Integration by parts : $\int u\,dv = uv - \int v\,du$

Date ....

(Exercise # 8.1) (1 - 50)

Q(1-25) → Integration by parts.

① $\int x \sin\frac{x}{2}\,dx$

let,

$\boxed{u = x}$

$\boxed{du = dx}$

$\int dv = \int \sin\frac{x}{2}\,dx$

$v = \dfrac{-\cos\frac{x}{2}}{1/2}$

$\boxed{v = -2\cos\frac{x}{2}}$

Using Formula ⇒

$\int u\,dv = uv - \int v\,du$

$= (x)\left(-2\cos\frac{x}{2}\right) - \int\left(-2\cos\frac{x}{2}\right)dx$

$-2x\cos\frac{x}{2} + 2\int \cos\frac{x}{2}\,dx$

$-2x\cos\frac{x}{2} + \dfrac{2\sin\frac{x}{2}}{1/2}$

$= \boxed{-2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + c}$

② $\int \theta\cos\pi\theta\,d\theta$

let,

$\boxed{u = \theta}$

$\boxed{du = d\theta}$

$\int dv = \int \cos\pi\theta\,d\theta$

$\boxed{v = \dfrac{\sin\pi\theta}{\pi}}$

Using Formula ⇒

$\int u\,dv = uv - \int v\,du$

$= (\theta)\left(\dfrac{\sin\pi\theta}{\pi}\right) - \int \dfrac{\sin\pi\theta}{\pi}\,d\theta$

$= \dfrac{\theta\,\overset{\sin}{\cancel{\cos}}\,\pi\theta}{\pi} + \dfrac{\overset{\cos}{\cancel{\sin}}\,\pi\theta}{\pi^2} + c$

$= \boxed{\dfrac{\theta\sin\pi\theta}{\pi} + \dfrac{\cos\pi\theta}{\pi^2} + c}$

③ $\int t^2\cos t\,dt$

let ~~$u = t^2$~~ By tabular Integration.

~~$du = dt$~~

Derivative		Integration
		$\cos t$
t^2	+	$\sin t$
$2t$	−	$-\cos t$
2	+	$-\sin t$
0	−	$\cos t$

~~$\int t^2\cos t\,dt = t^2\cos t + 2t\sin t - 2\cos t + \sin t + c$~~

$\boxed{\int t^2\cos t\,dt = t^2\sin t + 2t\cos t - 2\sin t + c}$.

④ $\int x^2\sin x\,dx$

		$\sin x$
x^2	+	$-\cos x$
$2x$	−	$-\sin x$
2	+	$\cos x$
0		

$\boxed{\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + c}$

Date

⑤ $\int_1^2 x \ln x \, dx$

Let:

$\boxed{\cancel{u=x}}$ $\boxed{u = \ln x}$

$\boxed{\cancel{du=dx}}$ $\boxed{du = \frac{dx}{x}}$

$\int dv = \int x \, dx$

$\boxed{V = \frac{x^2}{2}}$

Using Formula ⇒

$(\ln x)\left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right)\left(\frac{dx}{x}\right)$

$\frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx$

$\left[\frac{x^2 \ln x}{2}\right]_1^2 - \left[\frac{x^2}{4}\right]_1^2 \cancel{+c}$

$\left[\frac{(2)^2 \ln(2)}{2} - \frac{(1)^2 \cancel{\ln(1)}}{2}\right] - \left[\frac{(2)^2}{4} - \frac{(1)^2}{4}\right] \cancel{+c}$

$\frac{2\cancel{4} \ln 2}{\cancel{2}} - \left[\frac{3}{4}\right] \cancel{+c}$

$\boxed{2\ln 2 - \frac{3}{4}}$

⑥ $\int_1^e x^3 \ln x \, dx$

$\boxed{u = \ln x}$

$\boxed{du = \frac{dx}{x}}$

$\int dv = \int x^3 \, dx$

$\boxed{V = \frac{x^4}{4}}$

Using formula,

$(\ln x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4} \frac{dx}{x}$

$\left[\frac{x^4 \ln x}{4}\right]_1^e - \int \frac{x^3}{4} \, dx$

$\left[\frac{e^4 \cancel{\ln e}}{4} - \cancel{\frac{(1)^4 \ln(1)}{4}}\right] - \left[\frac{x^4}{16}\right]_1^e$

