Date: 19/12/2023

(Assignment # 4)

(CHAPTER # 04)

(Exercise # 4.1)

→ 1. let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $u = (u_1, u_2)$ and $v = (v_1, v_2)$.

$$u+v = (u_1 + v_1, u_2 + v_2), \quad ku = (0, ku_2).$$

a) Compute $u+v$ and ku for $u = (-1, 2)$, $v = (3, 4)$ and $k = 3$.

Sol: Given data: $u = (-1, 2)$, $v = (3, 4)$ and $k = 3$.

$$u+v = (-1, 2) + (3, 4)$$
$$u+v = (-1+3, 2+4)$$
$$u+v = (2, 6)$$
$$u+v = (2, 6)$$

Now, $ku = k(-1, 2)$ As we know rule: $\therefore ku = (0, ku_2)$

$$ku = (3)(-1, 2)$$
$$ku = (0, 3\times 2)$$
$$ku = (0, 6)$$
$$ku = (0, 6)$$

$$u+v = (2, 6) \; ; \; ku = (0, 6)$$

Date:

b) In words, explain why V is closed under addition and scalar multiplication?

Ans: For $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in V:

$$u+v = (u_1+v_1, u_2+v_2)$$

is an ordered pair of real numbers, hence $u+v$ is in V. As a result, V is closed under addition.

For any $u(u_1, u_2)$ in V and for any scalar k:

$$ku = (0, ku_2)$$

is an ordered pair of real number, hence ku is in V. As a result, V is closed under scalar multiplication.

c) Since addition on V is the standard addition operation on R^2, certain vector space axioms hold for V because they are known to hold for R^2, which axioms are they?

Ans: Axiom 1 – 5.

<u>Reason</u>: As first five axioms hold addition property.

d) Show that Axiom 7, 8, and 9 holds?

Sol: let $u = (u_1, u_2)$ and $v = (v_1, v_2)$

So, <u>Axiom #7</u> ⇒

↳ $k(u+v) = ku+kv$.

So, $k(u+v) = k(u_1+v_1, u_2+v_2)$

$k(u_1+v_1, u_2+v_2) = (ku_2+kv_2, 0)$

$k(u+v) = (ku_2, 0) + (kv_2, 0)$

So, $\boxed{k(u+v) = ku_1 + kv}$

Axiom ~~9~~8 :

$\hookrightarrow \quad (k+m)u = ku + mu.$

So, $(\because k, m \in R)$

$$(k+m)u = (k+m)(u_1, u_2)$$
$$\quad = (0, (k\cancel{u_2} + m)u_2)$$
$$\quad = (0, (k+m)u_2)$$
$$\quad = (0, ku_2) + (0, mu_2)$$
$$\boxed{(k+m)u = ku + mu}$$

Axiom 8

$\hookrightarrow \quad (km)u = k(mu)$

So, $(\because k, m \in R)$

$$(km)u = (km)(u_1, u_2)$$
$$\quad = (0, (km)u_2)$$
$$\quad = k(0, mu_2)$$
$$\boxed{(km)u = k(mu)}$$

e) Show that axiom 10 fails and hence that V is not a vector space under the given operations:

Sol: Axiom 10:

$\hookrightarrow \quad 1 \cdot u = u.$

So,

$$1u = (0, u_2) \neq u$$

So, axiom 10 does not hold, and hence V is not a vector space.

→ 2. Let V be the set of all ordered pairs of real numbers and consider of the following addition and scalar multiplication operation

Date:

on $u = (u_1, u_2)$ and $v = (v_1, v_2)$

$u+v = (u_1 + v_1 + 1, u_2 + v_2 + 1)$, $ku = (ku_1, ku_2)$

a) Compute $u+v$ and ku for ($u = (0,4)$, $v = (1,-3)$ and $k = 2$).

Sol:

Given data,

$u = (0,4)$, $v = (1,-3)$ and $k = 2$.

So,

$u+v = (0,4) + (1,-3)$

$\quad = (0+1+1, 4-3+1)$

$\quad = (2, 2)$

$$\boxed{u+v = (2,2)}$$

$ku = 2(0,4)$

$ku = (0\times2, 4\times2)$

$$\boxed{ku = (0,8)}$$

b) Show That $(0,0) \neq 0$

Sol: So, let $A = (0,0)$ So,

$u + A = u$

$(u_1, u_2) + (0,0) = (u_1, u_2)$

taking L.H.S ⇒

$= (u_1, u_2) + (0,0)$

So, as we know the rule:

$= (u_1 + 0 + 1, u_2 + 0 + 1)$

$= (u_1 + 1, u_2 + 1)$

So,

$$(u_1+1,\ v_1+1) \neq (v_1, v_2)$$

Hence,

$$(u_1, u_2) + (0,0) \neq (v_1, v_2)$$

So,

$$(0,0) \neq 0$$

So, A is not identity element.

c) Show that $(-1,-1) = 0$.

Sol. forming equation:

let $A = (-1,-1)$

$$u + A = v$$

$$(u_1, u_2) + (-1,-1) = (v_1, v_2)$$

So, as we know the rule ⇒

$$(u_1 \cancel{-1} \cancel{+1},\ u_2 \cancel{-1} \cancel{+1}) = (v_1, v_2)$$

$$(u_1, u_2) = (v_1, v_2)$$

So,

$$(u_1, u_2) + (-1,-1) = (u_1, u_2)$$

Hence,

$$(-1,-1) = \cancel{(u_1, u_2)} - \cancel{(u_1, u_2)}$$

$$(-1,-1) = 0$$

$$(-1,-1) = 0$$

So, A is an identity element.

d) Show that axiom 5 holds by producing a vector $-u$ such that $u + (-u) = 0$ for $u = (u_1, u_2)$.

Date:

let

$$-u = (-u_1 - 2,\ -u_2 - 2)$$

Such that:

$$u + (-u) = (u_1, u_2) + (-u_1 - 2, -u_2 - 2)$$

from rule:

$$u + (-u) = (\cancel{u_1} - \cancel{u_1} - 2 + 1,\ \cancel{u_2} - \cancel{u_2} - 2 + 1)$$

$$u + (-u) = (-1, -1)$$

$$\boxed{u + (-u) = 0} \qquad \therefore (-1,-1) = 0$$

Now evaluate $(-u) + u$

$$(-u) + u = (-u_1 - 2, -u_2 - 2) + (u_1, u_2)$$

$$(-u) + u = (-u_1 - 2 + u_1 + 1, -u_2 - 2 + u_2 + 1)$$

$$(-u) + u = (-1, -1)$$

$$\boxed{(-u) + u = 0}$$

So,

$$\boxed{u + (-u) = (-u) + u = 0}$$

Hence, Axiom 5 holds.

e) Find two vector space axioms that fails to hold.

let <u>Axiom 7</u>

$$\hookrightarrow \quad k(u+v) = ku + kv.$$

let $k = 2$, $u = (0, 4)$, $v = (1, -3)$. So,

L.H.S ⇒ $k(u+v) = 2((0,4) + (1,-3))$

$$'' = 2(0+1+1, 4-3+1) \qquad \therefore \text{using rule.}$$

$$'' = 2(2, 2)$$

$$\boxed{k(u+v) = (4, 4)} \quad \text{------ i)}$$

R.H.S ⇒

$$ku + kv = 2(0,4) + 2(1,-3)$$

$$Ku + Kv = (0,8) + (2,-6)$$

$$Ku + Kv = (0+2+1,\ 8-6+1) \quad \because \text{using Rule.}$$

$$\boxed{Ku + Kv = (3,3)} \quad \text{......ii)}$$

Hence from eq- i) and ii) we conclude That:

$$K(u+v) \neq Ku + Kv$$

So, vector space axiom 7 fails.

Let Axiom 9:

$$\hookrightarrow (K+m)u = Ku + mu.$$

Let $K=2$, $m=1$, $u=(0,4)$

So,

L.H.S ⇒

$$(K+m)u = (2+1)(0,4)$$

$$(K+m)u = (3)(0,4)$$

$$\boxed{(K+m)u = (0,12)} \quad \text{---- a)}$$

R.H.S ⇒

$$Ku + mu = 2(0,4) + 1(0,4)$$

$$Ku + mu = (0,8) + (0,4)$$

$$Ku + mu = (0+0+1,\ 8+4+1) \quad \because \text{using Formula:}$$

$$\boxed{Ku + mu = (1,13)} \quad \text{---- b)}$$

Hence from eq a) and b) we conclude:

$$(K+m)u \neq Ku + mu$$

So, vector space axiom 9 fails.

→ 3. The set of all real numbers with standard operations of addition and multiplication?

Sol:

we have $V = R \rightarrow$ set of all real numbers.

Axiom # 1 : According to defination axiom 1 : If u and v are objects in $V = R$ then $u+v$ is in $V = R$ So, $V = R$ is closed under addition.

Axiom # 2 : $u+v = v+u$ because in set of real numbers is commutativity. Hence commutative is true.

Axiom # 3 : If $u, v, w \in V = R$, Then $u+(v+w) = (u+v)+w$
Hence associativity is satisfied.

