(Exercise 3.2)

Date ....

Q(1-22)

"Derivatives as a Function".

⇒ Calculate Derivative of function from 1-6. Then find the value of the derivative specfied:

1) $f(x) = 4-x^2$; $f'(-3), f'(0), f'(1)$

Sol,

Formula used: $\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Given: $f(x) = 4-x^2$

$f(x+h) = 4-(x+h)^2$

find:

$f'(x) = ?$

Using Formula,

$$f'(x) = \lim_{h \to 0} \frac{4-(x+h)^2-(4-x^2)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{4} - \cancel{x^2} - h^2 - 2xh - \cancel{4} + \cancel{x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(-h - 2x)}{\cancel{h}}$$

$$f'(x) = \lim_{h \to 0} -h - 2x$$

Applying limit

$$\boxed{f'(x) = -2x}$$

$f'(-3) = -2(-3) = \boxed{+6}$

$f'(0) = -2(0) = \boxed{0}$

$f'(1) = -2(1) = \boxed{-2}$

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2) $f(x) = (x-1)^2+1$; $f'(-1)$, $f'(0)$, $f'(2)$

Sol,

Given:

$$f(x) = (x-1)^2+1$$

$$f'(x) = ?$$

$$f(x+h) = ((x+h)-1)^2+1$$

Using Formula ⇒

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h-1)^2+1-((x-1)^2+1))}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h-1)^2+1-(x^2-2x-1+1))}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h-1)^2+1-x^2+2x+1-1}{h}$$

$$\therefore (x+h-1)^2 = x^2+h^2+1^2+2xh+2h\mp 2x$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^2}+h^2+\cancel{1}+2xh\mp 2h-\cancel{2x}+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1}-\cancel{1}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h^2+2xh\mp 2h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(h+2x\mp 2)}{\cancel{h}}$$

$$f'(x) = \lim_{h\to 0} h+2x-2$$

Applying limit.

$$\boxed{f'(x) = 2x-2}$$

$$f'(-1) = 2(-1)-2 = \boxed{-4}.$$

$$f'(0) = 2(0)-2 = \boxed{-2}.$$

$$f'(2) = 2(2)-2 = \boxed{2}.$$

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③ $g(t) = \frac{1}{t^2}$; $g'(-1)$, $g'(2)$, $g'(\sqrt{3})$

Sol,

Given $g(t) = \frac{1}{t^2}$

$g(t+h) = \frac{1}{(t+h)^2}$

Using Formula:

$$g'(t) = \lim_{h\to 0} \frac{\frac{1}{(t+h)^2} - \frac{1}{t^2}}{h}$$

$$g'(t) = \lim_{h\to 0} \frac{\frac{t^2-(t+h)^2}{(t+h)^2 \cdot t^2}}{h}$$

$$g'(t) = \lim_{h\to 0} \frac{t^2-(t^2+h^2+2th)}{(t+h)^2 \cdot t^2 \cdot h}$$

$$g'(t) = \lim_{h\to 0} \frac{\cancel{t^2}-\cancel{t^2}-h^2-2th}{(t+h)^2 t^2 h}$$

$$g'(t) = \lim_{h\to 0} \frac{-h^2-2th}{(t+h)^2 t^2 h}$$

$$g'(t) = \lim_{h\to 0} \frac{\cancel{h}(-h-2t)}{(t+h)^2 t^2 \cancel{h}}$$

$$g'(t) = \lim_{h\to 0} \frac{-h-2t}{(t+h)^2 t^2}$$

Applying limit.

$$g'(t) = \lim_{h\to 0} \frac{-0-2t}{(t+0)^2 t^2}$$

$$g'(t) = \frac{-2t}{t^2 \cdot t^2}$$

$$g'(t) = -\frac{2\cancel{t}}{t^4}$$

$$g'(t) = -2/t^3$$

$$\boxed{g'(t) = -2/t^3}$$

$\therefore g'(t) = -2/t^3$

$g'(-1) = -2/(-1)^3 = \boxed{2}$

$g'(2) = -2/(2)^3 = \boxed{-\frac{1}{2}}$

$g'(\sqrt{3}) = -2/(\sqrt{3})^3 = \boxed{\frac{-2}{3\sqrt{3}}}$

④ $k(z) = \frac{1-z}{2z}$; $k'(-1)$, $k'(1)$, $k'(\sqrt{2})$.