$\frac{e^4}{4} - \left[\frac{e^4}{16} - \frac{(1)^4}{16}\right] \Rightarrow \frac{e^4}{4} - \frac{e^4}{16}$

$\frac{4e^4 - e^4}{16} = \boxed{\frac{3e^4}{16}}$

⑦ $\int x e^x \, dx$

Let: $u = x$

$du = dx$

$\int dv = \int e^x \, dx$

$\boxed{V = e^x}$

Using Formula,

$= (x)(e^x) - \int e^x \, dx \Rightarrow \boxed{xe^x - e^x + c}$

Date ....

⑧ $\int x e^{3x}\, dx$

let,

$u = x$

$du = dx$

$\int dv = \int e^{3x} dx$

$v = \dfrac{e^{3x}}{3}$

Using Formula ⇒

$(x)\left(\dfrac{e^{3x}}{3}\right) - \int \left(\dfrac{e^{3x}}{3}\right)(dx)$

$\dfrac{xe^{3x}}{3} - \dfrac{1}{3}\int e^{3x} dx$

$\boxed{\dfrac{xe^{3x}}{3} - \dfrac{e^{3x}}{9} + C}$

⑨ $\int x^2 e^{-x}\, dx$

		e^{-x}
x^2	+	$-e^{-x}$
$2x$	−	e^{-x}
2	+	$-e^{-x}$
0	−	e^{-x}

$\boxed{\int x^2 e^{-x} dx = -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c}$

⑩ $\int (x^2 - 2x + 1) e^{2x}\, dx$

		e^{2x}
$(x^2 - 2x + 1)$	+	$e^{2x}/2$
$2x - 2$	−	$e^{2x}/4$
2	+	$e^{2x}/8$
0		

$\int (x^2 - 2x + 1) e^{2x} dx = \dfrac{e^{2x}}{2}(x^2 - 2x + 1) - \dfrac{e^{2x}}{4}(2x-2) + \dfrac{e^{2x}}{\cancel{8}\,4}(\cancel{2})$

$= \boxed{\dfrac{e^{2x}}{2}(x^2 - 2x + 1) - \dfrac{e^{2x}}{2}(x-1) + \dfrac{e^{2x}}{4} + C}$

⑪ $\int \tan^{-1} y\, dy$

let,

$u = \tan^{-1} y$

$du = \dfrac{dy}{1+y^2}$

$dv = dy$

$\boxed{v = y}$

Using Formula,

$(\tan^{-1} y)(y) - \int y \dfrac{dy}{1+y^2}$

$y \tan^{-1} y - \dfrac{1}{2}\int \dfrac{y}{1+y^2} dy$

$\cancel{y \tan^{-1} y - \dfrac{1}{2} \ln \sqrt{1+y^2} + c}$ $\quad \boxed{y \tan^{-1} y - \dfrac{1}{2} \ln|1+y^2| + C}$

Date ....

(12) $\int \sin^{-1} y \, dy$.

let,

$u = \sin^{-1} y$

$du = \frac{dy}{\sqrt{1-y^2}}$

$dv = dy$

$v = y$

$(\sin^{-1} y)(y) - \int y \left(\frac{dy}{\sqrt{1-y^2}}\right)$

$y \sin^{-1} y - \int \frac{y}{\sqrt{1-y^2}} dy$.

$\boxed{y \sin^{-1} y - \sqrt{1-y^2} + c}$

(13) $\int x \sec^2 x \, dx$

Let,

$u = x$

$du = dx$

$\int dv = \int \sec^2 x \, dx$

$v = \tan x$

$(x)(\tan x) - \int (\tan x)(dx)$

$x \tan x - \int \tan x \, dx$

$x \tan x + \int \frac{1}{\cos x} dx$

$\boxed{x \tan x + \ln|\cos x| + c}$

(14) $\int 4x \sec^2 2x \, dx$

$\cancel{u = 4x}$

$\cancel{du = 4dx}$

$\cancel{\int dv = \int \sec^2 2x \, dx}$

$v = \tan 2x$

$\cancel{(4x)(\tan 2x)} - \int (\tan 2x) d^4$

Let, $y = 2x$

$\int 2y \sec^2 y \, dy$.

