CHAPTER TWO

(Exercise 2.2) Q (11-50)

Date

"Calculating limits using limit laws."

Q. Find the limits in Questions 11 - 22.

11) $\lim_{x\to -7}(2x+5)$

Sol. Putting value of x

$= 2(-7)+5$

$= -14+5$

$= \boxed{-9}$

12) $\lim_{x\to 2}(-x^2+5x-2)$

Sol. Putting value of x

$= -(2)^2+5(2)-2$

$= -4+10-2$

$= \boxed{4}$

13) $\lim_{t\to 6} 8(t-5)(t-7)$

Sol. Putting value of t.

$= 8(6-5)(6-7)$

$= 8(1)(-1)$

$= \boxed{-8}$

14) $\lim_{x\to -2}(x^3-2x^2+4x+8)$

Sol. Putting value of x

$= x^3-2x^2+4x+8$

$= (-2)^3-2(-2)^2+4(-2)+8$

$= -\cancel{8}-\cancel{8}-\cancel{8}+\cancel{8}$

$= \boxed{-16}$

15) $\lim_{x\to 2}\frac{x+3}{x+6}$

Putting values

$= \frac{x+3}{x+6}$

$= \frac{2+3}{2+6}$

$= \boxed{\frac{5}{8}}$

16) $\lim_{s\to 2/3} 3s(2s-1)$

Putting values.

$= \cancel{3}\left(\frac{2}{\cancel{3}}\right)\left(2\left(\frac{2}{3}\right)-1\right)$

$= 2\left(\frac{4-3}{3}\right)$

$= \boxed{\frac{2}{3}}$

17) $\lim_{x\to -1} 3(2x-1)^2$

Sol. Putting values

$= 3(2(-1)-1)^2$

$= 3(-3)^2$

$= 3(9)$

$= \boxed{27}$

18) $\lim_{y\to 2}\frac{y+2}{y^2+5y+6}$

Sol. Putting values.

$= \frac{2+2}{(2)^2+5(2)+6}$

$= \frac{4}{4+10+6}$

$= \frac{4}{20}$

$\Rightarrow \boxed{\frac{1}{5}}$

19) $\lim_{y\to -3}(5-y)^{4/3}$

Putting values.

$= (5-(-3))^{4/3}$

$= (8)^{4/3}$

$\approx \boxed{16}$

Note $(8^{1/3})^4 \Rightarrow 2^4 = \boxed{16}$

Date ....

20) $\lim_{z \to 0} (2z-8)^{1/3}$

Putting limit

$(2(0)-8)^{1/3}$

$(-8)^{1/3}$

$\boxed{-2}$

21) $\lim_{h \to 0} \frac{3}{\sqrt{3h+1}+1}$

Putting limit

$\frac{3}{\sqrt{3h+1}+1}$

$\frac{3}{\sqrt{3(0)+1}+1}$

$\boxed{\frac{3}{2}}$

22) $\lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h}$

Using Rationalization

$= \frac{\sqrt{5h+4}-2}{h} \times \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$

$= \frac{(\sqrt{5h+4})^2-(2)^2}{h(\sqrt{5h+4}+2)}$

$= \frac{5\cancel{h}+\cancel{4}-\cancel{4}}{\cancel{h}(\sqrt{5h+4}+2)}$

$= \frac{5}{\sqrt{5h+4}+2}$

Applying limit.

$= \frac{5}{\sqrt{5(0)+4}+2}$

$= \boxed{\frac{5}{4}}$

Limits of Quotients. Find the limits in Question 23-42.

