Exercise # 3.6 Date ....

Q (1 - 18)

"The Chain Rule"

"Outside - Inside" Rule:

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

* Given $y = f(u)$ and $u = g(x)$, find $dy/dx = f'(g(x))g'(x)$.

1) $y = 6u - 9$, $u = (1/2)x^4$

Sol ⇒

$f'(u) = \frac{dy}{du} = \frac{d}{du}6u - \frac{d}{du}9$

$\boxed{f'(u) = 6}$

$g'(x) = \frac{d}{dx}\frac{x^4}{2}$

$\boxed{g'(x) = 2x^3}$

$\frac{dy}{dx} = f'(g(x))g'(x)$

$= 6 \cdot 2x^3 \Rightarrow 12x^3$

$\boxed{f'(g(x)) \cdot g'(x) = 12x^3}$

2) $y = 2u^3$, $u = 8x - 1$.

$f(u) = 2u^3$

$f'(u) = 2\frac{d}{du}u^3$

$\boxed{f'(u) = 6u^2}$

$g'(x) = 8\frac{d}{dx}(x) - \frac{d}{dx}(1)$

$\boxed{g'(x) = 8}$

$f'(g(x)) \cdot g'(x) = 6 \cdot 8u^2$

$f'(g(x)) \cdot g'(x) = 48u^2$

$\boxed{f'(g(x)) \cdot g'(x) = 48(8x - 1)^2}$

3) $y = \sin u$, $u = 3x + 1$

$f(u) = \sin u$

$f'(u) = \frac{d}{du}\sin u$

$\boxed{f'(u) = \cos u}$

$g'(x) = 3\frac{d}{dx}(x) + \frac{d}{dx}(1)$

$\boxed{g'(x) = 3}$

$f'(g(x)) \cdot g'(x) = 3\cos u$

$\boxed{f'(g(x)) \cdot g'(x) = 3\cos(3x + 1)}$

4) $y = \cos u$, $u = -x/3$

$f'(u) = \frac{d}{du}\cos u$

$\boxed{f'(u) = -\sin u}$

$g'(x) = \frac{1}{3}\frac{d}{dx}\,x$

$\boxed{g'(x) = -\frac{1}{3}}$

$f'(g(x))\cdot g'(x) = (-\frac{1}{3})(-\sin u)$

$\boxed{f'(g(x))\cdot g'(x) = +\frac{\sin(-x/3)}{3}}$

5) $y = \cos u$, $u = \sin x$

$f'(u) = \frac{d}{du}\cos u$

$\boxed{f'(u) = -\sin u}$

$g'(x) = \frac{d}{dx}\sin x$

$\boxed{g'(x) = \cos x}$

$f'(g(x))\cdot g'(x) = -\sin u \cos x$

$\boxed{f'(g(x))\cdot g'(x) = -\cos x \sin(\sin x)}$

6) $y = \sin u$, $u = x - \cos x$

$f'(u) = \frac{d}{du}\sin u$

$\boxed{f'(u) = +\cos u}$

$g'(x) = \frac{d}{dx}(x) - \frac{d}{dx}\cos x$

$\boxed{g'(x) = 1 + \sin x}$

$f'(g(x))g'(x) = (\cos u + \sin x \cos u)$

$\boxed{f'(g(x))\cdot g'(x) = (1+\sin x)(\cos(x-\cos x))}$

7) $y = \tan u$, $u = 10x - 5$

$f'(u) = \frac{d}{du}\tan u$

$\boxed{f'(u) = \sec^2 u}$

$g'(x) = \frac{d}{dx}10x - \frac{d}{dx}5$

$\boxed{g'(x) = 10}$

$f'(g(x))\cdot g'(x) = 10\sec^2 u$

$\boxed{f'(g(x))\cdot g'(x) = 10\sec^2(10x-5)}$

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⑧ $y = -\sec u, u = x^2 + 7x$

$f'(u) = -\frac{d}{du}\sec u$

$\boxed{f'(u) = -\sec u \tan u}$

$g'(x) = \frac{d}{dx}(x)^2 + 7\frac{d}{dx}(x)$

$\boxed{g'(x) = 2x + 7}$

$f'(g(x)) \cdot g'(x) = (2x+7)(-\sec u \tan u)$

$= \boxed{(2x+7)(-\sec(x^2+7x)\tan(x^2+7x))}$

* Ex # 9-18, write function in form of $y = f(u)$ and $u = g(x)$, Then find $\frac{dy}{dx}$ as a Function of x:

9) $y = (2x+1)^5$

let,

$u = 2x + 1$ so, $y = u^5$

so, $f'(u) = \frac{d}{du}u^5$

$\frac{dy}{du} = \boxed{f'(u) = 5u^4}$

$\frac{du}{dx} = g'(x) = 2\frac{d}{dx}(x) + \frac{d}{dx}(1)$

$\frac{du}{dx} = \boxed{g'(x) = 2}$

$\frac{dy}{dx} = 10u^4 \Rightarrow \boxed{10(2x+1)^4}$

⑩ $y = (4-3x)^9$

Sol,

let, $u = 4 - 3x$

$y = u^9$

$\frac{dy}{du} = \frac{d}{du}u^9$

$\boxed{\frac{dy}{du} = 9u^8}$

$\frac{du}{dx} = \frac{d}{dx}4 - 3\frac{d}{dx}x$

$\boxed{\frac{du}{dx} = -3}$

$\boxed{\frac{dy}{dx} = -27(4-3x)^8}$

⑪ $y = \left(1 - \frac{x}{7}\right)^{-7}$

Sol;

let,

$u = 1 - \frac{x}{7}$

$y = u^{-7}$

$\frac{dy}{du} = \frac{d}{du}u^{-7}$

$\boxed{\frac{dy}{du} = -7u^{-8}}$

$\frac{du}{dx} = \frac{d}{dx}(1) - \frac{1}{7}\frac{d}{dx}x$

$\boxed{\frac{du}{dx} = -\frac{1}{7}}$

$\frac{dy}{dx} = (\cancel{-}\frac{1}{\cancel{7}})(\cancel{-}\cancel{7}u^{-8})$

$\boxed{\frac{dy}{dx} = \left(1 - \frac{x}{7}\right)^{-8}}$

⑫ $y = \left(\frac{x}{2} - 1\right)^{-10}$

let,

$u = \frac{x}{2} - 1$

$\frac{dy}{du} = \frac{d}{du}u^{-10}$

$\boxed{\frac{dy}{du} = -10u^{-11}}$

$\frac{du}{dx} = \frac{1}{2}\frac{d}{dx}(x) - \frac{d}{dx}(1)$

$\boxed{\frac{du}{dx} = \frac{1}{2}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = -5u^{-11}$

$\boxed{\frac{dy}{dx} = -5\left(\frac{x}{2} - 1\right)^{-11}}$

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(13) $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$

$u = \frac{x^2}{8} + x - \frac{1}{x}$

$y = u^4$

$\frac{dy}{du} = \frac{d}{du} u^4$

$\boxed{\frac{dy}{du} = 4u^3}$

$\frac{du}{dx} = \frac{1}{8}\frac{d}{dx}(x^2) + \frac{d}{dx}(x) - \frac{d}{dx}(x^{-1})$

$\boxed{\frac{dy}{dx} = \frac{x}{4} + 1 + \frac{1}{x^2}}$

$\boxed{\frac{dy}{dx} = \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)\left(4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3\right)}$

(14) $y = \sqrt{3x^2 - 4x + 6}$

$u = 3x^2 - 4x + 6$

$y = u^{1/2}$

$\frac{dy}{du} = \frac{d}{du} u^{1/2} \Rightarrow \frac{1}{2} u^{-1/2} \Rightarrow \frac{1}{2\sqrt{u}}$

$\frac{du}{dx} = 3\frac{d}{dx}x^2 - 4\frac{d}{dx}x + \frac{d}{dx}6$

$\boxed{\frac{dy}{dx} = 6x - 4}$

$\frac{dy}{dx} = (6x-4)\left(\frac{1}{2\sqrt{u}}\right)$

$\frac{dy}{dx} = \frac{\cancel{2}(3x-2)}{\cancel{2}\sqrt{3x^2-4x+6}}$

$\boxed{\frac{dy}{dx} = \frac{3x-2}{\sqrt{3x^2-4x+6}}}$

(15) $y = \sec(\tan x)$

$u = \tan x$

$y = \sec u$

$\frac{dy}{du} = \frac{d}{du}\sec u \Rightarrow \boxed{\sec u \tan u}$

$\frac{du}{dx} = \frac{d}{dx}\tan x \Rightarrow \boxed{\sec^2 x}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = \sec u \tan u \sec^2 x$

$\boxed{\frac{dy}{dx} = \sec^2 x \sec(\tan x)\tan(\tan x)}$

(16) $y = \cot\left(\pi - \frac{1}{x}\right)$

$u = \pi - \frac{1}{x}$

$y = \cot u$

$\frac{dy}{du} = \frac{d}{du}\cot u \Rightarrow \boxed{-\csc^2 u}$

$\boxed{\frac{du}{dx} = \frac{1}{x^2}}$

$\boxed{\frac{dy}{dx} = -\frac{\csc^2\left(\pi - \frac{1}{x}\right)}{x^2}}$

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17 $y = \sin^3 x$

$u = \sin x$

$y = u^3$

$\frac{dy}{du} = \frac{d}{du} u^3 \Rightarrow \boxed{3u^2}$

$\frac{du}{dx} = \frac{d}{dx} \sin x \Rightarrow \boxed{\cos x}$

$\boxed{\frac{dy}{dx} = 3\cos x (\sin^3 x)^2}$

18 $y = 5\cos^{-4} x$

$u = \cos x$

$y = 5u^{-4}$

$\frac{dy}{du} = 5 \frac{d}{du} u^{-4} \Rightarrow \boxed{-20u^{-5}}$

$\frac{du}{dx} = \frac{d}{dx} \cos x \Rightarrow \boxed{-\sin x}$

$\boxed{\frac{dy}{dx} = 20 \sin x (\cos x)^{-5}}$