$u = 2y$

$du = 2dx$

$\int dv = \int \sec^2 y \, dy$

$v = \tan y$.

$(2y)(\tan y) - \int (\tan y)(2dx)$

$\boxed{4x \tan 2x - 2\ln|\sec 2x| + c}$

(15) $\boxed{\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + c}$

x^3		e^x
x^3	+	e^x
$3x^2$	−	e^x
$6x$	+	e^x
6	−	e^x
0		e^x

(16) $\int p^4 e^{-p} dp = \boxed{-p^4 e^{-p} - 4p^3 e^{-p} - 12pe^{-p} - 12e^{-p} + c}$

p^4		e^{-p}
p^4	+	$-e^{-p}$
$4p^3$	−	e^{-p}
$12p^2$	+	$-e^{-p}$
$24p$	−	e^{-p}
24	+	$-e^{-p}$

correct it accordingly

(17) $\int (x^2 - 5x) e^x dx$

$x^2 - 5x$	+	e^x
$2x - 5$	−	e^x
2	+	e^x
0		

$\boxed{\int (x^2 - 5x) e^x dx = e^x(x^2 - 5x) - e^x(2x - 5) + 2e^x + c}$

Date ....

(18) $\int (r^2+r+1)e^r\,dr = \boxed{e^r(r^2+r+1) - e^r(2r+1) + 2e^r + c}$

r^2+r+1	$+$	e^r
$2r+1$	$-$	e^r
2	$+$	e^r
0		e^r

(19) $\int x^5 e^x\,dx = \boxed{x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120xe^x - 120e^x}$

x^5	$+$	e^x
$5x^4$	$-$	e^x
$20x^3$	$+$	e^x
$60x^2$	$-$	e^x
$120x$	$+$	e^x
120	$-$	e^x

Using Formulas

(20) $\int t^2 e^{4t}\,dt$

t^2	$+$	$\frac{e^{4t}}{4}$
$2t$	$-$	$\frac{e^{4t}}{16}$
2	$+$	$\frac{e^{4t}}{64}$

$= \boxed{\frac{t^2e^{4t}}{4} - \frac{te^{4t}}{8} + \frac{e^{4t}}{34} + c}$

(21) $\int e^\theta \sin\theta\,d\theta$

$I = \int e^\theta \sin\theta\,d\theta$

let,

$\boxed{u = \sin\theta}$

$\boxed{du = +\cos\theta\,d\theta}$

$\int dv = \int e^\theta d\theta$

$\boxed{v = e^\theta}$.

$(\sin\theta)(e^\theta) - \int (e^\theta)(\cos\theta\,d\theta)$

$\sin\theta e^\theta - \int e^\theta \cos\theta\,d\theta$

let,

$u = \cos\theta \qquad \int dv = \int e^\theta d\theta$

$\boxed{du = \sin\theta\,d\theta} \qquad \boxed{v = e^\theta}$.

$\sin\theta e^\theta - \left[(\cos\theta)(e^\theta) - \int (e^\theta)(\sin\theta\,d\theta)\right]$

$I = \sin\theta e^\theta - \cos\theta e^\theta - \underline{\int e^\theta \sin\theta\,d\theta}$

$2I = e^\theta \sin\theta - e^\theta\cos\theta$.

$\boxed{I = \frac{1}{2}\left[e^\theta\sin\theta - e^\theta\cos\theta\right) + c}$

(22) $I = \int e^{-y}\cos y\,dy$

let,

$\boxed{u = \cos y}$.

$\boxed{du = \sin y\,dy}$

$\int dv = \int e^{-y}dy$

$\boxed{v = -e^{-y}}$.

$(\cos y)(-e^{-y}) + \int e^{-y}\sin y\,dy$

$I = -e^{-y}\cos y + \int e^{-y}\sin y\,dy$

$u = \sin y \Rightarrow du = -\cos y\,dy$.