Sol,

Given,

$k(z) = \frac{1-z}{2z}$

$k(z+h) = \frac{1-(z+h)}{2(z+h)}$

$= \frac{1-z-h}{2z+2h}$

Using Formula,

$$K'(z) = \lim_{h \to 0} \frac{\frac{1-z-h}{2(z+h)} - \frac{1-z}{2z}}{h}$$

$$K'(z) = \lim_{h \to 0} \frac{(1-z-h)z - (1-z)(z+h)}{2(z+h)(zh)}$$

$$K'(z) = \lim_{h \to 0} \frac{z - z^2 - zh - (z+h-z^2-zh)}{2zh(z+h)}$$

$$K'(z) = \lim_{h \to 0} \frac{\cancel{z} - \cancel{z^2} - \cancel{zh} - \cancel{z} - h + \cancel{z^2} + \cancel{zh}}{2zh(z+h)}$$

$$K'(z) = \lim_{h \to 0} \frac{-\cancel{h}}{2z(z+h)\cancel{h}}$$

$$K'(z) = \lim_{h \to 0} \frac{-1}{2z(z+h)}$$

Applying limit

$$K'(z) = \frac{-1}{2z(z)}$$

$$\boxed{K'(z) = \frac{1}{2z^2}}$$

$$K'(-1) = \frac{1}{2(-1)^2} = \boxed{\frac{1}{2}}$$

$$K'(2) = \frac{1}{2(2)^2} = \boxed{\frac{1}{8}}$$

$$K'(1) = \frac{1}{2(1)^2} = \boxed{\frac{1}{2}}$$

$$K'(\sqrt{2}) = \boxed{\frac{1}{2(\sqrt{2})^2}} = \boxed{\frac{1}{4}}$$

⑤ $p(\theta) = \sqrt{3\theta}$, $p'(1)$, $p'(3)$, $p'(2/3)$.

Sol, Given data,

$$p(\theta) = \sqrt{3\theta}$$

$$p(\theta+h) = \sqrt{3(\theta+h)}$$

Using Formula,

$$p'(\theta) = \lim_{h \to 0} \frac{p(\theta+h) - p(\theta)}{h}$$

$$p'(\theta) = \lim_{h \to 0} \frac{\sqrt{3(\theta+h)} - \sqrt{3\theta}}{h}$$

$$p'(\theta) = \lim_{h \to 0} \frac{\sqrt{3(\theta+h)} - \sqrt{3\theta}}{h}$$

× $\cancel{p'(\theta) = \lim_{h \to 0} \frac{\sqrt{3\theta + 3h - 3\theta}}{h}}$

$$p'(\theta) = \lim_{h \to 0} \frac{\sqrt{3(\theta+h)} - \sqrt{3\theta}}{h} \times \frac{\sqrt{3(\theta+h)} + \sqrt{3\theta}}{\sqrt{3(\theta+h)} + \sqrt{3\theta}}$$

$$p'(\theta) = \lim_{h \to 0} \frac{\left(\sqrt{3(\theta+h)}\right)^2 - \left(\sqrt{3\theta}\right)^2}{h\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)}$$

$$p'(\theta) = \lim_{h \to 0} \frac{3(\theta+h) - 3\theta}{h\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)}$$

$$p'(\theta) = \lim_{h \to 0} \frac{\cancel{3\theta} + 3\cancel{h} - \cancel{3\theta}}{\cancel{h}\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)}$$

$$p'(\theta) = \lim_{h \to 0} \frac{3}{\sqrt{3\theta + 3h} + \sqrt{3\theta}} =$$ Applying limit

$$\boxed{p'(\theta) = \frac{3}{2\sqrt{3\theta}}}$$

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$$p'(1) = \frac{3}{2\sqrt{3\theta}} = \frac{3}{2\sqrt{3(1)}} \Rightarrow \boxed{\frac{3}{2\sqrt{3}}}$$

$$p'(3) = \frac{3}{2\sqrt{3\theta}} = \frac{3}{2\sqrt{3\times3}} = \frac{3}{2(3)} = \frac{3}{6} = \boxed{\frac{1}{2}}.$$

$$p'(\tfrac{2}{3}) = \frac{3}{2\sqrt{3\theta}} = \frac{3}{2(\sqrt{3(\frac{2}{3})}} = \boxed{\frac{3}{2\sqrt{2}}}.$$

