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Exercise 2.4 (21 - 42)

"One Sided limits and limits at infinity".

Q. Find the limits in Question (21 - 42) ?

(21) $\lim_{\theta \to 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$

Sol:

$\lim_{\theta \to 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$

$\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

So,

$\lim_{\theta \to 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} = \boxed{1}$

22) $\lim_{t \to 0} \frac{\sin kt}{t}$

Sol. (k constant)

$\lim_{t \to 0} \frac{\sin kt}{t}$

Multiplying & Dividing by k.

$\lim_{t \to 0} \frac{\sin kt}{t} \times \frac{k}{k}$

$k \cdot \left(\lim_{t \to 0} \frac{\sin kt}{kt}\right)$

$k \cdot 1 \Rightarrow \boxed{k}$

23) $\lim_{y \to 0} \frac{\sin 3y}{4y}$

Sol.

$\lim_{y \to 0} \frac{\sin 3y}{4y}$

Multiply & Divide by 3.

$\lim_{y \to 0} \frac{\sin 3y}{4y} \times \frac{3}{3}$

$\frac{3}{4}\left(\lim_{y \to 0} \frac{\sin 3y}{3y}\right)$

$\frac{3}{4}(1) \Rightarrow \boxed{\frac{3}{4}}$.

24) $\lim_{h \to 0^-} \frac{h}{\sin 3h}$

Sol:

$\lim_{h \to 0^-} \frac{h}{\sin 3h}$

Multiply & Divide by 3.

$\lim_{h \to 0^-} \frac{h}{\sin 3h} \times \frac{3}{3}$

$\frac{1}{3}\left(\lim_{h \to 0^-} \frac{3h}{\sin 3h}\right)$

$\frac{1}{3}\left(\lim_{h \to 0^-} \frac{1}{\frac{\sin 3h}{3h}}\right)$

$\frac{1}{3}\left(\frac{1}{1}\right)$

$\boxed{\frac{1}{3}}$.

25) $\lim_{x \to 0} \frac{\tan 2x}{x}$

Sol:

$\because \tan 2x = \frac{\sin 2x}{\cos 2x}$

$\lim_{x \to 0} \frac{\sin 2x}{x \cos 2x}$

$\left(\lim_{x \to 0} \frac{\sin 2x}{x}\right)\left(\lim_{x \to 0} \frac{1}{\cos 2x}\right)$

$2\left(\lim_{x \to 0} \frac{\sin 2x}{2x}\right)\left(\lim_{x \to 0} \frac{1}{\cos 2(0)}\right)$

$2(1)(1) \Rightarrow (2)$

26) $\lim_{t \to 0} \frac{2t}{\tan t}$

Sol.

$\lim_{t \to 0} \frac{2t \cos t}{\sin t}$

$\left(2 \lim_{t \to 0} \cos t\right)\left(\lim_{t \to 0} \frac{t}{\sin t}\right)$

$2(1)(1) \Rightarrow (2)$.

27) $\lim_{x\to 0} \frac{x \csc 2x}{\cos 5x}$

Sol,

$\lim_{x\to 0} \frac{x \csc 2x}{\cos 5x}$

$\therefore \csc 2x = \frac{1}{\sin 2x}$

So,

$\lim_{x\to 0} \frac{x}{\sin 2x (\cos 5x)}$

$\left(\lim_{x\to 0} \frac{x}{\sin 2x}\right)\left(\lim_{x\to 0} \frac{1}{\cos 5x}\right)$

$\frac{1}{2}\left(\lim_{x\to 0} \frac{2x}{\sin 2x}\right)\left(\lim_{x\to 0} \frac{1}{\cos 5x}\right)$

$\frac{1}{2}(1)(1) \Rightarrow \boxed{\frac{1}{2}}$

28) $\lim_{t\to 0} 6x^2 (\cot x)(\csc 2x)$

Sol,

$\lim_{t\to 0} 6x^2 (\cot x)(\csc 2x)$

$\therefore \cot x = \frac{\cos x}{\sin x}$, $\csc 2x = \frac{1}{\sin 2x}$

So,

$3\left(\lim_{t\to 0} \frac{2x \cdot x \cdot \cos x}{\sin x \cdot \sin 2x}\right)$

$3\left(\lim_{t\to 0} \frac{x}{\sin x}\right)\left(\lim_{t\to 0} \frac{2x}{\sin 2x}\right)\left(\lim_{t\to 0} \cos x\right)$

$3\left(\lim_{t\to 0} \frac{1}{\frac{\sin x}{x}}\right)\left(\lim_{t\to 0} \frac{1}{\frac{\sin 2x}{2x}}\right)\left(\lim_{t\to 0} \cos x\right)$

$3(1)(1)(1) \Rightarrow \boxed{3}$.

