"Measures of Variation" Lecture#11

1- Range

Range = maximum value − minimum value

$$\text{Range} = x_{max} - x_{min}$$

Good →	An overview
Drawback →	In case of outlier.

Q-1: find Range of the following values:

36, 7, 19, 26, 21, 57, 31, 48, 50, 73, 61, 68, 11, 40, 52.

Sol:-

$$\text{Range} = x_{max} - x_{min}$$

$$= 73 - 7$$

$$= 66$$

2- Interquartile Range:

Interquartile Range = 3^{rd} quartile − 1^{st} quartile

$$I.Q.R = Q_3 - Q_1$$

Good →	In case of outlier.

* **Quartile Deviation** → Semi - Interquartile Range.

$$Q.D = \frac{Q_3 - Q_1}{2}$$

. — .

Q-2:- 37, 7, 29, 12, 16, 8, 22, 11, 33, 30, 26, 307

find Quartile Deviation:

Sol:- Arranged data:

7, 8, 11, 12, 16, 22, 26, 29, 30, 33, 37, 307

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ value}$$

$$= \left(\frac{13}{4}\right)^{th} \text{ value} = 3.25^{th} \text{ value.}$$

$$\therefore Q_1 = 11 + 0.25(12-11)$$

$$= 11 + 0.25 = 11.25$$

$$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th} \text{ value}$$

$$= \left(\frac{3 \times 13}{4}\right)^{th} \text{ value} = 9.75^{th} \text{ value}$$

$$\therefore Q_3 = 30 + 0.75(33-30)$$

$$= 30 + 0.75 \times 3$$

$$= 30 + 2.25 = 32.25$$

$\therefore$ $Q.D = \dfrac{Q_3 - Q_1}{2}$

$= \dfrac{32.25 - 11.25}{2}$

$= 10.5$

Practice: Find Interquartile Range and Q.D of the following values:

43, 12, 8, 15, 6, 13, 27, 5, 31, 9, 17, 21, 38.

Sol:- $Q_1 = 8.5$

$Q_3 = 29.$

$\therefore$ $Q.D = \dfrac{Q_3 - Q_1}{2} = \dfrac{29 - 8.5}{2} = 10.25$

and

$I.Q.R = Q_3 - Q_1 = 29 - 8.5 = 20.5$

"Population Variance and Standard deviation"

denoted by

Variance $= \sigma^2$

Standard Deviation $= \sigma$.

If $S.D = 3$, then variance = 9.

$\therefore S.D = \sqrt{Variance}$

or

$Variance = S.D^2$

* The variance of 15 values having $S.D = +7$, is ________.

* The S.D of 11 values having Variance = 10 is ________.

Pop. Variance $= \sigma^2 = \frac{\sum(x-\bar{x})^2}{N}$

or $\sigma^2 = \frac{\sum x^2}{N} - \left[\frac{\sum x}{N}\right]^2$

$S.D = \pm\sqrt{Var}$

Sample Variance and Standard deviation)

denoted by

Sample variance = S^2

Sample Standard deviation = S

$\therefore$ $\boxed{\text{Sample variance}} = \boxed{\dfrac{\Sigma(x-\bar{x})^2}{n-1}}$

or $\quad S^2 = \dfrac{1}{n-1}\left[\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}\right]$

and $\quad \boxed{S.D = \pm\sqrt{Var}}$

Q-3: find Sample variance and sample S.D of the following values:

7, 9, 2, 6, 11, 8, 3

Sol.-

X	X^2	$(X-6.57)^2$
7	49	0.1849
9	81	5.9049
2	4	20.8849
6	36	0.3249
11	121	19.6249
8	64	2.0449
3	9	12.7449
46	364	61.7143

$$\bar{X} = \frac{\Sigma X}{n} = \frac{46}{7} = 6.57$$

Method I:-

Sample variance $= S^2$

$$= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]$$

$$= \frac{1}{7-1}\left[364 - \frac{(46)^2}{7}\right]$$

$$= \frac{1}{6}\left[61.71\right]$$

$$= 10.29$$

Method II:-

Sample variance $= S^2$

$$= \frac{\Sigma(X-\bar{X})^2}{n-1}$$

$$= \frac{61.7143}{7-1} = 10.29$$

$\therefore$ Sample S.D $= S = \sqrt{10.29}$

$\Rightarrow S = 3.21$

Activity:-

For the following data sets.

11, 9, 15, 3, 8, 17, 21, 25, 13.

Find following measures of variations:

i) Range

ii) Interquartile Range

iii) Quartile Deviation

iv) Population Variance

v) Population Standard deviation

vi) Sample Variance

vii) Sample Standard deviation.

Sol.

Required Calculations:

$\boxed{\text{Minimum}} = 3$, $\boxed{\text{Maximum}} = 25$

$\boxed{n} = 9$ $\boxed{\Sigma X} = 122$ $\boxed{\Sigma X^2} = 2024$

$\boxed{Q_1} = 8.5$ $\boxed{Q_3} = 19$

$\boxed{I.Q.R} = 10.5$

$\boxed{Q.D} = 5.25$

$\boxed{\Sigma(X-\bar{X})^2} = 370.22$

$\boxed{\sigma^2} = 41.14$

$\boxed{\sigma} = 6.41$

$\boxed{S^2} = 46.28$

$\boxed{S} = 6.8$