Exercise 7.7 (13-20)

(13) $y = 6\sinh\frac{x}{3}$

Sol,

$$y = 6\frac{d}{dx}\left(\sinh\frac{x}{3}\right)$$

$$y' = 6\left(\cosh\frac{x}{3}\right)\frac{1}{3}\frac{d}{dx}(x)$$

$$y' = {}^{2}\cancel{6}\left(\cosh\frac{x}{3}\right)\left(\frac{1}{\cancel{3}_{1}}\right)$$

$$\boxed{y' = 2\cosh\frac{x}{3}}$$

(14) $y = \frac{1}{2}\sinh(2x+1)$

Sol,

$$y = \frac{1}{2}\left(\sinh(2x+1)\right)$$

$$y' = \frac{1}{2}\frac{d}{dx}\left(\sinh(2x+1)\right)$$

$$y' = \frac{1}{2}\left(\cosh(2x+1)\frac{d}{dx}(2x) + \frac{d}{dx}(1)\right)$$

$$y' = \frac{1}{\cancel{2}}\left(\cosh(2x+1)(\cancel{2})\right)$$

$$\boxed{y' = \cosh(2x+1)}$$

(15)

$$y = 2\sqrt{t}\tanh\sqrt{t}$$

Using Formula (Product Rule)

$$y' = 2\left[(\sqrt{t})\frac{d}{dt}(\tanh\sqrt{t}) + (\tanh\sqrt{t})\frac{d}{dt}(\sqrt{t})\right]$$

$$y' = 2\left[(\sqrt{t})(\text{sech}^2\sqrt{t})\frac{d}{dt}(\sqrt{t}) + (\tanh\sqrt{t})\left(\frac{1}{2\sqrt{t}}\right)\right]$$

$$y' = \cancel{2}(\cancel{\sqrt{t}})(\text{sech}^2\sqrt{t})\left(\frac{1}{\cancel{2}\cancel{\sqrt{t}}}\right) + \cancel{2}(\tanh\sqrt{t})\left(\frac{1}{\cancel{2}\sqrt{t}}\right)$$

$$\boxed{y' = \text{sech}^2\sqrt{t} + \frac{\tanh\sqrt{t}}{\sqrt{t}}}$$

(16) $y = t^2\tanh\frac{1}{t}$

$$y = t^2\tanh t^{-1}$$

Product Rule.

$$y' = t^2\frac{d}{dt}(\tanh t^{-1}) + (\tanh t^{-1})\frac{d}{dt}t^2$$

$$y' = (t^2)(\text{sech}^2 t^{-1})\frac{d}{dt}t^{-1} + (\tanh t^{-1})(2t)$$

$$y' = (t^2)(\text{sech}^2 t^{-1})(-t^{-2}) + \ldots$$

$$y' = (\cancel{t^2})\left(\text{sech}^2\frac{1}{t}\right)\left(-\frac{1}{\cancel{t^2}}\right) + \left(\tanh\frac{1}{t}\right)(2t)$$

$$\boxed{y' = -\text{sech}^2\frac{1}{t} + 2t\tanh\frac{1}{2}}$$

(17) $y = \ln(\sinh z)$

$$y' = \frac{1}{\sinh z}\frac{d}{dz}\sinh z \Rightarrow y' = \frac{\cosh z}{\sinh z}$$

$$\boxed{y' = \coth z}$$

(18) $y = \ln(\cosh z)$

$$y' = \frac{1}{\cosh z}\frac{d}{dz}\cosh z \Rightarrow y' = -\frac{\sinh z}{\cosh z} \Rightarrow \boxed{y' = \tanh z}$$