Axiom # 4 : let 0 object in V, called zero vector for V,
Then $0+u = 0+u = u$, Hence Axiom 4 is satisfied $V = R$.

Axiom # 5 : let $-u$ object in $V = R$ called negative of u, such That :

$$u+(-u) = (-u)+u = 0$$

Hence, for every $u \in V = R$ there is inverse element $-u \in V = R$.

Axiom # 6 : let k is scalar in R. Then ku also be in R.

Axiom # 7 : let k is scalar, show that $k(u+v) = ku+kv$. because set of real numbers is closed for multiplication by a scalar. Axiom 7 is satisfied.

Axiom # 8 : let k, m are scalar, ~~k(mu)~~ $(k+m)u = ku+mu$.

Axiom # 9 : let k, m are scalar, so $k(mu) = (km)u$.

Axiom # 10 : Find $1u$, then $1u = u$.

Then, all 10 Axioms are satisfied.

Hence, V is a vector space.

Date: ________

Date:

4. The set of all pairs of real numbers of the form $(x, 0)$, ~~where $x \geq 0$~~, with standard operations on R^2.

Sol: we have

$$V = \{(x,0), x \in R\} \quad So, \; u=(x,0), \; v=(y,0)$$
and $w=(z,0)$ be in V.

Axiom #1: According to defination, If u and v are object in V, then $u+v$ is in V. V is closed under addition.

Axiom # 2:
$$u+v = (x,0)+(y,0)$$
$$=((x+y),0)$$
$$=((y+x),0)$$
$$u+v = (y,0)+(x,0)$$
$$u+v = v+u \quad \text{— True.}$$

Axiom # 3: If $u, v, w \in V$, Then
$$u+(v+w) = (x,0)+((y,0)+(z,0))$$
$$=(x+y+z,0)$$
$$=(x,0)+(y,0)+(z,0)$$
$$u+(v+w) = (u+v)+w \quad \text{— satisfied.}$$

Axiom # 4: let $0=(0,0)$ object in V, So,
$$0+u = (0,0)+(x,0)$$
$$0+u = (x,0)+(0,0)$$
$$0+u = (x,0) \quad \text{— satisfied.}$$

Axiom # 5: let $-u=(-x,0)$ object in $V=R$ called negative of u. So,
$$u+(-u) = (x,0)+(-x,0) \Rightarrow (x-x,0) = (0,0)$$
Hence, satisfied.

Date:

Axiom #6: let K be scalar of R, so,

$$K(x,0) = (Kx, 0)$$

$$K(x,0) = Kx \quad \text{— satisfied}$$

Axiom #7: let K be scalar so,

$$K(u+v) = K((x,0)+(y,0))$$

$$K(u+v) = K(x,0)+K(y,0)$$

$$K(u+v) = Ku+Kv \quad \text{— satisfied}$$

Axiom #8: let K, m are scalar. so,

$$(K+m)u = (K+m)(x,0)$$

$$\text{"} = (Kx,0)+(mx,0)$$

$$\text{"} = K(x,0)+m(x,0)$$

$$(K+m)u = Ku+Km \quad \text{— satisfied.}$$

Axiom #9: Let K, m are scalar, so,

$$K(mu) = K(m(x,0))$$

$$\text{"} = K(mx,0)$$

$$\text{"} = (Kmx,0)$$

$$\text{"} = (Km)(x,0)$$

$$K(mu) = (Km)u \quad \text{— satisfied}$$

Axiom # 10: $1u = u$, Then,

$$1u = 1(x,0)$$

$$= (1x,0)$$

$$1u = (x,0) \quad \text{— satisfied.}$$

Then,

All 10 Axioms are satisfied.

Hence,

V is a vector space.

Intro to LA

→ 5. The set of all pairs of real numbers to form (x,y), where $x \geq 0$, with the standard operation on R^2.

Sol. we have

$V = \{(x,y), x \geq 0\}$ as we know $u = (u_1, u_2)$
$v = (v_1, v_2)$
$w = (w_1, w_2)$

So,

Axiom #1: According to defination, if u and v are object in V, then $u+v$ is in V. Then,

$$u+v = (u_1, u_2) + (v_1, v_2)$$
$$= (u_1+v_1, u_2+v_2) \in R.$$

V is closed under addition.

Axiom #2: Checking. $u+v = v+u$.

$$u+v = (u_1, u_2) + (v_1, v_2)$$
$$'' = (u_1+v_1, u_2+v_2)$$
$$'' = (v_1+u_1, v_2+u_2)$$
$$u+v = v+u \text{ — satisfied.}$$

Axiom #3:

$$u+(v+w) = (u_1, u_2) + ((v_1, v_2) + (w_1, w_2))$$
$$'' = (u_1, u_2) + (v_1+w_1, v_2+w_2)$$
$$'' = (u_1, u_2) + (v_1+w_1, v_2+w_2)$$
$$'' = (u_1+v_1+w_1, u_2+v_2+w_2)$$
$$u+(v+w) = (u+v)+w \text{ — satisfied.}$$

Axiom #4: let $0 = (0,0)$ so,

$$u+0 = (u_1, u_2) + (0,0)$$

Date:

$$u+0 = (u_1, 0) + (u_2, 0)$$
$$= (u_1, u_2)$$
$$u+0 = u$$ — satisfied.

Axiom #5: let $-u = (-u_1, -u_2)$ object $V = R$
$$u+(-u) = -u+u = 0$$

Since $u_1 \geq 0$, ~~and~~ Then $u_1 \leq 0$, Hence, $-u$ isn't in V.

Axiom #6: let K be scalar, this axiom does not apply to all real numbers, like $\boxed{K = -2}$.
$$ku = -2(u_1, u_2)$$
$$= (-2u_1, -2u_2)$$

Since $u_1 \geq 0$, then $-2u_1 \leq 0$.

So, Axiom 6 not satisfied. and axiom 5 is also not satisfied. So,

V is not vector space.

→ 6. The set of all n-tuples of real number that have form $(x, x, \ldots x)$ with the standard op. R^n.

Sol, we have $V = \{(x, x, \ldots x), x \in R^n\}$. let $u = (u, u, \ldots u)$
$$v = (v, v, \ldots v)$$
So,
$$w = (w, w, \ldots w)$$

Axiom #1: from defination, If u and v are objects in V, then $u+v$ is in V.
$$u+v = (u, u, u, \ldots u) + (v, v, \ldots v)$$
$$= (u+v, u+v, \ldots u+v) \in V$$

V is closed under addition.

Axiom #2: $u+v = v+u$.

So,

Date:

$u+v = (u, u, \dots u) + (v, v, \dots v)$
$" = (u+v, u+v, \dots u+v)$
$" = (v+u, v+u, \dots v+u)$
$u+v = v+u$. — satisfied.

Axiom # 3:

$u+(v+w) = (u+v)+w$

$u+(v+w) = (u,u \dots u) + ((v,v, \dots v) + ((w, w, \dots w))$

$= (u, u, \dots u) + (v+w, v+w, \dots)$

$= (u+v+w, \dots u+v+w)$

$u+(v+w) = (u+v)+w$ — satisfy

Axiom #4. let $0 = (0, 0, \dots 0)$

So,
$u+0 = (u, u, \dots u) + (0, 0, \dots 0)$
$" = (u, 0) + (u, 0) + \dots + (u, 0)$
$" = (u, u, \dots u)$
$u+0 = 0+u = u$
Satisfied.

Axiom #5: let $-u = (-u, -u, \dots -u)$

So,
$u+(-u) = (u, u, \dots u) + (-u, -u, \dots -u)$
$= (u-u, u-u, \dots u-u)$
$u+(-u) = (0, 0, \dots 0)$

Hence satisfied

Axiom 6: let k is scalar in R,
$ku = k(u, u, \dots u)$
$= (ku, ku, \dots ku)$
$= ku$

Axiom #7: let k be scalar.

$k(u+v) = ku + kv$

$k(u+v) = k((u,u, \dots u) + (v, v, \dots v))$

$= (ku, ku, \dots ku) + (kv, \dots kv)$

$k(u+v) = ku + kv$ — satisfy.

Axiom 8: let k, m be scalar.
$(k+m)u = ku + mu$.
$(k+m)u = (k+m)(u, u, \dots u)$
$" = (ku, ku, \dots ku) + (mu, \dots mu)$
$" = k(u, u, u, \dots u) + m(u, u, \dots u)$
$(k+m)u = ku + mu$. — satisfied

Axiom #9: $k(mu) = (km)u$

$k(mu) = k(m(u, u, \dots u)) = k(mu, \dots mu)$

$= (km)(u, u, \dots u)$

$k(mu) = (km)u$ — satisfy.

Axiom 10: $1u = 1(u, u, \dots u)$
$= (u, u, u, \dots u)$
$1u = u$ — satisfied.

V is vector space.

Date:

→ 7. The set of all triples of real number with standard vector addition but with scalar multiplication defined by:

$$k(x,y,z) = (k^2x, k^2y, k^2z).$$

Sol: let $u = (x_1, y_1, z_1)$ and $w = (x_3, y_3, z_3)$

$v = (x_2, y_2, z_2)$ So,

→ All Axioms are satisfied except axiom 8.

