Sutskever. Robust speech recognition via large-scale weak supervision. In *International conference on machine learning*, pages 28492–28518. PMLR, 2023.
- [54] Yi Tay, Mostafa Dehghani, Samira Abnar, Yikang Shen, Dara Bahri, Philip Pham, Jinfeng Rao, Liu Yang, Sebastian Ruder, and Donald Metzler. Long range arena: A benchmark for efficient transformers. *arXiv preprint arXiv:2011.04006*, 2020.
- [55] Florin Gogianu, Tudor Berariu, Mihaela C Rosca, Claudia Clopath, Lucian Busoniu, and Razvan Pascanu. Spectral normalisation for deep reinforcement learning: an optimisation perspective. In *International Conference on Machine Learning*, pages 3734–3744. PMLR, 2021.
- [56] Chloe Wang, Oleksii Tsepa, Jun Ma, and Bo Wang. Graph-mamba: Towards long-range graph sequence modeling with selective state spaces. *arXiv preprint arXiv:2402.00789*, 2024.
- [57] Moein Heidari, Sina Ghorbani Kolahi, Sanaz Karimjafarbigloo, Bobby Azad, Afshin Bozorgpour, Soheila Hatami, Reza Azad, Ali Diba, Ulas Bagci, Dorit Merhof, et al. Computation-efficient era: A comprehensive survey of state space models in medical image analysis. *arXiv preprint arXiv:2406.03430*, 2024.
- [58] Hanwei Zhang, Ying Zhu, Dan Wang, Lijun Zhang, Tianxiang Chen, Ziyang Wang, and Zi Ye. A survey on visual mamba. *Applied Sciences*, 14(13):5683, 2024.
- [59] Ameen Ali, Itamar Zimerman, and Lior Wolf. The hidden attention of mamba models. *arXiv preprint arXiv:2403.01590*, 2024.
- [60] Xiuding Cai, Yaoyao Zhu, Xueyao Wang, and Yu Yao. Mambats: improved selective state space models for long-term time series forecasting. *arXiv preprint arXiv:2405.16440*, 2024.
- [61] Wenbing Li, Hang Zhou, Junqing Yu, Zikai Song, and Wei Yang. Coupled mamba: Enhanced multi-modal fusion with coupled state space model. *arXiv preprint arXiv:2405.18014*, 2024.
- [62] Yanyuan Qiao, Zheng Yu, Longteng Guo, Sihan Chen, Zijia Zhao, Mingzhen Sun, Qi Wu, and Jing Liu. Vl-mamba: Exploring state space models for multimodal learning. *arXiv preprint arXiv:2403.13600*, 2024.
- [63] Philipp Becker, Niklas Freymuth, and Gerhard Neumann. Kalmamba: Towards efficient probabilistic state space models for rl under uncertainty. *arXiv preprint arXiv:2406.15131*, 2024.
- [64] Badri N Patro and Vijay S Agneeswaran. Simba: Simplified mamba-based architecture for vision and multivariate time series. *arXiv preprint arXiv:2403.15360*, 2024.
- [65] Opher Lieber, Barak Lenz, Hofit Bata, Gal Cohen, Jhonathan Osin, Itay Dalmedigos, Erez Safahi, Shaked Meiron, Yonatan Belinkov, Shai Shalev-Shwartz, et al. Jamba: A hybrid transformer-mamba language model. *arXiv preprint arXiv:2403.19887*, 2024.
- [66] Arpita Vats, Rahul Raja, Mrinal Mathur, Vinija Jain, and Aman Chadha. Multilingual state space models for structured question answering in indic languages. *arXiv preprint arXiv:2502.01673*, 2025.
- [67] Paolo Glorioso, Minghan He, Yehonatan Rozen, Alex Kuefler, Omer Lieber, Brendan Millidge, Peter Battaglia, Aran Komatsuzaki, Aäron van den Oord, Alex Graves, et al. Zamba: A compact 7b ssm hybrid model. *arXiv preprint*

arXiv:2405.16712, 2024.