(19) $y = \operatorname{sech}\theta\,(1 - \ln \operatorname{sech}\theta)$

Sol,

$$y' = (\operatorname{sech}\theta)\frac{d}{d\theta}(1 - \ln\operatorname{sech}\theta) + (1 - \ln\operatorname{sech}\theta)\frac{d}{d\theta}(\operatorname{sech}\theta)$$

$$y' = (\operatorname{sech}\theta)\left(\frac{d}{d\theta}(1) - \frac{d}{d\theta}(\ln\operatorname{sech}\theta)\right) + (1 - \ln\operatorname{sech}\theta)(-\operatorname{sech}\theta\tanh\theta)$$

$$y' = (\operatorname{sech}\theta)\left(-\frac{1}{\operatorname{sech}\theta}\frac{d}{d\theta}\operatorname{sech}\theta\right) + (1 - \ln\operatorname{sech}\theta)(-\operatorname{sech}\theta\tanh\theta)$$

$$y' = (\cancel{\operatorname{sech}\theta})\left(-\frac{1}{\cancel{\operatorname{sech}\theta}}(-\operatorname{sech}\theta\tanh\theta)\right) + (1 - \ln\operatorname{sech}\theta)(-\operatorname{sech}\theta\tanh\theta)$$

$$y' = \operatorname{sech}\theta\tanh\theta - (\operatorname{sech}\theta\tanh\theta)(1 - \ln\operatorname{sech}\theta)$$

$$y' = \operatorname{sech}\theta\tanh\theta\,[1 - (1 - \ln\operatorname{sech}\theta)]$$

$$y' = \operatorname{sech}\theta\tanh\theta\,[\cancel{1} - \cancel{1} + \ln\operatorname{sech}\theta]$$

$$\boxed{y' = (\operatorname{sech}\theta\tanh\theta)(\ln\operatorname{sech}\theta)}$$

(20) $y = \operatorname{csch}\theta\,(1 - \ln\operatorname{csch}\theta)$

Sol,

$$y' = (\operatorname{csch}\theta)\frac{d}{d\theta}(1 - \ln\operatorname{csch}\theta) + (1 - \ln\operatorname{csch}\theta)\frac{d}{d\theta}(\operatorname{csch}\theta)$$

$$y' = (\operatorname{csch}\theta)\left(\frac{d}{d\theta}(1) - \frac{d}{d\theta}(\ln\operatorname{csch}\theta)\right) + (1 - \ln\operatorname{csch}\theta)(-\operatorname{csch}\theta\coth\theta)$$

$$y' = (\cancel{\operatorname{csch}\theta})\left(+\frac{(\cancel{-}\operatorname{csch}\theta\coth\theta)}{\cancel{\operatorname{csch}\theta}}\right) + (1 - \ln\operatorname{csch}\theta)(-\operatorname{csch}\theta\coth\theta)$$

$$y' = (\operatorname{csch}\theta\coth\theta) + (1 - \ln\operatorname{csch}\theta)(-\operatorname{csch}\theta\coth\theta)$$

$$y' = \operatorname{csch}\theta\coth\theta\,(1 - (1 - \ln\operatorname{csch}\theta))$$

$$y' = \operatorname{csch}\theta\coth\theta\,(\cancel{1} - \cancel{1} + \ln\operatorname{csch}\theta)$$

$$\boxed{y' = (\operatorname{csch}\theta\coth\theta)(\ln\operatorname{csch}\theta)}$$

Exercise 7.6 (21 - 26)

(21) $y = \cos^{-1}(x^2)$

Using Formula:

$$y' = -\frac{2x}{\sqrt{1-(x^2)^2}}$$

$$\boxed{y' = -\frac{2x}{\sqrt{1-x^4}}}$$

(22) $y = \cos^{-1}(1/x)$

Using Formula:

$$\cancel{y' = -\frac{1}{|x|\sqrt{1-x^2}}}$$

$$y' = \sec^{-1} x$$

$$\boxed{y' = \frac{1}{|x|\sqrt{x^2-1}}}$$

(23) $y = \sin^{-1}\sqrt{2}\,t$

Using Formula:

$$y' = \frac{\sqrt{2}}{\sqrt{1-(\sqrt{2}t)^2}}$$

$$\boxed{y' = \frac{\sqrt{2}}{\sqrt{1-2t^2}}}$$

(24) $y = \sin^{-1}(1-t)$

Using Formula:

$$y' = \frac{-1}{\sqrt{1-(1-t)^2}}$$

$$y' = \frac{-1}{\sqrt{1-(1-2t+t^2)}}$$

$$\boxed{y' = \frac{-1}{\sqrt{2t-t^2}}}$$

(25) $y = \sec^{-1}(2s+1)$

Using Formula:

