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Exercise 7.5 (1-50)

Use L'Hospital Rule to find the limit and then use method to evaluate:

(1) $\lim_{x\to -2} \frac{x+2}{x^2-4}$

$\frac{x+2}{(x+2)(x-2)}$

taking derivative

$\lim_{x\to 2} \frac{1}{2x}$

$\frac{1}{2x}\Big|_{x=2} \Rightarrow \boxed{\frac{1}{4}}$

OR

$\lim_{x\to 2} \frac{x+2}{(x+2)(x-2)}$

$\lim_{x\to 2} \frac{1}{x-2}$

$\boxed{\frac{1}{4}}$

(2) $\lim_{x\to 0} \frac{\sin 5x}{x}$

taking derivative

$\frac{5\cos 5x}{1}\Big|_{x\to 0}$

$5(1) = \boxed{5}$

OR

$5\lim_{x\to 0} \frac{\sin 5x}{5x}$

$5 \cdot 1 = \boxed{5}$

(3) $\lim_{x\to \infty} \frac{5x^2-3x}{7x^2+1}$

taking derivatives.

$\frac{10x-3}{14x}\Big|_{x\to\infty} = \frac{10}{14}\Big|_{x\to\infty}$

$\frac{10}{14} = \boxed{\frac{5}{7}}$. OR

$\lim_{x\to\infty} \frac{\frac{5x^2}{x^2}-\frac{3x}{x^2}}{\frac{7x^2}{x^2}+\frac{1}{x^2}} \Rightarrow \lim_{x\to\infty} \frac{5-3/x}{7+1/x^2} = \boxed{\frac{5}{7}}$

(4) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$

taking derivative

$\lim_{x\to 1} \frac{3x^2}{12x^2-1}$

Applying limit

$\boxed{\frac{3}{11}}$

OR $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$

$\lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(4x^2+4x+3)}$

$\lim_{x\to 1} \frac{x^2+x+1}{4x^2+4x+3} = \boxed{\frac{3}{11}}$.

(5) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\lim_{x\to 0} \frac{\sin x}{2x}$

$\lim_{x\to 0} \frac{\cos x}{2}$

$\boxed{\frac{1}{2}}$

OR $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\lim_{x\to 0} \left[\frac{(1-\cos x)}{x^2}\left(\frac{1+\cos x}{1+\cos x}\right)\right]$

$= \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)}$

$= \lim_{x\to 0} \left[\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)\left(\frac{1}{1+\cos x}\right)\right]$

$\boxed{\frac{1}{2}}$

(6) $\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1}$

taking derivative

$\lim_{x\to\infty} \frac{4x+3}{3x^2+1}$

$\lim_{x\to\infty} \frac{4}{6x} = \boxed{0}$.

OR: $\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1}$

$\lim_{x\to\infty} \frac{\frac{2}{x}+\frac{3}{x^2}}{1+\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0}{1} = \boxed{0}$.

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(7) $\lim_{x\to 2} \frac{x-2}{x^2-4}$

Applying derivation

$\lim_{x\to 2} \frac{1}{2x}$

$\lim_{x\to 2} \frac{1}{2x}$

Applying limit.

$\frac{1}{4}$

(8) $\lim_{x\to -5} \frac{x^2-25}{x+5}$

Derivation

$\lim_{x\to -5} \frac{2x}{1}$

Applying limit

$2(-5)$

-10

(9) $\lim_{t\to -3} \frac{t^3-4t+15}{t^2-t-12}$

Derivation

$\lim_{t\to -3} \frac{3t^2-4}{2t-1} = \frac{3(-3)^2-4}{2(-3)-1}$

$\lim_{t\to -3} \frac{6t}{2} \Rightarrow \frac{6(-3)}{2} \Rightarrow -9$.

$= -\frac{23}{7}$.

(10) $\lim_{t\to 1} \frac{t^3-1}{4t^3-t-3}$

$\lim_{t\to 1} \frac{3t^2}{12t^2-1}$

$\frac{3(1)^2}{12(1)^2-1}$

$\frac{3}{11}$

(11) $\lim_{x\to \infty} \frac{5x^3-2x}{7x^3+3}$

$\lim_{x\to \infty} \frac{15x^2-2}{21x^2+}$

$\lim_{x\to \infty} \frac{30x}{42x}$

$\frac{30}{42} = \frac{5}{7}$.

(12) $\lim_{x\to \infty} \frac{x-8x^2}{12x^2+5x}$

$\lim_{x\to \infty} \frac{-16x}{24x+5}$

$\lim_{x\to \infty} \frac{-16}{24}$

$-\frac{4}{3}$.

(13) $\lim_{t\to 0} \frac{\sin t^2}{t}$

$\lim_{t\to 0} \cos t^2 \cdot 2t$

$\cos(0)^2 \cdot 2(0)$

$1 \times 0 = 0$

(14) $\lim_{t\to 0} \frac{\sin 5t}{2t}$

$\lim_{t\to 0} \frac{\cos 5t \cdot 5}{2}$

$\frac{\cos 5(0) \cdot 5}{2} = \frac{(1)5}{2}$

$\frac{5}{2}$.

