D.E Exercise # 2.5 (1 to 22)

Date ....

Homogenous Mixture:

$M(x,y)$

$M(tx,ty) = t^{\alpha} M(x,y)$

$N(tx,ty) = t^{\alpha} N(x,y)$

$Mdx + Ndy = 0$

$$\boxed{\begin{aligned} y &= ux \\ dy &= udx + xdu \end{aligned}} \qquad \boxed{u = \frac{y}{x}}$$

$$\boxed{x = vy \quad dx = vdy + ydv \quad \boxed{v = \frac{x}{y}}}$$

every power should be equal.

Q#5: $(y^2 + xy)\,dx - x^2\,dy = 0$ → i)

(Hint: check powers as y have 2 power and should be equal to $x'y'$ ⇒ sum of both) which is 2. So, it is homogenous)

$M = y^2 + xy$, $N = -x^2$

$M(tx,ty) = t^2y^2 + t^2xy$

$= t^2 M(x,y)$

$N(tx,ty) = -t^2x^2$

$= t^2 N(x,y)$

$y = ux \Rightarrow dy = udx + xdu$.

↓ putting in eq-i)

$(u^2x^2 + x(ux))\,dx - x^2(udx + xdu) = 0$

$(u^2x^2 + x(ux))dx - x^2(udx + xdu) = 0$ → Divided by x^2

$(u^2 + u)\,dx - udx + xdu = 0$

$u^2dx + \cancel{udx} - \cancel{udx} + xdu = 0$

$u^2dx + xdu = 0$ → using Separation of variable.

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$$u^2 dx = x\,du$$

$$\int \frac{dx}{x} = \int \frac{du}{u^2} \Rightarrow \ln|x| + \ln c = \frac{u^{-2+1}}{-1}$$

$$\ln|x| + \ln c = \frac{u^{-1}}{1}$$

$$\ln c^3 x^3 = -u^{-3} \qquad \therefore \boxed{u = \frac{y}{x}}$$

$$\ln cx^3 = -\left(\frac{y}{x}\right)^{-1}$$

$$\ln cx^3 = -\frac{x^1}{y^1}$$

$$e^{\ln cx^3} = e^{-\left(\frac{x}{y}\right)^3}$$

$$\boxed{cx^3 = e^{-\left(\frac{x}{y}\right)^3}}$$

(9)

$$-y\,dx + (x + \sqrt{xy})\,dy = 0 \quad \text{--- a)}$$

$$\therefore M = -y, \quad N = (x + \sqrt{xy}).$$

$$M(tx, ty) = -ty = t^1 M(x,y) \qquad N(tx, ty) = tx + t\sqrt{xy} = t N(x,y)$$

let $x = vy$, $dx = v\,dy + y\,dv$.

Putting value in eq. - a)

$$-y(v\,dy + y\,dv) + ((vy) + \sqrt{(vy)y})\,dy = 0$$

$\frac{1}{(v)^{1/2}}$

$v^{-\frac{1}{2}+1}$

Date .... $\frac{-1+2}{2} = +\frac{1}{2}$

$$-yv\,dy + y^2 dv + vy + y\sqrt{v}\,dy = 0$$

— divide by y

$$-v\,dy - y\,dv + (v + y\sqrt{v})dy = 0$$

$$-\cancel{v\,dy} - y\,dv + \cancel{v\,dy} + \sqrt{v}\,dy = 0$$

$$-y\,dv + \sqrt{v}\,dy = 0$$

$$\sqrt{v}\,dy = y\,dv$$

$$\int \frac{dy}{y} = \int \frac{dv}{\sqrt{v}}$$

$$\ln|y| + c = \frac{v^{1/2}}{1/2}$$

$$\ln|y| + c = 2v^{1/2} \qquad \therefore \left(v = \frac{x}{y}\right)$$

$$\ln|y| + c = 2\left(\frac{x}{y}\right)^{1/2}$$

$$\ln|y| + c = 2\left(\frac{x}{y}\right)^{1/2}$$

$$e^{\ln|y| + c} = e^{2\left(\frac{x}{y}\right)^{1/2}}$$

$$\boxed{cy = e^{2\left(\frac{x}{y}\right)^{1/2}}}$$

Q 14

$$y\,dx + x(\ln x - \ln y - 1)\,dy = 0$$

$$M(x,y) = y \qquad N(x,y) = x(\ln x - \ln y - 1)$$

$$M(tx,ty) = ty = tM(x,y)$$

$$N(tx,ty) = tx(\ln(tx) - \ln(ty) - 1)$$
$$= tx(\cancel{\ln t} + \ln x - \cancel{\ln t} - \ln y - 1)$$
$$= tx(\ln x - \ln y - 1)$$
$$= tN(x,y)$$

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Assuming $\Rightarrow$ $\boxed{x = vy}$

