Exercise 4.5

(35) $y'' - 9y = 54$

$y'' - 9y = 54$

Auxilliary ⇒

$m^2 - 9 = 0$

~~$m(m-9)=0$~~

$(m-3)(m+3) = 0$

$\boxed{m_1 = 3}$, $\boxed{m_2 = -3}$

$y_c = c_1 e^{3x} + c_2 e^{-3x}$

Annihilator of 54 ⇒

Formula will be ⇒ D^n

So, D^1

Now,

$y'' - 9y = 54$

$D^2 - 9 = 54$

Multiplying by D' on b-s.

$D(D^2 - 9) = 0$

So,

$m(m^2 - 9) = 0$

$m((m+3)(m-3)) = 0$

$m_1 = 3,\ m_2 = -3,\ m_3 = 0$

So,

$y_p = c_1 e^{3x} + c_2 e^{-3x} + c_3 e^0$

$y_p = \underbrace{c_1 e^{3x} + c_2 e^{-3x}}_{\text{already in } y_c} + c_3$

So, $y_p = c_3$ — a)

let: $\boxed{y_p = A,\ y_p' = 0,\ y_p'' = 0}$ So,

Putting values in original equation

$y'' - 9y = 54$

$0 - 9A = 54$

$A = -\frac{54}{9}$

$\boxed{A = -6}$

So, putting value in eq-a)

$\boxed{y_p = -6}$

So,

$y = y_c + y_p$

$\boxed{y = c_1 e^{3x} + c_2 e^{-3x} - 6}$

Answer.

(36) $2y'' - 7y' + 5y = -28$ — i)

Auxillary

$2y'' - 7y' + 5y = -28$

$2m^2 - 7m + 5 = \cancel{-28}\ 0$

$2m^2 - 5m - 2m + 5 = 0$

$m(2m-5) - 1(2m-5) = 0$

$(2m-5)(m-1) = 0$

$\boxed{m_1 = \frac{5}{2}}$, $\boxed{m_2 = 1}$

$y_c = c_1 e^{5/2 x} + c_2 e^{x}$

Annihilate -28 ⇒

Formula will be D^n

So, D^1

Now,

$2y'' - 7y' + 5y = -28$

$2D^2 - 7D + 5 = -28$

Multiplying by D' on b-s

$D(2D^2 - 7D + 5) = 0$

So,

$m(2m^2 - 7m + 5) = 0$

$m((2m-5)(m-1)) = 0$

$m_1 = \frac{5}{2},\ m_2 = 1,\ m_3 = 0$

So,

$y_p = \overbrace{c_1 e^{5/2 x} + c_2 e^{x}}^{\text{same } y_c} + c_3$

So, $y_p = c_3$ — a)

let

$\boxed{y_p = A}$, $\boxed{y_p' = 0}$, $\boxed{y_p'' = 0}$

So, putting value in eq-i) (original equation)

$2y'' - 7y' + 5y = -28$

$2(0) - 7(0) + 5A = -28$

$5A = -28$

$\boxed{A = -\frac{28}{5}}$

So, putting value in eq-a)

~~$y_p = c_1 e^{5/2 x} + c_2 e^{x} + \left(-\frac{28}{5}\right)$~~

~~$y_p = c_1 e^{5/2 x} + c_2 e^{x} - \frac{28}{5}$~~

$y_p = -\frac{28}{5}$

So, $y = y_c + y_p$

$\boxed{y = c_1 e^{5/2 x} + c_2 e^{x} - \frac{28}{5}}$ Ans

(37) $y'' + y' = 3.$

Auxillary ⇒

$y'' + y' = 3$

$m^2 + m = 0$

$m(m+1) = 0$

$\boxed{m_1 = 0}$, $\boxed{m_2 = -1}$

$y_c = c_1 e^{0x} + c_2 e^{-x}$

$\boxed{y_c = c_1 + c_2 e^{-x}}$

Annihilate the 3 ⇒
Formula used ⇒

D^n

So, D^1.