$$y' = \frac{2}{|2s+1|\sqrt{(2s+1)^2-1}}$$

$$y' = \frac{2}{|2s+1|\sqrt{4s^2+4s+\cancel{1}-\cancel{1}}}$$

$$y' = \frac{\cancel{2}}{|2s+1|\,\cancel{2}\sqrt{s^2+s}}$$

$$\boxed{y' = \frac{1}{|2s+1|\sqrt{s^2+s}}}$$

(26) $y = \sec^{-1}5s$

Using Formula:

$$y' = \frac{5}{|5s|\sqrt{(5s)^2-1}}$$

$$y' = \frac{5}{|5s|\sqrt{25s^2-1}}$$

$$y' = \frac{\cancel{5}}{|5|\,\cancel{5}\sqrt{s^2-1}}$$

$$\boxed{y' = \frac{1}{|5|\sqrt{s^2-1}}}$$

Exercise 7.3 (5-28, 61-68)

(61) $y = (\cos\theta)^{\sqrt{2}}$

$y' = \sqrt{2}\,(\cos\theta)^{\sqrt{2}-1} \frac{d}{d\theta}(\cos\theta)$

$y' = \sqrt{2}\,(\cos\theta)^{\sqrt{2}-1}(-\sin\theta)$

$\boxed{y' = -\sqrt{2}(\cos\theta)^{\sqrt{2}-1}(\sin\theta)}$

(62) $y = (\ln\theta)^{\pi}$

$y' = \pi(\ln\theta)^{\pi-1} \frac{d}{d\theta}(\ln\theta)$

$y' = \pi(\ln\theta)^{\pi-1}\left(\frac{1}{\theta}\right)$

$\boxed{y' = \frac{\pi(\ln\theta)^{\pi-1}}{\theta}}$

(63) $y = 7^{\sec\theta}\ln 7$

$y' = (7^{\sec\theta}\ln 7)(\ln 7)\frac{d}{d\theta}(\sec\theta)$

$y' = 7^{\sec\theta}(\ln 7)^2(\sec\theta\tan\theta)$

$\boxed{y' = 7^{\sec\theta}(\ln 7)^2(\sec\theta\tan\theta)}$

(64) $y = 3^{\tan\theta}\ln 3$

$y' = (3^{\tan\theta}\ln 3)(\ln 3)\frac{d}{d\theta}(\tan\theta)$

$\boxed{y' = 3^{\tan\theta}(\ln 3)^2(\sec^2\theta)}$

(65) $y = 2^{\sin 3t}$

$y' = 2^{\sin 3t}(\ln 2)\frac{d}{dt}\sin 3t$

$y' = 2^{\sin 3t}(\ln 2)(\cos 3t)\frac{d}{dt}3t$

$\boxed{y' = 2^{\sin 3t}(\ln 2)(3\cos 3t)}$

(66) $y = 5^{-\cos 2t}$

$y' = 5^{-\cos 2t}(\ln 5)\frac{d}{dt}\cos 2t$

$y' = 5^{-\cos 2t}(\ln 5)(-\sin 2t)\frac{d}{dt}(2t)$

$y' = 5^{-\cos 2t}(\ln 5)(-\sin 2t)(2)$

$y' = (-2\sin 2t)(\ln 5)(5^{-\cos 2t})$

$\boxed{y' = (-2\sin 2t)(\ln 5)(5^{-\cos 2t})}$

(67) $y = \log_2 5\theta$

$y' = \frac{\log 5\theta}{\ln 2}$

$y' = \left(\frac{1}{\ln 2}\right)\left(\frac{1}{5\theta}\right)\frac{d}{d\theta}(5\theta)$

$y' = \left(\frac{1}{\ln 2}\right)\left(\frac{1}{\cancel{5}\theta}\right)(\cancel{5})$