$(-e^{-y}\cos y) + \left[(\sin y)(-e^{-y}) \mp \int \underline{e^{-y}\cos y}\right.$

$\boxed{I = \frac{1}{2}\left[\left(-e^{-y}\cos y - e^{-y}\sin y\right) + c\right]}$

Date ....

Q(21)

$$I = \int e^{\theta} \sin\theta \, d\theta$$

$\boxed{u = \sin\theta}$ | $\int dv = \int e^{\theta}$

$\frac{du}{d\theta} = +\cos\theta$ | $\boxed{v = e^{\theta}}$

$\boxed{du = \cos\theta \, d\theta}$

$$\boxed{I = \frac{1}{2}\left[\left(e^{\theta}\sin\theta - e^{\theta}\cos\theta\right) + C\right]}$$

$$I = (e^{\theta})(\sin\theta) - \int (e^{\theta})(\cos\theta \, d\theta)$$

$$I = e^{\theta}\sin\theta - \int \underline{e^{\theta}} \, \underline{\cos\theta} \, d\theta$$

$\boxed{u = \cos\theta}$ | $\int dv = \int e^{\theta}$

$\frac{du}{d\theta} = -\sin\theta$ | $\boxed{v = e^{\theta}}$

$du = -\sin\theta \, d\theta$

$$I = e^{\theta}\sin\theta - \left[(e^{\theta})(\cos\theta) - \int (e^{\theta})(-\sin\theta) d\theta\right.$$

$$I = e^{\theta}\sin\theta - e^{\theta}\cos\theta - \int \underline{e^{\theta}\sin\theta \, d\theta}$$

$$I = e^{\theta}\sin\theta - e^{\theta}\cos\theta - I + C$$

$$I + I = e^{\theta}\sin\theta - e^{\theta}\cos\theta + C$$

$$2I = e^{\theta}\sin\theta - e^{\theta}\cos\theta + C$$

let $= c' = e^{\theta}\sin\theta - e^{\theta}\cos\theta$.

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$$v\,du = v \cdot u - \int v\,du.$$

Date ....

(23) $I = \int e^{2x} \cos 3x\,dx.$

$u = \cos 3x$ | $\int dv = \int e^{2x}$

$\frac{du}{dx} = -3\sin 3x$ | $\boxed{V = \frac{1}{2}e^{2x}}$

$\boxed{du = -3\sin 3x\,dx}$ | $\boxed{V = \frac{e^{2x}}{2}}$.

$\int vu$

$v\,du$,

Using Formula $\Rightarrow$

$$I = (\cos 3x)\left(\frac{e^{2x}}{2}\right) - \int\left(\frac{e^{2x}}{2}\right)(\cancel{\cos}\,-3\sin 3x\,dx)$$

$$I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\int e^{2x}\sin 3x\,dx$$

$$I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\left[\left(\frac{e^{2x}}{2}\right)(\cancel{3}\cos 3x) - \frac{3}{2}\int\left(\frac{e^{2x}}{2}\right)\cos 3x\,dx\right]$$

$$I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\left[\frac{e^{2x}}{2}\sin 3x - \frac{3}{2}I\right]$$

$$I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}I + c$$

$$I + \frac{9}{4}I = \frac{e^{2x}\cos 3x}{2} + \frac{3e^{2x}\sin 3x}{4} + c$$

$$\boxed{I = \frac{13}{4}\left(\frac{e^{2x}\cos 3x}{2} + \frac{3e^{2x}\sin 3x}{4}\right) + c}$$

take ss of previous is incorrect

Date ....

