2- Poisson Distribution

Case 1:-

When average is given

average $= \lambda$

$$X \sim \text{Poisson}(\lambda)$$

with

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \dots$$

or

representing the no. of outcomes occuring in a given time interval or specified region denoted by t,

$$X \sim \text{Poisson}(\lambda t)$$

with

$$f(x) = \frac{e^{-\lambda t}(\lambda t)^{x}}{x!} \qquad x = 0, 1, 2, \dots$$

where λ is the average number of outcomes per unit time, distance, area, or volume.

$\therefore$ Average $= \lambda t$

Q-1: A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make

a) no errors?

Sol:-

$$\text{Average} = \lambda = 2.$$

$$\therefore \; X \sim \text{Poisson}(2)$$

with

$$P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

Thus

$$P(X=0) = ?$$

$$\Rightarrow \quad P(X=0) = \frac{e^{-2}\,2^{0}}{0!}$$

$$= 0.1353$$

b) 4 or more errors?

Sol:-

$$P(X \geq 4) = ?$$

$$\therefore \; P(X \geq 4) = 1 - P(X \leq 3)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!}\right]$$

$$= 1 - e^{-2}\left[1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6}\right]$$

$$= 1 - 0.1353\left[1 + 2 + 2 + 1.33\right]$$

$$= 1 - 0.856$$

$$= 0.144$$

Q-2:- The no. of customers arriving per hour at a certain automobile service facility is assumed to follow a poisson distribution with $\lambda = 7$.

a) Compute the probability that more than 5 customers will arrive in a 2-hour period.

Sol:-

$$\lambda = 7$$

$$t = 2.$$

$$\therefore \text{Average} = \lambda t$$

$$= 7 \times 2$$

$$= 14$$

$P(X > 5) = ?$

$$X \sim \text{Poisson}(14)$$

with

$$f(x) = \frac{e^{-14}\, 14^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

$$\therefore P(X>5) = 1 - P[X \leq 5]$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$$

$$= 1 - \left[\frac{e^{-14}\, 14^{0}}{0!} + \frac{e^{-14}\, 14^{1}}{1!} + \frac{e^{-14}\, 14^{2}}{2!} + \frac{e^{-14}\, 14^{3}}{3!} + \frac{e^{-14}\, 14^{4}}{4!} + \frac{e^{-14}\, 14^{5}}{5!} \right]$$

$$= 1 - e^{-14} [1 + 14 + 98 + 457.33 + 1600.67 + 4481.87]$$

$$= 1 - 8.32 \times 10^{-7} (6652.87)$$

$$= 1 - 0.0055$$

$$= 0.9945$$

"Mean and Variance of Poisson Distribution"

for $X \sim Poisson(\lambda)$

$$\text{Mean} = \lambda$$

$$\text{Variance} = \lambda$$

and

$$\text{Standard deviation} = \sqrt{\lambda}.$$

. — .

(b):- Q-1:- What is the mean no. of arrivals during a 2-hours period?

sol:- $\text{Mean} = \lambda t = 14$

. — .

Q-2:- A certain area of the eastern United States is, on average, hit by 6 hurricans a year. find the mean and variance of the r.v X, representing the no. of hurricanes per year to hit a certain area of the eastern US.

sol:- $\because$ Average $= \lambda = 6$.

$\therefore$ Mean $= 6$

& Variance $= 6$.

. — .

Poisson Distribution

Case 2:-

A limiting approximation of the binomial distribution, when p, the probability of success is very small but n, the number of trials is so large, that the product $np = \lambda$ is of a moderate size; that is,

$$n \to \infty$$

$$p \longrightarrow 0 .$$

$$\therefore \quad \lambda = np$$

We will use poisson distribution, rather than binomial distribution.

Q-1:- Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. If 10,000 returns are selected at random and examined, find the probability that 6, 7, or 8 of them contain an error.

Sol:-

$$p = 1/1000 \quad \text{(very small)}$$

$$n = 10,000 \quad \text{(very large)}$$

$\therefore$ Average $= \lambda = np$

$$= 10,000 \times \frac{1}{1000}$$

$$\lambda = 10.$$

Thus: $X \sim$ Poisson (10)

with

$$P(X) = \frac{e^{-10}\, 10^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

Here

$$P(6 \le X \le 8) = ?$$

$$P(6 \le X \le 8) = P(X=6) + P(X=7) + P(X=8)$$

$$= \frac{e^{-10}\, 10^{6}}{6!} + \frac{e^{-10}\, 10^{7}}{7!} + \frac{e^{-10}\, 10^{8}}{8!}$$

$$= e^{-10}\left[\frac{10^{6}}{720} + \frac{10^{7}}{5040} + \frac{10^{8}}{40320}\right]$$

$$= 0.0000454 \times 10^{6}\left[0.00139 + 0.00198 + 0.00248\right]$$

$$= 45.4 \times 0.00585$$

$$= 0.2656$$