(6) $r(s) = \sqrt{2s+1}$

Sol⇒

Given,

$$r(s) = \sqrt{2s+1}$$

$$r(s+h) = \sqrt{2(s+h)+1}$$

Using Formula ⇒

$$r'(s) = \lim_{h\to0} \frac{\sqrt{2(s+h)+1} - \sqrt{2s+1}}{h}$$

$$r'(s) = \lim_{h\to0} \frac{\sqrt{2(s+h)+1} - \sqrt{2s+1}}{h} \cdot \frac{\sqrt{2(s+h)+1} + \sqrt{2s+1}}{\sqrt{2(s+h)+1} + \sqrt{2s+1}}$$

$$r'(s) = \lim_{h\to0} \frac{(\sqrt{2(s+h)+1})^2 - (\sqrt{2s+1})^2}{h(\sqrt{2(s+h)+1} + \sqrt{2s+1})}$$

$$r'(s) = \lim_{h\to0} \frac{\cancel{2s} + 2\cancel{h} + \cancel{1} - \cancel{2s} - \cancel{1}}{\cancel{h}(\sqrt{2(s+h)+1} + \sqrt{2s+1})}$$

$$r'(s) = \lim_{h\to0} \frac{2}{\sqrt{2(s+h)+1} + \sqrt{2s+1}}$$

Applying limit

$$r'(s) = \frac{2}{\sqrt{2s+1} + \sqrt{2s+1}}$$

$$r'(s) = \frac{\cancel{2}}{\cancel{2}\sqrt{2s+1}}$$

$$\boxed{r'(s) = \frac{1}{\sqrt{2s+1}}}$$

$$r'(0) = \frac{1}{\sqrt{2(0)+1}}$$

$$\boxed{r'(0) = 1}$$

$$r'(1) = \frac{1}{\sqrt{2(1)+1}}$$

$$r'(1) = \boxed{\frac{1}{\sqrt{3}}}.$$

$$r'(\tfrac{1}{2}) = \boxed{\frac{1}{\sqrt{2}}}$$

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⇒ Find the indicated derivatives.

(7) $\frac{dy}{dx}$ if $y = 2x^3$

Sol,

$$f(x) = 2x^3.$$

$$f(x+h) = 2(x+h)^3$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{2(x+h)^3 - 2x^3}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{6x^2h+\cancel{2x^3}+6xh^2+2h^3-\cancel{2x^3}}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cancel{h}(6x^2+6xh+2h^2)}{\cancel{h}}$$

$$\frac{dy}{dx} = \lim_{h\to 0} 6x^2+6xh+2h^2$$

Applying limit

$$\boxed{\frac{dy}{dx} = 6x^2}$$

(8) $\frac{dr}{ds}$ if $r = s^3 - 2s^2 + 3$

Sol,

$$\frac{dr}{ds} = \lim_{h\to 0} \frac{((s+h)^3 - 2(s+h)^2+3) - (s^3-2s^2+3)}{h}$$

$$= \lim_{h\to 0} \frac{(s^3+3s^2h+3sh^2+h^3-2s^2-4sh-h^2+3-s^3+2s^2-3)}{h}$$

$$\frac{dr}{ds} = \lim_{h\to 0} \frac{(3s^2h+3sh^2+h^3-4sh-h^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3s^2+3sh+h^2-4s-h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (3s^2+h^2+3sh-4s-h)$$

Applying limit

$$\boxed{\frac{dr}{ds} = 3s^2-2s}$$

(8) $\frac{dr}{ds}$ if $r = s^3 - 2s^2 + 3.$

Sol. $r = s^3 - 2s^2 + 3$

$$\frac{dr}{ds} = \lim_{h\to 0} \frac{((s+h)^3 - 2(s+h)^2+3) - (s^3-2s^2+3)}{h}$$

$$\frac{dr}{ds} = \lim_{h\to 0} \frac{s^3+3s^2h+3sh^2+h^3-2s^2-4sh-h^2+3-s^3+2s^2-3}{h}$$

$$= \lim_{h\to 0} \frac{3s^2h+3sh^2+h^3-4sh-h^2}{h}$$

$$= \lim_{h\to 0} 3s^2+3sh+h^2-4s-h$$

$$= 3s^2-2s$$

$$\boxed{\frac{dr}{ds} = 3s^2-2s}$$

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(9) $\frac{ds}{dt}$ if $s = \frac{t}{2t+1}$