29) $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x}$

Sol,

$\lim_{x\to 0} \frac{x}{\sin x \cos x} + \frac{x \cos x}{\sin x \cos x}$

$\left(\lim_{x\to 0} \frac{x}{\sin x}\right)\left(\lim_{x\to 0} \frac{1}{\cos x}\right) + \left(\lim_{x\to 0} \frac{x}{\sin x}\right)$

$\left(\lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}\right)\left(\lim_{x\to 0} \frac{1}{\cos x}\right) + \left(\lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}\right)$

$(1)(1) + 1 \Rightarrow \boxed{2}$.

30) $\lim_{x\to 0} \frac{x^2 - x + \sin x}{2x}$

Sol,

$\lim_{x\to 0} \frac{x^2 - x + \sin x}{2x}$

$\lim_{x\to 0} \frac{x^2}{2x} + \frac{\sin x}{2x} - \frac{x}{2x}$

$\left(\lim_{x\to 0} \frac{x}{2}\right) + \frac{1}{2}\left(\lim_{x\to 0} \frac{\sin x}{x}\right) - \left(\lim_{x\to 0} \frac{1}{2}\right)$

$\frac{1}{2}\left(\lim_{x\to 0} x\right) + \frac{1}{2}\left(\lim_{x\to 0} \frac{\sin x}{x}\right) - \frac{1}{2}$

$0 + \frac{1}{2} - \frac{1}{2} \Rightarrow \boxed{0}$

CLASS ASSIGNMENT # 01 (Q-31 - 42)

31) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin 2\theta}$

Sol,

$\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin 2\theta}$

$\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin\theta\cos\theta}$

Multiply & Divide by $1+\cos\theta$.

$\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin\theta\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$\lim_{\theta \to 0} \frac{1-\cos^2\theta}{(2\sin\theta\cos\theta)(1+\cos\theta)}$

$\because 1-\cos^2\theta = \sin^2\theta$

$\lim_{\theta \to 0} \frac{\sin^2\theta}{(2\sin\theta\cos\theta)(1+\cos\theta)}$

$\lim_{\theta \to 0} \frac{\sin\theta}{(2\cos\theta)(1+\cos\theta)}$

Applying limit

$\lim_{\theta \to 0} \frac{\sin 0^\circ}{(2\cos 0^\circ)(1+\cos 0^\circ)}$

$\frac{0}{(2)(2)} \Rightarrow \boxed{0}$.

32) $\lim_{x \to 0} \frac{x-x\cos x}{\sin^2 3x}$

Sol,

$\lim_{x \to 0} \frac{x-x\cos x}{\sin^2 3x}$

$\lim_{x \to 0} \frac{x(1-\cos x)}{\sin^2 3x} \Rightarrow \lim_{x \to 0} \frac{x(1-\cos x)}{(\sin 3x)(\sin 3x)}$

$\lim_{x \to 0} \left(\frac{1}{3}\right)\left(\frac{3x}{\sin 3x}\right)\left(\frac{1-\cos x}{x}\right)\left(\frac{3x}{\sin 3x}\right)\left(\frac{1}{3}\right)$

$\left(\frac{1}{3}\right)(1)(0)(1)\left(\frac{1}{3}\right) \Rightarrow \boxed{0}$.

33) $\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

Sol,

$\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

$\because \lim_{t \to 0} \frac{\sin\theta}{\theta} = 1$.

So, here $\theta = 1-\cos t$

So, $\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

$\boxed{1}$.

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34) $\lim_{h\to 0} \frac{\sin(\sinh)}{\sinh}$

Sol,

$\lim_{h\to 0} \frac{\sin(\sinh)}{(\sinh)}$

$\because \lim_{h\to 0} \frac{\sin\theta}{\theta} = 1$

as $\theta = \sinh$.

so,

$\lim_{h\to 0} \frac{\sin(\sinh)}{\sinh} = \boxed{1}$

35) $\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta}$

Sol,

$\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta}$

Multiply & Divide by 2θ.

$\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta} \times \frac{2\theta}{2\theta}$

$\frac{1}{2}\left(\lim_{\theta\to 0} \frac{\sin\theta}{\theta}\right)\left(\lim_{\theta\to 0} \frac{2\theta}{\sin 2\theta}\right)$

$\frac{1}{2}(1)(1) = \boxed{\frac{1}{2}}$.

