"Hypothesis Testing about Single Population Mean"

Case I: $n \geq 30$, σ known.

A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kg with a standard deviation of 0.5 kg. Test the hypothesis that $\mu = 8$ kg against the alternative that $\mu \neq 8$ kg if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kg. Use 0.01 level of significance.

Sol:-

i) $H_0 : \mu = 8$

$H_A : \mu \neq 8$ two tailed test

ii) Level of Significance:

$\alpha = 0.01$

iii) Test-Statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

iv) Computations:

$\bar{X} = 7.8$, $n = 50$, $\sigma = 0.5$, $\mu = 8$

$$\therefore Z = \frac{7.8 - 8}{0.5/\sqrt{50}} = \frac{-0.2}{0.071} = -2.83$$

v) Critical region:

$$|Z| \geq Z_{1-\alpha/2}$$

$$> Z_{1-\frac{0.01}{2}}$$

$$> Z_{0.995}$$

$$|Z| > 2.58$$

$\Rightarrow$ $Z < -2.58$ or $Z > 2.58$

vi) Conclusion:

Because calculated value of $Z = -2.83$ falls in critical region (i.e; $-2.83 < -2.58$) So we reject H_0 and conclude that mean breaking strength is not equal to 8 kgs.

Q-2:- Use $H_A: \mu < 8$, then:

Sol:- i) $H_0: \mu = 8$

$H_A: \mu < 8$. (one tail)

ii) $\alpha = 0.01$

iii) $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

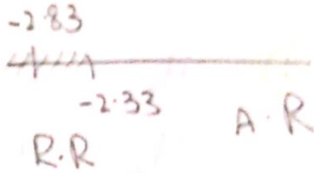
iv) $Z = -2.83$

v) $Z < -Z_{1-\alpha}$

$Z < -Z_{1-0.01}$

$Z < -Z_{0.99}$

$Z < -2.33$



vi) Conclusion: (same result)

- Reject H_0 and conclude that $\mu < 8$.

Q-3:- Use $H_A: \mu > 8$, then:

Sol:-

i) $H_0: \mu = 8$

$H_A: \mu > 8$ (one tail)

- ii) $\alpha = 0.01$

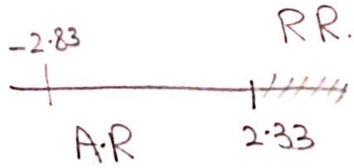
iii) $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

iv) $Z = -2.83$

v) $Z > Z_{1-\alpha}$

$Z > Z_{0.99}$

$Z > 2.33$



vi) Conclusion:

Accept H_0 and conclude that $\mu = 8$.

Case II: $n < 30$, σ known.

Q-4: $H_A: \mu < 0.31$, $n = 15$, $\sigma = 7$

$\bar{X} = 0.35$, $\alpha = 0.02$.

Sol:- i) $H_0: \mu = 0.31$

$H_A: \mu < 0.31$

ii) Level of Significance:

$$\alpha = 0.02$$

iii) Test-statistics:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

iv) Computation:

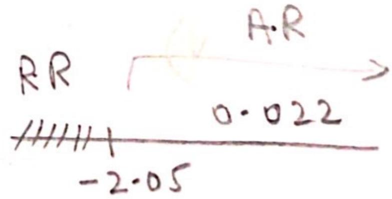
$$Z = \frac{0.35 - 0.31}{7/\sqrt{15}} = \frac{0.04}{1.81} = 0.022$$

v) Critical region:

$$Z < -Z_{1-\alpha}$$

$$Z < -Z_{0.98}$$

$$Z < -2.05$$



vi) Conclusion:

Because calculated value falls in Acceptance Region. Thus accept H_0 i.e; $\mu = 0.31$

Case III: $n \geq 30$, σ unknown. (i.e; s)

Q-5:- $H_A: \mu > 72$, $n = 100$, $S = 12$

$\bar{X} = 75$, $\alpha = 0.05$

Sol:-

i) $H_0: \mu = 72$

$H_A: \mu > 72$ (one tail)

ii) $\alpha = 0.05$

iii) $$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

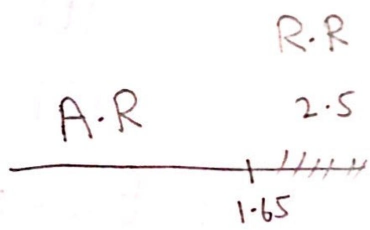
iv) $$Z = \frac{75 - 72}{12/\sqrt{100}} = \frac{3}{1.2} = 2.5$$

v) Critical region:

$Z > Z_{1-\alpha}$

$Z > Z_{0.95}$

$Z > 1.65$



vi) Conclusion:

falls in critical region. Thus Reject H_0 and conclude that $\mu > 72$.

Case IV: $n < 30$, σ unknown (s)

Q-6: $H_A: \mu \neq 25$, $n = 20$, $S = 0.75$

$\bar{x} = 22$, $\alpha = 0.05$

Sol:- i) $H_0: \mu = 25$

$H_A: \mu \neq 25$ (two tail)

ii) Level of Significance: $\alpha = 0.05$

iii) Test - statistics:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

iv) Computation:

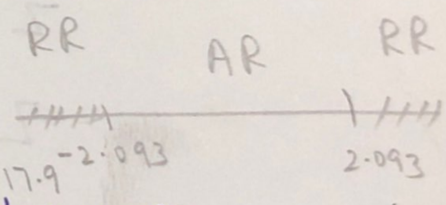
$$t = \frac{22 - 25}{0.75/\sqrt{20}} = -\frac{3}{0.17} = -17.9$$

v) Critical region:

$$|t| > t_{\alpha/2,\, n-1}$$

$$|t| > t_{0.025,19}$$

$$|t| > 2.093$$



$\therefore$ $t < -2.093$ and $t > 2.093$.

vi) Conclusion:

falls in rejection region.

i.e; Reject H_0.

1: $H_A: \mu < 11.6$, $n = 8$, $s = 4$

$\bar{x} = 12$. $\alpha = 0.01$

Sol :- i) $H_0: \mu = 11.6$

$H_A: \mu < 11.6$ (one tail)

ii) Level of Significance: $\alpha = 0.01$

iii) Test-statistic:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

iv) Computation:

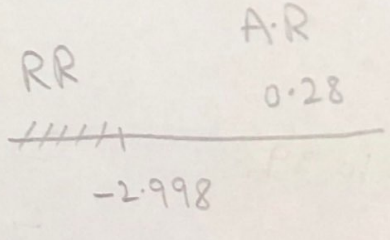
$$t = \frac{12 - 11.6}{4/\sqrt{8}} = \frac{0.4}{1.41} = 0.28$$

v) Critical region:

$$t < -t_{\alpha, n-1}$$

$$t < -t_{0.01, 7}$$

$$t < -2.998$$



v) Conclusion:

falls in acceptance region.

Thus accept H_0 i.e; $\mu = 11.6$.

Practice

Course pack:- Examples 7.2.1, 7.2.2,
7.2.3, 7.2.4, 7.2.5.

Exercises:- 7.2.1, to 7.2.19.

Helping Book:- 10.19, 10.20, 10.21, 10.22,
10.23, 10.24, 10.25, 10.26. 10.27,
10.28, 10.29.

Q-10.26, $t = 4.38$

$t > 1.729$.

Reject H_0.

Q-10.22, $Z = 3.33$ $\alpha = 0.05$

$Z > 1.65$

Reject H_0.

Q-10.29 $t = -1.98$

$t < -1.729$

Reject H_0.