Conditional Probability

Hint: given that

* The conditional probability of A, given B, denoted by $P(A/B)$, is defined by

$$P(A/B) = \frac{P(A \cap B)}{P(A)} \qquad \forall\ P(A) \neq 0.$$

Q-1:- The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(A \cap D) = 0.78$. Find the probability that a plane arrives on time, given that it departs at time.

Sol. $P(A/D) = ?$

$$\therefore P(A/D) = \frac{P(A \cap D)}{P(D)}$$

$$= \frac{0.78}{0.83} = 0.94$$

Q-2:-

Education	Male	female	Total
Elementary	38	45	83
Secondary	28	50	78
college	22	17	39
Total	88	112.	200

If a person is picked at random from this group, find the probability that

a) person is a female, given that she is a college educator.

sol:-

$$P(F/C) = ?$$

$$P(F/C) = \frac{P(F \cap C)}{P(C)}$$

$$= \frac{17/200}{39/200}$$

$$= \frac{17}{39} = 0.44$$

a) person is a secondary educator given that he is a male.

Sol:- $P(S/M) = ?$

$$P(S/M) = \frac{P(S \cap M)}{P(M)}$$

$$= \frac{28/200}{88/200}$$

$$= \frac{28}{88} = 0.32.$$

Q-3:-

	Age group (years)	Investment persentage E_5 <5%	E_6 5% – 10%	E_7 10% – 30%	E_8 30% +	Total
E_1	<30	70	240	270	80	660
E_2	31 to 50	90	300	630	1120	2140
E_3	51 to 65	110	305	780	530	1725
E_4	66 +	200	170	370	260	1000
	Total	470	1015	2050	1990	5525

If a person is selected at random, what is the probability of:

a) Seleted person is from age group of 51 to 65 years, E_3 who invested 5% to 10% E_6.

Sol:- $P(E_3 / E_6) = ?$

$$P(E_3 / E_6) = \frac{P(E_3 \cap E_6)}{P(E_6)}$$

$$= \frac{305/5525}{1015/5525}$$

$$= \frac{305}{1015} = 0.3$$

b) Selected person is who invested more than 30% E_8 given that person is of 31 to 50 years. E_2

Sol: $P(E_8 / E_2) = ?$

$$P(E_8 / E_2) = \frac{P(E_8 \cap E_2)}{P(E_2)}$$

$$= \frac{1120/5525}{2140/5525}$$

$$= 0.52$$

"Multiplication Rule"

from

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \overset{\times}{\cap} B) = P(A/B)\,P(B)$$

or $$P(A \cap B) = P(B/A)\,P(A)$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \leftarrow$$

In general;

$$P(A_1 \cap A_2 \cap \dots \cap A_K) = P(A_1)\,P(A_2/A_1)\,P(A_3/A_1 \cap A_2) \dots P(A_K/A_1 \cap A_2 \cap \dots A_{K-1})$$

Q-1:- 7 black, 9 Red, 11 White and 8 green balls. If 4 balls are randomly selected **without** replacement. What is the probability that:

a) first is Red, 2nd White, 3rd Red and 4th black

Sol:- $P(R_1 \cap W_2 \cap R_3 \cap B_4)$

$= P(R_1) \times P(W_2/R_1) \times P(R_3/R_1 \cap W_2) \times P(B_4/R_1 \cap W_2 \cap R_3)$

$= \frac{9}{35} \times \frac{11}{34} \times \frac{8}{33} \times \frac{7}{32}$

$$= \frac{99}{22440} = \frac{3}{680}$$

$$= 0.0044$$

b) $P(W_1 \cap W_2 \cap G_3 \cap B_4)$

$$= P(W_1) \times P(W_2/W_1) \times P(G_3/W_1 \cap W_2) \times P(B_4/W_1 \cap W_2 \cap \ldots)$$

$$= \frac{11}{35} \times \frac{10}{34} \times \frac{8}{33} \times \frac{7}{32}$$

$$= \frac{\quad}{204} = 0.0049$$

Practice: from table of Education level and Gende... If three persons are selected at random without replacement, then find the probability of:

a) first person is a college educator, 2nd is a Elementary educator and 3rd is again a college edu.

b) first person is a male, 2nd again a male and 3rd person is a female.

Probability: Independent Events

$$P(A/B) = P(A)$$

or $$P(B/A) = P(B)$$

Two events A and B are independent events if and only if

$$P(A \cap B) = P(A) \times P(B)$$

In general;

$$P(A_1 \cap A_2 \cap A_3 \cdots \cap A_k) = P(A_1) \times P(A_2) \times \cdots \times P(A_k)$$

Independent case

$\because P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(A/B)P(B)$

$\Rightarrow P(A \cap B) = P(A)P(B)$

Q-1: A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Sol:-

$$P(F) = 0.98$$

$$P(A) = 0.92$$

$$P(F \cap A) = ?$$

& Because of operating Independently

$$P(F \cap A) = P(F)\,P(A)$$

$$= (0.98)(0.92)$$

$$= 0.9016$$

Q-2:- A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and then a yellow marble?

Sol:-

$$P(G_1 \cap Y_2) = ?$$

$$P(G_1 \cap Y_2) = P(G_1) \times P(Y_2)$$

$$= \frac{5}{16} \times \frac{6}{16}$$

$$= \frac{30}{256} = 0.117$$

Ex:- A coin is tossed and a single 6-sided die is rolled. Find the probability of landing the head on side of the coin and rolling a 3 on the die.

Sol:-

$$P(H) = 1/2$$

$$P(3) = 1/6.$$

$$\therefore P(H \cap 3) = P(H) \times P(3)$$

$$= 1/2 \times 1/6$$

$$= 1/12.$$