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(Exercise # 7.2) Q(5-34)

Find the derivatives:

(5) $y = \ln 3x$

Sol:

$y' = \frac{1}{3x}\frac{d}{dx}(3x)$

$y' = \frac{1}{3x}(3)$

$\boxed{y' = 1/x}$

(6) $y = \ln Kx$, K constant

Sol:

$y' = \frac{1}{Kx}\frac{d}{dx}(Kx)$

$y' = \frac{1}{Kx}(K)$

$\boxed{y' = 1/x}$

(7) $y = \ln(t^2)$

Sol:

$y' = \frac{1}{t^2}\frac{d}{dt}(t^2)$

$y' = \frac{1}{t^2}(2t)$

$y' = \frac{2t}{t^2} \Rightarrow \boxed{y' = \frac{2}{t}}$

(8) $y = \ln(t^{3/2})$

Sol:

$y' = \frac{1}{t^{3/2}}\frac{d}{dt}(t^{3/2})$

$y' = \frac{1}{t^{3/2}}\left(\frac{3}{2}t^{1/2}\right)$

$y' = \frac{3}{2}t^{\frac{1}{2}-\frac{3}{2}}$

$y' = \frac{3}{2}t^{-1}$

$\boxed{y' = \frac{3}{2t}}$

(9) $y = \ln\frac{3}{x}$

Sol:

$y' = \frac{1}{3x^{-1}}\frac{d}{dx}(3x^{-1})$

$y' = \frac{x}{3}(-3x^{-2})$

$\boxed{y' = -\frac{1}{x}}$

(10) $y = \ln\frac{10}{x}$

Sol:

$y' = \frac{1}{10x^{-1}}\frac{d}{dx}(10x^{-1})$

$y' = \frac{x}{10}(-10x^{-2})$

$\boxed{y' = -\frac{1}{x}}$

(11) $y = \ln(\theta+1)$

$y' = \frac{1}{\theta+1}\frac{d}{d\theta}(\theta+1)$

$y' = \frac{1}{\theta+1}(1)$

$\boxed{y' = \frac{1}{\theta+1}}$

(12) $y = \ln(2\theta+2)$

Sol:

$y' = \frac{1}{2(\theta+1)}\frac{d}{d\theta}(2\theta+2)$

$y' = \frac{1}{2\theta+2}(2)$

$y' = \frac{1}{2(\theta+1)}(2)$

$\boxed{y' = \frac{1}{\theta+1}}$

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(13) $y = \ln x^3$

Sol,

$y' = \frac{1}{x^3}\frac{d}{dx}x^3$

$y' = \frac{1}{x^3}(3x^2)$

$\boxed{y' = \frac{3}{x}}$

(14) $y = (\ln x)^3$

Sol,

$y' = \frac{d}{dx}(\ln x)^3$

$y' = 3(\ln x)^2 \cdot \frac{d}{dx}x(\ln x)$

$\boxed{y' = \frac{3(\ln x)^2}{x}}$

(15) $y = t(\ln t)^2$

Sol,

$y = t\frac{d}{dt}(\ln t)^2 + (\ln t)^2\frac{d}{dt}(t)$

$y = 2t(\ln t)\cdot\frac{d}{dt}(\ln t) + (\ln t)^2$

$y' = 2t(\ln t)\frac{1}{t}\frac{d}{dt}(t) + (\ln t)^2$

$\boxed{y' = (\ln t)^2 + 2\ln t}$

(16) $y = t\sqrt{\ln t}$

$y = t(\ln t)^{1/2}$

$y' = t\frac{d}{dt}(\ln t)^{1/2} + (\ln t)^{1/2}\frac{d}{dt}(t)$

$y' = t\frac{1}{2}(\ln t)^{-1/2}\frac{d}{dt}(\ln t) + (\ln t)^{1/2}$

$y' = \frac{t(\ln t)^{-1/2}}{2t} + (\ln t)^{1/2}$

$\boxed{y' = (\ln t)^{1/2} + \frac{1}{2(\ln t)^{1/2}}}$

(17) $y = \frac{x^4}{4}\ln x - \frac{x^4}{16}$

Sol,

$y = \frac{1}{4}\left[x^4\frac{d}{dx}\ln x + \ln x\frac{d}{dx}x^4\right] - \frac{1}{16}\frac{d}{dx}x^4$

$y' = \frac{1}{4}\left[x^4\cdot\frac{1}{x} + \ln x\,4x^3\right] - \frac{4}{16}x^3$

