$\frac{1}{2}\int u^{-3}\,du$

$-\frac{1}{4u^2}+c$

$\boxed{-\frac{1}{4(x^2-4)^2}+c}$

$\int u^{-2}+\frac{4}{u^3}$

$\int u^{-2}du+4\int u^{-3}du$

$-\frac{1}{u}-\frac{2}{u^2}+c$

Exercise 7.2 (37–54)

37 $\int_{-3}^{-2}\frac{dx}{x}$

$\int_{-3}^{-2}\frac{1}{x}\,dx$

$[\ln|x|]_{-3}^{-2}$

$\ln|-2|-\ln|-3|$

$\boxed{\ln\frac{2}{3}}$

38 $\int_{-1}^{0}\frac{3\,dx}{3x-2}$

$[\ln|3x-2|]_{-1}^{0}$

$\ln|3(0)-2|-\ln|3(-1)-2|$

$\ln 2-\ln 5$

$\boxed{\ln\frac{2}{5}}$.

39 $\int\frac{2y}{y^2-25}\,dy$

$\boxed{\ln|y^2-25|+c}$

40 $\int\frac{8r}{4r^2-5}\,dr$

$\boxed{\ln|4r^2-5|+c}$

Date ....

(41) $\int_0^{\pi} \frac{\sin t}{2-\cos t}\, dt$

$\left[\ln|2-\cos t|\right]_0^{\pi}$

$\ln|2-\cos\pi| - \ln|2-\cos(0)|$

$\ln|2-(-1)| - \ln|2-1|$

$\ln 3 - \ln 1$

$\boxed{\ln 3}$.

(42) $\int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\, d\theta$

$\left[\ln|1-4\cos\theta|\right]_0^{\pi/3}$

$\ln|1-4\cos\frac{\pi}{3}| - \ln|1-4\cos(0)|$

$\ln|-1| - \ln|-3|$

$\boxed{\ln(\frac{1}{3})}$ OR $\boxed{-\ln 3}$.

(43) $\int_1^2 \frac{2\ln x}{x}\, dx$

$\cancel{\ln x}$ $u=\ln x$

$du = \frac{dx}{x}$

$\int_1^2 2u\, du$

$[u^2]_1^2$ $\cancel{+C}$

$\cancel{[(2)^2-(1)^2]}$

$[(\ln x)^2]_1^2$

$\boxed{(\ln 2)^2}$.

(44) $\int_2^4 \frac{dx}{x\ln x}$

$u=\ln x$

$du = \frac{dx}{x}$

$\int_2^4 \frac{1}{u}\, du$

$[\ln u]_2^4$

$[\ln|\ln x|]_2^4$

$\ln(\ln 4) - \ln(\ln 2)$

$\ln\left(\frac{\ln 4}{\ln 2}\right)$

$\ln\left(\frac{2\ln 2 \cdot \cancel{\ln 2}}{\cancel{\ln 2}}\right)$

$\ln\left(\frac{\cancel{\ln 2}}{2}\right) = \boxed{\ln 2}$.

(45) $\int_2^4 \frac{dx}{x(\ln x)^2}$

$u=\ln x$

$du = \frac{dx}{x}$

$\int_2^4 \frac{1}{u^2}\, du \Rightarrow \int_2^4 u^{-2}\, du$

$\left[-\frac{1}{u}\right]_2^4 \Rightarrow \left[-\frac{1}{\ln x}\right]_2^4$

$-\left[+\frac{1}{\ln 4} \mp \frac{1}{\ln 2}\right]$

$-\frac{1}{2\ln 2} + \frac{1}{\ln 2}$

$\frac{-1+2}{2\ln 2} \Rightarrow \frac{1}{2\ln 2}$

$\frac{1}{\ln 2^2} \Rightarrow \boxed{\frac{1}{\ln 4}}$.

Date ....

(46) $\int_2^{16} \frac{dx}{2x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{dx}{x}$

$\frac{1}{2}\int_2^{16} \frac{1}{\sqrt{u}}\, du$

$\frac{1}{2}\int_2^{16} u^{-1/2}\, du$

$\left[\frac{1}{\cancel{2}}\left(\cancel{2}u^{1/2}\right)\right]_2^{16}$

$\left[(\ln x)^{1/2}\right]_2^{16}$

$\sqrt{\ln 16} - \sqrt{\ln 2}$

$\sqrt{4\ln 2} - \sqrt{\ln 2}$

$2\sqrt{\ln 2} - \sqrt{\ln 2}$

$\boxed{\sqrt{\ln 2}}$.