Sol,

$s = \gamma(t) = t/_{2t+1}$

$$\frac{ds}{dt} = \lim_{h\to 0} \frac{\left(\frac{t+h}{2(t+h)+1}\right) - \left(\frac{t}{2t+1}\right)}{h}$$

$$\frac{ds}{dt} = \lim_{h\to 0} \left(\frac{(t+h)(2t+1) - t(2t+2h+1)}{(2t+2h+1)(2t+1)h}\right)$$

$$\frac{ds}{dt} = \lim_{h\to 0} \frac{\cancel{2t^2} + \cancel{t} + \cancel{2ht} + h - \cancel{2t^2} - \cancel{2ht} - \cancel{t}}{(2t+2h+1)(2t+1)h}$$

$$\frac{ds}{dt} = \lim_{h\to 0} \frac{\cancel{h}}{(2t+2h+1)(2t+1)\cancel{h}}$$

$$\frac{ds}{dt} = \lim_{h\to 0} \frac{1}{(2t+2h+1)(2t+1)}$$

Applying limit

$$\frac{ds}{dt} = \frac{1}{(2t+1)(2t+1)}$$

$$\boxed{\frac{ds}{dt} = \frac{1}{(2t+1)^2}}$$

(10) $\frac{dv}{dt}$ if $v = t - \frac{1}{t}$

Sol,

$$\frac{dv}{dt} = \lim_{h\to 0} \frac{\left[(t+h) - \frac{1}{t+h}\right] - \left(t - \frac{1}{t}\right)}{h}$$

$$= \lim_{h\to 0} \frac{h - \frac{1}{t+h} + \frac{1}{t}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\frac{h(t+h)t - t + (t+h)}{(t+h)t}}{h}\right)$$

$$= \lim_{h\to 0} \frac{ht^2 + h^2t + h}{h(t+h)t}$$

$$= \lim_{h\to 0} \frac{t^2 + ht + 1}{(t+h)t}$$

$$\frac{dv}{dt} = \frac{t^2+1}{t^2}$$

$$\boxed{\frac{dv}{dt} = 1 + \frac{1}{t^2}}$$

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(11) $\frac{dp}{dq}$ if $p = \frac{1}{\sqrt{q+1}}$

let

$$p = (q) = \frac{1}{\sqrt{q+1}}$$

$$p(q+h) = \frac{1}{\sqrt{(q+h)+1}}$$

Using Formula:

$$p'(q) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{(q+h)+1}} - \frac{1}{\sqrt{q+1}}}{h}$$

$$p'(q) = \lim_{h\to 0} \left(\frac{(\sqrt{q+1}) - (\sqrt{q+h+1})}{h(\sqrt{q+h+1})(\sqrt{q+1})}\right)$$

$$= \lim_{h\to 0} \frac{\sqrt{q+1} - \sqrt{q+h+1}}{h(\sqrt{q+h+1})(\sqrt{q+1})} \times \frac{\sqrt{q+1} + \sqrt{q+h+1}}{\sqrt{q+1} + \sqrt{q+h+1}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{q+1})^2 - (\sqrt{q+h+1})^2}{h((\sqrt{q+h+1})(\sqrt{q+1})(\sqrt{q+1}) + (\sqrt{q+h+1}))}$$

$$= \lim_{h\to 0} \frac{\cancel{q} + \cancel{1} - \cancel{q} - \cancel{h} - \cancel{1}}{\cancel{h}(\sqrt{q+h+1})(\sqrt{q+1})(\sqrt{q+1}) + (\sqrt{q+h+1})}$$

$$= \lim_{h\to 0} \frac{-1}{(\sqrt{q+h+1})(\sqrt{q+1})(\sqrt{q+1}) + (\sqrt{q+h+1})}$$

Applying limit:

$$= \frac{-1}{(\sqrt{q+1})(\sqrt{q+1})(\sqrt{q+1}) + (\sqrt{q+1})}$$

$$= \frac{-1}{(\sqrt{q+1})^2(\sqrt{q+1} + \sqrt{q+1})} = \boxed{\frac{-1}{2(q+1)\sqrt{q+1}}}$$

(12) $\frac{dz}{dw}$ if $z = \frac{1}{\sqrt{3w-2}}$

Sol.

