Area;

$$X \longrightarrow Z \longrightarrow \text{Area (Prob.)} \quad \boxed{Z = \dfrac{X - \mu}{\sigma}}$$

Inverse Area; $\Longrightarrow \boxed{X = \mu + \sigma Z}$

Area (Prob./ percentage) $\longrightarrow Z \longrightarrow X$

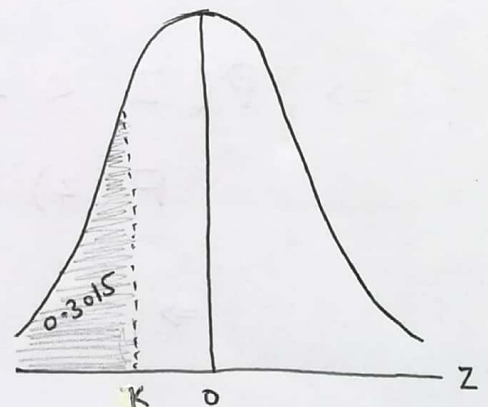**Q-1:** Given a standard normal distribution, find the value of K such that

a) $P(Z < K) = 0.3015$
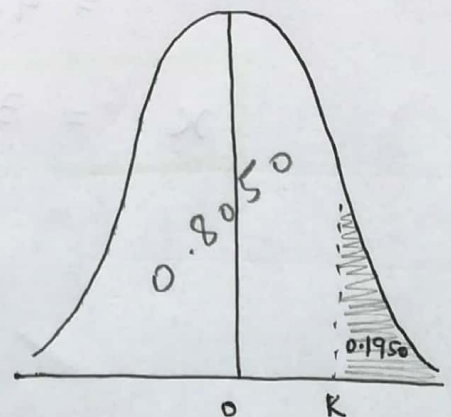
Sol:-

$\Longrightarrow P(Z < K) = 0.3015$

$\Longrightarrow F(K) = 0.3015$

$\Longrightarrow \boxed{K = -0.52}$

b) $P(Z > K) = 0.1950$

Sol:- $P(Z > K) = 0.1950$

$\Longrightarrow P(Z < K) = 1 - 0.1950$

$\quad F(K) = 0.8050$

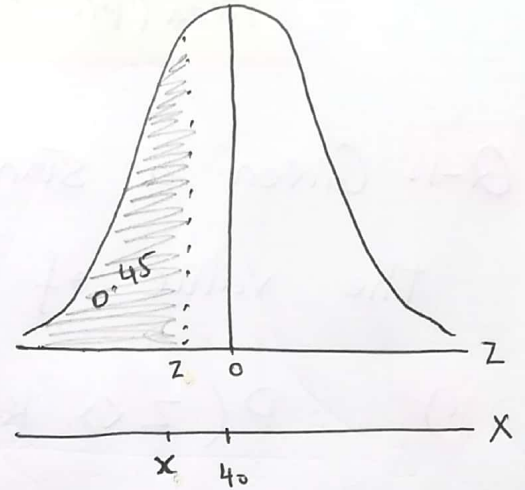$\Longrightarrow \boxed{K = 0.86}$

①

Q-2: Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has

a) 45% of the area to the left.

Sol:-

$$P(X < x) = 0.45$$

$$x = ?$$

with

$$X \sim N(40, 6^2)$$

$$\therefore \quad P(X < x) = 0.45$$

$$\Rightarrow \quad P(Z < z) = 0.45$$

$$F(z) = 0.45$$

$$\Rightarrow \quad Z = -0.13$$

$$\therefore \quad x = \mu + \sigma Z$$

$$= 40 + 6 \times (-0.13)$$

$$= 40 - 0.78$$

$$\boxed{x = 39.22}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$\Rightarrow \sigma Z = X - \mu$$

$$\Rightarrow X = \mu + \sigma Z$$

②

b) 14% of the area to the right.

Sol.- $\qquad P(X > x) = 0.14$

$$x = ?$$

with $\qquad X \sim N(40, 6^2)$

So

$$P(X > x) = 0.14$$

$$\Rightarrow \quad P(Z > z) = 0.14$$
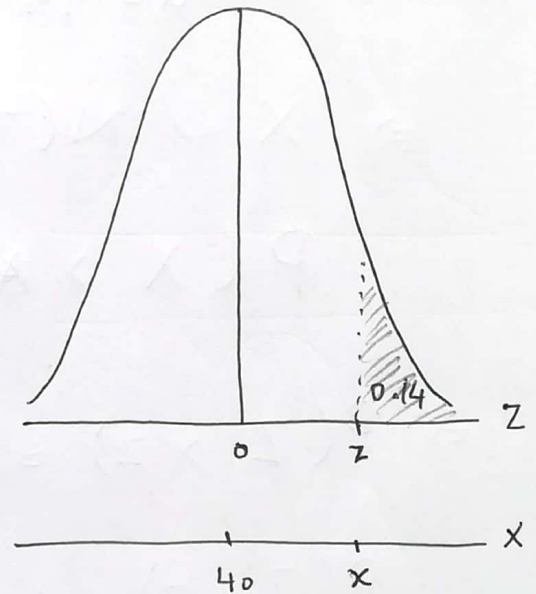
$$\Rightarrow \quad P(Z < z) = 1 - 0.14$$

$$F(z) = 0.86$$

$$\Rightarrow \quad Z = 1.08$$

$$\therefore \quad x = \mu + \sigma z$$

$$= 40 + 6 \times 1.08$$

$$= 40 + 6.48$$

$$\Rightarrow \quad \boxed{x = 46.48}$$

③

Q-3: The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given A's, and the grades are curred to follow a normal distribution, what is the lowest possible A and highest possible B?

Sol:-

$$\mu = 74, \qquad \sigma = 7$$

$$\Rightarrow \quad X \sim N(74, 49).$$

$$P(X > A) = 0.12$$

$$\Rightarrow \quad P(Z > z) = 0.12$$

$$P(Z < z) = 1 - 0.12$$

$$F(z) \quad = 0.88$$

$$\Rightarrow \quad Z = 1.18$$

$$\therefore \quad A = \mu + \sigma Z$$

$$= 74 + 7 \times 1.18$$

$$= 82.26 \quad \approx 83.$$

Therefore lowest A is 83.

and highest B is 82.

④