(12) $(x^2+2y^2)\dfrac{dx}{dy} = xy, \quad y(-1) = 1. \quad$ — a)

$$(x^2+2y^2)\frac{dx}{dy} = xy$$

$$(x^2+2y^2)\,dx - xy\,dy = 0$$

$M = x^2+2y^2 \qquad N = -xy.$

$M(x,y)$

$M(tx,ty) = t^2x^2 + 2t^2y^2 \qquad N(tx,ty) = -txy$

$= t^2M(x,y) \qquad = tN(x,y)$

$(y = ux, \quad dy = u\,dx + x\,du) \qquad \boxed{u = \frac{y}{x}}$

Putting value in eq- a)

$$(x^2+2(u^2x^2))\,dx - x(ux)(u\,dx + x\,du) = 0$$

$$(x^2+2u^2x^2)\,dx - x^2u(u\,dx + x\,du) = 0$$

— Divide by x^2

$$(1+2u^2)\,dx - u(u\,dx + x\,du) = 0$$

$$dx + 2x^2dx - u^2dx - ux\,du = 0$$

$$(1+2x^2-u^2)\,dx = ux\,dx.$$

$$(1+u^2)\,dx = ux\,du$$

$$\int \frac{dx}{x} = \int \frac{u\,du}{(1+u^2)}$$

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$$\ln|x| + \ln C = \cancel{2}\frac{u^{1+1}}{\cancel{2}} \cdot \ln|1+u^2|$$

$$\ln|x| + c_1 = \cancel{2}\frac{u^2}{\cancel{2}} \frac{1}{2} \ln|1+u^2|$$

$$\ln x^2 c_1^2 = \ln|1+u^2|$$

$$\cancel{e^{\ln}} x^2 c^2 = \cancel{e^{\ln}} |1+u^2|$$

$$x^2 c^2 = 1+u^2 \qquad \therefore \left(u = \frac{y}{x}\right)$$

$$x^2 c^2 = 1 + \left(\frac{y^2}{x^2}\right) \quad \text{— ii)}$$

$$y(-1) = 1 \quad \therefore \quad y = +1 \;;\; x = -1$$

$$(-1)^2 c^2 = 1 + \frac{(1)^2}{(-1)^2}$$

$$c^2 = 1 + 1$$

$$\boxed{c = \sqrt{2}}.$$

Putting in eq-ii)

$$x^2(2) = 1 + \left(\frac{y^2}{x^2}\right)$$

$$\boxed{2x^2 = 1 + \frac{y^2}{x^2}}$$

Assignment: 2.5 Q. 8, 10, 13, 14, 18, 22.

2.4 Q. 23, 25, 26.

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Mathematics (DE)

(Ex. 2.5) (1-22)

Q15-22 → Bernoulli's Equation :

$$\frac{dy}{dx} + P(x)y = f(x)y^n \rightarrow \text{(Jame Bernoulli)}$$

I.F: $e^{\int P(x)dx}$.

$$\frac{d(u\cdot y)}{dx} = u \cdot f(x)$$

$$u \cdot y = \int u f(x) dx.$$

let ⇒ $u = y^{1-n}$ ∴ $n \neq 0, 1$

$$y = u^{n-1}, \quad \frac{dy}{dx} = \frac{d(u^{n-1})}{dx} = (n-1)u^{n-2}\frac{du}{dx}$$

So, if $n=2$:

$$u = y^{1-n} = y^{1-2} = y^{-1}.$$

$$\boxed{y = u^{-1}}$$

So,

$$\frac{dy}{dx} = (-1)u^{-2}\frac{du}{dx}$$

$$\frac{dy}{dx} + \frac{1}{x}y = xy^2$$

$$(-1)u^{-2}\frac{du}{dx} + \frac{1}{x}u^{-1} = xu^{-2}$$ → Divide ~~Multiple~~ with $-u^{-2}$

$$\frac{du}{dx} - \frac{1}{x}\frac{u^{-1}}{u^{-2}} = -\frac{x\cancel{u^{-2}}}{\cancel{u^{-2}}}$$

$$\frac{du}{dx} - \frac{u}{x} = -x$$

$P(x) = -\frac{1}{x}$ $\quad$ $f(x) = -x$.

$$I.F = e^{-\int \frac{1}{x}dx} = e^{\ln|x|^{-1}} = \boxed{\frac{1}{x}}$$

∴ $\frac{d(u\cdot y)}{dx} = u \cdot f(x)$. So,

$$\frac{d\left(\frac{1}{x}\cdot u\right)}{dx} = \frac{1}{\cancel{x}}(-\cancel{x})$$

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$$\frac{d}{dx}\left(\frac{1}{x}\cdot u\right) = -1$$

Integration:

$$\frac{1}{x}u = \int -1\, dx$$

$$\frac{1}{x}u = -x + c$$

$$u = x(-x+c)$$

$$u = -x^2 + cx$$

$$u = -x^2 + cx \qquad \therefore u = y^{-1}$$

$$y^{-1} = -x^2 + cx$$

$$\boxed{y = \frac{1}{cx - x^2}}$$

⇒ Q No. 19 ⇒ $t^2 \frac{dy}{dt} + y^2 = ty$ — Divide by t^2

$$\frac{dy}{dt} + \frac{y^2}{t^2} = \frac{y}{t}$$

$$\frac{dy}{dt} - \frac{1}{t}y = -\frac{1}{t^2}y^2 \quad \text{— i)}$$

$n = 2$.

