Derivative Rules & Formulas

Power Rule, Product Rule and Quotient Rule: Ex. 3.3, 3.5, 3.6

Sr.	Derivative Formulas	Examples
1.	**Power Rule:** $\frac{d}{dx}(x^n) = n\,x^{n-1}$	1. $\frac{d}{dx}(x^3) = 3\,x^{3-1} = 3x^2,$ 2. $\frac{d}{dt}\left(\frac{1}{t^3}\right) = \frac{d}{dt}(t^{-3}) = -3t^{-4} = \frac{-3}{t^4}.$
2.	**Generalized Power Rule:** $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$	$\frac{d}{dx}(1-2x)^3 = 3(1-2x)^2\frac{d}{dx}(1-2x)$ $= 3(1-2x)^2 \times (-2)$ $= -6(1-2x)^2$
3.	$\frac{d}{dx}(x) = 1,$ *Derivative of "x" w.r.t. "x" is 1.*	$\frac{d}{d\theta}(\theta) = 1,\quad \frac{d}{ds}(s) = 1,\quad \frac{d}{dy}(y) = 1.$
4.	$\frac{d}{dx}(c) = 0,$ *Derivative of a constant is zero.*	$\frac{d}{dx}(2) = 0,\quad \frac{d}{dx}\left(\frac{1}{5}\right) = 0,\quad \frac{d}{dx}(\pi) = 0.$
5.	$\frac{d}{dx}(cx^n) = c\frac{d}{dx}(x^n) = cnx^{n-1}$	$\frac{d}{dx}(2x^5) = 2\frac{d}{dx}(x^5) = 2(5x^4) = 10x^4.$
Let u and v be functions of x, i.e., $u = u(x),\ v = v(x)$. Then,		
6.	**Product Rule:** $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$y = (x-1)(2x^3-7x)$ $\frac{dy}{dx} = (x-1)\frac{d}{dx}(2x^3-7x) + (2x^3-7x)\frac{d}{dx}(x-1)$ $= (x-1)(6x^2-7) + (2x^3-7x)(1-0)$ $= 6x^3 - 7x - 6x^2 + 7 + 2x^3 - 7x$ $= 8x^3 - 6x^2 - 14x + 7$
7.	**Quotient Rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$y = \frac{x-1}{2x^3-7x}$ $\frac{dy}{dx} = \frac{(2x^3-7x)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(2x^3-7x)}{(2x^3-7x)^2}$ $= \frac{(2x^3-7x)(1-0) - (x-1)(6x^2-7)}{(2x^3-7x)^2}$ $= \frac{-4x^3+6x^2-7}{(2x^3-7x)^2}$

Derivatives of Trigonometric Functions: Ex. 3.5, 3.6

Sr.	Formula	Generalization
1.	$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin ax) = \cos ax \frac{d}{dx}(ax) = a\cos ax$
2.	$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos ax) = -\sin ax \frac{d}{dx}(ax) = -a\sin ax$
3.	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan ax) = \sec^2 ax \frac{d}{dx}(ax) = a\sec^2 ax$
4.	$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot ax) = -\csc^2 ax \frac{d}{dx}(ax) = -a\csc^2 ax$
5.	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec ax) = \sec ax \tan ax \frac{d}{dx}(ax) = a\sec ax \tan ax$
6.	$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc ax) = -\csc ax \cot ax \frac{d}{dx}(ax) = -a\csc ax \cot ax$

Derivatives of Logarithmic & Exponential Functions: Ex. 7.2, 7.3

Sr.	Formula	Generalization
1.	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}\ln(ax+b) = \frac{1}{ax+b} \times \frac{d}{dx}(ax+b)$ $= \frac{1}{ax+b}(a+0) = \frac{a}{ax+b}$
2.	$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$	$\frac{d}{dx}\log_a(cx+d) = \frac{1}{(cx+d)\ln a} \times \frac{d}{dx}(cx+d)$ $= \frac{c}{(cx+d)\ln a}$
3.	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(e^{ax+b}) = e^{ax+b} \times \frac{d}{dx}(ax+b)$ $= e^{ax+b} \times a = ae^{ax+b}$
4.	$\frac{d}{dx}(a^x) = a^x \ln a$	$\frac{d}{dx}(a^{cx+d}) = a^{cx+d} \ln a \times \frac{d}{dx}(cx+d)$ $= a^{cx+d} \ln a \times c = c \ln a \,.\, a^{cx+d}$

Derivatives of Inverse Trigonometric Functions: Ex. 7.6

Sr.	Formula	Generalization
1.	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{1}{\sqrt{1-(ax)^2}} \times \frac{d}{dx}(ax) = \frac{a}{\sqrt{1-a^2x^2}}$
2.	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-1}{\sqrt{1-(ax)^2}} \times \frac{d}{dx}(ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
3.	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{1}{1+(ax)^2} \times \frac{d}{dx}(ax) = \frac{a}{1+a^2x^2}$
4.	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1} ax) = \frac{-1}{1+(ax)^2} \times \frac{d}{dx}(ax) = \frac{-a}{1+a^2x^2}$
5.	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\lvert x\rvert \sqrt{x^2-1}}$	$\frac{d}{dx}(\sec^{-1} ax) = \frac{1}{\lvert ax\rvert \sqrt{(ax)^2-1}} \times \frac{d}{dx}(ax) = \frac{a}{\lvert ax\rvert \sqrt{a^2x^2-1}}$
6.	$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{\lvert x\rvert \sqrt{x^2-1}}$	$\frac{d}{dx}(\csc^{-1} ax) = \frac{-1}{\lvert ax\rvert \sqrt{(ax)^2-1}} \times \frac{d}{dx}(ax) = \frac{-a}{\lvert ax\rvert \sqrt{a^2x^2-1}}$

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