- [68] Eric Nguyen, Karan Goel, Albert Gu, Gordon Downs, Preey Shah, Tri Dao, Stephen Baccus, and Christopher Ré. S4nd: Modeling images and videos as multidimensional signals with state spaces. *Advances in neural information processing systems*, 35:2846–2861, 2022.
- [69] Ankit Gupta, Albert Gu, and Jonathan Berant. Diagonal state spaces are as effective as structured state spaces. *Advances in Neural Information Processing Systems*, 35:22982–22994, 2022.
- [70] Ramin Hasani, Matthias Lechner, Tsun-Hsuan Wang, Makram Chahine, Alexander Amini, and Daniela Rus. Liquid structural state-space models. *arXiv preprint arXiv:2209.12951*, 2022.
- [71] Michael Poli, Stefano Massaroli, Eric Nguyen, Daniel Y Fu, Tri Dao, Stephen Baccus, Yoshua Bengio, Stefano Ermon, and Christopher Ré. Hyena hierarchy: Towards larger convolutional language models. In *International Conference on Machine Learning*, pages 28043–28078. PMLR, 2023.
- [72] Albert Gu, Karan Goel, Ankit Gupta, and Christopher Ré. On the parameterization and initialization of diagonal state space models. *Advances in Neural Information Processing Systems*, 35:35971–35983, 2022.
- [73] Xuezhe Ma, Chunting Zhou, Xiang Kong, Junxian He, Liangke Gui, Graham Neubig, Jonathan May, and Luke Zettlemoyer. Mega: Moving average equipped gated attention. *arXiv preprint arXiv:2209.10655*, 2022.
- [74] Bo Peng, Eric Alcaide, Quentin Anthony, Alon Albalak, Samuel Arcadinho, Stella Biderman, Huanqi Cao, Xin Cheng, Michael Chung, Matteo Grella, et al. Rwkv: Reinventing rnns for the transformer era. *arXiv preprint arXiv:2305.13048*, 2023.
- [75] J. Ma, F. Li, and B. Wang. U-mamba: Enhancing long-range dependency for biomedical image segmentation. *arXiv preprint arXiv:2401.04722*, 2024.
- [76] Xinyu Xie, Yawen Cui, Tao Tan, Xubin Zheng, and Zitong Yu. Fusionmamba: Dynamic feature enhancement for multimodal image fusion with mamba. *Visual Intelligence*, 2(1):37, 2024.
- [77] Zhiyuan Li, Tingyu Xia, Yi Chang, and Yuan Wu. A survey of rkwk. *arXiv preprint arXiv:2412.14847*, 2024.
- [78] Zhenrui Yue, Yueqi Wang, Zhankui He, Huimin Zeng, Julian McAuley, and Dong Wang. Linear recurrent units for sequential recommendation. In *Proceedings of the 17th ACM international conference on web search and data mining*, pages 930–938, 2024.
- [79] Zhenhui Ye, Xiaohong Jiang, Guanghua Song, and Bowei Yang. Soft hierarchical graph recurrent networks for many-agent partially observable environments. *arXiv preprint arXiv:2109.02032*, 2021.
- [80] Yixiang Ren, Zhenhui Ye, Yining Chen, Xiaohong Jiang, and Guanghua Song. Soft-hgrns: soft hierarchical graph recurrent networks for multi-agent partially observable environments. *Frontiers of Information Technology & Electronic Engineering*, 24(1):117–130, 2023.
- [81] Harsh Mehta, Ankit Gupta, Ashok Cutkosky, and Behnam Neyshabur. Long range language modeling via gated state spaces. *arXiv preprint arXiv:2206.13947*, 2022.
- [82] Zhen Qin, Xiaodong Han, Weixuan Sun, Bowen He, Dong Li, Dongxu Li, Yuchao Dai, Lingpeng Kong, and Yiran Zhong. Toeplitz neural network for sequence modeling. *arXiv preprint arXiv:2305.04749*, 2023.
- [83] Federico Arangath Joseph, Kilian Konstantin Haefeli, Noah Liniger, and Caglar Gulcehre. Hippo-prophecy: State-space models can provably learn dynamical systems in context. *arXiv preprint arXiv:2407.09375*, 2024.
- [84] Haohao Qu, Liangbo Ning, Rui An, Wenqi Fan, Tyler Derr, Hui Liu, Xin Xu, and Qing Li. A survey of mamba. *arXiv preprint arXiv:2408.01129*, 2024.
- [85] Daniel Y Fu, Tri Dao, Khaled K Saab, Armin W Thomas, Atri Rudra, and Christopher Ré. Hungry hungry hippos: Towards language modeling with state space models. *arXiv preprint arXiv:2212.14052*, 2022.

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