Axiom 8: $(k+m)u = ku + mu.$

L.H.S ⇒ $(k+m)u = (k+m)(x_1, y_1, z_1)$

" $= ((k+m)^2x_1, (k+m)^2y_1, (k+m)^2z_1)$

" $= (k+m)^2x_1, (k+m)^2y_1, (k+m)^2z_1)$ — i)

R.H.S: $ku + mu = k(x_1, y_1, z_1) + m(x_1, y_1, z_1)$

$= (k^2x_1, k^2y_1, k^2z_1) + (m^2x_1, m^2y_1, m^2z_1)$

$= ((k^2+m^2)x_1, (k^2+m^2)y_1, (k^2+m^2)z_1)$ — ii)

So, L.H.S ≠ R.H.S

$((k+m)^2x_1, (k+m)^2y_1, (k+m)^2z_1) \neq (k^2+m^2)x_1, (k^2+m^2)y_1, (k^2+m^2)z_1)$

Hence, V is not vector space.

→ 8. The set of all 2x2 invertible matrices with the standard matrix addition and scalar multiplication.

Sol ⇒ let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $B = \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}$

Axiom #1: If u, v in V, Then,

$$u + v = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}$$

$$= \begin{bmatrix} a-a & 0 \\ 0 & b-b \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, V is not vector space.

Since zero matrix is not invertible matrix So, Axiom 1 is not fulfilled.

→ 9. The set of all 2x2 matrices of the form: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

let

$$u = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}, \quad v = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

Axiom #1:

$$u+v = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

$$'' = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

'' = Since u, v is in R then $u+v$ in V. V is closed under addition.

Axiom #3:

$$u+(v+w) = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \left(\begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix}\right)$$

$$u+(v+w) = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & 0 \\ 0 & b_2+b_3 \end{bmatrix}$$

$$u+(v+w) = \begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix}$$

$$u+(v+w) = \begin{bmatrix} a_2+a_1 & 0 \\ 0 & b_2+b_1 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix}$$

$$u+(v+w) = \left(\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}\right) + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix}$$

$$u+(v+w) = (u+v)+w$$

↳ Satisfied

Axiom 2:

$$u+v = v+u$$

$$u+v = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

$$'' = \begin{bmatrix} a_2+a_1 & 0 \\ 0 & b_2+b_1 \end{bmatrix}$$

$$'' = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$$u+v = v+u.$$

Axiom #4: let $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$u+0 = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$'' = \begin{bmatrix} a+0 & 0 \\ 0 & b_1+0 \end{bmatrix}$$

$$'' = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$u+0 = u$ — Satisfied

Axiom #5: let $-u = \begin{bmatrix} -a_1 & 0 \\ 0 & -b_1 \end{bmatrix}$

$$u+(-u) = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} -a_1 & 0 \\ 0 & -b_1 \end{bmatrix}$$

$$'' = \begin{bmatrix} a_1-a_1 & 0 \\ 0 & b_1-b_1 \end{bmatrix}$$

$u+(-u) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ — Satisfied

Date:

Axiom 6: let K be scalar in R,

$$Ku = K\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \Rightarrow \begin{bmatrix} Ka_1 & 0 \\ 0 & Kb_1 \end{bmatrix}$$

Axiom 7: let K is scalar,

Show That $K(u+v) = Ku + Kv$.

$$K(u+v) = K\left(\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}\right)$$

$$'' = K\begin{bmatrix} Ka_1 + Ka_2 & 0 \\ 0 & Kb_1 + Kb_2 \end{bmatrix}$$

$$'' = \begin{bmatrix} Ka_1 & 0 \\ 0 & Kb_1 \end{bmatrix} + \begin{bmatrix} Ka_2 & 0 \\ 0 & Kb_2 \end{bmatrix}$$

$$'' = K\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + K\begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

$K(u+v) = Ku + Kv$. — satisfied.

Axiom 8:

$$(K+m)u = (K+m)\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$$'' = \begin{bmatrix} (K+m)a_1 & 0 \\ 0 & (K+m)b_1 \end{bmatrix}$$

$$'' = \begin{bmatrix} Ka_1 + ma_1 & 0 \\ 0 & Kb_1 + mb_1 \end{bmatrix}$$

$$'' = \begin{bmatrix} Ka_1 & 0 \\ 0 & Kb_1 \end{bmatrix} + \begin{bmatrix} ma_1 & 0 \\ 0 & mb_1 \end{bmatrix}$$

$$'' = K\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + m\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$(K+m)u = Ku + mu$ —satisfied.

Axiom 9: let K, m are scalars,

$$K(mu) = (Km)u$$

$$K(mu) = K\left(m\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}\right)$$

$$K(mu) = K\begin{bmatrix} ma_1 & 0 \\ 0 & mb_1 \end{bmatrix}$$

$$K(mu) = \begin{bmatrix} (Km)a_1 & 0 \\ 0 & (Km)b_1 \end{bmatrix}$$

$K(mu) = (Km)u$. —satisfy.

Axiom 10:

$1u = u$, Then,

$$1u = 1\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$$1u = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$1u = u$ — satisfy.

Hence, V is a vector space.

10. The set of all real-valued functions f defined everywhere on the real line and such that $f(1) = 0$ with the operations used in Example 6.

Date:

~~let~~ → operation → $(f+g)(x) = f(x)+g(x)$

$(kf)(x) = kf(x)$

So,

Axiom-1: let $f(1)=0$ & $g(1)=0$,

Then =

$f(1)+g(1) = (f+g)(1) = 0$

Axiom-2: $f+g = g+f$.

$(f+g)(x) = f(x)+g(x)$

$\quad " \quad = g(x)+f(x)$

$(f+g)(x) = (g+f)(x)$ — satisfied

Axiom-3: $(f+g)+h = f+(g+h)$

$((f+g)+h)(x) = (f+g)x + hx$

$\quad " \quad = (f(x)+g(x))+h(x)$

$\quad " \quad = f(x)+(g(x)+h(x))$

$\quad = f(x)+(g+h)(x)$

$((f+g)+h)(x) = (f+(g+h))(x)$ — sa

Axiom 4: let $z(1)=0$.

$(f+z)(x) = f(x)+z(x)$

Since,

$f(x)+z(x) = z(x)+f(x)$ — S.

Axiom-5: For each u. there is $-u$.

$(f+(-f))(x) = (f(x)+(-f(x)))$

$\quad " \quad = f(x)-f(x)$

$\quad " \quad = 0$

$((-f)+f)(x) = (-f(x))+f(x)$

$= -f(x)+f(x)$

$= 0$ — S.

Axiom-6: let k be scalar.

$kf = k(f(x))$

$kf = kf(x)$

Axiom-7: $k(u+v) = ku+kv$

$k(f+g))(1) = k(f+g)(1))$

$= k(f(1)+g(1))$

$= k(f(1))+k(g(1))$

$k(u+v) = ku+kv$. — S

Axiom-8.

$(k+m)u = ku+mu$.

$kf+mf = (kf+mf)(x)$

$\quad " \quad = kf(x)+mf(x)$

$\quad " \quad = (k+m)f(x)$

$kf+mf = (k+m)f$ — S

Axiom-9: $k(mf) = (km)f$

$(km)f = (km)(f)(x)$

$\quad " \quad = k(mf(x))$

$\quad " \quad = kmf(x)$ → $km(f)(x)$

$= (km)f$

So,

$k(mf) = (km)f$ — S.

Date:

Axiom-10.

$1f = (1f)(x)$
$= 1f(x)$
$1f = 1f$ — s

So, all 10 axioms are satisfied, Hence, V is a vector space.

→ 11: The set of all pairs of real numbers of the form $(1,x)$ with operations:

$$(1,y)+(1,y') = (1+y+y') \text{ and } k(1,y) = (1,ky)$$

So,

let $u=(1,y)$, $v=(1,y')$, $w=(1,y'')$ and k and V are scalar.

Axiom-1: According to Defination, let $u=(1,y)$, $v=(1,y')$ be in V then

$u+v = (1,y)+(1,y')$
$u+v = (1,y+y')$ — s

Axiom-2: $u+v = v+u$

$u+v = (1,y)+(1,y')$
$u+v = (1,y+y')$
$u+v = (1,y')+(1,y)$
$u+v = v+u.$ — s

Axiom-4: let $0=(1,0)$ So,

$u+0 = (1,y)+(1,0)$
$u+0 = (1,y+0)$
$u+0 = u$ — s.

Axiom-7: let k is scalar →

$k(u+v) = ku+kv$
$k(u+v) = k((1,y)+(1,y'))$ → $= k(1,y+y')$ → $(1,ky+ky')$
$(1,ky)+(1,ky')$ → $k(1,y)+k(1,y')$
$ku+kv$ — s

Axiom-3: If $u,v,w \in V$, then

$u+(v+w) = (1,y)+((1,y')+(1+y''))$
" $= (1,y)+((1,y'+y''))$
" $= (1,y)+(1,y'+y'')$
$u+(v+w) = (1,y+y'+y'')$
$= ((1,y)+(1,y'))+(1,y'')$
$u+(v+w) = (u+v)+w$ — s

Axiom-5: let $u=(1,y)$ and $v=(1,y')$

$u+(-v) = (1,y)+(1,-y')$
$u+v = (1,y-y')$
$u+(-v) = (1,0)$ — S

Axiom-6: let K be scalar in R,

$Ku = K(1,y)$
$Ku = (1,ky)$ — s.

