Topic:

FUNDAMENTAL THEOREM OF CALCULUS...

Date .....

Exercise 5.4 (1-28)

Evaluate the Integrals:

① $\int_{-2}^{0} (2x+5)\,dx$

Sol:

$\int_{-2}^{0} (2x+5)\,dx$

Integrating:

$[x^2+5x]_{-2}^{0}$

$[(0)^2+5(0)]-[(-2)^2+5(-2)]$

$-[4-10] \Rightarrow \boxed{6}$ Answer.

② $\int_{-3}^{4} (5-\frac{x}{2})\,dx$

Sol:

$\int_{-3}^{4} (5-\frac{x}{2})\,dx$

$[5x-\frac{x^2}{4}]_{-3}^{4}$

$[5(4)-\frac{(4)^2}{4}]-[5(-3)-\frac{(-3)^2}{4}]$

$[20-4]-[-15-\frac{9}{4}]$

$16-(-\frac{69}{4})$

$\frac{64+69}{4}$

$= \boxed{\frac{133}{4}}$ Answer.

③ $\int_{0}^{2} x(x-3)\,dx$

$\int_{0}^{2} (x^2-3x)\,dx$

$[\frac{x^3}{3}-\frac{3x^2}{2}]_{0}^{2}$

$[\frac{(2)^3}{3}-\frac{3(2)^2}{2}]-[\frac{(0)^3}{3}-\frac{3(0)}{2}]$

$[\frac{8}{3}-\frac{12}{2}] \Rightarrow \boxed{-\frac{10}{3}}$ Answer.

④ $\int_{-1}^{1} (x^2-2x+3)\,dx$

$\int_{-1}^{1} (x^2-2x+3)\,dx$

$[\frac{x^3}{3}-\frac{2x^2}{2}+3x]_{-1}^{1}$

$[\frac{(1)^3}{3}-(1)^2+3(1)]-[\frac{(-1)^3}{3}-(-1)^2+3(-1)]$

$[\frac{3(4)^2}{2}-\frac{(4)^4}{16}]-(0)$

$[\frac{48}{2}-\frac{256}{16}] \Rightarrow \boxed{8}$ Answer.

$[\frac{1-3+9}{3}]-[\frac{-1-3-9}{3}]$

$= \boxed{\frac{20}{3}}$ Answer.

⑤ $\int_{0}^{4} (3x-\frac{x^3}{4})\,dx$

$\int_{0}^{4} (3x-\frac{x^3}{4})\,dx$

$[\frac{3x^2}{2}-\frac{x^4}{16}]_{0}^{4}$

⑥ $\int_{-2}^{2} (x^3-2x+3)\,dx.$

$\int_{-2}^{2} (x^3-2x+3)\,dx$

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$\left[\frac{x^4}{4} - x^2 + 3x\right]_{-2}^{2}$ $\left[\left[\frac{16}{4} - 4 + 6\right] - \left[\frac{16}{4} - 4 - 6\right]\right]$

$6+6$

$\left[\frac{(2)^4}{4} - (2)^2 + 3(2)\right] - \left[\frac{(-2)^4}{4} - (-2)^2 + 3(-2)\right]$ $\boxed{12}$ Answer

⑦ $\int_0^1 (x^2 + \sqrt{x})\,dx$

$\int_0^1 (x^2 + (x)^{1/2})\,dx$

$\left[\frac{x^3}{3} + \frac{2x^{3/2}}{3}\right]_0^1$

$\left[\frac{1}{3} + \frac{2(1)^{3/2}}{3}\right]$

$\frac{3}{3} \Rightarrow \boxed{1}$ Answer

⑧ $\int_1^{32} x^{-6/5}\,dx$

$\int_1^{32} x^{-6/5}\,dx$

$\left[-5x^{-1/5}\right]_1^{32}$

$\left[-5(32)^{-1/5}\right] - \left[-5(1)^{-1/5}\right]$

$\left[-\frac{5}{2}\right] - (-5)$

$\boxed{\frac{5}{2}}$ Answer.

⑨ $\int_0^{\pi/3} 2\sec^2 x\,dx$

$\int_0^{\pi/3} 2\sec^2 x\,dx$

$\left[2\tan x\right]_0^{\pi/3}$

$\left[2\tan\left(\frac{\pi}{3}\right)\right]$

$\boxed{2\sqrt{3}}$ Answer.

