Exercise 6.3

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→ Imp are 3×3 variables.

Q 29: $u_1 = (1,1,1)$
$u_2 = (-1,1,0)$
$u_3 = (1,2,1)$

Solution→

$$v_1 = u_1$$

$$\boxed{v_1 = (1,1,1)}$$

$$v_2 = u_2 - \frac{\langle u_2, v_1\rangle}{\|v_1\|^2} v_1$$

$$v_2 = (-1,1,0) - \frac{(\cancel{-1}+\cancel{1}+0)}{3}(1,1,1)$$

$$v_2 = (-1,1,0) - 0$$

$$\boxed{v_2 = (-1,1,0)}$$

$$v_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2} v_2$$

$$v_3 = (1,2,1) - \frac{1+2+1}{3}(1,1,1) - \frac{-1+2+0}{2}(-1,1,0)$$

$$v_3 = (1,2,1) - (4/3, 4/3, 4/3) - (-\tfrac{1}{2}, \tfrac{1}{2}, 0)$$

$$\cancel{v_3 = (1,2,1) - \left(\tfrac{4}{3}+\tfrac{1}{2}, \tfrac{4}{3}-\tfrac{1}{2}, \tfrac{4}{3}-0\right)}$$

$$\cancel{v_3 = (1,2,1) - \left(\tfrac{8+3}{6}, \tfrac{8-3}{6}, 4/3\right)}$$

$$\cancel{v_3 = (1,2,1) - \left(\tfrac{11}{6}, \tfrac{5}{6}, \tfrac{4}{3}\right)}$$

$$\cancel{v_3 = \left(1-\tfrac{11}{6}, 2-\tfrac{5}{6}, 1-\tfrac{4}{3}\right)}$$

$$\cancel{v_3 = \left(\tfrac{6-11}{6}, \tfrac{12-5}{6}, \tfrac{3-4}{3}\right)}$$

$$\cancel{v_3 = \left(\tfrac{-5}{6}, \tfrac{7}{6}, \tfrac{-1}{3}\right)}$$

$$v_3 = (1,2,1) - \left(\tfrac{4}{3}, \tfrac{4}{3}, 4/3\right) - \left(-\tfrac{1}{2}, 1/2, 0\right)$$

$$v_3 = (1-4/3, 2-4/3, 1-4/3) - \left(-\tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

$$v_3 = \left(3-4/3, \tfrac{6-4}{3}, 3-4/3\right) - \left(-\tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

$$v_3 = (-1/3, 2/3, -1/3) - \left(-\tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

$$v_3 = \left(\tfrac{-2+3}{6}, \tfrac{4-3}{6}, -1/3\right)$$

$$\boxed{v_3 = (+1/6, 1/6, -1/3)}$$

Now, orthro-normals ⇒

$$\|v_1\| = \sqrt{1^2+1^2+1^2} = \boxed{\sqrt{3}}$$

$$\|v_2\| = \sqrt{(-1)^2+1^2+0} = \boxed{\sqrt{2}}$$

$$\|v_3\| = \sqrt{\tfrac{1}{36}+\tfrac{1}{36}+\tfrac{1}{9}}$$

$$= \sqrt{\tfrac{1+1+4}{36}}$$

$$= \frac{\sqrt{6}}{6} \Rightarrow \frac{\cancel{\sqrt{6}}}{\sqrt{6}\cdot\cancel{\sqrt{6}}}$$

$$\|v_3\| = \boxed{\frac{1}{\sqrt{6}}}$$

Now,

$$q_1 = \frac{v_1}{\|v_1\|} = \frac{(1,1,1)}{\sqrt{3}}$$

$$\boxed{q_1 = \left(\tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}\right)}$$

$$q_2 = \frac{v_2}{\|v_2\|} = \frac{(-1,1,0)}{\sqrt{2}}$$

$$\boxed{q_2 = \left(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)}$$

$$q_3 = \frac{v_3}{\|v_3\|}$$

$$q_3 = \frac{(1/6, 1/6, -1/3)}{1/\sqrt{6}}$$

$$q_3 = \left(\frac{1}{6}\times\sqrt{6}, \frac{1}{6}\times\sqrt{6}, -\frac{1}{3}\sqrt{6}\right)$$

$$\boxed{q_3 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{\sqrt{6}}{3}\right)}$$