Axiom 8: let K, m are scalars, show that $(K+m)u = Ku + mu$

$(K+m)u = (K+m)(1,y)$
$= (1, (K+m)y)$
$= (1, Ky + my)$
$= K(1,y) + m(1,y)$
$= Ku + mu$ — S.

Axiom 9. let K, m are scalars, show that $K(mu) = (Km)(u)$.

$K(mu) = K(m(1,y))$
$\quad = K(1, my)$
$K(mu) = (1, Kmy)$
$K(mu) = (Km)(1,y)$
$K(mu) = (Km)u$ — S

Axiom 10: $1u = 1(1,y)$
$= (1,y)$
$1u = u$ — S

So, all 10 axioms are satisfied. Hence, V is the vector's space.

→ 12. The set of polynomials of the form $a_0 + a_1x$ with the operations:

$$(a_0 + a_1x) + (b_0 + b_1x) = (a_0 + b_0) + (a_1 + b_1)x$$

and

$$K(a_0 + a_1x) = (Ka_0) + (Ka_1)x$$

we have V in which:

$u = (a_0 + a_1x)$, $v = (b_0 + b_1x)$, $w = (c_0 + c_1x)$ and K, m are scalar.

→ Axiom - 1: let $u = (a_0 + a_1x)$ $v = (b_0 + b_1x)$ be in V then:

$u + v = (a_0 + a_1x) + (b_0 + b_1x)$
$= (a_0 + b_0) + (a_1 + b_1)x$

Thus, V is closed under closed addition.

Axiom #2:

$u + v = v + u$.

$u + v = (a_0 + a_1x) + (b_0 + b_1x)$
$\quad = (a_0 + b_0) + (a_1 + b_1)x$
$\quad = ((b_0 + a_0), (b_1 + a_1)x)$
$\quad = (b_0 + b_1x) + (a_0 + a_1x)$

Date:

$u+v = v+u$ — satisfied

Axiom 3: The next axiom is associativity.

$u+(v+w) = (a_0+a_1x)+((b_0+b_1x)+(c_0+c_1x))$

$" = (a_0+(b_0+c_0))+(a_1+(b_1+c_1)x)$

$= ((a_0+b_0)+c_0)+((a_1+b_1)+c_1)x)$

$= ((a_0+a_1x)+(b_0+b_1x))+(c_0+c_1x)$

$u+(v+w) = (u+v)+w$ —— s.

Axiom 4: let $O=(0,0)$ be zero vector in V, when ⇒

$u+O = (a_0+a_1x)+(0,0)$

$" = ((a_0+0)+(a_1+0)x)$

$" = (a_0+a_1x)$

$" = u$

$O+u = (0,0)+(a_0+a_1x)$

$" = (0+a_0)+(0+a_1)x$

$" = (a_0+a_1x)$

$" = u$

Hence, satisfied

Axiom 5: let $-u=(-a_0 \mp a_1x)$

$u+(-u) = (a_0+a_1x)+(-a_0-a_1x)$

$= (a_0+(-a_0))+(a_1+(-a_1)x)$

$= 0$

$u+(-u) = (-u)+u = 0$ — s

Axiom 6: let k be scalar in R,

$ku = k(a_0+a_1x)$

$ku = (ka_0)+(ka_1)x$

$u \in V$ —— satisfied

Axiom 7: let k is scalar,

$k(u+v) = ku+kv$

$k(u+v) = k((a_0+b_0x)+(a_1+b_1)x)$

$" = (ka_0+b_0x)k, k(a_1+b_1)x)$

$" = (ka_0+kb_0, ka_1x+kb_1x)$

$" = (ka_0, ka_1x)+(kb_0, kb_1x)$

$= k(a_0+a_1x)+k(b_0+b_1)x$

$k(u+v) = ku+kv$ —— s

Axiom 8: $(k+m)u = ku+mu.$

$(k+m)u = (k+m)(a_0+a_1x)$

$" = ((k+m)a_0)+((k+m)a_1)x$

$" = ((ka_0+ma_0),(ka_1+ma_1)x)$

$" = (ka_0, ka_1x)+(ma_0+ma_1x)$

$= k(a_0+a_1x)+m(a_0+a_1x)$

$(k+m)u = ku+km$ —— s.

Axiom 9: let k, m are scalars,

$k(mu) = (km)u$

$k(mu) = k(m(a_0+a_1x))$

$k(mu) = k(ma_0+(ma_1)x)$

$= (kma_0+(kma_1)x)$

$= (km)(a_0+a_1x)$

$k(mu) = (km)u$ —— s.

Axiom 10. $1u = u$

$1u = 1(a_0 + a_1 x)$

$\quad = (1a_0) + (1a_1 x)$

$\quad = (a_0 + a_1 x)$

$1u = u$ —— satisfied.

So, all the 10 Axioms are satisfied, So, Hence, V is a vector space.

→ 13. Verify Axioms 3, 7, 8, and 9 for the vector space given in Example 4. (we have matrices 2×2).

$$u+v = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix}$$

$$ku = k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix}$$

Axiom 3: $u + (v+w) = (u+v) + w$ —— i)

As we know:

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, \quad v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}, \quad w = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$u + (v+w) = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \left(\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}\right)$$

$$\quad = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \left(\begin{bmatrix} v_{11}+w_{11} & v_{12}+w_{12} \\ v_{21}+w_{21} & v_{22}+w_{22} \end{bmatrix}\right)$$

$$\quad = \begin{bmatrix} u_{11}+v_{11}+w_{11} & u_{12}+v_{12}+w_{12} \\ u_{21}+v_{21}+w_{21} & u_{22}+v_{22}+w_{22} \end{bmatrix}$$

$$\quad = \left(\begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix}\right) + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$\quad = \left(\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}\right) + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

Date:

$$u+(v+w) = (u+v)+w.$$

Hence Axiom 3 satisfied.

Axiom 7. $k(u+v) = ku+kv$

$$k(u+v) = k\left(\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}\right)$$

$$'' = k\begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} ku_{11}+kv_{11} & ku_{12}+kv_{12} \\ ku_{21}+kv_{21} & ku_{22}+kv_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} + \begin{bmatrix} kv_{11} & kv_{12} \\ kv_{21} & kv_{22} \end{bmatrix}$$

$$'' = k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + k\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$k(u+v) = ku+kv$ —— satisfy.

Hence, Axiom 7 is satisfied.

Axiom 8: let k, m be scalar ⇒

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$(k+m)u = ku+mu$$

$$(k+m)u = (k+m)\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} u_{11}(k+m) & u_{12}(k+m) \\ u_{21}(k+m) & u_{22}(k+m) \end{bmatrix}$$

$$(k+m)u = \begin{bmatrix} ku_{11}+mu_{11} & ku_{12}+mu_{12} \\ ku_{21}+mu_{21} & ku_{22}+mu_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} + \begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix}$$

$$'' = k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + m\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$(k+m)u = ku+mu$ —— satisfy.

Axiom 9: let k, m be scalars and.

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$k(mu) = (km)(u)$$

$$k(mu) = k\left(m\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}\right)$$

$$'' = k\begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} kmu_{11} & kmu_{12} \\ kmu_{21} & kmu_{22} \end{bmatrix}$$

$$'' = \begin{bmatrix} (km)u_{11} & (km)u_{12} \\ (km)u_{21} & (km)u_{22} \end{bmatrix}$$

$$'' = (km)\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$k(mu) = (km)u$ —— satisfied.

Date:

Axioms 3, 7, 8 and 9 are satisfied.

14. Verify Axioms 1, 2, 3, 7, 8, 9 and 10 for the vector space given in Example 6?

Sol: **Axiom-1:** For $f, g \in V$.

$f(x) \in R$ for all $x \in R$

$g(x) \in R$ for all $x \in R$

$f(x) + g(x) \in R$ for all $x \in R$

$f + g \in V$. — s

Axiom-2:

$(f+g)(x) = f(x) + g(x)$

$= g(x) + f(x)$

$(f+g)(x) = (g+f)(x)$ — s.

Axiom-3:

$(f+(g+h))(x) = f(x) + (g+h)(x)$

$\quad = f(x) + (g(x) + h(x))$

$\quad = (f(x) + g(x) + h(x))$

$\quad = (f+g)(x) + h(x)$

$(f+(g+h))(x) = ((f+g)+h)(x) \rightarrow s$

Axiom 7:

$(k(f+g))(x) = k(f+g)(x)$

$\quad = k(f(x) + g(x))$

$\quad = (kf(x) + kg(x))$

$\quad = (kf)(x) + (kg)(x)$

$(k(f+g))(x) = (kf + kg)(x) \rightarrow s.$

Axiom 8:

$((k+m)f)(x) = (k+m)f(x)$

$\quad = kf(x) + mf(x)$

$\quad = (kf)(x) + (mf)(x)$

$\quad = (kf + mf)(x)$

$((k+m)f)(x) = (kf + mf)(x)$ — s

Axiom 9.