$$u = y^{1-n} = y^{1-2} = y^{-1} \quad \text{So, } \boxed{u = y^{-1}}$$

$$\boxed{y = u^{-1}}$$

$$\frac{dy}{dt} = \cancel{(n-1)u^{n-2}\frac{du}{dt}} \; -u^{-2}\frac{du}{dt} \quad \text{— a)}$$

So, putting values in eq-i)

$$-u^{-2}\frac{du}{dt} - \frac{1}{t}(u^{-1}) = -\frac{1}{t^2}(u^{-2})$$

— Divide by $-u^{-2}$

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$$\frac{du}{dt} + \frac{1}{t}(u) = \frac{1}{t^2}$$

$$P(t) = \frac{1}{t} \qquad f(t) = \frac{1}{t^2}$$

I.F: $e^{\int P(t)\,dt} = e^{\int \frac{1}{t} dt} = e^{\ln|t|} = \boxed{t}$

$$\frac{d(t \cdot u)}{dt} = \frac{1}{t^2}(t)$$

Integration:

$$tu = \int \frac{1}{t}\,dt$$

$$tu = \ln|t| + c$$

$$tu = \ln|t| + c$$

$$t(y^{-1}) = \ln|t| + c$$

$$\frac{t}{y} = \ln|t| + c$$

$$e^{t/y} = e^{\ln|t| + c}$$

$$\boxed{e^{t/y} = tc}$$

Q NO. 22:

$y(0) = 4$; $\quad y^{1/2}\frac{dy}{dx} + y^{3/2} = 1$ — i)

— Divide by $y^{1/2}$

$$\frac{dy}{dx} + y^{3/2 - 1/2} = y^{-1/2}$$

$$\frac{dy}{dx} + y' = y^{-1/2} \quad \text{— i)}$$

So,

$$\boxed{n = -1/2}$$

$\frac{2}{3} - \frac{3}{3}$

$-\frac{1}{3}$

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$\boxed{n = -1/2}$.

$u = y^{1-n} = y^{1+\frac{1}{2}} = y^{3/2}$; $\boxed{u = y^{3/2}}$

$\boxed{y = u^{2/3}}$

So,

$$\frac{dy}{dx} = \left(\frac{2}{3}\right) u^{-1/3} \frac{dy}{dx} \quad \text{— i)}$$

Putting in eq-i,

$$\left(\frac{2}{3}\right) u^{-1/3} \frac{dy}{dx} + (u^{2/3}) = (u^{2/3})^{-1/2}$$

Multiple by $3/2$

$$u^{-1/3} \frac{dy}{dx} + \frac{3}{2}(u^{2/3}) = \frac{3}{2}(u^{-1/3})$$

2nd method:

$$u = y^{3/2}$$

$$\frac{du}{dy} = \frac{3}{2} y^{1/2} \quad \& \quad \frac{dy}{du} = \frac{2}{3y^{1/2}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{2}{3y^{1/2}} \cdot \frac{du}{dx} \quad \text{— i)}$$

Puting in eq - 1,

$$y^{1/2}\left[\frac{2}{3y^{1/2}} \cdot \frac{du}{dx}\right] + u = 1.$$

$$\frac{2}{3} \cdot \frac{du}{dx} + u = 1.$$

— Multiply $3/2$.

$$\frac{du}{dx} + \frac{3}{2}u = \frac{3}{2}$$

$P(x) = 3/2$; $f(x) = \frac{3}{2}$.

$$I.F = e^{\int(\frac{3}{2})dx} = \boxed{e^{3/2 x}}_{,1}.$$

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$$\therefore e^{3/2}\left[\frac{du}{dx} + \frac{3}{2}x\right] = e^{3/2x}\left[\frac{3}{2}\right]$$

$$\frac{d}{dx}\left[e^{3/2x} \cdot u\right] = \frac{3}{2}e^{3/2x}$$

$$e^{3/2x}u = \int \frac{3}{2}\frac{e^{3/2x}}{3/2}dx$$

$$e^{3/2x}u = \cancel{\frac{3}{2}}\frac{e^{3/2x}}{\cancel{3/2}} + c$$

$$e^{3/2x}u = e^{3/2x} + c$$

$$u = \cancel{\frac{e^{3/2x}}{e^{3/2x}}} + \frac{c}{e^{3/2x}}$$

$$u = 1 + ce^{-3/2x} \quad \text{--- ii)}$$

$$\boxed{u = y^{3/2}}$$

$$\therefore y^{3/2} = 1 + ce^{-3/2x}$$

$$(4)^{3/2} = 1 + ce^{-3/2(0)}$$

$$8 = 1 + c(1)$$

$$c = 8 - 1$$

$$\boxed{c = 7}$$

Putting value in eq-ii)

$$y^{3/2} = 1 + 7e^{-3/2x}$$