Now,

$y'' + y' = 3$

$D^2 + D = 3$

Multiplying b.s by D'

$D(D^2+D) = 0$

$D(D^2+D) = 0$

$D(D(D+1)) = 0$

$\cancel{D_1 = D}$,

$m(m(m+1)) = 0$

$\boxed{m_1 = 0,\ m_2 = -1,\ m_3 = 0}$

So,

$y_p = \underbrace{c_1 + c_2 e^{-x}}_{y_c} + c_3 x$

$y_p = c_3 x$

So, let,

$y_p = A$

$y_p'' = 0$

$y_p' = A$

Hence,
Putting value in original equation ⇒

$y'' + y' = 3$

$0 + A = 3$

$\boxed{A = 3}$.

So, $\boxed{y_p = 3x}$

So, $\boxed{y_p = c_1 + c_2 e^{-x} + 3x}$

Hence,

$y = y_c + y_p$

$\boxed{y = c_1 + c_2 e^{-x} + 3x}$

Answer.

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(39) $y'' + 4y' + 4y = 2x + 6$

$y'' + 4y' + 4y = 2x + 6$

Auxillary:

$m^2 + 4m + 4 = 0$

$m^2 + 2m + 2m + 4 = 0$

$m(m+2) + 2(m+2) = 0$

$(m+2)(m+2) = 0$

$\boxed{m_{1,2} = -2}$

$y_c = c_1 e^{-2x} + c_2 x e^{-2x}$

Annihilate the $2x+6$ ⇒
As Formula used will be: D^n

D^2

So,

$y'' + 4y' + 4y = 2x + 6$

$D^2 + 4D + 4 = 2x + 6$

Multiplying by D^2 on b.s.

$D^2(D^2 + 4D + 4) = 0$

$D^2(D^2 + 4D + 4) = 0$

$m^2(m^2 + 4m + 4) = 0$

$m^2((m+2)^2) = 0$

$\boxed{m_{1,2} = -2}$, $\boxed{m_{3,4} = 0}$

So,

$y_p = c_1 e^{-2x} + c_2 x e^{-2x} + c_3 e^0 + c_4 x e^0$

$y_p = \underbrace{c_1 e^{-2x} + c_2 x e^{-2x}}_{y_c} + c_3 + c_4 x$

$\boxed{y_p = c_3 + c_4 x}$

Let:

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

So, putting value in original equation:

$y'' + 4y' + 4y = 2x + 6$

$0 + 4(Ax + B) + 4A = 2x + 6$

$4A + 4Ax + 4B = 2x + 6$

Making equation:

$4A = 2$ — i) ⇒ $\boxed{A = 1/2}$

$4A + 4B = 6$ — ii)

Putting value of B in ii)

$\cancel{4A + \frac{1}{2} = 6}$

$\cancel{4A = 6 - \frac{1}{2}} \Rightarrow \cancel{\frac{12-1}{2}}$

$\cancel{4A = \frac{11}{2}}$

$\cancel{A = -\frac{11}{8}}$

$4A + 4B = 6$

$4(A+B) = 6 \Rightarrow A + B = \frac{3}{2}$

as we know $A = 1/2$.

$\frac{1}{2} + B = \frac{3}{2}$

$B = \frac{3}{2} - \frac{1}{2} \Rightarrow B = \cancel{\frac{2}{2}}$

$\boxed{B = 1}$, $\boxed{A = 1/2}$

So,

$y_p = \frac{1}{2}x + 1$

So, Hence,

$y = y_c + y_p$

$\boxed{y = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{1}{2}x + 1}$

Answer

(40) $y'' + 3y' = 4x - 5$

$y'' + 3y' = 4x - 5$

Auxillary equation:

