(Exercise # 2.6) Q(23-36) Date .....

"Limits involving infinity"

limits as $x \to \infty$ or $x \to -\infty$

Find the limits in Q.23-36?

(23) $\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$

Sol,

Divide all values with highest power of denonimator.

$$\lim_{x\to\infty} \sqrt{\frac{\frac{8x^2}{x^2}-\frac{3}{x^2}}{\frac{2x^2}{x^2}+\frac{x}{x^2}}}$$

* Some Rules:

$\frac{\infty}{\infty} = \infty$, $\frac{1}{\infty} = 0$,

$\frac{\infty}{0} = -\infty$ or ∞

$$\lim_{x\to\infty} \sqrt{\frac{8-3/x^2}{2-1/x}}$$

Applying limit

$$\sqrt{\frac{8-0}{2-0}}$$

$$\sqrt{\frac{8}{2}4} = \sqrt{4} = \boxed{2}$$

(24) $\lim_{x\to-\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3}$

Sol

Divide all values with highest power of denonimator.

$$\lim_{x\to-\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3}$$

$$\lim_{x\to-\infty} \left(\frac{x^2/x^2 + x/x^2 - 1/x^2}{8x^2/x^2 - 3/x^2}\right)^{1/3}$$

$$\lim_{x\to-\infty} \left(\frac{1+1/x-1/x^2}{8-3/x^2}\right)^{1/3}$$

Applying limit

$$\left(\frac{1+0-0}{8-0}\right)^{1/3}$$

$$\left(\frac{1}{8}\right)^{1/3} \Rightarrow \boxed{\frac{1}{2}}$$

(25) $\lim_{x\to-\infty} \left(\frac{1-x^3}{x^2-7x}\right)^5$

Sol,

$$\lim_{x\to-\infty} \left(\frac{1/x^2 - x^3/x^2}{x^2/x^2 - 7x/x^2}\right)^5$$

$$\lim_{x\to-\infty} \left(\frac{1/x^2 - x}{1 - 7/x}\right)^5$$

Applying limit

$$\left(\frac{0+\infty}{1-0}\right)^5 \Rightarrow \boxed{\infty}$$

Date .....

(26) $\lim_{x\to\infty} \sqrt{\frac{x^2-5x}{x^3+x-2}}$

Sol,

$$\lim_{x\to\infty} \sqrt{\frac{x^2-5x}{x^3+x-2}}$$

$$\lim_{x\to\infty} \sqrt{\frac{x^2/x^3 - 5x/x^3}{x^3/x^3 + x/x^3 - 2/x^3}}$$

$$\lim_{x\to\infty} \sqrt{\frac{1/x - 5/x^2}{1 + 1/x^2 - 2/x^3}}$$

Applying limit

$$\sqrt{\frac{0-0}{1+0-0}} = \sqrt{0} = \boxed{0}$$

(27) $\lim_{x\to\infty} \frac{2\sqrt{x} + x^{-1}}{3x-7}$

Sol,

$$\lim_{x\to\infty} \frac{2\sqrt{x} + x^{-1}}{3x-7}$$

$$\lim_{x\to\infty} \frac{\frac{2\sqrt{x}}{x} + \frac{1}{x\cdot x}}{3x/x - 7/x}$$

$$\lim_{x\to\infty} \frac{\frac{2}{\sqrt{x}} + \frac{1}{x^2}}{3 - 7/x}$$

Applying limit

$$\frac{0+0}{3-0} \Rightarrow \boxed{0}$$

(28) $\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}}$

Sol,

$$\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}}$$

$$\lim_{x\to\infty} \frac{2/\sqrt{x} + 1}{2/\sqrt{x} - 1}$$

Applying limit

$$\frac{0+1}{0-1} \Rightarrow \boxed{-1}$$

$\therefore \sqrt{x}\cdot\sqrt{x} = x$

$\therefore \frac{\sqrt{x}}{\sqrt{x}} = 1$

(29) $\lim_{x\to-\infty} \frac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}}$

Sol,

$$\lim_{x\to-\infty} \frac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$\lim_{x\to-\infty} \frac{\frac{\sqrt[3]{x}}{\sqrt[3]{x}} - \frac{\sqrt[5]{x}}{\sqrt[3]{x}}}{\frac{\sqrt[3]{x}}{\sqrt[3]{x}} + \frac{\sqrt[5]{x}}{\sqrt[3]{x}}}$$

$$\lim_{x\to-\infty} \frac{1 - \sqrt[5]{x}/\sqrt[3]{x}}{1 + \sqrt[5]{x}/\sqrt[3]{x}}$$