(47) $\int \frac{3\sec^2 t}{6+3\tan t}\, dt$

$u = 6 + 3\tan t$

$du = 3\sec^2 t\, dt$

$\int \frac{1}{u}\, du$

$\ln|u| + C$

$\boxed{\ln|6+3\tan t| + C}$

(48) $\int \frac{\sec y \tan y}{2+\sec y}\, dy$.

$u = 2 + \sec y$

$du = \sec y \tan y\, dy$

$\int \frac{1}{u}\, du$

$\ln|u| + C$

$\boxed{\ln|2+\sec y| + C}$

(49) $\int_0^{\pi/2} \tan\frac{x}{2}\, dx \Rightarrow \int_0^{\pi/2} \frac{\sin(\pi/2)}{\cos(\pi/2)}\, dx$.

$u = \cos\frac{x}{2}$

$du = \frac{1}{2}\sin\frac{x}{2}\, dx$

$2du = \sin\frac{x}{2}\, dx$

$\int_0^{\pi/2} \frac{2}{u}\, du \Rightarrow \left[2\ln|u|\right]_0^{\pi/2}$

$\left[2\ln\left(\cos\frac{x}{2}\right)\right]_0^{\pi/2}$

$2\left[\ln\left(\cos\frac{\pi/2}{2}\right) - \ln(\cos 0)\right]$

$-2\ln\frac{1}{\sqrt{2}}$

$2\ln\sqrt{2}$

$\boxed{\ln 2}$.

(50) $\int_{\pi/4}^{\pi/2} \cot t\, dt$

$\int_{\pi/4}^{\pi/2} \frac{\cos t}{\sin t}\, dt$

$u = \sin t$

$du = \cos t\, dt$

$\int_{\pi/4}^{\pi/2} \frac{1}{u}\, du$

$\left[\ln|u|\right]_{\pi/4}^{\pi/2}$

$\ln|\sin(\frac{\pi}{2})| - \ln|\sin(\frac{\pi}{4})|$

$\cancel{\ln 1} - \ln\frac{1}{\sqrt{2}} \Rightarrow \boxed{\ln\sqrt{2}}$

(51) $\int_{\pi/2}^{\pi} 2\cot\frac{\theta}{3}\, d\theta \Rightarrow \int_{\pi/2}^{\pi} 2\frac{\cos\theta/3}{\sin\theta/3}\, d\theta$

$u = \sin\frac{\theta}{3}$

$du = \frac{1}{3}\cos\frac{\theta}{3}\, d\theta \Rightarrow 3du = \cos\frac{\theta}{3}\, d\theta$

$6\int_{\pi/2}^{\pi} \frac{1}{u}\, du \Rightarrow 6\int_{1/2}^{\sqrt{3}/2} \frac{1}{u}\, du$

$\cancel{6[\ln|0| - \ln|1|]}$

$6\ln\sqrt{3} = \boxed{\ln 27}$.

Date ....

(52) $\int_0^{\pi/12} 6\tan 3x\,dx$

$\int_0^{\pi/12} 6\left(\frac{\sin 3x}{\cos 3x}\right)dx$

$u = \cos 3x$

$-2du = \cancel{6}\sin 3x\,dx$

$-2\int_0^{\pi/12} \frac{\cancel{6}\sin 3x}{\cos 3x}\,dx$

$-2\int_1^{\sqrt{2}/2} \frac{1}{u}\,du$

$-2\left[\ln|u|\right]_1^{\sqrt{2}/2}$

$-2\left[\ln\left(\frac{1}{\sqrt{2}}\right) - \ln|1|\right]$

$-2\ln\left(\frac{1}{\sqrt{2}}\right)$

$2\ln\sqrt{2}$

$\cancel{2}\left(\frac{1}{\cancel{2}}\right)\ln 2$

$\boxed{\ln 2}$.

(53) $\int \frac{dx}{2\sqrt{x}+2x} \Rightarrow \int \frac{dx}{2\sqrt{x}(1+\sqrt{x})}$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\int \frac{1}{u}\,du \Rightarrow \boxed{\ln|u| + C} \Rightarrow \boxed{\ln|1+\sqrt{x}| + C}$

(54) $\int \frac{\sec x\,dx}{\sqrt{\ln(\sec x + \tan x)}}$

let:

$u = \sec x + \tan x$

$du = \cancel{\sec x + \tan x}(\sec x\tan x + \sec^2 x)\,dx$

$du = (\sec x)(\tan x + \sec x)\,dx$

$\frac{du}{\cancel{ta}\ u} = \sec x\,dx.$

$\int \frac{du}{u\sqrt{\ln u}} \Rightarrow \int (\ln u)^{-1/2}\cdot\frac{1}{u}\,du$

$2(\ln u)^{1/2} + C \Rightarrow \boxed{2\sqrt{\ln(\sec x + \tan x)} + C}$