23) $\lim_{x \to 5} \frac{x-5}{x^2-25}$

Sol,

$\lim_{x \to 5} \frac{x-5}{(x)^2-(5)^2}$

$\lim_{x \to 5} \frac{\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}$

$\lim_{x \to 5} \frac{1}{x+5}$

Applying limit

$\frac{1}{5+5} \Rightarrow \boxed{\frac{1}{10}}$

24) $\lim_{x \to 3} \frac{x+3}{x^2+4x+3}$

Sol,

$\frac{x+3}{x^2+3x+x+3}$

$\frac{\cancel{x+3}}{(x+1)\cancel{(x+3)}}$

$\lim_{x \to 3} \frac{1}{x+1}$

$\frac{1}{3+1} \Rightarrow \boxed{\frac{1}{4}}$

25) $\lim_{x \to -5} \frac{x^2+3x-10}{x+5}$

Sol,

$\lim_{x \to -5} \frac{x^2+3x-10}{x+5}$

$\frac{x^2+5x-2x-10}{x+5}$

$\frac{x(x+5)-2(x+5)}{(x+5)}$

$\frac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}}$

$\lim_{x \to -5} (x-2)$

Applying limit

$-5-2 \Rightarrow \boxed{-10}$

26) $\lim_{x \to 2} \frac{x^2-7x+10}{x-2}$

$= \frac{x^2-5x-2x+10}{x-2}$

$= \frac{x(x-5)-2(x-5)}{x-2}$

$= \frac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}}$

$\lim_{x \to 2} (x-5)$

Applying limit

$2-5 \Rightarrow \boxed{-3}$.

27) $\lim_{t \to 1} \frac{t^2+t-2}{t^2-1}$

Sol,

$\frac{t^2+t-2}{t^2-1}$

$= \frac{t^2+2t-t-2}{(t)^2-(1)^2}$

$= \frac{\cancel{(t-1)}(t+2)}{\cancel{(t-1)}(t+1)}$

$\lim_{t \to 1} \frac{t+2}{t+1}$

Applying Limit

$\frac{1+2}{1+1} \Rightarrow \boxed{\frac{3}{2}}$

28) $\lim_{t \to -1} \frac{t^2+3t+2}{t^2-t-2}$

Sol,

$\frac{t^2+3t+2}{t^2-t-2}$

$= \frac{t^2+2t+t+2}{t^2-2t+t-2}$

$= \frac{t(t+2)+1(t+2)}{t(t-2)+1(t-2)}$

$= \frac{\cancel{(t+1)}(t+2)}{(t-2)\cancel{(t+1)}}$

$= \frac{t+2}{t-2}$

$\lim_{t \to -1} \frac{t+2}{t-2}$

Applying limit

$\frac{-1+2}{-1-2} \Rightarrow \boxed{-\frac{1}{3}}$

29) $\lim_{x \to -2} \frac{-2x-4}{x^3+2x^2}$

Sol,

$\frac{2(-x-2)}{x(x+2)}$

$\frac{-2\cancel{(x+2)}}{x^2\cancel{(x+2)}}$

$\lim_{x \to -2} \frac{-2}{x^2}$

Applying limit

$\frac{-2}{(-2)^2} = (1)$

$\boxed{-\frac{1}{2}}$

30) $\lim_{x \to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$

$\frac{\cancel{y^2}(5y+8)}{\cancel{y^2}(3y^2-16)}$

Applying limit

$\frac{5(0)+8}{3(0)^2-16} = \boxed{-\frac{1}{2}}$

31) $\lim_{x \to 1} \frac{\frac{1}{x}-1}{x-1}$

Sol,

$\frac{1/x-1}{x-1} \Rightarrow \frac{1-x/x}{x-1}$

$\frac{1-x}{x^2-x} \Rightarrow \frac{-\cancel{(x-1)}}{x\cancel{(x-1)}}$

$-\frac{1}{x}$ Applying limit

$-\frac{1}{1} = (-1)$.

32) $\lim_{x \to 0} \frac{\frac{1}{x-1}+\frac{1}{x+1}}{x}$

Sol,

$= \frac{\frac{(x+1)+(x-1)}{(x-1)(x+1)}}{x}$

$= \lim_{x \to 0} \left(\frac{2x}{(x-1)(x+1)} \cdot \frac{1}{x}\right)$

Applying limit

$\frac{2}{x^2+x-x-1} = \frac{2}{(0)^2-1} = \boxed{-2}$

33) $\lim_{u \to 1} \frac{u^4-1}{u^3-1}$

Sol,

$= \frac{u^4-1}{u^3-1}$

$= \frac{(u^2)^2-(1)^2}{(u)^3-(1)^3}$

$\because a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}$

$\frac{\cancel{(u-1)}(u+1)(u^2+1)}{\cancel{(u-1)}(u^2+u+1)}$

$\frac{(u+1)(u^2+1)}{(u^2+u+1)}$

Applying limit.

