"Mean and Variance of Binomial Distribution"

For $X \sim B(n, p)$

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

and

$$\text{standard deviation} = \sqrt{npq}$$

Q-1:- The probability that a patient recovers from a rare blood disease is 0.4. If 150 people are known to have contracted this disease,

a) What is average no. of people who survive?

Sol:- $X \sim B(150, 0.4)$

$\Rightarrow n = 150, \quad p = 0.4$

$\therefore$ Average no. of survival people $= np$

$= 150 \times 0.4$

$= 60.$

b) What is the variance value of recoverd people?

Sol:- Variance $= npq$

$= 150 \times 0.4 \times 0.6$

$= 36$

2- Poisson Distribution

Case 1:-

When average is given

average $= \lambda$

$$X \sim \text{Poisson}(\lambda)$$

with

$$f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

or

representing the no. of outcomes occuring in a given time interval or specified region denoted by t,

$$X \sim \text{Poisson}(\lambda t)$$

with

$$f(x) = \frac{e^{-\lambda t} (\lambda t)^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

where λ is the average number of outcomes per unit time, distance, area, or volume.

$\therefore$ Average $= \lambda t$

Q-1: A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make

a) no errors?

Sol:-

$$\text{Average} = \lambda = 2.$$

$$\therefore \quad X \sim \text{Poisson}(2)$$

with

$$P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

Thus

$$P(X=0) = ?$$

$$\Rightarrow \quad P(X=0) = \frac{e^{-2}\,2^{0}}{0!}$$

$$= 0.1353$$

b) 4 or more errors?

Sol:-

$$P(X \geq 4) = ?$$

$$\therefore \quad P(X \geq 4) = 1 - P(X \leq 3)$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3) \right]$$

$$= 1 - \left[\frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!} + \frac{e^{-2} 2^{3}}{3!}\right]$$

$$= 1 - e^{-2}\left[1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6}\right]$$

$$= 1 - 0.1353\left[1 + 2 + 2 + 1.33\right]$$

$$= 1 - 0.856$$

$$= 0.144$$

Q-2:- The no. of customers arriving per hour at a certain automobile service facility is assumed to follow a poisson distribution with $\lambda = 7$.

a) Compute the probability that more than 5 customers will arrive in a 2-hour period.

Sol:-

$$\lambda = 7$$

$$t = 2.$$

$$\therefore \text{Average} = \lambda t$$

$$= 7 \times 2$$

$$= 14$$

$P(X > 5) = ?$

$$X \sim \text{Poisson}(14)$$

with

$$f(x) = \frac{e^{-14}\, 14^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

$$\therefore P(X>5) = 1 - P[X \leq 5]$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$$

$$= 1 - \left[\frac{e^{-14}\, 14^{0}}{0!} + \frac{e^{-14}\, 14^{1}}{1!} + \frac{e^{-14}\, 14^{2}}{2!} + \frac{e^{-14}\, 14^{3}}{3!} + \frac{e^{-14}\, 14^{4}}{4!} + \frac{e^{-14}\, 14^{5}}{5!} \right]$$

$$= 1 - e^{-14} [1 + 14 + 98 + 457.33 + 1600.67 + 4481.87]$$

$$= 1 - 8.32 \times 10^{-7} (6652.87)$$

$$= 1 - 0.0055$$

$$= 0.9945$$

"Mean and Variance of Poisson Distribution"

for $X \sim \text{Poisson}(\lambda)$

$$\boxed{\text{Mean} = \lambda}$$

$$\boxed{\text{Variance} = \lambda}$$

and

$$\boxed{\text{Standard deviation} = \sqrt{\lambda}}$$

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(b):- Q-1:- What is the mean no. of arrivals during a 2-hours period?

Sol:- $\text{Mean} = \lambda t = 14$

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Q-2:- A certain area of the eastern United states is, on average, hit by 6 hurricans a year. find the mean and variance of the r.v X, representing the no. of hurricanes per year to hit a certain area of the eastern US.

Sol:- $\because$ Average $= \lambda = 6$

$\therefore$ Mean $= 6$

& Variance $= 6$.

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