$$z(w) = \frac{1}{\sqrt{3w-2}}$$

$$z(w+h) = \frac{1}{\sqrt{3(w+h)-2}}$$

Using Formula ⇒

$$z'(w) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{3(w+h)-2}} - \frac{1}{\sqrt{3w-2}}}{h}$$

$$z'(w) = \lim_{h\to 0} \frac{\sqrt{3w-2} - (\sqrt{3w+3h-2})}{h(\sqrt{3(w+h)-2})(\sqrt{3w-2})}$$

$$= \lim_{h\to 0} \frac{(\sqrt{3w-2})^2 - (\sqrt{3w+3h-2})^2}{h(\sqrt{3(w+h)-2})(\sqrt{3w-2})(\sqrt{3w-2} + \sqrt{3w+3h-2})}$$

$$= \lim_{h\to 0} \frac{3\cancel{w} - \cancel{2} - 3\cancel{w} - 3\cancel{h} + \cancel{2}}{\cancel{h}(\sqrt{3w+3h-2})(\sqrt{3w-2})(\sqrt{3w-2} + \sqrt{3w+3h-2})}$$

$$= \lim_{h\to 0} \frac{-3}{(\sqrt{3w+3h-2})(\sqrt{3w-2})(\sqrt{3w-2} + \sqrt{3w+3h-2})}$$

Applying limit.

$$\frac{-3}{(\sqrt{3w-2})(\sqrt{3w-2})(\sqrt{3w-2} + \sqrt{3w-2})}$$

$$= \boxed{\frac{-3}{2(3w-2)\sqrt{3w-2}}}$$

Answer

Formulas $\Rightarrow$ 3.2, 3.3

used in exercise 3.2 Question 13 onwards

① Power Rule : $\frac{d}{dx} = x^n \Rightarrow n x^{n-1} \frac{d}{dx}(x)$

② Quotient Rule : $f\left(\frac{u}{v}\right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{(v)^2}$

used in exercise 3.3 from 1 to 40

③ Product Rule $y(uv) = \frac{dy}{dx} = u \frac{d}{dx} v + v \frac{d}{dx} u.$

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Find the slope of the tangent line.

13) $f(x) = x + \frac{9}{x}$, $x = -3$

Sol,

$$\frac{df}{dx} = f'(x) = x + \frac{9}{x}$$

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}\left(x + \frac{9}{x}\right)$$

$$f'(x) = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{9}{x}\right)$$

$$f'(x) = \frac{d}{dx}(x) + 9\frac{d}{dx}\left(x^{-1}\right)$$

$$f'(x) = 1 + 9(-1x^{-2})\left[\frac{d}{dx}(x)\right]$$

$$f'(x) = 1 - 9x^{-2}(1)$$

$$\boxed{f'(x) = 1 - \frac{9}{x^2}}$$

when $x = -3$

$$f'(-3) = 1 - \frac{9}{(-3)^2}$$

$$f'(3) = 1 - \frac{\cancel{9}}{\cancel{9}}$$

$$f'(3) = 1 - 1 = \boxed{0} \rightarrow \text{slope.}$$

14) $K(x) = \frac{1}{2+x}$, $x = 2$

Sol,

$$K(x) = \frac{1}{2+x}$$

$$\frac{dK}{dx} = K'(x) = \frac{d}{dx}\left(\frac{1}{2+x}\right)$$

$$K'(x) = \frac{d}{dx}(2+x)^{-1} = (1).$$

$$K'(x) = -(2+x)^{-2}\left[\frac{d}{dx}(2) + \frac{d}{dx}(x)\right]$$

$$\boxed{K'(x) = -\frac{1}{(2+x)^2}}$$

when $x = 2$:

$$K'(2) = -\frac{1}{(2+2)^2}$$

$$K'(2) = -\frac{1}{(4)^2}$$

$$\boxed{K'(2) = -1/16} \rightarrow \text{slope.}$$

15) $s = t^3 - t^2$, $t = -1$

Sol,

$$\frac{d}{dt}(s) = \frac{d}{dt}(t^3 - t^2)$$

$$\frac{d}{dt}(s) = \frac{d}{dt}(t^3) - \frac{d}{dt}(t^2)$$

$$s'(t) = 3t^2\frac{d}{dt}[t] - 2t\frac{d}{dt}(t)$$

$$s'(t) = 3t^2(1) - 2t(1)$$

$$\boxed{s'(t) = 3t^2 - 2t}$$

when $t = -1$

$$s'(-1) = 3(-1)^2 - 2(-1)$$

$$s'(-1) = 3 + 2 = \boxed{5} \rightarrow \text{slope.}$$

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(16) $y = \dfrac{x+3}{1-x}$, $x = -2$

