Lecture

Continuous Probability Distributions

1 - Normal probability Distribution:

One of the most important continuous probability distribution is normal distribution.

* Probability density function of normal distribution is:

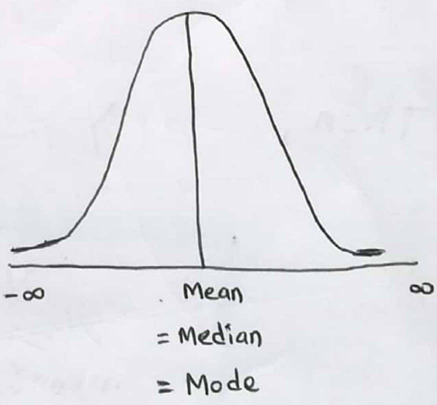
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty$$

Properties:

i) It is a bell shaped distribution.

ii) It is unimodal; i.e; it peaks at a single value.*



iii) It is symmetrical; about a vertical axis through the mean μ.

iv) It has two parameters; Mean = μ and Variance = σ^2. It is denoted by:

* Modal value

$X \sim N(\mu, \sigma^2)$.

v) It ranges from $-\infty$ to ∞.

vi) The mean, median and mode are equal.

vii) The total area under the normal curve (above the horizontal axis) is always equal to 1.

viii) If $X \sim N(\mu, \sigma^2)$ and

if $Y = a + bX$

then, $Y \sim N(a + b\mu, b^2\sigma^2)$

$$Y = a + bX$$

$$\text{mean}(Y) = a + b\,\text{mean}(X)$$

$$\text{mean}(Y) = a + b\mu$$

$$Y = a + bX$$

$$Var(Y) = Var(a) + Var(bX)$$

$$= 0 + b^2 Var(X)$$

$$\therefore Var(Y) = b^2\sigma^2$$

* If $X \sim N(16, 100)$, and if $Y = 3X + 9$,

then $Y \sim N\underline{(50, 900)}$.

$\mu = 16$

$\sigma^2 = 100$

$\text{mean}(Y) = a + b\mu$

$= 9 + 3 \times 16$

$= 50$

$\text{Var}(Y) = b^2 \sigma^2$

$= 3^2 \times 100$

$= 900$

ix) The sum of independent normal variables is a normal variable. i.e;

If $X_1 \sim N(\mu_1, \sigma_1^2)$

and $X_2 \sim N(\mu_2, \sigma_2^2)$

then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

* If $X_1 \sim N(7, 36)$ and $X_2 \sim N(12, 64)$

also, $Y = X_1 + X_2$, then $Y \sim \underline{N(19, 100)}$.

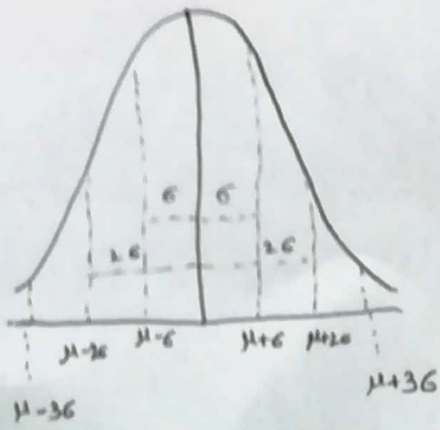
Revision:

x) Areas under normal curve remains in certain fixed proportions within a specified number of standard deviations on either side of μ.

a) $\mu \pm \sigma$ will contain 68.26%

b) $\mu \pm 2\sigma$ will contain 95.44%

c) $\mu \pm 3\sigma$ will contain 99.73%



"Standard Normal Distribution"

The distribution of a normal random variable with mean "0" and variance "1" is called a standard normal distribution. It is denoted by Z.

i.e; $Z \sim N(0, 1)$.

where
$$Z = \frac{X - \mu}{\sigma}$$

*** Understanding of tables of cumulative normal distribution.**