$(k(mf))(x) = k(mf)(x)$

$= (kmf)(x)$

$= (km)f(x)$

$(k(mf))(x) = ((km)f)(x)$ — s

Axiom 10:

$(1f)(x) = 1 \cdot f(x)$

$(1f)(x) = f(x)$ — s

Hence, all Axiom 1, 2, 3, 7, 8, 9 and 10 are satisfied.

(Exercise 4.2)

1. Use the Subspace Test to determine which of the sets are subspaces of R^3.

a) All vectors of the form $(a,0,0)$.

Sol: Consider that: $W=(a,0,0)$ that all vectors in R^3 with last two components equal to zero. Let,

$$u=(u_1,0,0) \quad , \quad v=(v_1,0,0)$$

* Vector Addition:

$$u+v = (u_1,0,0)+(v_1,0,0)$$
$$u+v = (u_1+v_1,0,0)$$

satisfy, as there are two zeros at the last two components.

* Scalar Multiplication:

$$ku = k(u_1,0,0)$$
$$ku = (ku_1,0,0)$$

Satisfy, as result also have zero at the last two components.

* Result:

All the vectors of the form $(a,0,0)$ is a subspace of R^3.

b) All vectors of the form $(a,1,1)$.

Sol, Consider that $w=(a,1,1)$ that all vectors in R^3 with last two component equal to 1, let:

$$u=(u_1,1,1) \quad , \quad v=(v_1,1,1)$$

* Vector Addition:

$$u+v = (u_1,1,1)+(v_1,1,1)$$
$$u+v = (u_1+v_1,2,2) \notin W$$

Thus, the resulting vector $u+v$ is not in the

Date:

defined form and hence not in W.

Result: All vectors to the form $(a,1,1)$ is not a subspace of R^3.

c) All vectors of the form (a,b,c) where $b=a+c$.

Sol: Consider that $W=(a,b,c)$

let $u=(a_1,b_1,c_1)$

$v=(a_2,b_2,c_2)$

be two vector spaces in R^3, such that:

$b_1 = a_1+c_1$ — i)

$b_2 = a_2+c_2$ — ii)

* Vector Addition:

$u+v = (a_1,b_1,c_1)+(a_2,b_2,c_2)$

$u+v = (a_1+a_2,\ b_1+b_2,\ c_1+c_2)$ — satisfy.

where:

$b_1+b_2 = (a_1+c_1)+(a_2+c_2)$ — from eq -i) and ii)

* Scalar Multiplication:

$Ku = K(a_1,b_1,c_1)$

$Ku = (Ka_1,Kb_1,Kc_1)$

where:

$Kb_1 = Ka_1+Kc_1$ — satisfy.

* Result:

All vectors to the form (a,b,c), where $b=a+c$, is a subspace of R^3.

2. a) All vectors of the form (a,b,c), where $b=a+c+1$.

Sol, $u=(a_1,b_1,c_1)$

$v=(a_2,b_2,c_2)$.

Date:

be two vector spaces in R^3 such That :

$$b_1 = a_1 + c_1 + 1 \quad \text{——— i)}$$

$$b_2 = a_2 + c_2 + 1 \quad \text{——— ii)}$$

* Vector Addition:

$$u+v = (a_1, b_1, c_1) + (a_2, b_2, c_2)$$

$$u+v = (a_1+a_2, b_1+b_2, c_1+c_2)$$

where, from eq- i) and ii)

$$b_1 + b_2 = (a_1 + c_1 + 1) + (a_2 + c_2 + 1)$$

$$b_1 + b_2 = (a_1 + c_1) + (a_2 + c_2) + 2.$$

Thus, The resulting vector is not in The defined form and hence $u+v \notin W$.

Result: All vectors of The form (a,b,c), where $b = a+c+1$, is not a subspace of R^3.

b) All vectors of The form $(a, b, 0)$.

Sol: Consider That $W = (a, b, 0)$

let $u = (a_1, b_1, 0)$

$v = (a_2, b_2, 0)$

* Vector Addition:

$$u+v = (a_1, b_1, 0) + (a_2, b_2, 0)$$

$$u+v = (a_1+a_2, b_1+b_2, 0)$$

Thus, the resulting vector is The defined form and hence $u+v \in W$.

* Scalar Multiplication:

$$ku = k(a_1, b_1, 0)$$

$$ku = (ka_1, kb_1, 0)$$

Thus, resultant vector is also in The defined form.

Result:

All vectors of the form $(a, b, 0)$, is a subspace of R^3.

c) All vectors of the form (a,b,c) for which $a+b=7$.

Sol. Consider $W = (a,b,c)$

let $u = (a_1, b_1, c_1)$

$v = (a_2, b_2, c_2)$

be two vectors space in R^3 such that:

$a_1 + b_1 = 7$ ——— i)

$a_2 + b_2 = 7$ ——— ii)

* **Vector Addition:**

$u + v = (a_1, b_1, c_1) + (a_2, b_2, c_2)$

$u + v = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

where

$(a_1 + b_1) + (a_2 + b_2) = 7 + 7$ from eq - i) & ii)

$(a_1 + a_2) + (b_1 + b_2) = 14$ Simplify.

Thus, resultant vector is not defined form.

Hence,

Result: All vectors of the form (a,b,c), where $a+b=7$, is not a subspace of R^3.

→ 3. Use the subspace Test to determine which of the sets are subspaces of M_{nn}.

a) The set of all diagonal $n \times n$ matrices.

Consider W is vector and A and B are two elements.

$$A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & 0 & \cdots & a_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ \vdots & b_{22} & \cdots & 0 \\ \vdots & \cdots & \cdots & b_{nn} \end{bmatrix}$$

* Vector Addition:

$$A+B = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ \cdots & a_{22} & \cdots & \\ \cdots & \cdots & \cdots & a_{nn} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ \vdots & b_{22} & \cdots & 0 \\ 0 & \vdots & \cdots & b_{nn} \end{bmatrix}$$

$$A+B = \begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ 0 & a_{22}+b_{22} & \cdots & 0 \\ \vdots & \vdots & & a_{nn}+b_{nn} \end{bmatrix}$$

Therefore the resulting matrix is $n \times n$ diagonal matrix. So, it satisfy.

* Scalar Multiplication:

$$KA = K \begin{bmatrix} a_{11} & \cdots & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & a_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} Ka_{11} & \cdots & \cdots & 0 \\ 0 & Ka_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & Ka_{nn} \end{bmatrix}$$

Thus, the resulting matrix is $n \times n$ diagonal. So, it satisfy the condition.

Result: The set of all diagonal $n \times n$ matrices is a subspace of M_{nn}.

b) The set of all $n \times n$ matrices A such that $\det(A) = 0$.

Sol, Consider W, is vector space and A is element. As,

Date:

$$A = \begin{bmatrix} a_{11} & \cdots & \cdots & 0 \\ \vdots & a_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & a_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & \cdots & \cdots & 0 \\ 0 & b_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & b_{nn} \end{bmatrix}$$

such that $\det(A) = 0$ and $\det(B) = 0$, Now finding the sum of the two matrices.

* Vector Addition:

$$A+B = \begin{bmatrix} a_{11} & \cdots & 0 \\ 0 & a_{22} & 0 \\ \vdots & \vdots & a_{nn} \end{bmatrix} + \begin{bmatrix} b_{11} & \cdots & 0 \\ 0 & b_{22} & 0 \\ \vdots & \vdots & b_{nn} \end{bmatrix}$$

$$A+B = \begin{bmatrix} a_{11}+b_{11} & \cdots & \cdots & 0 \\ \vdots & a_{22}+b_{22} & \cdots & 0 \\ 0 & 0 & \cdots & a_{nn}+b_{nn} \end{bmatrix}$$

Since, determinant is not distributive, $\det(A+B) \neq \det(A) + \det(B)$. Therefore,

$$\det(A+B) \neq 0$$

c) The set of all $n \times n$ matrices A such that $tr(A) = 0$

let

$$A = [a_{ij}]$$
$$B = [b_{ij}]$$

be two $n \times n$ matrices such that:

$$tr(A) = a_{11} + a_{22} + \cdots + a_{nn} = 0. \quad \text{— i)}$$
$$tr(B) = b_{11} + b_{22} + \cdots + b_{nn} = 0 \quad \text{— ii)}$$

So,

$$tr(A+B) = (a_{11}+a_{22}+\dots a_{nn}) + (b_{11}+b_{22}+\dots b_{nn})$$

from eq-i) and ii)

$$tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \dots + (a_{nn}+b_{nn})$$

$$tr(A+B) = 0+0$$

$$\boxed{tr(A+B) = 0} \quad \text{— satisfy.}$$

* Scalar Multiplication:

$$tr(KA) = Ka_{11} + Ka_{22} + \dots + Ka_{nn}$$

$$tr(KA) = K(a_{11}+a_{22}+\dots+a_{nn})$$

$$tr(KA) = K \times 0$$

$$\boxed{tr(KA) = 0} \quad \text{— satisfy.}$$

Result: The set of all $n \times n$ matrices A such that $tr(A)=0$ is a subspace of M_{nn}.

d) The set of all symmetric $n\times n$ matrices:

let W be the set of all symmetric $n \times n$ matrices

let A and B be two symmetric $n \times n$ matrices in W.

$$A^T = A \quad \text{—— i)}$$

$$B^T = B \quad \text{—— ii)}$$

Adding two matrices ⇒

$$(A+B)^T = A^T + B^T$$

$$(A+B)^T = A+B \quad \text{from eq-i) \& ii)}$$

Scalar Multiplication ⇒

$$(KA)^T = KA^T$$

$$(KA)^T = KA \quad \text{from eq-i)}$$

Thus, The resulting matrix is in defined form.