$\frac{(2)(2)}{3} \Rightarrow \boxed{\frac{4}{3}}$

CamScanner

34) $\lim_{v \to 2} \frac{v^3-8}{v^4-16}$

Sol.

$$\frac{v^3-8}{v^4-16}$$

$$= \frac{(v)^3-(2)^3}{(v^2)^2-(4)^2}$$

$$= \frac{(v-2)(v^2+2v+4)}{(v^2-4)(v^2+4)}$$

$$= \frac{\cancel{(v-2)}(v^2+2v+4)}{\cancel{(v-2)}(v+2)(v^2+4)}$$

Applying limit

$$= \frac{(2)^2+2(2)+4}{(2+2)((2)^2+4)}$$

$$= \frac{4+4+4}{(4)(8)} \Rightarrow \frac{12}{32} \Rightarrow \boxed{\frac{3}{8}}$$

35) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

Sol.

$$\frac{\sqrt{x}-3}{x-9} \times \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$= \frac{(\sqrt{x})^2-(3)^2}{(x-9)(\sqrt{x}+3)}$$

$$= \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$$

$$= \frac{1}{\sqrt{x}+3}$$

Applying limit

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3} \Rightarrow \boxed{\frac{1}{6}}$$

36) $\lim_{x \to 4} \frac{4x-x^2}{2-\sqrt{x}}$

Sol.

$$\frac{4x-x^2}{2-\sqrt{x}} \times \frac{2+\sqrt{x}}{2+\sqrt{x}}$$

$$\frac{(4x-x^2)(2+\sqrt{x})}{(2)^2-(\sqrt{x})^2}$$

$$\frac{x\cancel{(4-x)}(2+\sqrt{x})}{\cancel{(4-x)}}$$

Applying limit

$$x(2+\sqrt{x})$$

$$4(2+\sqrt{4})$$

$$4(2+2)$$

$$4(4) \Rightarrow \boxed{16}$$

37) $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$

Sol.

$$\frac{x-1}{\sqrt{x+3}-2} \times \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$= \frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3})^2-(2)^2}$$

$$= \frac{(x-1)(\sqrt{x+3}+2)}{x+3-4}$$

$$= \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x-1)}}$$

Applying limit

$$\sqrt{x+3}+2$$

$$\sqrt{1+3}+2$$

$$\sqrt{4}+2$$

$$2+2$$

$$\boxed{4}$$

38) $\lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1}$

Sol.

$$\frac{\sqrt{x^2+8}-3}{x+1} \times \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$$

$$\frac{(\sqrt{x^2+8})^2-(3)^2}{(x+1)(\sqrt{x^2+8}+3)}$$

$$\frac{x^2+8-9}{(x+1)(\sqrt{x^2+8}+3)}$$

$$\frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$$

$$\frac{(x-1)}{(\sqrt{x^2+8}+3)}$$

Applying limit

$$\frac{-2}{6} \Rightarrow \boxed{-\frac{1}{3}}$$

39) $\lim_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$

Sol.

$$\frac{\sqrt{x^2+12}-4}{x-2}$$

$$\frac{\sqrt{x^2+12}-4}{x-2} \times \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$$

$$= \frac{(\sqrt{x^2+12})^2-(4)^2}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \frac{x^2+12-16}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \frac{(x^2-4)}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(\sqrt{x^2+12}+4)}$$

$$= \frac{(x+2)}{(\sqrt{x^2+12}+4)}$$

Applying limit

$$\frac{2+2}{(\sqrt{(2)^2+12}+4)}$$

$$\frac{4}{8} \Rightarrow \boxed{\frac{1}{2}}$$

40) $\lim_{x \to -2} \dfrac{x+2}{\sqrt{x^2+5}-3}$

Sol.

$$\frac{x+2}{\sqrt{x^2+5}-3} \times \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$

$$= \frac{(x+2)(\sqrt{x^2+5}+3)}{(\sqrt{x^2+5})^2-(3)^2}$$

$$= \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2+5-9}$$

$$= \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4}$$

$$= \frac{(\sqrt{x^2+5}+3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$= \frac{\sqrt{x^2+5}+3}{(x-2)}$$

Applying limit

$$\frac{3+3}{-4} = \boxed{\frac{-6}{4}} \Rightarrow \boxed{-\frac{3}{2}}$$

41) $\lim_{x \to -3} \dfrac{2-\sqrt{x^2-5}}{x+3}$

Sol.

