CHAPTER ONE

(Exercise # 1.1) Q(1-6, 15-20) Date ....

"DOMAIN & RANGE OF FUNCTIONS"

* In Exercise 1-6, Find The domain and range of each following functions:

1) $f(x) = 1 + x^2$

Sol, $1+x^2$]- linear function

$\boxed{\text{Domain} = (-\infty, \infty)}$

As we know The value of x^2 is always non-negative, That is since The square of any quantity is non-negative.

let ⇒

x	$y = f(x) = 1 + x^2$	Output
1	$1 + (1)^2$	2
2	$1 + (2)^2$	5
0	$1 + (0)^2$	1
-2	$1 + (-2)^2$	5
-1	$1 + (-1)^2$	2

So,

$$x^2 \geq 0$$

$$1 + x^2 \geq 0 + 1 \Rightarrow 1 + x^2 \geq 1$$

So,

$\boxed{\text{Range} = [1, \infty)}$

2) $f(x) = 1 - \sqrt{x}$

Sol, Function $= 1 - \sqrt{x}$

The value inside The radical Takes real value only When The value is non-negative.

So, $1 - \sqrt{x}$ function takes values only for non-negative values x.

So,

$\boxed{\text{Domain} = [0, \infty)}$

So, As we take non-negative values x. So,

$$\sqrt{x} \geq 0$$

Moreover, let say:

x	$y = f(x) = 1 - \sqrt{x}$	Output
2	$1 - \sqrt{2}$	-0.41
1	$1 - \sqrt{1}$	0
0	$1 - \sqrt{0}$	1
+3	$1 - \sqrt{+3}$	-0.73
+4	$1 - \sqrt{+4}$	-1

So,

$$-\sqrt{x} \leq 0$$

$$1 - \sqrt{x} \leq 1 - 0$$

$$1 - \sqrt{x} \leq 1$$

So,

$\boxed{\text{Range} = (-\infty, 1]}$

3) $f(x) = \sqrt{5x + 10}$

Sol,

$$\sqrt{5x + 10}$$

Date ....

So, we are taking,

$$5x+10 \geq 0$$

$$5x \geq -10$$

$$x \geq -10/5$$

$$\boxed{x \geq -2}$$

So,

$$\boxed{\text{Domain} = [-2, \infty)}$$

let suppose,

x	$y=f(x)=\sqrt{5x+10}$		Output
-2	$\sqrt{5(-2)+10}$	=	0
-1	$\sqrt{5(-1)+10}$	=	$\sqrt{5}$
0	$\sqrt{5(0)+10}$	=	$\sqrt{10}$
1	$\sqrt{5(1)+10}$	=	$\sqrt{15}$
2	$\sqrt{5(2)+10}$	=	$\sqrt{20}$

So,

$$\boxed{\text{Range} = [0, \infty)}$$

4) $g(x) = \sqrt{x^2-3x}$

let:

$$x^2 - 3x \geq 0$$

using completing square

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 \geq \left(\frac{3}{2}\right)^2$$

$$\sqrt{\left(x-\frac{3}{2}\right)^2} \geq \sqrt{\left(\frac{3}{2}\right)^2}$$

$$\left(x-\frac{3}{2}\right) \geq \pm\frac{3}{2}$$

So,

$x-\frac{3}{2} \geq \frac{3}{2}$	$x-\frac{3}{2} \leq -\frac{3}{2}$
$x \geq \frac{3}{2}+\frac{3}{2}$	$x \leq \frac{3}{2}-\frac{3}{2}$
$\therefore \boxed{x \geq 3}$	$\boxed{x \leq 0}$

So,

$$\boxed{\text{Domain} = (-\infty, 0] \cup [3, \infty)}$$

So, as the square root of any number x will be positive. So, the range will be non-negative.

let

x	$y=g(x)=\sqrt{x^2-3x}$	Output
-2	$\sqrt{(-2)^2-3(-2)}$	$\sqrt{10}$
-1	$\sqrt{(-1)^2-3(-1)}$	2
0	$\sqrt{(0)^2-3(0)}$	0
4	$\sqrt{(4)^2-3(4)}$	2
3	$\sqrt{(3)^2-3(3)}$	0

So,

$$\boxed{\text{Range} = [0, \infty)}$$

5) $f(t) = \dfrac{4}{3-t}$

Sol, First we check that which values make it undefined.