Result ⇒

The set of all symmetric $n\times n$ matrices is a subspace of M_{nn}.

Date:

→ 4. a) The set of all $n \times n$ matrices A such that $A^T = -A$.

let:

$$A^T = -A$$

$$B^T = -B$$

Adding two matrices ⇒

$$(A+B)^T = A^T + B^T$$

$$= (-A) + (B-)$$

$$(A+B)^T = -(A+B)$$

Thus, the resultant matrix A+B is defined form. etc.

Scalar Multiplication:

$$(KA)^T = KA^T$$

$$" = K(-A)$$

$$(KA)^T = -(KA) \text{ — satisfy.}$$

Result: The set of all $n \times n$ matrices A such that $A^T = -A$ is a subspace of M_{nn}.

b) The set of all $n \times n$ matrices of A for which $Ax=0$ has only the trivial solution:

let =

$$A = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

As we know $Ax=0$, $Bx=0$ has only the trivial solution, Now adding the two matrices we get the matrix A+B as represent below:

$$A+B = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1+1 & -3+3 \\ 2+2 & -4+4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} \qquad (\det(A+B) = 0)$$

The matrix equations $Ax = 0$ and $Bx = 0$ has trivial solution because the determinant of A and determinant of B are not equal.

As.

$$(A+B)x = 0$$

has infinity many solutions as the determinant of $A+B$ is equal to zero. Therefore, resultant matrix $A+B$ is not in defined form.

Result: The set of all $n \times n$ matrices A for which $Ax = 0$ has only the trivial solution is not a subspace of M_{nn}.

c) The set of all $n \times n$ matrices A such that $AB = BA$ for some fixed $n \times n$ matrix B.

Sol. Consider That

$A + B$ are two vectors such that $AB = BA$.

$$(A+B)C = AC + BC$$

$$'' = CA + CB$$

$$(A+B)C = C(A+B)$$

thus, The resultant matrix $A+B$ is defined form. Therefore, W is closed under addition.

Date:

Scalar Multiplication

$$(KA)C = K(AC)$$
$$\quad " = K(CA)$$
$$\quad " = (KC)A$$
$$(KA)C = C(KA)$$

Thus, the resulting matrix KA is in the defined form.

Result: The set of all $n \times n$ matrices A such that $AB = BA$ for some fixed $n \times n$ matrix B is a subspace of M_{nn}.

d) The set of all investible $n \times n$ matrix ?

Let:

$$A = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix} \qquad (\det A = 2)$$

$$B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad (\det B = -2)$$

Adding two matrices $\Rightarrow$

$$A+B = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} \qquad (\det(A+B) = 0)$$

Hence, the det(A) and det(B) are investible because their det(A) and det(B) are not equal. But the $\det(A+B) = 0$, so $A+B$ is not investible. Thus, $A+B$ matrix is not defined form.

Result:

The set of all investible $n \times n$ matrices is not a subspace of M_{nn}.

→ Q 5. Use the subspace test to determine which of the sets are subspaces of P_3.

a) All polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 = 0$.

let f and g are two polynomial in W as represented below:

$$f = a_0 + a_1x + a_2x^2 + a_3x^3$$
$$g = b_0 + b_1x + b_2x^2 + b_3x^3$$

where $a_0 = 0$ and $b_0 = 0$. Adding two polynomials:

$$f+g = (a_0 + a_1x + a_2x^2 + a_3x^3) + (b_0 + b_1x + b_2x^2 + b_3x^3)$$
$$f+g = (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$$

Since $a_0 = 0$ & $b = 0$

So, $a_0 + b_0 = 0$ Hence $f+g \in W$.

* Scalar Multiplication:

For any scalar k, consider the scalar multiple of the polynomial f in W as:

$$kf = k(a_0 + a_1x + a_2x^2 + a_3x^3)$$
$$kf = (ka_0) + (ka_1)x + (ka_2)x^2 + (ka_3)x^3$$

Since $a_0 = 0$ we have $ka_0 = 0$. Therefore $kf \in W$.

Thus,

Result: The set of all polynomial $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 = 0$ is a subspace of P_3.

b) All polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 + a_1 + a_2 + a_3 = 0$.

let f and g are two functions;

$$f = a_0 + a_1x + a_2x^2 + a_3x^3.$$

Date:

$g = b_0 + b_1 x + b_2 x^2 + b_3 x^3 \in P.$

Then, by defination $\Rightarrow$

$a_0 + a_1 + a_2 + a_3 = 0$ & $b_0 + b_1 + b_2 + b_3 = 0$

Now,

\# Vector Addition:

$f+g = (a_0 + a_1 x + a_2 x^2 + a_3 x^3) + (b_0 + b_1 x + b_2 x^2 + b_3 x^3)$

$f+g = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3.$

Since $\Rightarrow$

$(a_0+b_0) + (a_1+b_1) + (a_2+b_2) + (a_3+b_3) = (a_0+a_1+a_2+a_3) + (b_0+b_1+b_2+b_3)$

$= 0$

Scalar Multiplication:

let k be any scalar and f we know. So,

$kf = k(a_0 + a_1 x + a_2 x^2 + a_3 x^3)$

$= (ka_0) + (ka_1)x + (ka_2)x^2 + (ka_3)x^3$

$= k(a_0 + a_1 + a_2 + a_3) \Rightarrow 0$

So,

Result: Since $(ka_0) + (ka_1) + (ka_2) + (ka_3) = k(a_0 + a_1 + a_2 + a_3) = 0$ is a Sub-space of Vector space P_3.

$\rightarrow$ 6. a) All polynomials of the form $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ in which a_0, a_1, a_2 and a_3 are rational numbers?

let f and g are two polynomials in W:

$f = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$g = b_0 + b_1 x + b_2 x^2 + b_3 x^3$

in which $a_0, a_1, a_2, a_3, b_0, b_1, b_2$ and b_3 are rational numbers. So, Adding two numbers $\Rightarrow$

$$f+g = (a_0+a_1x+a_2x^2+a_3x^3)+(b_0+b_1x+b_2x^2+b_3x^3)$$

$$f+g = (a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2+(a_3+b_3)x^3$$

Since,

$$(a_0+b_0)+(a_1+b_1)+(a_2+b_2)+(a_3+b_3)$$

are also rational numbers. therefore $f+g \in W$.

Scalar Multiplication:

$$kf = k(a_0+a_1x+a_2x^2+a_3x^3)$$

$$=(ka_0)+(ka_1)x+(ka_2)x^2+(ka_3)x^3$$

Since product of a irrational number and a rational number is irrational. So,

ka_0, ka_1, ka_2, ka_3 are irrational

Therefore, $kf \notin W$.

Result: The set of all polynomials of the form $a_0+a_1x+a_2x^2+a_3x^3$ in which a_0, a_1, a_2 and a_3 are rational numbers is not a subspace of P_3.

b) All the polynomials of the form a_0+a_1x where a_0 and a_1 are real numbers?

let f and g are two polynomial in W, having values

$$f = a_0+a_1x$$

$$g = b_0+b_1x$$

where a_0 and a_1 are real numbers.

Vector Addition:

$$f+g = (a_0+a_1x)+(b_0+b_1x)$$

$$f+g = (a_0+b_0)+(a_1+b_1)x$$

Since $a_0, a_1, b_0,$ and b_1 are real numbers. So, their sums (a_0+b_0) and (a_1+b_1) are also

Date:

real numbers. Therefore $f+g \in W$.

Scalar Multiplication: For any scalar k, consider the scalar multiple of the polynomial. So,

$$kf = k(a_0 + a_1 x)$$
$$kf = (ka_0) + (ka_1)x$$

Since product of any scalar and a real number is a real number. So,

Result: The set of all polynomial of the form $a_0 + a_1 x$, where a_0 and a_1 are real numbers is a subspace of P_3.

→ Q 7. Use the subspace test to determine whether which of the sets are subspace of $F(-\infty, \infty)$

a) All functions f in $F(-\infty,\infty)$ for which $f(0) = 0$.

let f and g are two functions of W.

$$f(0) = 0 \quad \text{— i)}$$
$$g(0) = 0 \quad \text{— ii)}$$

* Adding two function:

$$f+g = f(0) + g(0)$$
$$= 0 + 0 \quad \text{from eq - i) \& ii)}$$
$$(f+g)(0) = 0 \quad \text{— s}$$

* Scalar Multiplication:

$$(kf)(0) = k(f(0))$$
$$= k(0)$$
$$(kf)(0) = 0$$

Thus,

Vector is a subspace of $F(-\infty, \infty)$.