Sol,

Using Formula.

$$\frac{u}{v} = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{(v)^2}$$

$$\frac{dy}{dv} = \frac{d}{dx}\left(\frac{x+3}{1-x}\right)$$

$$y'(x) = \frac{(1-x)\frac{d}{dx}(x+3) - (x+3)\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$y'(x) = \frac{(1-x)\left(\frac{d}{dx}(x) + \frac{d}{dx}(3)\right) - (x+3)\left(\frac{d}{dx}(1) - \frac{d}{dx}(x)\right)}{(1-x)^2}$$

$$y'(x) = \frac{(1-x)(1+0) - (x+3)(0-1)}{(1-x)^2}$$

$$y'(x) = \frac{(1-x) - (-x-3)}{(1-x)^2}$$

$$y'(x) = \frac{1 - \cancel{x} + \cancel{x} + 3}{(1-x)^2}$$

$$\boxed{y'(x) = \frac{4}{(1-x)^2}}$$

when $x = -2$

$$y'(-2) = \frac{4}{(1-(-2))^2}$$

$$\boxed{y'(-2) = \frac{4}{9}}$$

(17) Differentiate the function. Then find an equation of the tangent line.

(17) $y = f(x) = \dfrac{8}{\sqrt{x-2}}$, $(x, y) = (6, 4)$

Sol,

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}\left(\frac{8}{\sqrt{x-2}}\right)$$

$$= \frac{(\sqrt{x-2})\frac{d}{dx}(8) - 8\frac{d}{dx}(\sqrt{x-2})}{(\sqrt{x-2})^2}$$

$$= 8\frac{d}{dx}(\sqrt{x-2})^{-1}$$

$$= 8\frac{d}{dx}(x-2)^{-1/2}$$

$$= 8\left[-\frac{1}{2}(x-2)^{-3/2}\left[\frac{d}{dx}(x-2)\right]\right]$$

$$= 8\left[-\frac{1}{2}(x-2)^{-3/2}\left[\frac{d}{dx}(x) - \frac{d}{dx}(2)\right]\right]$$

$$= 8\left[-\frac{1}{2}(x-2)^{-3/2}\left[1-0\right]\right]$$

$$= -\frac{8}{2(x-2)^{3/2}}$$

$\therefore (x-2)^{3/2} = (x-2)\sqrt{x-2}$

$$\boxed{f'(x) = -\frac{4}{(x-2)\sqrt{x-2}}}$$

$$f'(6) = -\frac{4}{(6-2)\sqrt{6-2}} \Rightarrow \boxed{-\frac{1}{2}}$$

equation,

$$y - 4 = -\frac{1}{2}(x-6)$$

$$y = -\frac{1}{2}x + 3 + 4 \Rightarrow \boxed{y = -\frac{1}{2}x + 7}$$

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(18) $w = g(z) = 1+\sqrt{4-z}$, $(z, w) = (3, 2)$

Sol,

Using Formula,

$$\frac{d}{dz} = g'(z) = \frac{d}{dz}\left(1+\sqrt{4-z}\right)$$

$$g'(z) = \frac{d}{dz}(1) + \frac{d}{dz}(4-z)^{1/2}$$

$$g'(z) = 0 + \frac{1}{2}(4-z)^{-1/2}\left[\frac{d}{dz}(4-z)\right]$$

$$g'(z) = 0 + \frac{1}{2}(4-z)^{-1/2}\left[\frac{d}{dz}(4) - \frac{d}{dz}(z)\right]$$

$$g'(z) = 0 + \frac{1}{2}(4-z)^{-1/2}\left[0-1\right]$$

$$\boxed{g'(z) = -\frac{1}{2\sqrt{4-z}}}$$

$$g'(3) = -\frac{1}{2\sqrt{4-3}} \Rightarrow \boxed{-\frac{1}{2}}$$

forming equation ⇒

$$w-2 = -\frac{1}{2}(z-3)$$

$$w = -\frac{1}{2}z + \frac{3}{2} + 2$$

$$\boxed{w = -\frac{1}{2}z + \frac{7}{2}}.$$

(19) find the values of derivatives.