$m^2 + 3m = 0$

$m(m+3) = 0$

$\boxed{m_1 = -3}, \boxed{m_2 = 0}$

So,

$y_c = c_1 e^{-3x} + c_2 e^0$

$\boxed{y_c = c_1 e^{-3x} + c_2}$

Annihilate of $4x-5 \Rightarrow$

Using Formula $\Rightarrow$

$D^n \xrightarrow{so} D^2$

So,

$y'' + 3y' = 4x - 5$

$D^2 + 3D = 4x - 5$

Multiplying b.s by D^2

$D^2(D^2 + 3D) = 0$

$D^2(D(D+3)) = 0$

$m^2(m(m+3)) = 0$

$\boxed{m_1 = -3}, \boxed{m_{2,3,4} = 0}$

$y_p = c_1 e^{-3x} + c_2 + c_3 x e^0 + c_4 x^2 e^0$

$y_p = \underbrace{c_1 e^{-3x} + c_2}_{y_c} + c_3 x + c_4 x^2$

$y_p = c_3 x + c_4 x^2$

let:

$y_p = Ax^2 + Bx$

So, $y_p' = 2Ax + B$

$y_p'' = 2A$

Putting value in original equation

$y'' + 3y' = 4x - 5$

$2A + 3(2Ax + B) = 4x - 5$

$2A + 6Ax + 3B = 4x - 5$

Rearrange & Making equation

$6A = 4$ —i)

$\boxed{A = 2/3}$

eq-ii) $2A + 3B = -5$

Putting value of A in eq-ii)

$2\left(\frac{2}{3}\right) + 3B = -5$

$3B = -5 - \frac{4}{3}$

$3B = \frac{-15-4}{3}$

$3B = \frac{-19}{3}$

$\boxed{B = -\frac{19}{9}}$

So,

$\boxed{y_p = \frac{2}{3}x^2 - \frac{19}{9}x}$

Hence,

$y = y_c + y_p$

$\boxed{y = c_1 e^{-3x} + c_2 + \frac{2}{3}x^2 - \frac{19}{9}x}$

(42) $y'' - 2y' + y = x^3 + 4x$

Auxillary:

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$\boxed{m_{1,2} = 1}$

$\boxed{y_c = c_1 e^x + c_2 x e^x}$

Annihilate $x^3 + 4x \Rightarrow$

Formula $\Rightarrow D^n$

Here $\Rightarrow$

D^4

So, $y'' - 2y' + y = x^3 + 4x$

$D^2 - 2D + 1 = x^3 + 4x$

Multiply by D^4 on b.s $\Rightarrow$

$D^4(D^2 - 2D + 1) = 0$

$m^4(m^2 - 2m + 1) = 0$

$m^4((m-1))^2 = 0$

So,

$\boxed{m_{1,2} = 1}, \boxed{m_{3,4,5,6} = 0}$

$\boxed{y_p = \underbrace{c_1 e^x + c_2 x e^x}_{} + c_3 + c_4 x + c_5 x^2 + c_6 x^3}$

$y_p = c_3 + c_4 x + c_5 x^2 + c_6 x^3$

let $\Rightarrow$

$y_p = Ax^3 + Bx^2 + Cx + D$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6Ax + 2B$

Putting value in original equation $\Rightarrow$

$y'' - 2y' + y = x^3 + 4x$

$(6Ax + 2B) - 2(3Ax^2 + 2Bx + C) + (Ax^3 + Bx^2 + Cx + D) = x^3 + 4x$

$6Ax + 2B - 6Ax^2 - 4Bx - 2C + Ax^3 + Bx^2 + Cx + D = x^3 + 4x$

Forming equation $\Rightarrow$

$x^3 \Rightarrow A = 1$ —i)

$x^2 \Rightarrow -6A + B = 0$ —ii)

$x \Rightarrow 6A - 4B + C = 4$ —iii)

$x^0 \Rightarrow 2B - 2C + D = 0$ —iv)

So, as we know =>

$\boxed{A=1}$.

2nd equation =>

$B-6A=0$

$B-6(1)=0$

$B-6=0$

$\boxed{B=6}$.