Date:

b) All functions f in $F(-\infty,\infty)$ for which $f(0)=1$

let f and g are two functions in W. So,

$$f(0) = 1 \quad \text{— i)}$$

$$g(0) = 1 \quad \text{— ii)}$$

Adding two functions:

$$f+g = f(0)+g(0)$$

$$= 1+1 \qquad \text{from eq – i) \& ii)}$$

$$(f+g)(0) = 2$$

So, The resulting vector is not defined. $f+g \notin W$.

So, V is not subspace of $F(-\infty,\infty)$.

8. a) All functions f in $F(-\infty,\infty)$ for which $f(-x)=f(x)$.

let f and g are two functions:

$$f(-x) = f(x)$$

$$g(-x) = g(x)$$

Adding two functions:

$$f+g = f(-x)+g(-x)$$

$$f+g = f(x)+g(x)$$

$$(f+g)(-x) = (f+g)(x) \text{ — S.}$$

Scalar Multiplication:

$$(kf)(-x) = k(f(-x))$$

$$\text{"} = k(f(x))$$

$$(kf)(-x) = (kf)(x) \text{ — S.}$$

Thus,

Result: The set of all functions f in $F(-\infty,\infty)$ for which $f(-x)=f(x)$ is a subspace of $F(-\infty,\infty)$.

Date:

b) All polynomials of degree 2.

Let f and g are two polynomials.

$$f = 1 + 2x + 3x^2$$

$$g = 1 + 2x - 3x^2$$

Adding.

$$f+g = (1+2x+3x^2) + (1+2x-3x^2)$$

$$= (1+1) + (2+2)x + (3+(-3))x^2$$

$$f+g = 2+4x$$

Thus, the resulting polynomial is not defined. So,

Result: The set of all polynomials of degree 2 is not a subspace of $F(-\infty, \infty)$

→ Q 9. Use subspace test to determine which of the sets are subspaces of R^∞.

a) All sequences v in R^∞ of the form $v = (v, 0, v, 0, v, 0, \ldots)$

let $v = (v, 0, v, 0, \ldots)$

$w = (w, 0, w, 0, \ldots)$

in W.

Adding.

$$v+w = (v,0,v,0,\ldots) + (w,0,w,0,\ldots)$$

$$v+w = (v+w, 0, v+w, 0, \ldots)$$

Thus, satisfied.

Scalar Multiplication:

$$kv = k(v,0,v,0,\ldots) \Rightarrow (kv, 0, kv, 0, \ldots)$$

Thus, satisfied.

W is ~~vector~~ sub space

b) All sequences v in R^∞ of the form $v = (v, 1, v, 1, v, \dots)$

let v and w:

$$V = (v, 1, v, 1, v \dots)$$

$$w = (w, 1, w, 1, \dots)$$

Thus, Adding,

$$v+w = (v, 1, v, 1, v \dots) + (w, 1, w, 1, \dots)$$

$$v+w = (v+w, 2, v+w, 2, \dots)$$

Hence not satisfied, Now scalar Multiplication

$$\cancel{kv = k(v, 1, v, 1, \dots) \Rightarrow (kv, 1, kv, 1, kv \dots)}$$

Hence, not satisfied. W is not subspace.

10. a) All sequences of v in R^∞ of the form $v = (v, 2v, 4v, \dots)$

let $v = (v, 2v, 4v, 6v, \dots)$ & $w = (w, 2w, 4w, 6w \dots)$.

Adding:

$$v+w = (v, 2v, 4v, 6v, \dots) + (w, 2w, 4w, \dots)$$

$$v+w = (v+w, 2v+2w, 4v+4w, \dots)$$

$$v+w = (v+w, 2(v+w) + 4(v+w)).$$ — satisfied.

Scalar Multiplication:

$$kv = k(v, 2v, 4v, 6v, \dots)$$

$$= (kv, 2(kv), 4(kv) \dots)$$

Thus, is defined, Hence, it is subspace.

b) All sequences in R^∞ whose components are 0 from some point on.

let $v = (v_1, v_2, 0, 0, 0, \dots)$, $w = (w_1, w_2, 0, 0, 0, \dots)$

Adding:

$$v+w = (v_1, v_2, 0, 0, 0, \dots) + (w_1, w_2, 0, 0, 0, \dots)$$

$$v+w = (v_1+w_1, v_2+w_2, 0, 0, 0,)$$ — S.

Date:

Scalar multiple ⇒

$$kv = k(v_1, v_2, 0, 0, 0, \ldots)$$

$$= (kv_1, kv_2, 0, 0, 0) \text{ — } s$$

Thus, the set of all sequences in R^∞ whose component are 0 from some point on is a subspace of R^∞.

→ Q 11. Use the subspace test to determine which of the sets are subspace of M_{22}.

a) All matrices of the form $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$

let A and B two matrices.

$$A = \begin{bmatrix} a_1 & 0 \\ b_1 & 0_1 \end{bmatrix}$$

$$B = \begin{bmatrix} a_2 & 0 \\ b_2 & 0 \end{bmatrix}$$

Adding,

$$A + B = \begin{bmatrix} a_1 & 0 \\ b_1 & 0 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ b_2 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_1 + a_2 & 0 \\ b_1 + b_2 & 0 \end{bmatrix} \text{ — satisfy}$$

Scalar multiple:

$$kA = k \begin{bmatrix} a_1 & 0 \\ b_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} ka_1 & 0 \\ kb_1 & 0 \end{bmatrix} \text{ — satisfy}$$

thus, the set of all matrices of the form $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$ is subspace of M_{22}.

b) All 2x2 matrices A such that:

$$A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} \quad \text{— i)}$$

Consider:

$$B = \begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} \quad \text{— ii)}$$

Adding two matrices ⇒

$$A+B = A\begin{bmatrix}1\\-1\end{bmatrix} + B\begin{bmatrix}1\\-1\end{bmatrix}$$

$$(A+B)\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} + \begin{bmatrix}2\\0\end{bmatrix}$$

$$(A+B)\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix}$$

Thus, the resulting matrix is not defined form. So, $A+B \notin W$ It is not subspace of M_{22}.

→Q 12: b) All 2x2 matrices A such that

$$A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A$$

Consider two 2x2 matrix,

$$A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A$$

$$B\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}B$$

Adding,

$$A+B = A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} + B\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}$$

$$A+B = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A + \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}B$$

$$(A+B)\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}(A+B) \quad \text{— S.}$$

Date:

Scalar multiple:

$$(KA)\begin{bmatrix}0 & 2\\ -2 & 1\end{bmatrix} = K\left(A\begin{bmatrix}0 & 2\\ -2 & 1\end{bmatrix}\right)$$

$$'' \quad = K\begin{bmatrix}0 & 2\\ -2 & 1\end{bmatrix}A$$

$$(KA)\begin{bmatrix}0 & 2\\ -2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\ -2 & 1\end{bmatrix}(KA)$$ — satisfied.

Thus, is a subspace of M_{22}.

→ Q 13: Use the subspace test to determine which of the sets are subspace of R^4.

a) All vectors of the form (a, a^2, a^3, a^4)

let $V = (1, 1, 1, 1, \cancel{1})$

$w = (2, 4, 8, 16)$

Adding element ⇒

$$V + w = (1,1,1,1) + (2,4,8,16)$$

$$V + w = (1+2, 1+4, 1+8, 1+16)$$

$$V + w = (3, 5, 9, 17)$$

Thus, $V + w \notin W$ as the resulting vector is not defined form. (a, a^2, a^3, a^4).

b) All the vector of the form $(a, 0, b, 0)$

let $V = (a_1, 0, b_1, 0)$

$w = (a_2, 0, b_2, 0)$

Adding two vectors ⇒

$$V + w = (a_1, 0, b_1, 0) + (a_2, 0, b_2, 0)$$

$$V + w = (a_1 + a_2, 0, b_1 + b_2, 0)$$

Thus the resulting vector is defined.

Scalar Multiplication,

$$kv = k(a_1, 0, b_1, 0)$$

$$kv = (ka_1, 0, kb_1, 0)$$

Thus, $kv \in W$. Thus, the set of all vectors of the form $(a, 0, b, 0)$ is a subspace of R^4.

→ Q 14 # a) All vector x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

let ⇒

$$Au = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Adding,

$$Au + Av = A(u+v)$$

$$" = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$" = \begin{bmatrix} 0+0 \\ 1+1 \end{bmatrix}$$

$$Au + Av = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Thus, $Au + Av \notin W$. Hence, the vector x in R^4 is not subspace.

b) All the vector x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where A is as in (a)

The answer ~~same~~ and method is same in (a).

Date:

→ 15. a) All polynomials of degree less than or equal to 6.

Let W be set of all polynomials of degree less than or equal to 6.

→ Adding any two polynomials of degree less than or equal to 6 in W will only result in polynomial. So, W is under closed addition.