$y' = \frac{x^3}{4} + \frac{4\ln x\cdot x^3}{4} - \frac{1}{4}x^3$

$\boxed{y' = x^3\ln x}$

(18) $y = (x^2\ln x)^4$

$y' = 4(x^2\ln x)^3\left[\frac{d}{dx}(x^2\ln x)\right]$

$y' = 4(x^2\ln x)^3\left[x^2\frac{d}{dx}\ln x + \ln x\frac{d}{dx}x^2\right]$

$y' = 4(x^2\ln x)^3\left[x^2\frac{1}{x} + \ln x\,2x\right]$

$y' = 4x^6\ln^3(x + 2x\ln x)$

$y' = 4x^6(\ln^3)(x + 2x\ln x)$

$\boxed{y' = 4x^7(\ln x)^3 + 8x^7(\ln x)^4}$

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(19) $y = \frac{\ln t}{t}$

$y' = \frac{t \frac{d}{dt} \ln t - \ln t \frac{d}{dt} t}{t^2}$

$y' = \frac{t(\frac{1}{t}) - \ln t}{t^2}$

$\boxed{y' = \frac{1 - \ln t}{t^2}}$

(20) $y = \frac{1 + \ln t}{t}$

$y' = \frac{+\frac{d}{dt}(1+\ln t) - (1+\ln t)\frac{d}{dt} t}{t^2} \Rightarrow \frac{t(\frac{1}{t}) - (1+\ln t)}{t^2}$

$y' = \frac{1 - 1 - \ln t}{t^2} \Rightarrow \boxed{y' = -\frac{\ln t}{t^2}}$

(21) $y = \frac{\ln x}{1 + \ln x}$

$y' = \frac{(1+\ln x)\frac{d}{dx}\ln x - \ln x \frac{d}{dx}(1+\ln x)}{(1+\ln x)^2}$

$y' = \frac{(1+\ln x)(\frac{1}{x}) - \ln x(\frac{1}{x})}{(1+\ln x)^2}$

$y' = \frac{1 + \ln x - \ln x}{x(1+\ln x)^2}$

$\boxed{y' = \frac{1}{x(1+\ln x)^2}}$

(22) $y = \frac{x \ln x}{1 + \ln x}$

$y' = \frac{(1+\ln x)\left[x(\frac{1}{x}) + \ln x\right] - x \ln x (0 + \frac{1}{x})}{(1+\ln x)^2}$

$y' = \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$

$y' = \frac{(1+\ln x)^2}{(1+\ln x)^2} - \frac{\ln x}{(1+\ln x)^2}$

$\boxed{y' = -\frac{\ln x}{(1+\ln x)^2}}$

(23) $y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \frac{d}{dx} \ln x$

$y' = \frac{1}{\ln x} \frac{1}{x} \frac{d}{dx}(x)$

$\boxed{y' = \frac{1}{x \ln x}}$

(24) $y = \ln(\ln(\ln x))$

$y' = \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx} \ln(\ln x))$

$y' = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

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$$\boxed{y' = \frac{1}{x(\ln x)\ln(\ln x)}}$$

(25) $y = \theta(\sin(\ln\theta) + \cos(\ln\theta))$

$$y' = \theta\left[\frac{d}{d\theta}\sin(\ln\theta) + \frac{d}{d\theta}\cos(\ln\theta)\right] + (\sin(\ln\theta) + \cos(\ln\theta))\frac{d}{d\theta}\theta$$

$$y' = \theta\left[\cos(\ln\theta)\frac{1}{\theta} + (-\sin(\ln\theta)\frac{1}{\theta})\right] + \sin(\ln\theta) + \cos(\ln\theta)$$

$$y' = \cos(\ln\theta) - \sin(\ln\theta) + \sin(\ln\theta) + \cos(\ln\theta)$$

$$\boxed{y' = 2\cos(\ln\theta)}$$

(26) $y = \ln(\sec\theta + \tan\theta)$

$$y' = \frac{1}{\sec\theta + \tan\theta}\frac{d}{d\theta}(\sec\theta + \tan\theta)$$

$$y' = \frac{1}{\sec\theta + \tan\theta}\left[\sec\theta\tan\theta + \sec^2\theta\right]$$

$$y' = \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta\tan\theta}$$

$$y' = \frac{\sec\theta\tan\theta}{\sec\theta\tan\theta} + \frac{\sec^2\theta}{\sec\theta\tan\theta}$$

$$y' = \frac{\sec\theta}{\tan\theta} \Leftarrow \boxed{\sec\theta}$$

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(27) $y = \ln \frac{1}{x\sqrt{x+1}}$

