"The Binomial Probability Distribution"

Properties:

i) The experiment consists of repeated trials.

ii) Each trial results in an outcome that may be classified as a success or a failure.

iii) The probability of success, denoted by p, remains constant from trial to trial.

iv) The repeated trials are independent.

p = prob. of success

q = probability of failure.

$$\therefore \boxed{p + q = 1}$$

$$\Rightarrow \boxed{q = 1 - p}$$

with

total no. of successes $= \boxed{n}$

It is denoted by

$$X \sim B(n, p).$$

Binomial function:

Probability distribution of the binomial random variable X, with prob. of success "p" and prob. of failure "q", the no. of successes in n independent trials, is:

$$\boxed{P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}} \qquad \boxed{x = 0, 1, 2, \ldots, n}$$

NOTE:- There are two parameters of Binomial distribution, i.e; n and p.

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Mean:

Mean of the binomial distribution.

$$\boxed{\mu_x = E(x) = \Sigma x\, p(x)}$$

Variance and Standard deviation:

Variance of the binomial distribution

$$\boxed{Var(X) = \sigma_x^2 = npq}$$

and

$$\boxed{S.D(X) = \sigma_x = \sqrt{npq}}$$

* next lecture

Q-1:- The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that:

i) at least 12 survive?

Sol:-

$$P(X \geq 12) = ? \qquad q = 0.6$$

$\because$ $p = 0.4$ and $n = 15$

$\therefore$ we will use binomial distribution with

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} \qquad x = 0, 1, 2, \dots, 15$$

Thus

$$P(X \geq 12) = P(X=12) + P(X=13) + P(X=14) + P(X=15)$$

$$= {}^{15}C_{12}\,(0.4)^{12}(0.6)^{15-12} + {}^{15}C_{13}\,(0.4)^{13}(0.6)^{15-13}$$

$$+ {}^{15}C_{14}\,(0.4)^{14}(0.6)^{15-14} + {}^{15}C_{15}\,(0.4)^{15}(0.6)^{15-15}$$

$$= 455 \times (0.000017)(0.216) + 105 \times (0.0000067)(0.36)$$

$$+ 15 \times (0.00000268)(0.6) + 1 \times (0.00000107) \times 1$$

$$= 0.00165 + 0.000254 + 0.000024 + 0.00000107$$

$$= 0.00193$$

ii) Exactly 7 will survive?

sol:- $P(X=7) = ?$

$$X \sim B(15, 0.4)$$

with
$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x} \qquad x = 0, 1, 2, \ldots, 15$$

$$\therefore P(X=7) = {}^{15}C_{7}\,(0.4)^{7}(0.6)^{15-7}$$

$$= 6435 \times 0.0016384 \times 0.0168$$

$$= 0.177$$

iii) at most 2 will survive?

$P(X \leq 2) = ?$

sol:-

$$\therefore P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{15}C_{0}(0.4)^{0}(0.6)^{15} + {}^{15}C_{1}(0.4)^{1}(0.6)^{14} + {}^{15}C_{2}(0.4)^{2}(0.6)^{13}$$

$$= 0.00047 + 0.0047 + 0.022$$

$$= 0.0271$$

Q-2: A nationwide survey of college seniors by the university of Michigan revealed that almost 70% disapprove of daily pot smoking, according to a report in Parade. If 12 seniors are selected at random and asked their opinion. Find the probability that the number who disapprove of smoking pot daily is:

a) any where from $\boxed{7 \text{ to } 9}$;

sol:-

$$P(7 \le X \le 9) = ?$$

where

$$p = 0.70 \implies q = 1 - 0.70$$
$$= 0.30$$

and

$$n = 12$$

$$\therefore X \sim B(12, 0.70)$$

with p.d.f as:

$$P(X) = {}^{n}C_{x}\, p^{x}\, q^{n-x} \qquad X = 0, 1, 2, \dots, 12.$$

Thus

$P(7 \le X \le 9) = P(X=7) + P(X=8) + P(X=9)$

$= {}^{12}C_7 (0.7)^7 (0.3)^5 + {}^{12}C_8 (0.7)^8 (0.3)^4 + {}^{12}C_9 (0.7)^9 (0.3)^3$

$= 0.1585 + 0.2311 + 0.2397$

$= 0.6293$

b) at most 5;

sol:- $P(X \le 5) = ?$

$P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= {}^{12}C_0 (0.7)^0 (0.3)^{12} + {}^{12}C_1 (0.7)^1 (0.3)^{11} + {}^{12}C_2 (0.7)^2 (0.3)^{10}$
$+ {}^{12}C_3 (0.7)^3 (0.3)^9 + {}^{12}C_4 (0.7)^4 (0.3)^8 + {}^{12}C_5 (0.7)^5 (0.3)^7$

$= 5.3 \times 10^{-7} + 1.5 \times 10^{-5} + 1.9 \times 10^{-4} + 1.5 \times 10^{-3} + 7.8 \times 10^{-3} + 0.0291$

$= 0.0386$

c) not less than 8;

sol:- $P(X \geq 8) = ?$

or

$1 - P(X \le 7) = ?$

$P(X \geq 8) = P(X=8) + P(X=9) + P(X=10) + P(X=11) + P(X=12).$

$= 0.7237$

"Mean and Variance of Binomial Distribution"

For $X \sim B(n, p)$

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

and

$$\text{standard deviation} = \sqrt{npq}$$

Q-1:- The probability that a patient recovers from a rare blood disease is 0.4. If 150 people are known to have contracted this disease,

a) What is average no. of people who survive?

sol:- $X \sim B(150, 0.4)$

$\Rightarrow n = 150, \quad p = 0.4$

$\therefore$ Average no. of survival people $= np$

$= 150 \times 0.4$

$= 60.$

b) What is the variance value of recoverd people?

sol:- $\text{Variance} = npq$

$= 150 \times 0.4 \times 0.6$

$= 36$