

(4.3)

Q \rightarrow 1-6 (linear comb)

- \rightarrow Simultaneous equation
- \rightarrow Reduced Echelon

\rightarrow spanning vector
(Q7) \rightarrow determine

\vec{v}	2	4	
	3		
	4		

$$\det = 0$$

\rightarrow Not span
 \rightarrow ~~in~~ dependent.

Q8-10

- \rightarrow Simultaneous equations
- \rightarrow Reduced Echelon
- \rightarrow Determine

$$\det \neq 0$$

\rightarrow span
 \rightarrow independent.

4.2

Two Rules.

$$\rightarrow (u+v)$$

$$\rightarrow (ku)$$

P_{∞} - (16) a) \rightarrow solve $\rightarrow f = a_0 + a_1x + a_2x^2 + a_3x^3$
 $g = b_0 + b_1x + b_2x^2$
 b) \rightarrow Subspace
 c) \rightarrow Not solve.

(15) a) Subspace x^5
 b) Not x^6 $f = x^3 - x^0$
 c) Not x^7 $g = x^0$

R^4 - (14) a) \rightarrow Not Subspace
 b) \rightarrow Not Subspace

(13) a) \rightarrow Not subspace. $f = (1, 1, 1, 1)$
 $g = (2, 4, 8, 16) \rightarrow k, a^2, a^3, a^4$
 b) \rightarrow Subspace.

M_{22} - (12) a) Subspace.
 b) Subspace.
 c) Not subspace. $\rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$.

(11) a) Subspace.
 b) Not subspace.
 c) Not Subspace.

- \mathbb{R}^∞ →
- 10) a) Subspace → $V = (v, 2v, 4v, 8v, \dots)$
 $W = (w, 2w, 4w, \dots)$
 - b) Subspace → $V \in (v_1, v_2, 0, 0, 0, \dots)$
 $W = (w_1, w_2, 0, 0, \dots)$
 - 9) a) Subspace
 - b) Not subspace.

- $F(-\infty, \infty)$ →
- 8) a) Subspace → $f(-x) = f(x)$
 $g(-x) = g(x)$
 - b) Notsubspace → $f = 1 + 2x + 3x^2$
 $g = 1 + 2x - 3x^2$
 - 7) a) Subspace → $f(0) = 0$
 $g(0) = 0$
 - b) Notsubspace → $f(0) = 1$
 $g(0) = 1$

- P_3 →
- 6) a) Subspace → $f = a_0 + a_1x$ (Real Numbers)
 $g = b_0 + b_1x$
 - a) Notsubspace → (Rational Numbers)
 - 5) a) subspace
 - b) Subspace

- M_{nn} →
- 4) a) Subspace → $A = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (Invertible/Trivial) → $\det(A) \neq 0$
 - b) Notsubspace → $(A+B)C = AC + BC \neq (KA)C = K(AC)$
 - c) Subspace → $A = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (Invertible)
 - d) Notsubspace → $(\det(A) = 0)$
 - 3) a) Subspace → $\begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & a_{nn} \end{bmatrix}$ (Since determinant is not distributive)
 - b) Notsubspace → $\text{tr}(A) = a_{11} + a_{22} + \dots + a_{nn} = 0$
 - c) Subspace → $\text{tr}(B) = b_{11} + b_{22} + \dots + b_{nn} = 0 \therefore (\text{tr}(A) = 0)$
 - d) Subspace → $AT = A$
 $BT = B$ (Symmetric) → $AT = A$

- \mathbb{R}^3 →
- 2) a) Notsubspace
 - b) Subspace
 - c) Notsubspace → $u = (a_1, b_1, c_1)$
 $v = (a_2, b_2, c_2)$
condition: $(a + b = 7)$

→ Axioms List

i) Closure law / Closed property.

$$u + v \in V$$

$$kv \in V$$

ii) Commutative Property.

$$u + v = v + u.$$

iii) Associative Property.

$$u + (v + w) = (u + v) + w$$

iv) Additive Property.

There should exist $v = 0 \in V$

$$\text{such that } 0 + v = v + 0 = v$$

v) Additive Inverse

$$u + (-u) = -u + u = 0$$

vi) Scalar Multiplication.

$$ku \in V$$

vii) Distributive Property

$$k(u + v) = ku + kv$$

ix) ~~viii)~~ Distributive property

$$k(mu) = (km)u$$

viii) ~~ix)~~ Associative Property

$$(k + m)u = ku + mu$$

x)

$$1 \cdot u = u$$

Some Extensions →

i) $Du = 0$

ii) $CO = 0$

iii) $-u = (-1)u.$

