Normal Distribution: Area

Lecture 28

Area ⟶ Probability

Q-1: Given a standard normal distribution, find the area under the curve that lies

a) to the right of $z = 1.84$

sol:

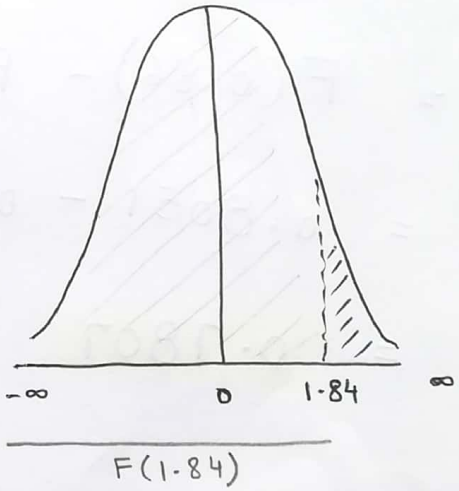
$$P(z > 1.84) = ?$$

$$\therefore P(z > 1.84)$$

$$= 1 - P(z < 1.84)$$

$$= 1 - 0.9671$$

$$= 0.0329$$



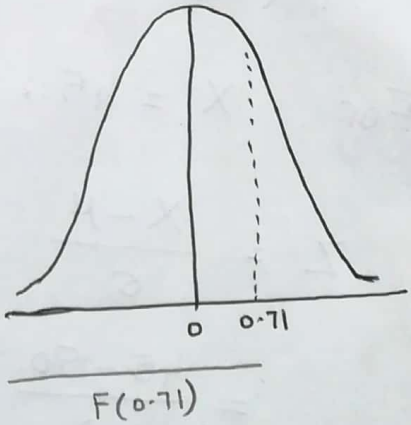
b) to the left of $z = 0.71$

sol:

$$P(z < 0.71) = ?$$

$$\therefore P(z < 0.71)$$

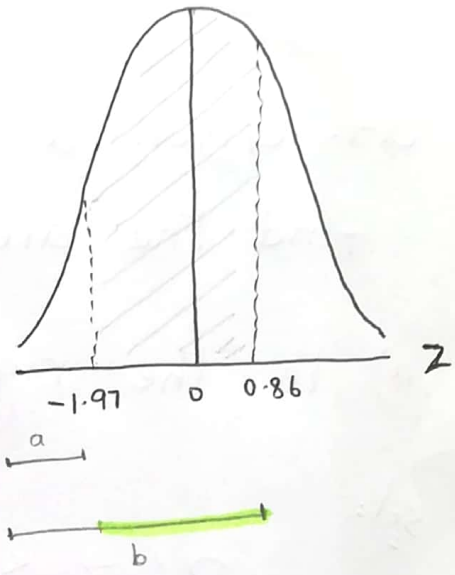
$$= 0.7611$$



c) between $Z = -1.97$ and $Z = 0.86$

Sol:-

$$P(-1.97 < Z < 0.86) = ?$$



$$P(-1.97 < Z < 0.86)$$

$$= P(Z < 0.86) - P(Z < -1.97)$$

$$= F(0.86) - F(-1.97)$$

$$= 0.8051 - 0.0244$$

$$= 0.7807$$

Q-2: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value

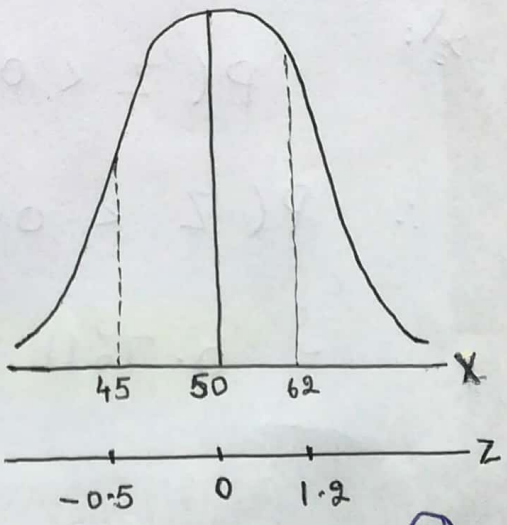
a) between 45 and 62.

Sol:- $P(45 < X < 62) = ?$

For $X = 45$;

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{45 - 50}{10} = -0.5$$



For $X = 62$;