(23) $\int e^{2x}\cos 3x\,dx$

$u = \cos 3x$

$du = -\sin 3x\,dx$

$\int dv = \int e^{2x}\,dx$

$\boxed{v = \frac{1}{2}e^{2x}}$

$(\cos 3x)\left(\frac{1}{2}e^{2x}\right) + \int \frac{1}{2}e^{2x}(\sin 3x\,dx)$

$\frac{1}{2}e^{2x}\cos 3x + \frac{1}{2}\int e^{2x}(\sin 3x\,dx)$

$u = \sin 3x$

$du = +\cos 3x\,dx$

$dv = e^{2x}\,dx$

$v = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x}\cos 3x + \frac{1}{2}\left[(\sin 3x)\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}\cos 3x\,dx\right]$

$I = \frac{1}{2}e^{2x}\cos 3x + \frac{1}{4}e^{2x}\sin 3x - I$

$\boxed{I = \frac{1}{2}\left[\left(\frac{1}{2}e^{2x}\cos 3x + \frac{1}{4}e^{2x}\sin 3x\right) + C\right]}$

(24) $\int e^{-2x}\sin 2x\,dx$

$u = \sin 2x$

$\boxed{du = 2\cos 2x\,dx}$

$\int dv = \int e^{-2x}\,dx$

$v = -\frac{e^{-2x}}{2}$

$(\sin 2x)\left(-\frac{e^{-2x}}{2}\right) + \int\left(\frac{e^{-2x}}{\cancel{2}}\right)(\cancel{2}\cos 2x\,dx)$

$-\frac{e^{-2x}\sin 2x}{2} + \int e^{-2x}\cos 2x\,dx$

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$\int dv = \int e^{-2x}\,dx$

$v = -\frac{e^{-2x}}{2}$

$-\frac{e^{-2x}\sin 2x}{2} + \left[(\cos 2x)\left(-\frac{e^{-2x}}{2}\right) - \int\left(\frac{e^{-2x}}{2}\right)(2\ldots\right.$

$-\frac{e^{-2x}\sin 2x}{2} - \frac{e^{-2x}\cos 2x}{2} - \left[\int e^{-2x}\cos 2x\,dx\right]$

$I = \frac{1}{2}\left[\left(-\frac{e^{-2x}\sin 2x}{2} - \frac{e^{-2x}\cos 2x}{2}\right) + C\right]$

$3/2 + \frac{1}{2}$

Date $\frac{5}{2}$

Using Substitution ⇒ (25 – 30)

(25) $\int e^{\sqrt{3s+9}}\,ds$

let:

$x = \sqrt{3s+9}$

$x^2 = 3s+9$

$3ds = x^2 dx$

$\boxed{ds = \frac{2}{3}x\,dx}$

So, $\int e^x \cdot \frac{2}{3}x\,dx$

$\frac{2}{3}\int e^x x\,dx$

Suppose: $\boxed{u = x}$ | $\int dv = \int e^x dx$

$du = dx$ | $\boxed{V = e^x}$.

$\frac{2}{3}\left[(x)(e^x) - \int (e^x)(dx)\right]$

$\frac{2}{3}(xe^x - e^x + c)$

$\boxed{\frac{2}{3}\left[\left(\sqrt{3s+9}\,e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right) + c\right]}$

(26) $\int_0^1 x\sqrt{1-x}\,dx$

let: $u = x$ | $\int dv = \int \sqrt{1-x}\,dx$

$du = dx$ | $V = -\frac{2}{3}(\sqrt{1-x})^3$

$\cancel{\left[(x)\left(-\frac{2}{3}\sqrt{(1-x)^3}\right)\right]_0^1} + \int \frac{2}{3}\sqrt{(1-x)^3}\,dx$

$\frac{2}{3}\left[\int \sqrt{(1-x)^3}\,dx\right]_0^1 = \frac{2}{3}\left[-\frac{2}{5}(1-x)^{5/2}\right]_0^1 \Rightarrow \boxed{\frac{4}{15}}$.

(27) $\int_0^{\pi/3} x\tan^2 x\,dx$

let,

$u = x$

$du = dx$

$\int dv = \int \tan^2 x\,dx$

$\int dv = \int \frac{\sin^2 x}{\cos^2 x}\,dx$

$\int dv = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx \Rightarrow \int \frac{1}{\cos^2 x}\,dx - \int dx$

$\boxed{V = \tan x - x}$

$(x)(\tan x - x) - \int (\tan x - x)(dx)$

$\left[x(\tan x - x)\right]_0^{\pi/3} - \int \tan x - \int x\,dx$.