3rd equation =>

$6A-4B+C=4$

$6(1)-4(6)+C=4$

$6-24+C=4$

$-18+C=4$

$C=4+18$

$\boxed{C=22}$.

4th equation =>

$2B-2C+D=0$

$2(6)-2(22)+D=0$

$12-44+D=0$

$-32+D=0$

$\boxed{D=32}$

So, putting values =>

$y_p = Ax^3+Bx^2+Cx+D$

$y_p = x^3+6x^2+22x+32$

So,

$y = y_c + y_p$

$$\boxed{y = c_1e^{x}+c_2xe^{x}+x^3+6x^2+22x+32}$$

43) $y''-y'-12y = e^{4x}$

Auxiliary

$m^2-m-12=0$

$m^2+3m\mp 4m-12=0$

$m^2-3m+4m-12=0$

$m(m+3)\mp 4(m+3)=0$

$(m+3)(m\mp 4)=0$

$\boxed{m_1=-3}$, $\boxed{m_2=+4}$.

$$\boxed{y_c = c_1e^{-3x}+c_2e^{+4x}}$$

Annihilate the e^{4x},

Formula used => $(D-\alpha)^n$

here $\alpha=4,\ n=1$

So, $(D-4)^1$

So,

$y''-y'-12y=e^{4x}$

$D^2-D-12=e^{4x}$

Multiplying on b.s

$(D-4)(D^2-D-12)=0$

$(m-4)(m^2-m-12)=0$

$(m-4)(m-4)(m+3)=0$

$\boxed{m_1=-3}$; $\boxed{m_{2,3}=4}$

So,

$y_p = \underbrace{c_1e^{-3x}+c_2e^{4x}}_{y_c}+c_3xe^{4x}$

$y_p = c_3xe^{4x}$

let =>

$y_p = Axe^{4x}$

$y_p' = 4Axe^{4x}+Ae^{4x}$

$y_p'' = 16Axe^{4x}+4Ae^{4x}+4Ae^{4x}$

$y_p'' = 16Axe^{4x}+8Ae^{4x}$

Putting values in eq-i) (original equation).

$y''-y'-12y = e^{4x}$

$(8Ae^{4x}+16Axe^{4x})-(4Axe^{4x}+Ae^{4x})-12(Axe^{4x}) = e^{4x}$.

$8Ae^{4x}+\cancel{16Axe^{4x}}-\cancel{4Axe^{4x}}-Ae^{4x}-\cancel{12xe^{4x}A} = e^{4x}$

$7Ae^{4x}=e^{4x}$

Making equation =>

$7A=1$

$\boxed{A=1/7}$

So, putting values =>

$y_p = Axe^{4x}$

$y_p = \frac{1}{7}xe^{4x}$

So,

$y = y_c + y_p$

$$\boxed{y = c_1e^{-3x}+c_2e^{4x}+\frac{1}{7}xe^{4x}}$$

~~45) $y^4 + 25y^2 = 6\sin x$.~~

44) $y'' + 2y' + 2y = 5e^{6x}$

$y'' + 2y' + 2y = 5e^{6x}$

Auxillary:

$m^2 + 2m + 2 = 0$

Using quadratic equation.

$b = 2,\ a = 1,\ c = 2$

$$m_{1,2} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2}$$

$$\boxed{m_{1,2} = -1 \pm i}$$

$$\boxed{y_c = c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}} \text{ — a)}$$

Annihilate, $5e^{6x}$ ⇒

Using Formula ⇒

$(D - \alpha)^n$

here $\alpha = 6,\ n = 1$

$(D-6)^1$

Multiply b.s of original eq.

$y'' + 2y' + 2y = 5e^{6x}$

$(D-6)(D^2 + 2D + 2) = (5e^{6x})(D-6)$

$(D-6)(D^2+2D+2) = 0$

$(m-6)(m^2+2m+2) = 0$.