$\boxed{y' = \frac{1}{\theta\ln 2}}$

68

$$y = \log_3(1+\theta \ln 3)$$

$$y = \frac{\log(1+\theta \ln 3)}{(\ln 3)}$$

$$y = \frac{1}{\ln 3}\left[\log(1+\theta \ln 3)\right]$$

$$y' = \frac{1}{\ln 3}\left[\frac{1}{(1+\theta\ln 3)}\frac{d}{d\theta}(1+\theta \ln 3)\right]$$

$$y' = \frac{1}{\cancel{\ln 3}}\left[\frac{1}{1+\theta\ln 3}(\cancel{\ln 3})\right]$$

$$\boxed{y' = \frac{1}{1+\theta \ln 3}}$$

28

$$\tan y = e^x + \ln x$$

$$\frac{dy}{dx}\operatorname{Tan} y = \frac{dy}{dx}e^x + \frac{dy}{dx}\ln x.$$

$$(\sec^2 y)\frac{dy}{dx} = e^x + \frac{1}{x}$$

$$y' = \frac{e^x + \frac{1}{x}}{\sec^2 y}$$

$$\therefore \frac{1}{\sec^2 y} = \cos^2 y$$

$$y' = \left(e^x + \frac{1}{x}\right)\cos^2 y$$

$$\boxed{y' = \frac{(xe^x+1)\cos^2 y}{x}}$$

27

$$e^{2x} = \sin(x+3y)$$

$$\frac{dy}{dx}e^{2x} = \frac{d}{dx}\sin(x+3y)$$

$$2e^{2x} = \cancel{\cos}\left(1+3\frac{dy}{dx}\right)\cos(x+3y)$$

$$\frac{2e^{2x}}{\cos(x+3y)} = (1+3y')$$

$$\boxed{y' = \frac{2e^{2x}-1}{3\cos(x+3y)}}$$

26

$$\ln xy = e^{x+y}$$

$$\ln x + \ln y = e^{x+y}$$

$$\frac{dy}{dx}(\ln x) + \frac{dy}{dx}(\ln y) = \frac{dy}{dx}(e^{x+y})$$

$$\frac{1}{x} + \frac{1}{y}y' = e^{x+y}\frac{d}{dx}(x+y)$$

$$\frac{1}{x} + \frac{1}{y}y' = e^{x+y}(1+y')$$

$$\cancel{y'} = \cancel{e^{x+y}}\, y'\left(\frac{1}{y} - e^{x+y}\right) = e^{x+y} - \frac{1}{x}$$

$$y'\left(\frac{1-ye^{x+y}}{y}\right) = \frac{xe^{x+y}-1}{x}$$

$$\boxed{y' = \frac{y(xe^{x+y}-1)}{x(1-ye^{x+y})}}$$

(Exercise 7.3) Q(5-28, 55-82)

Date ....

Q5: find the derivative of y with respect to x, t or θ, as appropriate:

⑤ $y = e^{-5x}$

$y' = e^{-5x} \frac{d}{dx}(-5x)$

$\boxed{y' = -5e^{-5x}}$

⑥ $y = e^{2x/3}$

$y' = e^{2x/3} \frac{d}{dx} \frac{2x}{3}$

$\boxed{y' = \frac{2}{3} e^{2x/3}}$

⑦ $y = e^{5-7x}$

$y' = e^{5-7x} \left(\frac{d}{dx} 5 - \frac{d}{dx} 7x\right)$

$\boxed{y' = -7e^{5-7x}}$

⑧ $y = e^{(4\sqrt{x}+x^2)}$

$y' = e^{(4\sqrt{x}+x^2)} \frac{d}{dx} 4x^{1/2} + \frac{d}{dx} x^2$

$y' = \left(\frac{\cancel{4}^{2}}{\cancel{2}\sqrt{x}} + 2x\right) e^{(4\sqrt{x}+x^2)}$

$\boxed{y' = \left(\frac{2}{\sqrt{x}} + 2x\right) e^{(4\sqrt{x}+x^2)}}$

⑨ $y = xe^{x} - e^{x}$

$y' = \cancel{e^{x} + xe^{x}}\ x \frac{d}{dx}(xe^{x}) - \frac{d}{dx} e^{x}$

$y' = \cancel{xe^{x}} \pm x \frac{d}{dx} e^{x} + e^{x} \frac{d}{dx} x - \frac{d}{dx} e^{x}$