Date ....

$$\left[\frac{\pi}{3}\left(\tan\frac{\pi}{3}-\frac{\pi}{3}\right)\right]-\left[\ln|\cos x|+\frac{x^2}{2}\right]_0^{\pi/3}$$

$$\left[\frac{\pi}{3}\left(\sqrt{3}-\frac{\pi}{3}\right)\right]+\left[\ln\frac{1}{2}+\frac{\pi^2}{18}\right] \Rightarrow \boxed{\frac{\pi\sqrt{3}}{3}-\ln 2-\frac{\pi^2}{18}}$$

(28) $\int \ln(x+x^2)\,dx$

Sol:

$u=\ln(x+x^2)$ | $\int dv=\int x\,dx$

$du=\frac{dx\,(2x+1)}{(x+x^2)}$ | $\boxed{V=x}$.

$(\ln(x+x^2)(x)-\int x\,\frac{(2x+1)}{x+x^2}\,dx$

$x\ln(x+x^2)-\int\frac{2x+1}{x+1}\,dx$

$$\boxed{x\ln(x+x^2)-2x+\ln|x+1|+C}$$

(29) $\int \sin(\ln x)\,dx$.

let

$u=\ln x$ | ~~$dv=\sin$~~

$du=\frac{dx}{x}$

$dx=e^u\,du$

$\int(\sin u)e^u\,du$

same as (21)

$I=\frac{1}{2}\left[-x\cos(\ln x)+x\sin(\ln x)\right.+C$

(30) $\int z(\ln z)^2\,dz$

let

$u=\ln z$

$du=\frac{dz}{z}$

$dz=e^u du$.

$\int e^u\cdot u^2\cdot e^u\,du$.

$\int e^{2u}u^2\,du$.

$u^2 \qquad e^{2u}$

$u^2 \quad + \quad e^{2u}/2$

$2u \quad - \quad \frac{e^{2u}}{4}$

$2 \quad + \quad \frac{e^{2u}}{8}$

$$\int e^{2u}u^2\,du=\frac{u^2e^{2u}}{2}-\frac{2ue^{2u}}{4}+\frac{2e^{2u}}{8}$$

$$\frac{e^{2(u)}}{4}\left[2u^2-2u+e^{2u}\right]$$

$$\boxed{\frac{z^2}{4}\left[2(\ln z)^2-2\ln z+1\right]+C}$$

Date ....

(31) $\int x \sec x^2 \, dx$

let

$u = x^2$

$\sqrt{u} = x$

$\frac{dx}{du} = \frac{1}{2\sqrt{u}}$

$dx = \frac{du}{2\sqrt{u}}$

$\int \cancel{\sqrt{u}} \sec u \frac{du}{2\cancel{\sqrt{u}}}$

$\frac{1}{2}\int \sec u \, du$

$\frac{1}{2} \ln|\sec u + \tan u| + c$

$\boxed{\frac{1}{2} \ln|\sec x^2 + \tan x^2| + c}$.

(32) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

let

$t = \sqrt{x}$

$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

$dt = \frac{dx}{2\sqrt{x}}$

$2dt = \frac{dx}{\sqrt{x}}$

$\int \cos t \, 2dt$

$2\int \cos t \, dt$

$2 \sin t + c$

$\boxed{2 \sin\sqrt{x} + c}$

(33) $\int x (\ln x)^2 dx$.

let II I

$u = (\ln x)^2$ $\quad \int dv = \int x$

$\frac{du}{dx} = 2(\ln x)\left(\frac{1}{x}\right)$ $\quad \boxed{v = \frac{x^2}{2}}$.

$du = \frac{2 \ln x}{x} dx$.

$((\ln x)^2\left(\frac{x^2}{2}\right) - \int \left(\frac{x^{\cancel{2}}}{\cancel{2}}\right)\left(\frac{\cancel{2}\ln x}{\cancel{x}}\right) dx$

$\frac{x^2(\ln x)^2}{2} - \int x \ln x \, dx$.

(34) $\int \frac{1}{x(\ln x)^2} dx$.

let

$u = \ln x$

$du = \frac{dx}{x}$

$\int \frac{1}{u^2} du$

$\int u^{-2} du = -u^{-1} du + c$

$\boxed{-\frac{1}{u} + c}$.