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{62 - 50}{10} = 1.2$$

$$\therefore P(45 < X < 62)$$

$$= P(-0.5 < Z < 1.2)$$

$$= F(1.2) - F(-0.5)$$

$$= 0.8849 - 0.3085$$

$$= 0.5764$$

b) greater than 70

Sol:-

$$P(X > 70) = ?$$

For $X = 70$;

$$Z = \frac{X - \mu}{\sigma}$$

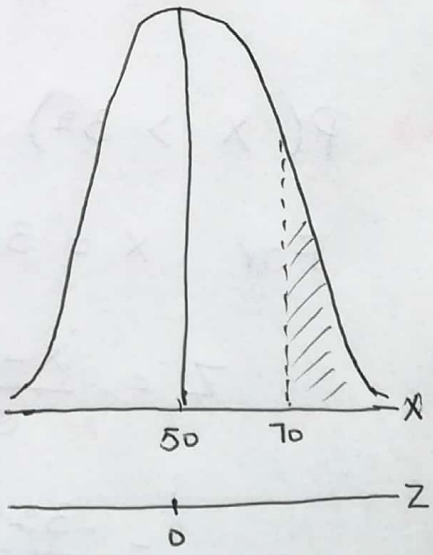
$$= \frac{70 - 50}{10} = 2.0$$

$$\therefore P(X > 70)$$

$$= P(Z > 2.0)$$

$$= 1 - F(2.0)$$

$$= 1 - 0.9772 = 0.0228$$



Q-3: A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assume that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live

a) more than 32 months

Sol:-

$$\mu = 40$$

$$\sigma = 6.3$$

$$\rightarrow \quad X \sim N(40, 6.3^2).$$

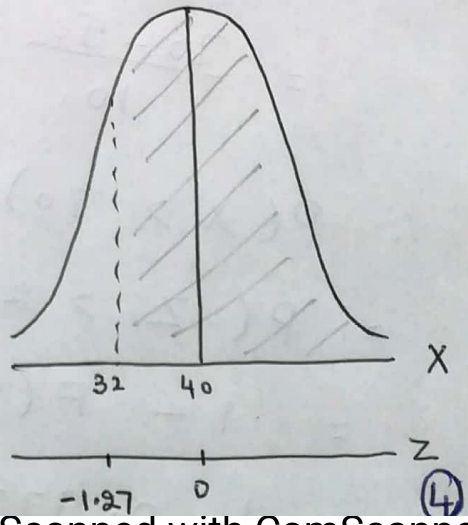
$$\rightarrow P(X > 32) = ?$$

For $X = 32$;

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{32 - 40}{6.3}$$

$$= -1.27$$



$\therefore P(X > 32)$

$= P(Z > -1.27)$

$= 1 - F(-1.27)$

$= 1 - 0.1020$

$= 0.898$

$\Rightarrow \boxed{P(X > 32) = 0.898}$

b) less than 28 months;

sol:- $P(X < 28) = ?$

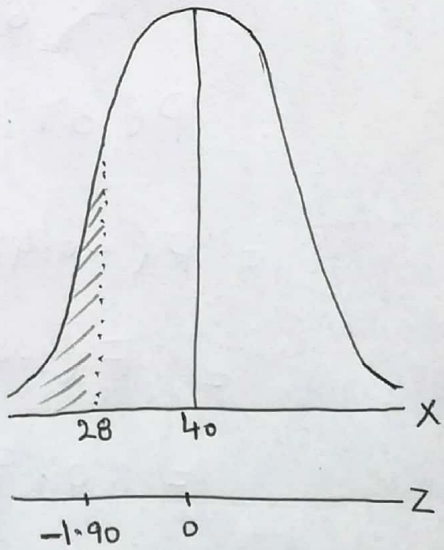
with $X \sim N(40, 6.3^2)$

For $X = 28$;

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{28 - 40}{6.3}$$

$$= -1.90$$



$\therefore P(X < 28) = P(Z < -1.90)$

$\boxed{P(X < 28) = 0.0287}$

c) between 37 and 49 months.

Sol:- $P(37 < X < 49) = ?$

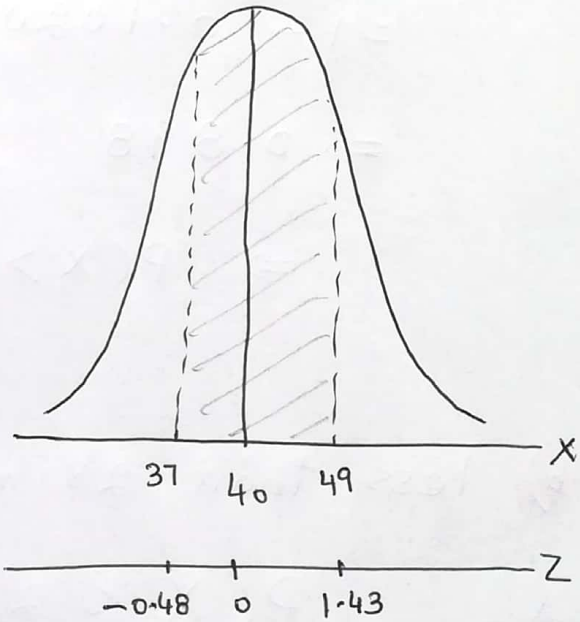
→ with $X \sim N(40, 6.3^2)$

→ for $X = 37$;

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{37 - 40}{6.3}$$

$$= -0.48$$



→ for $X = 49$;

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{49 - 40}{6.3} = 1.43$$

→ $\therefore P(37 < X < 49)$

$= P(-0.48 < Z < 1.43)$

$= F(1.43) - F(-0.48)$

$= 0.9236 - 0.3156$

$\Rightarrow$ $\boxed{P(37 < X < 49) = 0.6080}$