So,

$$3-t=0$$

$$\boxed{t=3}$$

If $t=3$, the function will undefined.

So,

$$\boxed{\text{Domain} = \mathbb{R} - \{3\}}$$ OR $(-\infty, 3) \cup (3, \infty)$

Here:

$\mathbb{R}$ = Real Number.

Date ....

As we know, let,

$$\frac{4}{3-t} = y$$

$$4 = y(3-t)$$

$$\frac{4}{y} = 3-t$$

$$t = 3 - \frac{4}{y}$$

$$t = \frac{3y-4}{y}$$

$$t = \frac{3\cancel{y}}{\cancel{y}} - \frac{4}{y}$$

$$t = 3 - \frac{4}{y}$$

$$\boxed{t = \frac{3y-4}{y}}$$

So,
in This case
only zero make
This function
undefine. So,

$$\boxed{\text{Range} = \mathbb{R} - \{0\}}$$

OR

$$(-\infty, 0) \cup (0, \infty)$$

6) $G(t) = \dfrac{2}{t^2-16}$

Sol,

$$G(t) = \frac{2}{t^2-16}$$

We will check which makes the function undefined.

$$t^2 - 16 = 0$$

$$\sqrt{t^2} = \sqrt{16}$$

$$\boxed{t = \pm 4}$$

So, the value $+4$ and -4 makes the function undefined.

$$\boxed{\text{Domain} = \mathbb{R} - \{-4, 4\}}$$

OR

$$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$$

let,

$$\frac{2}{t^2-16} = y \Rightarrow \frac{2}{y} = t^2 - 16$$

$$t^2 = \frac{2+16y}{y}$$

$$\boxed{t = \sqrt{\frac{2}{y}} + 2}$$

So, Here only zero makes the function undefined. So,

$$\boxed{\text{Range} = \mathbb{R} - \{0\}}$$

OR

$$(-\infty, 0) \cup (0, \infty)$$

Find The Domain & Graph the functions in Exercise 15-20.

15) $f(x) = 5 - 2x$

Sol,

$$f(x) = 5 - 2x$$

Date

As we this function is linear. So,
The graph will be linear/straight.
So,

Domain = $(-\infty, \infty)$

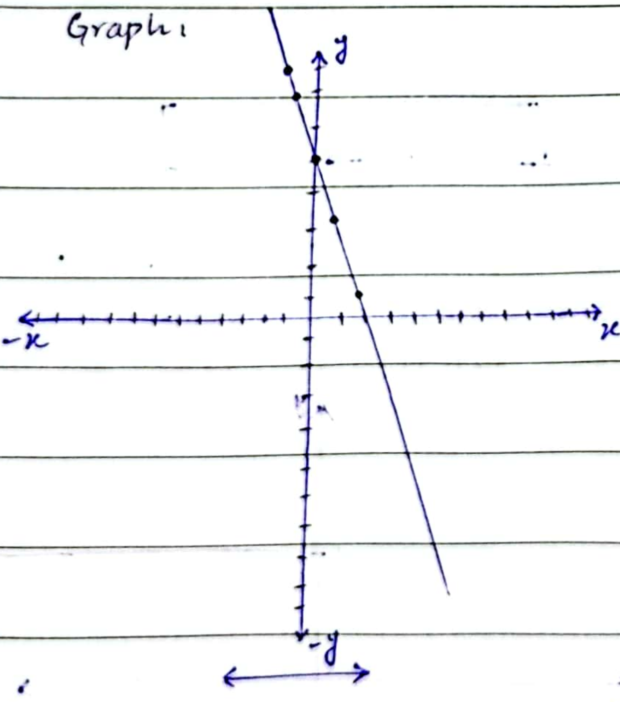
Let,

x	$y=f(x)=5-2x$	Output
0	$5-2(0)$	5
1	$5-2(1)$	3
2	$5-2(2)$	1
-2	$5-2(-2)$	9
-3	$5-2(-3)$	11

So, making set,

(0,5), (1,3), (2,1), (-2,9), (-3,11)

Graph:



16) $f(x) = 1-2x-x^2$

Sol,
As this function is Quadratic Function So, The graph is parabolic graph.
So,
Domain = $(-\infty, \infty)$
Using Quadratic Formula,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=-1$, $b=-2$, $c=+1$

$$x = \frac{+2 \pm \sqrt{(-2)^2-4(1)(-1)}}{2(-1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{-2}$$

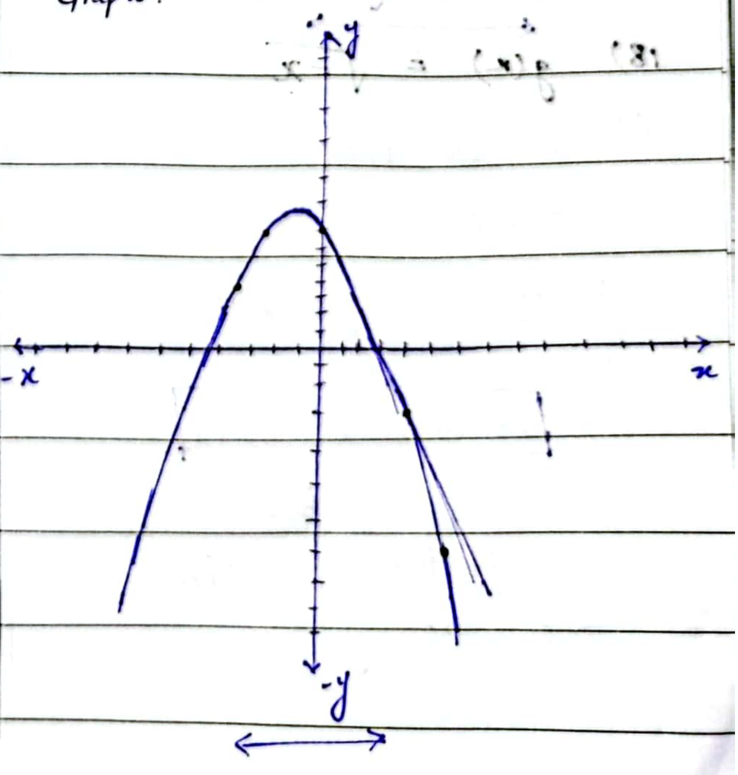
$$\boxed{x = -1 \pm \sqrt{2}}$$

Let,

x	$y=f(x)=1-2x-x^2$	Output
0	$1-2(0)-(0)^2$	+1
1	$1-2(1)-(1)^2$	-2
2	$1-2(2)-(2)^2$	-7
~~4~~ -2	$1-2(-2)-(-2)^2$	1
-1	$1-2(-1)-(-1)^2$	2

(0,1), (1,-2), (2,-7), (-2,1), (-1,2)

Graph:



Date ....

17) $g(x) = \sqrt{|x|}$

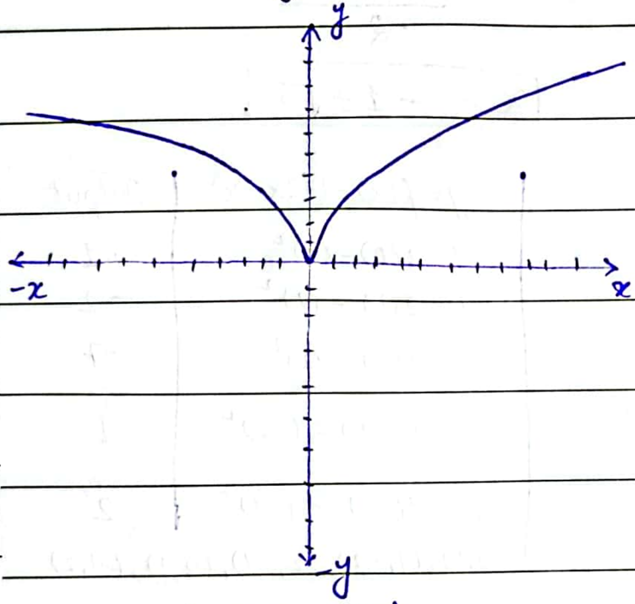
Sol:

$\boxed{\text{Domain} = (-\infty, \infty)}$

As

$\sqrt{x} \geq 0$

~~The Graph looks like~~ $\sqrt{x}$, when $x \geq 0$ and is symmetrical with respect to y axis