⑩ $\int_0^{\pi} (1 + \cos x)\,dx$

$\int_0^{\pi} (1 + \cos x)\,dx$

$\left[x + \sin x\right]_0^{\pi}$

$(\pi + \sin\pi)$

$(0 + \sin 0) = \boxed{\pi}$ Answer

⑪ $\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\,d\theta$

$\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\,d\theta$

$\left[-\csc\theta\right]_{\pi/4}^{3\pi/4}$

$\left[-\csc\left(\frac{3\pi}{4}\right)\right] - \left[-\csc\left(\frac{\pi}{4}\right)\right]$

$-\sqrt{2} - (-\sqrt{2}) = \boxed{0}$ Answer

⑫ $\int_0^{\pi/3} 4\sec u\tan u\,du$

$\int_0^{\pi/3} 4\sec u\tan u\,du.$

$\left[4\sec u\right]_0^{\pi/3} \Rightarrow -\left[4\sec(0)\right]$

$+\left[4\sec\left(\frac{\pi}{3}\right)\right] \Rightarrow 4(2) - 4(1)$

$4(2) - 4(1)$

$\boxed{4}$ Answer

⑬ $\int_{\pi/2}^{0} \frac{1+\cos 2t}{2}\,dt$

$\int_{\pi/2}^{0} \frac{1+\cos 2t}{2}\,dt$

$\left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_{\pi/2}^{0} \Rightarrow \left[\frac{1}{2}(0) + \frac{1}{4}\sin 2(0)\right] - \left[\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin 2\left(\frac{\pi}{2}\right)\right]$

$\left[\frac{\pi}{4} + \frac{1}{4}\sin\pi\right] \Rightarrow \boxed{-\frac{\pi}{4}}$ Answer.

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(14) $\int_{-\pi/3}^{\pi/3} \frac{1-\cos 2t}{2}\, dt$

$\int_{-\pi/3}^{\pi/3} \left(\frac{1}{2} - \frac{\cos 2t}{2}\right) dt$

$\left[\frac{1}{2}t - \frac{1}{4}\sin 2t\right]_{-\pi/3}^{\pi/3}$

$\left[\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{4}\sin 2\left(\frac{\pi}{3}\right)\right] - \left[\frac{1}{2}\left(-\frac{\pi}{3}\right) - \frac{1}{4}\sin 2\left(-\frac{\pi}{3}\right)\right]$

$\left[\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3}\right] - \left[-\frac{\pi}{6} - \frac{1}{4}\sin\left(-\frac{2\pi}{3}\right)\right]$

$\boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{4}}$ Answer

(15) $\int_0^{\pi/4} \tan^2 x\, dx$

$\int_0^{\pi/4} \tan^2 x\, dx$

$\int_0^{\pi/4} (\sec^2 x - 1)\, dx$

$\left[\tan x - x\right]_0^{\pi/4}$

$\left[\tan\left(\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right)\right] - \left[\tan 0 - 0\right]$

$\boxed{1 - \frac{\pi}{4}}$ Answer

(16) $\int_0^{\pi/6} (\sec x \tan x)^2\, dx$

$\int_0^{\pi/6} (\sec^2 x + 2\sec x\tan x + \tan^2 x)\, dx$

$\int_0^{\pi/6} (2\sec^2 x + 2\sec x\tan x - 1)\, dx$

$\left[2\tan x + 2\sec x - x\right]_0^{\pi/6}$

$\left[2\tan\left(\frac{\pi}{6}\right) + 2\sec\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{6}\right)\right]$

$2\sqrt{3} + 2(-1) - \frac{\pi}{6}$

$\boxed{2\sqrt{3} - 2 - \frac{\pi}{6}}$ Answer

(17) $\int_0^{\pi/8} \sin 2x\, dx$

$\int_0^{\pi/8} \sin 2x\, dx$

$\left[-\frac{1}{2}\cos 2x\right]_0^{\pi/8}$

$\left[-\frac{1}{2}\cos 2\left(\frac{\pi}{8}\right)\right] - \left[-\frac{1}{2}\cos 2(0)\right]$

$\boxed{\frac{2-\sqrt{2}}{4}}$ Answer

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(18) $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

Sol:

$\int_{-\pi/3}^{-\pi/4} (4\sec^2 t + \pi t^{-2}) dt$

$\left[4\tan t - \pi t^{-1}\right]_{-\pi/3}^{-\pi/4}$

$\left[4\tan\left(\frac{-\pi}{4}\right) - \pi/\left(\frac{-\pi}{4}\right)\right] - \left[4\tan\left(\frac{-\pi}{3}\right) - \pi/\left(\frac{-\pi}{3}\right)\right]$

$\left[4(-1)+4\right] - \left[4(-\sqrt{3})+3\right]$

$\boxed{4\sqrt{3}-3}$ Answer

(19) $\int_{1}^{-1} (r+1)^2 \, dr$

Sol:

$\int_{1}^{-1} (r+1)^2 dr$

$\int_{1}^{-1} (r^2+2r+1) dr \Rightarrow \left[\frac{r^3}{3} + r^2 + r\right]_{1}^{-1}$

$\left(\frac{(-1)^3}{3} + (-1)^2 + (-1)\right) - \left(\frac{(1)^3}{3} + (1)^2 + (1)\right)$

$= \boxed{\frac{-8}{3}}$ Answer.