Applying limit

$$\frac{1-0}{1+0} = \boxed{1}$$

(30) $\lim_{x\to\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$

Sol,

$$\lim_{x\to\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$$

$$\lim_{x\to\infty} \frac{x^x\frac{1}{x} + \frac{1}{x^4}xx^x}{x^x\frac{1}{x^2} - \frac{1}{x^3}xx^x}$$

$$\lim_{x\to\infty} \frac{x + \frac{1}{x^2}}{1 - 1/x}$$

Applying limit

$$\frac{\infty + 0}{1 - 0} = \boxed{\infty}$$

(31) $\lim_{x\to\infty} \frac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + x^{1/2}}$

Sol,

$$\lim_{x\to\infty} \frac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + x^{1/2}}$$

$$\lim_{x\to\infty} \frac{\frac{2x^{5/3}}{x^{8/5}} - \frac{x^{1/3}}{x^{8/5}} + 7/x^{8/5}}{\frac{x^{8/5}}{x^{8/5}} + \frac{3x}{x^{8/5}} + \frac{x^{1/2}}{x^{8/5}}}$$

$$\lim_{x\to\infty} \frac{2x^{7/15} - \frac{1}{x^{19/15}} + \frac{7}{x^{8/5}}}{1 + 3/x^{3/5} + \frac{1}{x^{11/10}}}$$

Applying limit

$$\frac{\infty - 0 + 0}{1 + 0 + 0} \Rightarrow \boxed{\infty}$$

Date .....

(32) $\lim_{x \to -\infty} \frac{\sqrt[3]{x} - 5x + 3}{2x + x^{2/3} - 4}$

Sol,

$\lim_{x \to -\infty} \frac{\sqrt[3]{x} - 5x + 3}{2x + x^{2/3} - 4}$

$\lim_{x \to -\infty} \frac{(x)^{1/3}}{x} - \frac{5x}{x} + \frac{3}{x}$

$\frac{2x}{x} + \frac{x^{2/3}}{x} - \frac{4}{x}$

$\lim_{x \to -\infty} \frac{\frac{1}{x^{2/3}} - 5 + \frac{3}{x}}{2 + \frac{1}{x^{1/3}} - \frac{4}{x}}$

Applying limit

$\frac{0 - 5 + 0}{2 + 0 - 0} \Rightarrow \boxed{\frac{-5}{2}}$

(33) $\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1}$

Sol,

$\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1}$

$\lim_{x \to \infty} \frac{\sqrt{x^2+1}/\sqrt{x^2}}{x+1/\sqrt{x^2}}$

$\lim_{x \to \infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{1}{x^2}}}{\frac{x}{x} + \frac{1}{\sqrt{x^2}}}$

$\lim_{x \to \infty} \frac{\sqrt{1 + 1/x}}{1 + 1/\sqrt{x^2}}$

Applying limit

$\frac{\sqrt{1+0}}{1+0} = \boxed{1}$

(34) $\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1}$

Same as in (33)

(35) $\lim_{x \to \infty} \frac{x-3}{\sqrt{4x^2+25}}$

Sol,

$\lim_{x \to \infty} \frac{x-3}{\sqrt{4x^2+25}}$

$\lim_{x \to \infty} \frac{\frac{x}{\sqrt{x^2}} - \frac{3}{\sqrt{x^2}}}{\sqrt{\frac{4x^2}{x^2} + \frac{25}{x^2}}}$

$\lim_{x \to \infty} \frac{1 - 3/x}{\sqrt{4 + 25/x^2}}$

Applying limit

$\frac{(1-0)}{\sqrt{4+0}} = \boxed{\frac{1}{2}}$

(36) $\lim_{x \to -\infty} \frac{4 - 3x^3}{\sqrt{x^6+9}}$

Sol,

$\lim_{x \to -\infty} \frac{4 - 3x^3}{\sqrt{x^6+9}}$

$\lim_{x \to -\infty} \frac{\frac{4}{\sqrt{x^6}} - \frac{3x^3}{\sqrt{x^6}}}{\sqrt{\frac{x^6}{x^6} + \frac{9}{x^6}}}$

$\lim_{x \to -\infty} \frac{-4/x^3 + 3}{\sqrt{1 + 9/x^6}}$

Applying limit

$\frac{0+3}{\sqrt{1+0}} = (3)$