18) $g(x) = \sqrt{-x}$

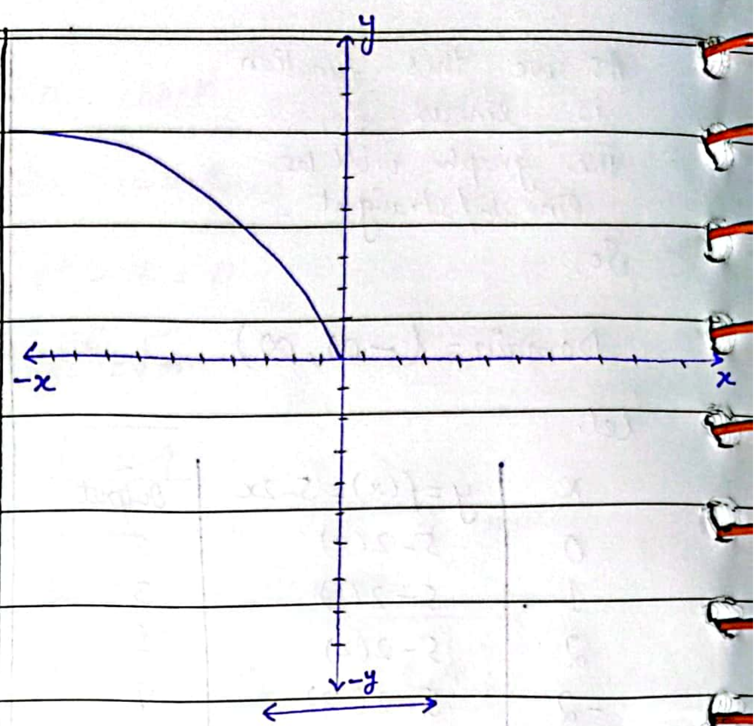
So, As we know

$x \leq 0$

As of -ve sign since the

$\boxed{\text{Domain} = (-\infty, 0]}$

The Graph of function is reflected of $y = \sqrt{x}$ over the y axis.



19) $F(t) = t/|t|$

As we know the function $t/|t|$. So, the zero can make it undefined. So,

$\boxed{\text{Domain} = \mathbb{R} - \{0\}}$

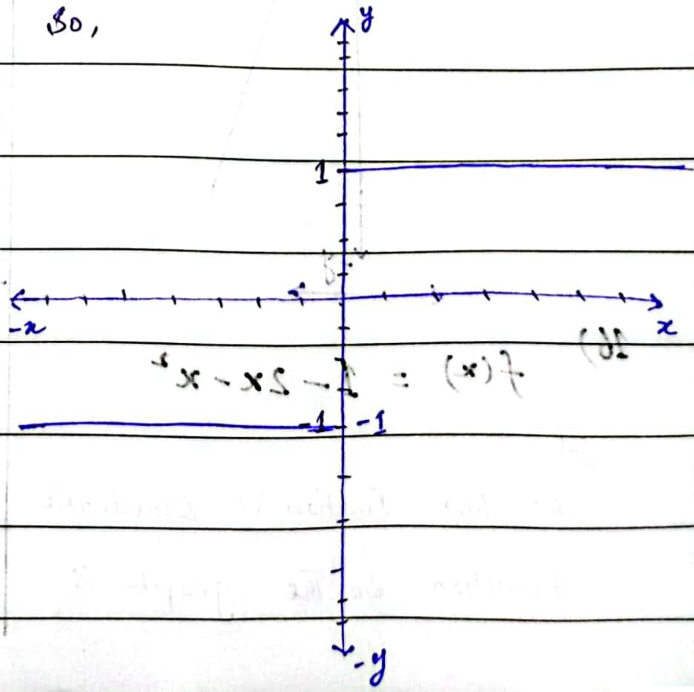
OR

$(-\infty, 0) \cup (0, \infty)$

when $t < 0$, $|t| = -t$ value $= -1$

~~when $t < 0$, $|t| = t$~~ value $= 1$

So,



Date ....

20) $G(t) = 1/|t|$

Same as in 19 question.

Denominator should be non-zero.

So,

$\boxed{\text{Domain} = \mathbb{R} - \{0\}}$

OR

$(-\infty, 0) \cup (0, \infty)$

Graph of function is $1/t$, when it is positive, when its negative, graph reflects its directions.