$\boxed{m_{1,2} = -1 \pm i}$, $\boxed{m_3 = 6}$

eq-a)

$y_c = c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}$

$y_c = c_1 e^{-x}(\cos x + \sin x) + c_2 e^{-x}(\cos x - \sin x)$

$y_c = (c_1 + c_2)e^{-x}\cos x + (c_1 - c_2)e^{-x}\sin x$

$$\boxed{y_c = k_1 e^{-x}\cos x + k_2 e^{-x}\sin x}$$

$$\boxed{y_p = \underbrace{k_1 e^{-x}\cos x + k_2 e^{-x}\sin x}_{y_c} + k_3 e^{6x}}$$

$y_p = k_3 e^{6x}$

let,

$y_p = Ae^{6x}$ —— i)

$y_p' = 6Ae^{6x}$

$y_p'' = 36Ae^{6x}$

Putting values in original equation ⇒

$y'' + 2y' + 2y = 5e^{6x}$

$36Ae^{6x} + 2(6Ae^{6x}) + 2(Ae^{6x}) = 5e^{6x}$

$36Ae^{6x} + 12Ae^{6x} + 2Ae^{6x} = 5e^{6x}$

$50A\cancel{e^{6x}} = 5\cancel{e^{6x}}$

$50A = 5 \Rightarrow \boxed{A = 1/10}$

Putting value in eq- i)

$y_p = \frac{1}{10} e^{6x}$.

Thus,

$y = y_c + y_p$

$$\boxed{y = k_1 e^{-x}\cos x + k_2 e^{-x}\sin x + \frac{1}{10}e^{6x}}$$

45) $y'' - 2y' - 3y = 4e^x - 9$.

$y'' - 2y' - 3y = 4e^x - 9$

Auxillary,

$m^2 - 2m - 3 = 0$

$m^2 - 3m + m - 3 = 0$

$m(m-3) + 1(m-3) = 0$

$(m+1)(m-3) = 0$

$\boxed{m_1 = -1}$ $\boxed{m_2 = +3}$

$$\boxed{y_c = c_1 e^{-x} + c_2 e^{3x}}$$

Annihilate $4e^x - 9$ ⇒

Using Formula ⇒

$(D-\alpha)^n$ here $\alpha = 1, n = 1$.

D^n ⇒ here $n = 1$.

So, $D(D-1)$

So, original equation ⇒

$y'' - 2y' - 3y = 4e^x - 9$

$(D^2 - 2D - 3) = 4e^x - 9$

Multiply $D(D-1)$ on b-s

$D(D-1)(D^2 - 2D - 3) = 0$

$m(m-1)(m^2 - 2m - 3) = 0$

$\boxed{m_1 = -1}$, $\boxed{m_2 = 3}$, $\boxed{m_3 = 1}$, $\boxed{m_4 = 0}$

$\boxed{y_p = \underbrace{c_1 e^{-x} + c_2 e^{3x}}_{y_c} + c_3 e^x + c_4}$

$y_p = c_3 e^x + c_4$

let,

$y_p = Ae^x + B$

$y_p' = Ae^x$

$y_p'' = Ae^x$

So, original equation,

$y'' - 2y' - 3y = 4e^x - 9$

$Ae^x - 2Ae^x - 3(Ae^x + B) = 4e^x - 9$

$Ae^x - 2Ae^x - 3Ae^x - 3B = 4e^x - 9$

$-4Ae^x - 3B = 4e^x - 9$

Forming equations,

$-4A\cancel{e^x} = 4\cancel{e^x}$ —i)