36) $\lim_{x\to 0} \frac{\sin 5x}{\sin 4x}$

Sol,

$\lim_{x\to 0} \frac{\sin 5x}{\sin 4x} \times \frac{4x}{5x} \times \frac{5}{4}$

$\frac{5}{4}\left(\lim_{x\to 0} \frac{\sin 5x}{5x}\right)\left(\lim_{x\to 0} \frac{4x}{\sin}\right.$

$\frac{5}{4}(1)(1)$

$\boxed{\frac{5}{4}}$

37) $\lim_{\theta\to 0} \theta\cos\theta$

Putting value

$\lim_{\theta\to 0} \theta\cos\theta$

$\lim_{\theta\to 0} (0)(\cos 0^\circ)$

$(0)(1) = \boxed{0}$.

38) $\lim_{\theta\to 0} \sin\theta \cot 2\theta$

Sol,

$\lim_{\theta\to 0} \sin\theta \left(\frac{\cos 2\theta}{\sin 2\theta}\right)$

$\lim_{\theta\to 0} \frac{\sin\theta\cos 2\theta}{2\sin\theta\cos\theta}$

$\lim_{\theta\to 0} \frac{\cos 2(0^\circ)}{2\cos(0^\circ)} \Rightarrow \boxed{\frac{1}{2}}$

39) $\lim_{x\to 0} \frac{\tan 3x}{\sin 8x}$

Sol:

$\lim_{x\to 0} \frac{\tan 3x}{\sin 8x}$

$\lim_{x\to 0} \frac{\sin 3x}{\cos 3x \sin 8x}$

$\lim_{x\to 0} \frac{\sin 3x}{\cos 3x \sin 8x} \times \frac{8x}{3x} \times \frac{3}{8}$

$\frac{3}{8}\left(\lim_{x\to 0} \frac{\sin 3x}{3x}\right)\left(\lim_{x\to 0} \frac{8x}{\sin 8x}\right)\left(\lim_{x\to 0} \frac{1}{\cos 3x}\right)$

$\frac{3}{8}(1)\left(\frac{1}{1}\right)(1)$

$\boxed{\frac{3}{8}}$.

40) $\lim_{y\to 0} \frac{\sin 3y \cot 5y}{y\cot 4y}$

$\lim_{y\to 0} \frac{\sin 3y \cos 5y \sin 4y}{y \sin 5y \cos 4y}$

$\left(\lim_{y\to 0} \frac{\sin 3y}{y}\right)\left(\lim_{y\to 0} \frac{\cos 5y}{\cos 4y}\right)$

$\left(\lim_{y\to 0} \frac{\sin 4y}{\sin 5y}\right)$

$\frac{4}{5}(1)(1)(1)$

$\boxed{\frac{4}{5}}$ Answer.

In 40 question, Multiply & Divide by $\frac{4}{5}$.

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41) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta^2 \cot 3\theta}$

Sol

$\lim_{\theta \to 0} \frac{\tan\theta}{\theta^2 \cot 3\theta}$

$\lim_{\theta \to 0} \frac{\sin\theta \sin 3\theta}{\theta^2 \cos\theta \cos 3\theta}$

$\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right)\left(\lim_{\theta \to 0} \frac{\sin 3\theta}{3\theta}\right)\left(\lim_{\theta \to 0} \frac{3}{\cos\theta \cos 3\theta}\right)$

$(1)(1)\left(\frac{3}{1 \cdot 1}\right) \Rightarrow \boxed{3}$

42) $\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta}$

Sol

$\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta}$

$\lim_{\theta \to 0} \frac{\theta \cos 4\theta \sin^2 2\theta}{\sin^2\theta \cos^2 2\theta \sin 4\theta}$

$\lim_{\theta \to 0} \frac{\theta \cos 4\theta (2\sin\theta\cos\theta)^2}{\sin^2\theta \cos^2 2\theta \sin 4\theta}$

$\lim_{\theta \to 0} \frac{\theta \cos 4\theta (4 \sin^2\theta \cos^2\theta)}{\sin^2\theta \cos^2 2\theta \sin 4\theta}$

$\lim_{\theta \to 0} \frac{4\theta \cos 4\theta \cos^2\theta}{\cos^2 2\theta \sin 4\theta}$

$\lim_{\theta \to 0} \left(\frac{4\theta}{\sin 4\theta}\right)\left(\frac{\cos 4\theta \cos^2\theta}{\cos^2 2\theta}\right)$

Applying limit

$(1)\left(\frac{1 \cdot 1^2}{1^2}\right) = \boxed{1}$