$\boxed{-\frac{1}{\ln x} + c}$.

(35) $\int \frac{\ln x}{x^2} dx$

let

$u = \ln x$ $\quad \int dv = \int \frac{1}{x^2} dx$

$du = \frac{dx}{x}$ $\quad \boxed{v = -\frac{1}{x}}$.

$(\ln x)\left(-\frac{1}{x}\right) + \int \frac{1}{x} \frac{dx}{x}$

$-\frac{\ln x}{x} + \int x^{-2} dx \Rightarrow \boxed{-\frac{\ln x}{x} - \frac{1}{x} + c}$

Date ....

(36) $\int \frac{(\ln x)^3}{x} dx$

Sol,

let,

$u = \ln x$

$du = \frac{dx}{x}$

$\int u^3 du$

$\frac{u^4}{4} + c$

$\boxed{\frac{(\ln x)^4}{4} + c}$.

(37) $\int x^3 e^{x^4} dx$

Sol,

let,

$\boxed{u = x^4}$

$du = 4x^3 dx$

$\boxed{\frac{1}{4} du = x^3 dx}$

$\int \frac{1}{4} e^u du$

$\frac{1}{4} \int e^u du$

$\frac{1}{4} e^u + C$

$\boxed{\frac{1}{4} e^{x^4} + C}$

(38) $\int x^5 e^{x^3} dx$

let,

$u = x^3$

$du = 3x^2 dx$

$\int dv = \int x^2 e^{x^3} dx$

$\boxed{v = \cancel{\frac{x^4}{4}}}$. $v = \frac{1}{3} e^{x^3}$

~~$(x^3)\left(\frac{x^4}{4}\right) - \int \left(\frac{x^4}{4}\right)(3x^2 dx)$~~

$(x^3)\left(\frac{1}{3} e^{x^3}\right) - \int \left(\frac{1}{3} e^{x^3}\right)(3x^2 dx)$

$\frac{1}{3} x^3 e^{x^3} - \frac{1}{3} \int e^{x^3} 3x^2 dx$

$\boxed{\frac{1}{3} x^3 e^{x^3} - \frac{1}{3} e^{x^3} + C}$.

(39) $\int x^3 \sqrt{x^2+1}\, dx$

$u = x^2$

$\boxed{du = 2x dx}$

$\int dv = \int x \sqrt{x^2+1}\, dx$

$\boxed{v = \frac{1}{3} (x^2+1)^{3/2}}$

$(x^2)\left(\frac{1}{3}(x^2+1)^{3/2}\right) - \int \left(\frac{1}{3}(x^2+1)^{3/2}\right)(2x dx)$.

$\frac{1}{3} (x^2)(x^2+1)^{3/2} - \frac{2}{2 \times 3} \int 2x (x^2+1)^{3/2} dx$.

$\frac{x^2(x^2+1)^{3/2}}{3} - \frac{1}{3}\left[\frac{2}{5}(x^2+1)^{5/2}\right] + c \Rightarrow \boxed{\frac{1}{3} x^2 (x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + c}$.

derivation | Integration
sin → cos | cos → sin
cos → − sin | sin → − cos

Date ....

(40) $\int x^2 \sin x^3 \, dx$

$u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

So,

$\frac{1}{3}\int \sin u \, du$

$\frac{1}{3}(-\cos u) + c$

$-\frac{\cos u}{3} + c \Rightarrow \boxed{-\frac{\cos x^3}{3} + c}$

(41) $\int \sin 3x \cos 2x \, dx$

$\boxed{u = \sin 3x}$

$\boxed{du = 3\cos 3x \, dx}$

$\int dv = \int \cos 2x \, dx$

$\boxed{v = \frac{1}{2}\sin 2x}$

$(\sin 3x)\left(\frac{1}{2}\sin 2x\right) - \int\left(\frac{1}{2}\sin 2x\right)(3\cos 3x \, dx)$