(19) $\left.\frac{ds}{dt}\right|_{t=-1}$ if $s = 1-3t^2$

Sol,

$$s(t) = 1-3t^2$$

$$\frac{ds}{dt} = s'(t) = \frac{d}{dt}(1-3t^2).$$

$$s'(t) = \frac{d}{dt}(1) - \frac{d}{dt}(3t^2)$$

$$s'(t) = 0 - \frac{d}{dt}(3t^2)$$

$$s'(t) = -2(3)t\left[\frac{d}{dt}t\right]$$

$$s'(t) = -6t(1) \Rightarrow \boxed{s'(t) = -6t}.$$

$$\left.\frac{ds}{dt}\right|_{t=-1} = -6(-1) \Rightarrow \boxed{6}.$$

(20) $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}}$ if $y = 1-\frac{1}{x}$

Sol,

$$\left.\frac{dy}{dx}\right|_{\cancel{x=\sqrt{3}}} = y'(x) = \frac{d}{dx}\left(1-\frac{1}{x}\right)$$

$$y'(x) = \frac{d}{dx}(1) - \frac{d}{dx}(x)^{-1} \Rightarrow y'(x) = 0-(-1)x^{-2}(1)$$

$$y'(x) = x^{-2} \Rightarrow \boxed{y'(x) = \frac{1}{x^2}}.$$

$$\left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = y = \frac{1}{x^2} \Rightarrow \frac{1}{(\sqrt{3})^2} \Rightarrow \boxed{\frac{1}{3}}.$$

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(21) $\left.\frac{dr}{d\theta}\right|_{\theta=0}$ if $r = \frac{2}{\sqrt{4-\theta}}$

Sol:

$$\frac{dr}{d\theta} = r'(\theta) = \frac{d}{d\theta}\left(\frac{2}{\sqrt{4-\theta}}\right)$$

$$r'(\theta) = \frac{d}{d\theta}\, 2(\sqrt{4-\theta})^{-1}$$

$$r'(\theta) = 2\,\frac{d}{d\theta}(\sqrt{4-\theta})^{-1/2}$$

$$r'(\theta) = 2\left(\frac{-1}{2}\right)(\sqrt{4-\theta})^{-3/2}\left[\frac{d}{d\theta}(4-\theta)\right]$$

$$r'(\theta) = -\frac{\cancel{2}}{\cancel{2}}(\sqrt{4-\theta})^{-3/2}(-1)$$

$$r'(\theta) = \frac{+\cancel{2}}{\cancel{2}(4-\theta)^{3/2}}$$

$$\boxed{r'(\theta) = +\frac{\cancel{2}\,1}{(4-\theta)\sqrt{4-\theta}}}$$

$$\left.\frac{d\theta}{d\theta}\right|_{\theta=0} = +\frac{1}{(4-0)\sqrt{4-0}}$$

$$\left.\frac{dr}{d\theta}\right|_{\theta=0} = +\frac{1}{(4)(2)}$$

$$\left.\frac{dr}{d\theta}\right|_{\theta=0} = +\frac{\cancel{2}}{\cancel{8}\cancel{4}}\frac{1}{8}$$

$$\boxed{\left.\frac{dr}{d\theta}\right|_{\theta=0} = +\frac{1}{8}}$$

(22) $\left.\frac{dw}{dz}\right|_{z=4}$ if $w = z + \sqrt{z}$

$$\frac{dw}{dz} = w'(z) = \frac{d}{dz}(z + \sqrt{z})$$

$$w'(z) = \frac{d}{dz}(z) + \frac{d}{dz}(z)^{1/2}$$

$$w'(z) = 1 + \frac{1}{2}z^{-1/2}(1)$$

$$\boxed{w'(z) = 1 + \frac{1}{2\sqrt{z}}}$$

$$\left.\frac{dw}{dz}\right|_{z=4} = 1 + \frac{1}{2\sqrt{4}}$$

$$\left.\frac{dw}{dz}\right|_{z=4} = 1 + \frac{1}{4}$$

$$\left.\frac{dw}{dz}\right|_{z=4} = \frac{4+1}{4}$$

$$\boxed{\left.\frac{dw}{dz}\right|_{z=4} = \frac{5}{4}}$$