Q30: $u_1 = (1,0,0)$

$u_2 = (3,7,-2)$

$u_3 = (0,4,1)$

Solution:

$$V_1 = u_1$$

$$\boxed{V_1 = (1,0,0)}$$

$$V_2 = u_2 - \frac{\langle u_2, V_1\rangle}{\|V_1\|^2} V_1$$

$$V_2 = (3,7,-2) - \frac{3}{1}(1,0,0)$$

$$V_2 = (3,7,-2) - (3,0,0)$$

$$\boxed{V_2 = (0,7,-2)}$$

$$V_3 = u_3 - \frac{\langle u_3, V_1\rangle}{\|V_1\|^2} V_1 - \frac{\langle u_3, V_2\rangle}{\|V_2\|^2} V_2$$

$$V_3 = (0,4,1) - 0 - \frac{28-2}{53}(0,7,-2)$$

$$V_3 = (0,4,1) - \frac{26}{53}(0,7,-2)$$

$$V_3 = (0,4,1) - \left(0, \frac{182}{53}, \frac{-52}{53}\right)$$

$$V_3 = \left(0, \frac{212-182}{53}, \frac{53+52}{53}\right)$$

$$\boxed{V_3 = \left(0, \frac{30}{53}, \frac{105}{53}\right)}$$

Now the ortho-normal of basis.

$$\|V_1\| = \boxed{1}$$

$$\|V_2\| = \sqrt{49+4} = \boxed{\sqrt{53}}$$

$$\|V_3\| = \sqrt{\frac{900}{2809} + \frac{11025}{2809}}$$

$$= \sqrt{\frac{11925}{2809}}$$

$$\|V_3\| = \boxed{\frac{15\sqrt{53}}{53}}$$

$$q_1 = \frac{V_1}{\|V_1\|} = (1,0,0)$$

$$\boxed{q_1 = (1,0,0)}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \frac{(0,7,-2)}{\sqrt{53}}$$

$$\boxed{q_2 = \left(0, \frac{7}{\sqrt{53}}, \frac{-2}{\sqrt{53}}\right)}$$

$$q_3 = \frac{V_3}{\|V_3\|} = \frac{\left(0, \frac{30}{53}, \frac{105}{53}\right)}{15\sqrt{53}/53}$$

$$q_3 = \left(0 \times \frac{53}{15\sqrt{53}}, \frac{\cancel{30}^{2}}{\cancel{53}} \times \cancel{\frac{53}{15\sqrt{53}}}, \frac{\cancel{53}}{\cancel{15}\sqrt{53}}, \frac{105}{\cancel{53}} \times \frac{\cancel{53}}{\cancel{15}\sqrt{53}}\right)$$

$$\boxed{q_3 = \left(0, \frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}}\right)}$$

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Example # 04

$u_1 = (0,1,0)$

$u_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$u_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

Solution:

$v_1 = u_1$

$\boxed{v_1 = (0,1,0)}$

$v_2 = u_2 - \frac{\langle u_2, v_1\rangle}{\|v_1\|^2} v_1$

$v_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) - 0$

$\boxed{v_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)}$

$v_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2} v_2$

$v_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) - 0 - \frac{1/2 - 1/2}{\frac{1}{2}+\frac{1}{2}} \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$\boxed{v_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)}$

Now, calculate ortho-normals:

$\|v_1\| = 1$

$\|v_2\| = \frac{1}{2} + \frac{1}{2} \Rightarrow 1$

$\|v_3\| = \frac{1}{2} + \frac{1}{2} \Rightarrow 1$

$q_1 = \frac{v_1}{\|v_1\|} = \frac{(0,1,0)}{1}$

$\boxed{q_1 = (0,1,0)}$

$q_2 = \frac{v_2}{\|v_2\|} = \frac{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)}{1}$

$\boxed{q_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)}$

$q_3 = \frac{v_3}{\|v_3\|} = \frac{\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)}{1}$

$\boxed{q_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)}$

Example # 5

$u_1 = (0,1,0)$
$u_2 = (-4/5, 0, 3/5)$
$u_3 = (3/5, 0, 4/5)$

$\therefore$ Note in example # 4,5 The answer will be same as question.