$\frac{1}{2}\sin 3x \sin 2x - \frac{3}{2}\int(\sin 2x)(\cos 3x)\, dx$

$u = \cos 3x$

$\boxed{du = -3\sin 3x}$

$\int dv = \int \sin 2x \, dx$

$\boxed{v = -\frac{1}{2}\cos 2x}$

$\frac{1}{2}(\sin 3x)(\sin 2x) - \frac{3}{2}\left[(\cos 3x)\left(-\frac{1}{2}\cos 2x\right) + \int \frac{1}{2}\cos 2x(-3\sin 3x)\, dx\right.$

I

$\frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x - \frac{3}{2}\boxed{\int \sin 3x \cos 2x \, dx}$

$I = \frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x - \frac{3}{2} I.$

$\boxed{I = \frac{2}{5}\left(\frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x + C\right)}$

$I =$

Date ....

42 same as 41.

43 $\int e^x \sin e^x \, dx$

let,

$u = e^x$

$du = e^x \, dx$

So, $\int \sin u \, du$

$-\cos u + C$

$\boxed{-\cos e^x + C}$

44 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

let,

$u = \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

$2 du = \frac{dx}{\sqrt{x}}$

$2\int e^u du \rightarrow \boxed{2e^u + C}$

$\boxed{2e^{\sqrt{x}} + C}$

45 $\int \cos\sqrt{x} \, dx$

let, $u = \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

$dx = 2u \, du$

So,

$\int \cos u \, 2u \, du$

$\int 2u \cos u \, du$

let, $y = \cancel{2y}\ 2u$

$dy = 2du$.

So,

$\int dv = \int \cos u \, du$

$\boxed{v = \sin u}$

$\int$

$(2u)(\sin u) - \int (\sin u)(2du)$

$2u \sin u - 2\int \sin u \, du$

$\boxed{2u \sin u + 2\cos u + C}$

$\boxed{2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C}$

46 $\int \sqrt{x} e^{\sqrt{x}} dx$

let, $u = \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

$2u \, du = dx$

$2u^2$	+	e^u
$4u$	−	e^u
4	+	e^u
0		e^u

$\int u e^u 2u \, du = \int 2u^2 e^u du \Rightarrow \boxed{2u^2 e^u - 4ue^u + 4e^u + C}$

$\int \sqrt{x} e^{\sqrt{x}} dx = \boxed{2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4e^{\sqrt{x}} + c}$

$\pi/6$, $\pi/4$, ~~$\pi/8$~~ , $\pi/3$, $\pi/2$, π
30 45 60 90 180

Date ....

(47) $\int_0^{\pi/2} \theta^2 \sin 2\theta \, d\theta = \left[-\frac{\theta^2}{2}\sin 2\theta + \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\sin 2\theta + c \right]_0^{\pi/2}$

D.	Integration
θ^2	$\sin 2\theta$
$\theta^2 \xrightarrow{+}$	$-\frac{1}{2}\sin 2\theta$
$2\theta \xrightarrow{-}$	$-\frac{1}{4}\sin 2\theta$
$2 \xrightarrow{+}$	$\frac{1}{8}\sin 2\theta$
0	

$= \left[-\frac{(\pi/2)^2}{2} \sin 2\left(\frac{\pi}{2}\right) + \frac{1}{2}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin 2\left(\frac{\pi}{2}\right) \right]$

$= -\frac{\pi^2}{8}(-1) +$

Then value put Just.

(48) $\int_0^{\pi/2} x^3 \cos 2x \, dx = \frac{1}{2}x^3 \sin 2x - \frac{3}{4}x^2 \cos 2x - \frac{1}{2} x \sin 2x - \frac{6}{16}\cos 2x +$

x^3	$\cos 2x$
x^3 +	$\frac{1}{2}\sin 2x$
$3x^2$ −	$-\frac{1}{4}\cos 2x$
$6x$ +	$-\frac{1}{8}\sin 2x$
6 −	$\frac{1}{16}\cos 2x$
0	

(49) $\int_{2/\sqrt{3}}^{2} t \sec^{-1} t \, dt$

$u = \sec^{-1} t$

$du = \frac{1}{t\sqrt{t^2-1}}$

same as previous