$$\frac{2-\sqrt{x^2-5}}{x+3} \times \frac{2+\sqrt{x^2-5}}{2+\sqrt{x^2-5}} \Rightarrow \frac{(2)^2-(\sqrt{x^2-5})^2}{(x+3)(2+\sqrt{x^2-5})}$$

$$\frac{4-x^2+5}{(x+3)(2+\sqrt{x^2-5})} \Rightarrow \frac{-x^2+9}{(x+3)(2+\sqrt{x^2-5})} \Rightarrow \frac{3-x}{2+\sqrt{x^2-5}} \Rightarrow \text{Applying limit}$$

$$\frac{6}{2+\sqrt{4}} \Rightarrow \frac{6}{2+2} \Rightarrow \frac{6}{4} \Rightarrow \boxed{\frac{3}{2}}$$

42) $\lim_{x\to 4} \dfrac{4-x}{5-\sqrt{x^2+9}}$

Sol.

$$\frac{4-x}{5-\sqrt{x^2+9}}$$

$$\frac{4-x}{5-\sqrt{x^2+9}} \times \frac{5+\sqrt{x^2+9}}{5+\sqrt{x^2+9}}$$

$$\frac{(4-x)(5+\sqrt{x^2+9})}{(5)^2-(\sqrt{x^2+9})^2}$$

$$\frac{(4-x)(5+\sqrt{x^2+9})}{25-x^2-9}$$

$$\frac{\cancel{(4-x)}(5+\sqrt{x^2+9})}{\cancel{(4-x)}(4+x)}$$

$$\frac{5+\sqrt{x^2+9}}{(4+x)}$$

Applying limit

$$\frac{5+\sqrt{(4)^2+9}}{4+4}$$

$$\frac{10}{8} \Rightarrow \boxed{\frac{5}{4}}$$

Limits with trigonometric Functions. Find the limits in Question 43-50.

43) $\lim_{x\to 0} (2\sin x - 1)$

Putting values

$2\sin(0)-1 \quad \therefore \sin 0 = 0$

$2(0)-1 \Rightarrow \boxed{-1}$.

44) $\lim_{x\to 0} \sin^2 x$

Putting value

$(\sin 0)^2 \Rightarrow 0^2 \Rightarrow 0$.

45) $\lim_{x\to 0} \sec x$.

$$\lim_{x\to 0} \frac{1}{\cos x}$$

Putting value

$$= \frac{1}{\cos 0} \quad \therefore \cos 0 = 1$$

$$= \boxed{1}$$

46) $\lim_{x\to 0} \tan x$

$\lim_{x\to 0} \dfrac{\sin x}{\cos x} \Rightarrow$ Putting values.

$$\frac{\sin 0}{\cos 0} = \frac{0}{1} \Rightarrow \boxed{0}$$

47) $\lim_{x\to 0} \dfrac{1+x+\sin x}{3\cos x}$

Putting value:

$$\frac{1+0+\sin 0}{3\cos 0}$$

$$\frac{1+0+0}{3(1)} \Rightarrow \boxed{\frac{1}{3}}.$$

48) $\lim_{x\to 0} (x^2-1)(2-\cos x)$

Putting values

$(0^2-1)(2-\cos 0) \Rightarrow (-1)(2-1) \Rightarrow (-1)(1) \Rightarrow \boxed{-1}$.

49) $\lim_{x\to -\pi} \sqrt{x+4}\cos(x+\pi)$

$\lim_{x\to-\pi} \sqrt{x+4} \cdot \lim_{x\to -\pi} \cos(x+\pi)$

$(\sqrt{-\pi+4})(\cos(\cancel{-\pi}+\cancel{\pi})) \Rightarrow (\sqrt{-x+4})(\cos 0)$

$\sqrt{-\pi+4} \cdot 1$

$\boxed{\sqrt{4-\pi}}$ Ans.

50) $\lim_{x\to 0} \sqrt{7+\sec^2 x}$

$$\sqrt{\lim_{x\to 0}(7+\sec^2 x)}$$

$$\sqrt{7+\lim_{x\to 0}(\sec^2 x)}$$

$$\sqrt{7+(\sec 0)^2}$$

$$\sqrt{7+(1)^2}$$

$$\sqrt{8}$$

$$\boxed{2\sqrt{2}}$$

THE END