Example # 8: IMP VIP

$u_1 = (1,1,1)$

$u_2 = (0,1,1)$

$u_3 = (0,0,1)$

Solution:

$v_1 = u_1$

$\boxed{v_1 = (1,1,1)}$

$v_2 = u_2 - \frac{\langle u_2, v_1\rangle}{\|v_1\|^2} v_1$

$v_2 = (0,1,1) - \frac{2}{3}(1,1,1)$

$v_2 = \left(0 - \frac{2}{3}, 1 - \frac{2}{3}, 1 - \frac{2}{3}\right)$

$\boxed{v_2 = \left(-2/3, 1/3, 1/3\right)}$

$$V_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2} v_2$$

$$V_3 = (0,0,1) - \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) - \frac{1/3}{2/9}\left(0, 1/3, 1/3\right)$$

$$V_3 = \left(0-\frac{1}{3}, 0-\frac{1}{3}, 1-\frac{1}{3}\right) - \frac{1}{\cancel{3}} \times \frac{\cancel{9}^{3}}{2}\left(0, \frac{1}{3}, \frac{1}{3}\right)$$

$$V_3 = \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right) - \left(0, \frac{3}{6}, \frac{3}{6}\right)$$

$$V_3 = \left(-\frac{1}{3} - 0, -\frac{1}{3} - \frac{3}{6}, \frac{2}{3} - \frac{3}{6}\right)$$

$$V_3 = \left(-1/3, -5/6, 1/6\right)$$

$$V_3 = \left(-1/3, -5/6, 1/6\right)$$

$$V_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2} - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2} v_2$$

$$V_3 = (0,0,1) - \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) - \frac{1/\cancel{3}}{2/\cancel{3}}\left(-2/3, 1/3, 1/3\right)$$

$$V_3 = \left(-\frac{1}{3}, -\frac{1}{3}, 2/3\right) - \left(-\frac{2}{6}, \frac{1}{6}, \frac{1}{6}\right)$$

$$V_3 = \left(-\cancel{\frac{1}{3}} + \cancel{\frac{1}{3}}, -\frac{1}{3} - \frac{1}{6}, \frac{2}{3} - \frac{1}{6}\right)$$

$$\boxed{V_3 = \left(0, -1/2, +1/2\right)}$$

Now, orthro-normals,

$$\|V_1\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$\|V_2\| = \sqrt{\frac{4}{9}+\frac{1}{9}+\frac{1}{9}} = \sqrt{\frac{2}{3}}$$

$$\|V_3\| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$$

Now, Applying on Formula,

$$q_1 = \frac{V_1}{\|V_1\|} = \frac{(1,1,1)}{\sqrt{3}}$$

$$\boxed{q_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \frac{\left(-\frac{2}{3}, 1/3, 1/3\right)}{\sqrt{\frac{2}{3}}}$$

$$q_2 = \left(-\frac{2}{3} \times \frac{\sqrt{3}}{\sqrt{2}}, \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}}, \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}}\right)$$

$$q_2 = \left(\frac{\overset{-2}{\cancel{-\sqrt{2}\cdot\sqrt{2}}}}{\sqrt{3}\cdot\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{2}}}, \frac{1}{\sqrt{3}\cdot\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\sqrt{2}}, \frac{1}{\sqrt{3}\cdot\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\sqrt{2}}\right)$$

$$q_2 = \left(-\frac{\sqrt{2}}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$\boxed{q_2 = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)}$$

$$q_3 = \frac{V_3}{\|V_3\|} = \frac{\left(0, -1/2, 1/2\right)}{1/\sqrt{2}}$$

$$q_3 = \left(0, -\frac{1}{\sqrt{2}\cdot\cancel{\sqrt{2}}} \times \cancel{\sqrt{2}}, \frac{1}{\sqrt{2}\cdot\cancel{\sqrt{2}}} \times \cancel{\sqrt{2}}\right)$$

$$\boxed{q_3 = \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$$