$y' = xe^{x} + \cancel{e^{x}} - \cancel{e^{x}} \Rightarrow \boxed{y' = xe^{x}}$

⑩ $y = (1+2x)e^{-2x}$

$y' = (1+2x)\frac{d}{dx} e^{-2x} + e^{-2x} \frac{d}{dx}(1+2x)$

$y' = (1+2x)(-2e^{-2x}) + (e^{-2x})(2)$

$y' = -\cancel{2e^{-2x}} - 4xe^{-2x} + \cancel{2e^{-2x}}$

$\boxed{y' = -4xe^{-2x}}$

⑪ $y = (x^2 - 2x + 2)e^{x}$

$y' = (x^2-2x+2)\frac{d}{dx} e^{x} + e^{x} \frac{d}{dx}(x^2-2x+2)$

$y' = e^{x}x^2 - 2xe^{x} + 2e^{x} + e^{x}(2x-2)$

$y' = e^{x}x^2 - \cancel{2xe^{x}} + \cancel{2e^{x}} + \cancel{2xe^{x}} - \cancel{2e^{x}}$

$\boxed{y' = x^2 e^{x}}$

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(12) $y = (9x^2 - 6x + 2)e^{3x}$

$y' = (9x^2 - 6x + 2)\frac{d}{dx}e^{3x} + e^{3x}\frac{d}{dx}(9x^2 - 6x + 2)$

$y' = (9x^2 - 6x + 2)(3e^{3x}) + e^{3x}(18x - 6)$

$y' = 27x^2e^{3x} - \cancel{18xe^{3x}} + \cancel{6e^{3x}} + \cancel{18xe^{3x}} - \cancel{6e^{3x}}$

$\boxed{y' = 27x^2e^{3x}}$

(13) $y = e^\theta(\sin\theta + \cos\theta)$

$y' = e^\theta\frac{d}{dx}(\sin\theta + \cos\theta) + (\sin\theta + \cos\theta)\frac{d}{dx}e^\theta.$

$y' = e^\theta(\cos\theta - \sin\theta) + e^\theta(\sin\theta + \cos\theta)$

$y' = e^\theta(\cos\theta - \cancel{\sin\theta} + \cancel{\sin\theta} + \cos\theta)$

$\boxed{y' = 2e^\theta\cos\theta}$.

(14) $y = \ln(3\theta e^{-\theta})$

$y' = \ln 3 + \ln\theta + \ln e^{-\theta}$

$y' = 0 + \frac{1}{\theta} - 1$

$y' = \frac{1}{\theta} - 1$

(15) $y = \cos(e^{-\theta^2})$

$y' = -\sin(e^{-\theta^2})\frac{d}{d\theta}(e^{-\theta^2})$

$y' = -\sin(e^{-\theta^2})(e^{-\theta^2})\frac{d}{d\theta}(-\theta^2)$

$\boxed{y' = 2\theta e^{-\theta^2}\sin(e^{-\theta^2})}$

(16) $y = \theta^3 e^{-2\theta}\cos 5\theta$

$y' = (3\theta^2)(e^{-2\theta}\cos 5\theta) +$
$(\theta^3\cos 5\theta)(-2e^{-2\theta}) +$
$(\theta^3 e^{-2\theta})(-\sin 5\theta \ldots$

$y' = \ldots$

$\boxed{y' = \theta^2 e^{-2\theta}(3\cos 5\theta - 2\theta\cos 5\theta - 5\theta\sin 5\theta)}$

(17) $y = \ln(3te^{-t})$

$y' = \ln 3 + \ln t + \ln e^{-t}$

$\ln 3 + \ln t - t$

$y' = 0 + \frac{1}{t} - 1$

$\boxed{y' = \frac{1}{t} - 1}$

(18) $y = \ln(2e^{-t}\sin t)$

$y' = \ln 2 + \ln e^{-t} + \ln\sin t$

$y' = \ln 2 - t + \ln\sin t$

$y' = 0 - 1 + \frac{1}{\sin t}(\cos t)$

$\boxed{y' = -1 + \frac{\cos t}{\sin t}}$

(19) $y = \ln\left(\frac{e^\theta}{1 + e^\theta}\right)$

$y = \ln e^\theta - \ln(1 + e^\theta)$

$y = \theta - \ln(1 + e^\theta)$

$y' = 1 - \frac{1}{1 + e^\theta}\frac{d}{d\theta}(1 + e^\theta)$

$y' = 1 - \frac{1}{1 + e^\theta}(e^\theta)$

$y' = \frac{1 + \cancel{e^\theta} - \cancel{e^\theta}}{1 + e^\theta}$

$\boxed{y' = 1/1 + e^\theta}$.