→ For any scalar, scalar multiplication of the polynomial results in a polynomial. So,

The set of all polynomials of degree less than or equal to 6 is subspace of R_∞.

b) All polynomials of degree equal to 6.

$f = x^3 - x^6$ $\quad f+g = (x^3 - x^6) + (x^6)$

$g = x^6$ $\quad f+g = x^3$

So, $f+g \notin W$. (Not subspace).

c) All polynomial of degree greater than or equal to 6.

$f = x^3 - x^7$ $\quad f+g = (x^3 - x^7) + (x^7)$

$g = x^7$ $\quad f+g = x^3$

So, $f+g \notin W$ (Not subspace).

→ 16. a) All polynomials with even co-efficients.

is subspace.

b) All polynomials with whose co-efficient sum to 0.

let $f = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$g = b_0 + b_1 x + b_2 x^4$

where $a_0 + a_1 + a_2 + a_3 = 0$

$b_0 + b_1 + b_2 = 0$.

Adding,

$$f+g = (a_0+a_1x+a_2x^2+a_3x^3)+(b_0+b_1x+b_2x^4)$$

$$f+g = (a_0+b_0)+(a_1+b_1)x+(a_2x^2+b_2x^4)+a_3x^3$$

$$f+g = (a_0+b_0)+(a_1+b_1)+a_2+a_3+b_2$$

$$'' = (a_0+a_1+a_2+a_3)+(b_0+b_1+b_2)$$

$$'' = 0+0$$

$$f+g = 0$$

Hence, satisfy

Scalar Multiple:

$$kf = k(a_0+a_1x+a_2x^2+a_3x^3)$$

$$kf = (ka_0)+(ka_1)x+(ka_2)x^2+(ka_3)x^3$$

$$'' = k(a_0+a_1+a_2+a_3)$$

$$'' = k(0)$$

$$kf = 0$$

Hence, it is subspace.

c) All polynomial of even degree.
is subspace.

(Exercise 4.3)

→ Q 1. Which of the following are linear combination of $u=(0,-2,2)$ and $v=(1,3,-1)$?

a) $(2,2,2)$:

let k_1 and k_2 are scalar. So,

Date:

$k_1(0,-2,2) + k_2(1,3,-1) = (2,2,2)$

forming equations:

$(0)k_1 + k_2 = 2$ —— i)

$-2k_1 + 3k_2 = 2$ —— ii)

$2k_1 - k_2 = 2$ —— iii)

from eq - i)

$\boxed{k_2 = 2}$

Putting in eq - 3

$2k_1 - k_2 = 2$

$2k_1 - 2 = 2$

$2k_1 = 4$

$k_1 = 4/2$

$\boxed{k_1 = 2}$

Putting k_2 & k_1 in eq - ii)

$-2(2) + 3(2) = 2$

$-4 + 6 = 2$

$\boxed{2 = 2}$

Hence, answer is same ~~as~~, so k_1 and k_2 must exist. $(2,2,2)$ is a linear combination of $u = (0,-2,2)$ and $v = (1,3,-1)$

b) $(0,4,5)$

$k_1(0,-2,2) + k_2(1,3,-1) = (0,4,5)$

forming equation:

$0k_1 + k_2 = 0$ —— i)

$-2k_1 + 3k_2 = 4$ —— ii)

$2k_1 - k_2 = 5$ —— iii)

from eq - i)

$\boxed{k_2 = 0}$

in eq - iii)

$2k_1 - 0 = 5 \Rightarrow \boxed{k_1 = 5/2}$

Putting k_2 & k_1 in eq - ii)

$-2(5/2) + 3(0) = 4$

$\boxed{-5 = 4}$

Thus, it is not linear combination.

c) $(0,0,0)$:

$$k_1(0,-2,2) + k_2(1,3,-1) = (0,0,0)$$

It is linear combination.

→ Q. 2. Express the following as linear combinations of $u=(2,1,4)$, $v=(1,-1,3)$ and $w=(3,2,5)$.

a) $(-9,-7,-15)$

$$(-9,-7,-15) = a_1(2,1,4) + a_2(1,-1,3) + a_3(3,2,5)$$

forming equations,

$$2a_1 + a_2 + 3a_3 = -9 \quad \text{—i)}$$

$$a_1 - a_2 + 2a_3 = -7 \quad \text{—ii)}$$

$$4a_1 + 3a_2 + 5a_3 = -15 \quad \text{—iii)}$$

~~Forming Matrix of Co-efficient ⇒~~

~~From eq-(ii)~~

$$\cancel{a_1 = a_2 - 2a_3 - 7} \quad \text{—iv)}$$

$$\cancel{a_2 = a_1 + 2a_3 + 7} \quad \text{—v)}$$

~~So, putting in eq-i)~~

$$\cancel{2(a_2 - 2a_3 - 7) + (a_1 + 2a_3 + 7) + 3a_3 = -7}$$

Forming Matrix of Co-efficient,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{vmatrix} \Rightarrow 2(-5-6) - 1(5-8) + (3+4)$$

$$|A| = -22 + 3 + 12$$

$$\boxed{|A| = -7}$$

Date:

Solve it, answer will,

$a_1 = -2 \quad , a_2 = 1 \; , \; a_3 = -2.$

b) $(6,11,6)$.

$(6,11,6) = a_1(2,1,4) + a_2(1,-1,3) + a_3(3,2,5)$

forming Equation ⇒

$2a_1 + a_2 + 3a_3 = 6$ —— i)

$a_1 - a_2 + 2a_3 = 11$ —— ii)

$4a_1 + 3a_2 + 5a_3 = 6.$ —— iii)

Adding eq - i) & ii)

$2a_1 + \cancel{a_2} + 3a_3 = 6$

$a_1 - \cancel{a_2} + 2a_3 = 11$

$3a_1 + 5a_3 = 17$

$a_1 = \dfrac{17 - 5a_3}{3}$

Putting eq - ii)

$\left(\dfrac{17 - 5a_3}{3}\right) - a_2 + 2a_3 = 11$

$\dfrac{17 - 5a_3 - 3a_2 + 6a_3}{3} = 11$

$17 - 3a_2 + a_3 = 33$

$a_3 = 33 - 17 + 3a_2$

$a_3 = 16 + 3a_2$

Solve it

$a_1 = 4, \; a_2 = -5, \; a_3 = 1.$

c) $(0,0,0)$

$a_1 = 0 \; , a_2 = 0 , a_3 = 0$

Date:

→Q 3. Which of following are linear combination of:

$$A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

a) $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = k_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = \begin{bmatrix} 4k_1 & 0 \\ -2k_1 & -2k_1 \end{bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ 2k_2 & 3k_2 \end{bmatrix} + \begin{bmatrix} 0 & 2k_3 \\ k_3 & 4k_3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = \begin{bmatrix} 4k_1 & 0 \\ -2k_1 & -2k_1 \end{bmatrix} + \begin{bmatrix} k_2 & -k_2+2k_3 \\ 2k_2+k_3 & 3k_2+4k_3 \end{bmatrix}$$

$4k_1 + k_2 = 6$ —— i)

$-k_2 + 2k_3 = -8$ —— ii)

$-2k_1 + 2k_2 + k_3 = -1$ —— iii)

$-2k_1 + 3k_2 + 4k_3 = -8$ —— iv)

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ -2 & 3 & 4 & -8 \end{array}\right] \xrightarrow[R_4+\frac{1}{2}R_1]{R_3+\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ 0 & 5/2 & 1 & 2 \\ 0 & 7/2 & 4 & -5 \end{array}\right] \xrightarrow[R_3-5/7R_2]{\text{swipe } R_2 \,\&\, R_4} \left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & 7/2 & 4 & -5 \\ 0 & 0 & -13/7 & 39/7 \\ 0 & -1 & 2 & -8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & 7/2 & 4 & -5 \\ 0 & 0 & -13/7 & 39/7 \\ 0 & -1 & 2 & -8 \end{array}\right] \xrightarrow[R_4+\frac{13}{22}R_3]{R_4+2/7R_2} \left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & 7/2 & 4 & -5 \\ 0 & 0 & 22/7 & -66/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solvecomplete

$k_1 = 1, \; k_2 = 2, \; k_3 = -3.$

Thus, It is linear combination.

→ Q 4. Determine whether the polynomial is a linear combination of

$p_1 = 2+x+x^2$; $p_2 = 1-x^2$, $p_3 = 1+2x$.

a) $1+x$

$$k_1p_1 + k_2p_2 + k_3p_3 = 1+x$$

$$k_1(2+x+x^2) + k_2(1-x^2) + k_3(1+2x) = 1+x.$$

forming equation ⇒

$$2k_1 + 2k_2 + k_3 = 1 \quad \text{— i)}$$

$$k_1 + 2k_3 = 1. \quad \text{— ii)}$$

$$k_1 - k_2 = 0 \quad \text{— iii)}$$

~~eq i)~~ & ~~iii) × 2~~.

$$|A| = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & -1 & 0 \end{vmatrix}$$

$$|A| = 2(0+2) - 2(0-2) + 1(-1-0)$$

$$|A| = 4+4-1$$

$$|A| = 7$$