(20) $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4) dt$

$\int_{-\sqrt{3}}^{\sqrt{3}} (t^3+t^2+4t+4) dt$

$\left[\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right]_{-\sqrt{3}}^{\sqrt{3}}$

$\left[\frac{(\sqrt{3})^4}{4} + \frac{(\sqrt{3})^3}{3} + 2(\sqrt{3})^2 + 4(\sqrt{3})\right]$

$- \left[\frac{(-\sqrt{3})^4}{4} + \frac{(-\sqrt{3})^3}{3} + 2(-\sqrt{3}) + 4(-\sqrt{3})\right]$

$\Rightarrow \boxed{10\sqrt{3}}$ → Answer.

(21) $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du.$

$\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du$

$\left[\frac{u^8}{16} - \frac{1}{4u^4}\right]_{\sqrt{2}}^{1} \Rightarrow \left[\frac{(1)^8}{16} - \frac{1}{4(1)^4}\right] -$

$\left[\frac{(\sqrt{2})^8}{16} - \frac{1}{4(\sqrt{2})^4}\right]$

$\Rightarrow \boxed{\frac{-3}{4}}$ Answer.

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(22) $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\, dy$

$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\, dy$

$\int_{-3}^{-1} \frac{y^5}{y^3} - \frac{2y}{y^3}\, dy$

$\left[\frac{y^3}{3} + 2y^{-1}\right]_{-3}^{-1}$

$\left[\frac{(-1)^3}{3} + 2(-1)^{-1}\right] - \left[\frac{(-3)^3}{3} + 2(-3)^{-1}\right]$

$\boxed{\frac{22}{3}}$ Answer

(23) $\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

$\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

$\int_{1}^{\sqrt{2}} (1 + s^{-3/2})\, ds$

$\left[s - \frac{2}{\sqrt{s}}\right]_{1}^{\sqrt{2}} \Rightarrow \left[\sqrt{2} - \frac{2}{\sqrt{\sqrt{2}}}\right] - \left[1 - \frac{2}{\sqrt{1}}\right]$

$\sqrt{2} - 2^{3/4} + 1 \Rightarrow \boxed{\sqrt{2} - \sqrt[4]{8} + 1}$ Answer.

(24) $\int_{1}^{8} \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}}\, dx$

$\int_{1}^{8} \frac{2x^{1/3} - x + 2 - x^{2/3}}{x^{1/3}}\, dx$

$\int_{1}^{8} (2 - x^{2/3} + 2x^{-1/3} - x^{1/3})\, dx$

$\left[2x - \frac{3}{5}x^{5/3} + 3x^{2/3} - \frac{3}{4}x^{4/3}\right]_{1}^{8}$

$\left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3}\right)$

After calculation:

$\boxed{-\frac{137}{20}}$ Answer

(25) $\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x}\, dx$

$\int_{\pi/2}^{\pi} \frac{2\sin x \cos x}{2\sin x}\, dx$

$\int_{\pi/2}^{\pi} \cos x\, dx \Rightarrow \left[\sin x\right]_{\pi/2}^{\pi}$

$\left(\sin(\pi)\right) - \left(\sin\left(\frac{\pi}{2}\right)\right)$

$\boxed{-1}$ Answer

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(26) $\int_0^{\pi/3} (\cos x + \sec x)^2 \, dx$.

$\int_0^{\pi/3} (\cos^2 x + 2 + \sec^2 x) \, dx$

$\int_0^{\pi/3} \left(\frac{\cos 2x + 1}{2} + 2 + \sec^2 x \right) dx$

$\int_0^{\pi/3} \left(\frac{1}{2}\cos 2x + \frac{5}{2} + \sec^2 x \right) dx$

$\left[\frac{1}{4}\sin 2x + \frac{5}{2}x + \tan x \right]_0^{\pi/3}$

$\left(\frac{1}{4}\sin 2\left(\frac{\pi}{3}\right) + \frac{5}{2}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right) \right)$

$\boxed{\frac{5\pi}{6} + \frac{9\sqrt{3}}{8}}$ Answer

(27) $\int_{-4}^{4} |x| \, dx$

$\int_{-4}^{4} |x| \, dx$

$\int_{-4}^{0} |x| \, dx + \int_{0}^{4} |x| \, dx$

$-\int_{-4}^{0} x \, dx + \int_{0}^{4} x \, dx$

$\left[-\frac{x^2}{2} \right]_{-4}^{0} + \left[\frac{x^2}{2} \right]_0^4$

$\left(-\frac{0^2}{2} + \frac{(-4)^2}{2} \right) + \left(\frac{4^2}{2} - \frac{0^2}{2} \right)$

$\boxed{16}$ Answer

(28) $\int_0^{\pi} \frac{1}{2} (\cos x + |\cos x|) \, dx$

$\int_0^{\pi/2} \frac{1}{2} (\cos x + \cos x) \, dx + \int_{\pi/2}^{\pi} \frac{1}{2} (\cos x - \cos x) \, dx$

$\int_0^{\pi/2} \frac{1}{2}(2\cos x) \, dx \Rightarrow \left[\sin x \right]_0^{\pi/2} \Rightarrow \left[\sin\frac{\pi}{2} - \sin 0 \right]$

$\boxed{1}$ Answer.