$-4A = 4$

$\boxed{A = -1}$

$-3B = -9$

$\boxed{B = 3}$

So,

$y_p = Ae^x + B$

$\boxed{y_p = -e^x + 3}$

Hence,

$y = y_c + y_p$

$\boxed{y = c_1 e^{-x} + c_2 e^{3x} - e^x + 3}$

46) $y'' + 6y' + 8y = 3e^{-2x} + 2x$

$y'' + 6y' + 8y = 3e^{-2x} + 2x$

Auxillary,

$m^2 + 6m + 8 = 0$

$m^2 + 4m + 2m + 8 = 0$

$m(m+4) + 2(m+4) = 0$

$(m+2)(m+4) = 0$

$\boxed{m_1 = -2}$, $\boxed{m_2 = -4}$

$\boxed{y_c = c_1 e^{-2x} + c_2 e^{-4x}}$

Annihilate $3e^{-2x} + 2x$

$(D-\alpha)^n \qquad D^n$

$\alpha = -2 \qquad n = 2$

$n = 1$

So,

$D^2(D+2)$ —i)

So, original equation ⇒

$y'' + 6y' + 8y = 3e^{-2x} + 2x$

$D^2 + 6D + 8 = 3e^{-2x} + 2x$

Multiply eq-i) on b-s

$D^2(D+2)(D^2 + 6D + 8) = 0$

$m^2(m+2)(m^2 + 6m + 8) = 0$

$\boxed{m_1 = -2}$, $\boxed{m_2 = -4}$, $\boxed{m_3 = -2}$, $\boxed{m_4 = 0}$, $\boxed{m_5 = 0}$

$\boxed{y_p = \underbrace{c_1 e^{-2x} + c_2 e^{-4x}}_{y_c} + c_3 x e^{-2x} + c_4 + c_5 x}$

$y_p = c_3 x e^{-2x} + c_4 + c_5 x$

let ⇒

$\boxed{y_p = Axe^{-2x} + B + Cx}$

So,

$y_p' = -2Axe^{-2x} + Ae^{-2x} + C$

$y_p'' = -4Axe^{-2x} + Ae^{-2x} + Ae^{-2x}$

$y_p'' = +4Ae^{-2x} + 4Ae^{-2x}$

So, putting value

$y'' + 6y' + 8y = 3e^{-2x} + 2x$

$+4Ae^{-2x} + 4Ae^{-2x} + 6(-2Axe^{-2x} + Ae^{-2x} + C) + 8(Axe^{-2x} + B + Cx) = 3e^{-2x} + 2x$

$+4Ae^{-2x} + 4Ae^{-2x} - 12Axe^{-2x} + 6Ae^{-2x} + 6C + 8Axe^{-2x} + 8B + 8Cx = 3e^{-2x} + 2x$

⇒ Forming equation ⇒

e^{-2x} ⇒

$4A + 4A + 6A - 12A = 3$

$14A - 12A = 3$

$2A = 3$

$\boxed{A = 3/2}$

x →

$8C = 2$

$\boxed{C = \frac{1}{4}}$

$6C + 8B = 0$

$\cancel{6}^{3}\left(\frac{1}{\cancel{4}_{2}}\right) + 8B = 0$

$\frac{3}{2} + 8B = 0$

$B = -\frac{3}{2 \times 8}$

$\boxed{B = -\frac{3}{16}}$

So,

$y_p = Axe^{-2x} + B + Cx$

$y_p = \frac{3}{2}xe^{-2x} + (-\frac{3}{16}) + \frac{1}{4}x$

$\boxed{y_p = \frac{3}{2}xe^{-2x} - \frac{3}{16} + \frac{1}{4}x}$

So,

$y = y_c + y_p$

$\boxed{y = c_1e^{-2x} + c_2e^{-4x} + \frac{3}{2}xe^{-2x} + \frac{1}{4}x - \frac{3}{16}}$