$y = \ln (x^{-1}(x+1)^{-1/2})$

$y' = -\ln(x)' - \frac{1}{2}\ln(x+1)$

$y' = -\frac{1}{x} - \frac{1}{2(x+1)} \frac{d}{dx}(x+1)$

$y' = -\frac{1}{x} - \frac{1}{2(x+1)}$

$y' = \frac{-2(x+1) - x}{2x(x+1)}$

$y' = \frac{-2x - 2 - x}{2x(x+1)}$

$$\boxed{y' = \frac{-(3x+2)}{2x(x+1)}}$$

(28) $y = \frac{1}{2}\ln \frac{1+x}{1-x}$

$y' = \frac{1}{2}\left[\ln(1+x) - \ln(1-x)\right]$

$y' = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]$

$y' = \frac{1}{2}\left[\frac{(1-x) + (1+x)}{(1+x)(1-x)}\right]$

$y' = \frac{1}{2}\left[\frac{1 - x + 1 + x}{(1+x)(1-x)}\right]$

$y' = \frac{2}{2(1+x)(1-x)}$

$$\boxed{y' = \frac{1}{(1+x)(1-x)}}$$

$$\boxed{y' = \frac{1}{1-x^2}}$$

(29) $y = \frac{1+\ln t}{1-\ln t}$

Sol

$y' = \frac{(1-\ln t)(\frac{1}{t}) - (1+\ln t)(-\frac{1}{t})}{(1-\ln t)^2}$

$y' = \frac{1 - \ln t + 1 + \ln t}{t(1-\ln t)^2}$

$$\boxed{y' = \frac{2}{t(1-\ln t)^2}}$$

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(30) $y = \sqrt{\ln\sqrt{t}}$

$y = (\ln t^{1/2})^{1/2}$

$y' = \frac{1}{2}(\ln t^{1/2})^{-1/2} \frac{d}{dt}(\ln t^{1/2})$

$y' = \frac{1}{2}(\ln t^{1/2})^{-1/2} \cdot \frac{1}{t^{1/2}} \frac{d}{dt} t^{1/2}$

$y' = \frac{1}{2}(\ln t^{1/2})^{-1/2} \cdot \frac{1}{t^{1/2}} \left(\frac{1}{2} t^{-1/2}\right)$

$y' = \frac{1}{t\sqrt{\ln\sqrt{t}}}$

$$\boxed{y' = \frac{1}{4t\sqrt{\ln\sqrt{t}}}}$$

(31) $y = \ln(\sec(\ln\theta))$

$y = \frac{1}{\sec(\ln\theta)} \frac{d}{d\theta}(\sec(\ln\theta))$

$y' = \frac{1}{\sec(\ln\theta)} \sec(\ln\theta)\tan\ln\theta \frac{d}{d\theta}\ln\theta$

$y' = \frac{1}{\sec(\ln\theta)} \sec(\ln\theta)\tan(\ln\theta)\frac{1}{\theta}$

$$\boxed{y' = \frac{\tan(\ln\theta)}{\theta}}$$

(32) $y = \ln\left(\frac{\sqrt{\sin\theta\cos\theta}}{1+2\ln\theta}\right)$

$y = \ln(\sin\theta\cos\theta)^{1/2} - \ln(1+2\ln\theta)$

$y' = \frac{1}{2}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\sin\theta + \frac{1}{\cos\theta}\frac{d}{d\theta}\cos\theta\right) - \frac{1}{1+2\ln\theta}\frac{d}{d\theta}(1+2\ln\theta)$

$y' = \frac{1}{2}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) - \frac{1}{1+2\ln\theta}\left(2\frac{1}{\theta}\right)$

$y' = \frac{1}{2}(\cot\theta - \tan\theta) - \frac{2}{\theta(1+2\ln\theta)}$

$$\boxed{y' = \frac{1}{2}\left[\cot\theta - \tan\theta - \frac{4}{\theta(1+2\ln\theta}\right]}$$

(33) $y = \ln\left(\frac{(x^2+1)^5}{\sqrt{1-x}}\right)$

$y' = \ln(x^2+1)^5 - \ln(1-x)^{1/2}$

$y' = 5\ln(x^2+1) - \frac{1}{2}\ln(1-x)$

$y' = \frac{5}{x^2+1}\frac{d}{dx}(x^2+1) - \frac{1}{2(1-x)}\frac{d}{dx}(1-x)$

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$$y' = \frac{5}{x^2+1}(2x) - \frac{1}{2(1-x)}(-1)$$

$$\boxed{y' = \frac{10x}{x^2+1} + \frac{1}{2(1-x)}}$$

(34) $y = \ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}}$

Sol,

$$y' = \ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)^{1/2}$$

$$y' = \frac{1}{2}\left[\ln(x+1)^5 - \ln(x+2)^{20}\right]$$

$$y' = \frac{1}{2}\left[5\ln(x+1) - 20\ln(x+2)\right]$$

$$y' = \frac{1}{2}\left[\frac{5}{x+1}\frac{d}{dx}(x+1) - \frac{20}{x+2}\frac{d}{dx}(x+2)\right]$$

$$y' = \frac{1}{2}\left[\frac{5}{x+1} - \frac{20}{x+2}\right] \Rightarrow \frac{1}{2}\left[\frac{5(x+2) - 20(x+1)}{(x+1)(x+2)}\right]$$

$$y' = \frac{1}{2}\left[\frac{-15x-10}{(x+1)(x+2)}\right]$$

$$\boxed{y' = -\frac{5}{2}\left[\frac{3x+2}{(x+1)(x+2)}\right]}$$