(20) $y = \ln\left(\frac{\sqrt{\theta}}{1 + \sqrt{\theta}}\right)$

$y' = \ln\sqrt{\theta} - \ln(1 + \sqrt{\theta})$

$y' = \frac{1}{\sqrt{\theta}}\frac{d}{d\theta}\sqrt{\theta} - \frac{1}{1 + \sqrt{\theta}}\frac{d}{d\theta}(1 + \sqrt{\theta})$

$y' = \left(\frac{1}{\sqrt{\theta}}\right)\left(\frac{1}{2\sqrt{\theta}}\right) - \left(\frac{1}{1 + \sqrt{\theta}}\right)\left(\frac{1}{2\sqrt{\theta}}\right)$

$\boxed{y' = \frac{1}{2\theta(1 + \theta^{1/2})}}$

Leibniz Rule:

$$y = \int_{u(x)}^{v(x)} f(t)\,dt \Rightarrow \frac{dy}{dx} = f(v(x))\frac{dv}{dx} - f(u(x))\frac{du}{dx}$$

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(21) $y = e^{(\cos t + \ln t)}$

$y' = e^{\cos t} \cdot e^{\ln t}$

$y' = t e^{\cos t}$

$y' = t \frac{d}{dt} e^{\cos t} + e^{\cos t} \frac{d}{dt} t$

$y' = t e^{\cos t}(-\sin t) + e^{\cos t}$

$y' = e^{\cos t} - t \sin t\, e^{\cos t}$

$$\boxed{y' = e^{\cos t}(1 - t\sin t)}$$

(22) $y = e^{\sin t}(\ln t^2 + 1)$

$y' = e^{\sin t} \frac{d}{dt}(\ln t^2 + 1) + (\ln t^2 + 1)\frac{d}{dt} e^{\sin t}$

$y' = e^{\sin t}\left(\frac{2}{t}\right) + (\ln t^2 + 1)\, e^{\sin t}(\cos t)$

$$\boxed{y' = e^{\sin t}\left[\frac{2}{t} + (\ln t^2 + 1)(\cos t)\right]}$$

(23) $\int_0^{\ln x} \sin e^t\, dt$

$u(x) = 0$

$v(x) = \ln x.$

Leibniz' Rule:

$\frac{dv}{dx} = \frac{1}{x}.$

$\frac{du}{dx} = 0$

$f(u(x)) = \sin e^0$

$= \sin(0)$

$f(v(x)) = \sin e^{\ln x}$ ~~t~~

$\boxed{f(u(x)) = 0}$ $\boxed{f(v(x)) = \sin x}$

Putting values:

$\frac{dy}{dx} = (\sin x)\left(\frac{1}{x}\right) - (0)(0)$

$$\boxed{\frac{dy}{dx} = \frac{\sin x}{x}}$$

(24) $y = \int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t\, dt.$

$u(x) = e^{4\sqrt{x}}$

$v(x) = e^{2x}$

$\frac{dv}{dx} = 2e^{2x}$

$\frac{du}{dx} = \frac{2}{\sqrt{x}} e^{4\sqrt{x}}$

~~$f(v(x)) = \ln e^{2x}$~~

$f(v(x)) = 2x$

$f(u(x)) = 4\sqrt{x}$

$\frac{dy}{dx} = (2x)(2e^{2x}) - (4\cancel{\sqrt{x}})\left(\frac{2}{\cancel{\sqrt{x}}} e^{4\sqrt{x}}\right)$

$$\boxed{\frac{dy}{dx} = 4xe^{2x} - \frac{8x}{\cancel{\sqrt{x}}} e^{4\sqrt{x}}}$$

(25) $\ln y = e^y \sin x$

$\frac{d}{dx} \ln y = \frac{d}{dx} e^y \sin x$

$\frac{1}{y} y' = e^y \frac{d}{dx} \sin x + \sin x \frac{d}{dx} e^y$

$\frac{1}{y} y' = \cos x\, e^y + \sin x\, e^y\, y'$

$y' - \sin x\, y e^y y' = \cos x\, y e^y$

$$\boxed{y' = \frac{y e^y \cos x}{1 - \sin x\, y e^y}}$$