(47): $y'' + 25y = 6\sin x.$

$y'' + 25y = 6\sin x$

Auxillary ⇒

$m^2 + 25 = 0$

$(m+5)^2 = 0$

$\boxed{m_{1,2} = \pm 5i}$

$\boxed{y_c = c_1e^{-5x} + c_2xe^{-5x}}$

Annhilate equation $6\sin x$ ⇒

Using Formula ⇒

$(D^2 - 2\alpha D + (\alpha^2 + \beta^2))^n$

Here ⇒ $\alpha = 0, \beta = 1, n = 1.$

$(D^2 - 2(0)D + ((0)^2 + (1)^2))'$

$(D^2+1)'$

So,

$y'' + 25y = 6\sin x$

$D^2 + 25 = 6\sin x$

Multiply (D^2+1) on b.s

$(D^2+1)(D^2+25) = 0$

$\boxed{m_{1,2} = -5}$, $\boxed{m_{3,4} = -1}$

$\boxed{y_p = \underbrace{c_1e^{-5x} + c_2xe^{-5x}}_{y_c} + c_3e^{-x} + c_4xe^{-x}}$

$y_p = c_3e^{-x} + c_4xe^{-x}$

Let ⇒

$\boxed{y_p = Ae^{-x} + Bxe^{-x}}$

$y_p' = Ae^{-x} + (-Bxe^{-x}) + Be^{-x}$

$y_p' = Ae^{-x} - Bxe^{-x} + Be^{-x}$

$y_p'' = (Ae^{-x} + Bxe^{-x} - Be^{-x} + Be^{-x}$

$y_c = c_1(\cos 5x + \sin 5x) + c_2(\cos 5x - \sin 5x)$

$y_c = (c_1 + c_2)\cos 5x + (c_1 - c_2)\sin 5x$

$\boxed{y_c = k_1\cos 5x + k_2\sin 5x.}$

Annhilate the $6\sin x$ ⇒

Using Formula ⇒

$(D^2 - 2\alpha D + (\alpha^2 + \beta^2))^n$

Here ⇒ $\alpha = 0, \beta = 1, n = 1.$

$(D^2 - 2(0)D + ((0)^2 + (1)^2))'$

$\boxed{(D^2+1)'}$

So, $y'' + 25y = 6\sin x$

$D^2 + 25 = 6\sin x$

$(D+1)(D^2+25) = 0$

$(m+1)(m^2+25) = 0$

So,

$\boxed{m_{1,2} = \pm 5i}$ & $\boxed{m_{3,4} = \pm 1i}$

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So,

$$y_p = \underbrace{K_1 \cos 5x + K_2 \sin 5x}_{y_c} + K_3 \cos x + K_4 \sin x$$

$$y_p = K_3 \cos x + K_4 \sin x$$

let,

$$y_p = A\cos x + B\sin x$$

$$y_p' = -A\sin x + B\cos x$$

$$y_p'' = -A\cos x - B\sin x$$

So, The original value ⇒

$$y'' + 25y = 6\sin x$$

$$-A\cos x - B\sin x + 25(A\cos x + B\sin x) = 6\sin x$$

$$-A\cos x - B\sin x + 25A\cos x + 25B\sin x = 6\sin x$$

$$24A\cos x + 24B\sin x = 6\sin x$$

Then we have ⇒

cos →

$$24A = 0 \quad \text{— i)}$$

$$\boxed{A = 0}$$

sin →

$$24B = 6 \quad \text{— ii)}$$

$$B = 6/24$$

$$\boxed{B = 1/4}$$

Then,

$$y_p = A\cos x + B\sin x$$

$$y_p = (0)\cos x + \frac{1}{4}\sin x$$

$$\boxed{y_p = \frac{1}{4}\sin x}$$

Hence,

$$y = y_c + y_p.$$

$$\boxed{y = K_1 \cos 5x + K_2 \sin 5x + \frac{1}{4}\sin x}$$

(48) $y'' + 4y = 4\cos x + 3\sin x - 8$

$$y'' + 4y = 4\cos x + 3\sin x - 8$$

Auxillary,

$$m^2 + 2m = 0$$

$$m(m+2) = 0$$

$$\boxed{m_1 = 0}, \boxed{m_2 = -2}$$

Annihilate $\underbrace{4\cos x}_{} + \underbrace{3\sin x}_{} - 8$

$$(D^2 - 2\alpha D + (\alpha^2 + \beta^2))^n$$

Do Rest of Questions yourself.