(15) $\lim_{x\to 0} \frac{8x^2}{\cos x - 1}$

$\lim_{x\to 0} \frac{16x}{-\sin x}$

$\lim_{x\to 0} \frac{16}{-\cos x}$

-16

(16) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

$\lim_{x\to 0} \frac{\cos x - 1}{3x^2}$

$\lim_{x\to 0} \frac{-\sin x}{6x}$

$\lim_{x\to 0} \frac{-\cos x}{6}$

$-1/6$

(17) $\lim_{\theta\to \pi/2} \frac{2\theta-\pi}{\cos(2\pi-\theta)}$

$\lim_{\theta\to \pi/2} \frac{2}{-\sin(2\pi-\theta)\cdot 1}$

$\frac{2}{\sin(2\pi-\frac{\pi}{2})}$

$\frac{2}{\sin(\frac{3\pi}{2})} \Rightarrow -2$

(18) $\lim_{\theta\to \pi/3} \frac{3\theta+\pi}{\sin(\theta+\frac{\pi}{3})}$

$\lim_{\theta\to \pi/3} \frac{3}{\cos(\theta+\frac{\pi}{3})}$

$\frac{3}{\cos(\frac{2\pi}{3})} \Rightarrow 3$.

(19) $\lim_{\theta\to \pi/2} \frac{1-\sin\theta}{1+\cos 2\theta}$

$\lim_{\theta\to \pi/2} \frac{-\cos\theta}{4\sin 2\theta (2)}$

$\lim_{\theta\to \pi/2} \frac{-\sin\theta}{4\cos 2\theta} \Rightarrow \frac{+1(+1)}{(-4)(-1)} = \frac{1}{4}$

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(20) $\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x}$

$\lim_{x\to 1} \frac{1}{1/x - \cos \pi x \cdot \pi}$

$\frac{1}{1/1 - \cos \pi(1)\cdot \pi} \Rightarrow \frac{1}{1 - \pi \cos \pi}$

$\frac{1}{1-\pi(-1)} = \boxed{\frac{1}{1+\pi}}$

(21) $\lim_{x\to 0} \frac{x^2}{\ln(\sec x)}$

$\lim_{x\to 0} \frac{2x}{\frac{1}{\sec x} \times \sec x \tan x}$

$\lim_{x\to 0} \frac{2x}{\tan x}$

$\lim_{x\to 0} \frac{2}{\sec^2 x} \Rightarrow$

$\frac{2}{\sec^2(0)} \Rightarrow (2)$.

(22) $\lim_{x\to \pi/2} \frac{\ln(\csc x)}{(x-\pi/2)^2}$

$\lim_{x\to \pi/2} \frac{\frac{1}{\csc x} \times \csc x \cot x}{2(x-\pi/2)}$

$\lim_{x\to \pi/2} \frac{-\cot x}{2(x-\pi/2)}$

$\lim_{x\to \pi/2} \frac{\csc^2 x}{2}$

$\frac{\csc^2(\frac{\pi}{2})}{2} = \boxed{\frac{1}{2}}$

(23) $\lim_{t\to 0} \frac{t(1-\cos t)}{t - \sin t}$

$\lim_{t\to 0} \frac{t(\sin t) + (1-\cos t)}{(1-\cos t)}$

$\lim_{t\to 0} \frac{t\cos t + \sin t + \sin t}{\sin t}$

$\lim_{t\to 0} \frac{-t\sin t + \cos t + \cos t + \cos t}{\cos t}$

Applying limit

$\frac{0+1+1+1}{1} = \boxed{3}$

(24) $\lim_{t\to 0} \frac{t\sin t}{1-\cos t}$

$\lim_{t\to 0} \frac{t\cos t + \sin t}{\sin t}$

$\lim_{t\to 0} \frac{-t\sin t + \cos t + \cos t}{\cos t}$

$\frac{0+1+1}{1} = \boxed{2}$

(25) $\lim_{x\to \pi/2} (x - \frac{\pi}{2}) \sec x$

$\lim_{x\to \pi/2^-} \frac{(x-\pi/2)}{\cos x}$

$\lim_{x\to \pi/2^-} \frac{1}{-\sin x}$

$= -\frac{1}{1} = \boxed{-1}$

(26) $\lim_{x\to (\pi/2)^-} (\frac{\pi}{2} - x) \tan x$

$\lim_{x\to (\pi/2)^-} \frac{(\pi/2 - x)}{\cot x}$

$\lim_{x\to \pi/2} \frac{-1}{+\csc^2 x}$

$\lim_{x\to \pi/2} \sin^2 x$

$\sin^2(\frac{\pi}{2})$

$\boxed{1}$

(27) $\lim_{\theta\to 0} \frac{3^{\sin\theta} - 1}{\theta}$

$\lim_{\theta\to 0} \frac{3^{\sin\theta}(\ln 3)(\cos\theta)}{1}$

$3^{\sin(0)}(\ln 3)(\cos 0)$

$1(\ln 3)(1)$

$\boxed{\ln 3}$

(28) $\lim_{\theta\to 0} \frac{(\frac{1}{2})^\theta - 1}{\theta}$

$\lim_{\theta\to 0} (\frac{1}{2})^\theta (\ln(\frac{1}{2}))(1)$

$(1)\ln(\frac{1}{2})(1)$

$\boxed{\ln(\frac{1}{2}) = -\ln 2}$

(29) $\lim_{x\to 0} \frac{x2^x}{2^x - 1}$

$\lim_{x\to 0} \frac{(1)(2^x) + x(2^x)(\ln 2)}{(2^x)(\ln 2)}$

$\frac{(1)(2^0) + 0}{(2^0)(\ln 2)} = \boxed{\frac{1}{\ln 2}}$

(30) $\lim_{x\to 0} \frac{3^x - 1}{2^x - 1}$

$\lim_{x\to 0} \frac{(3^x)(\ln 3)}{(2^x)(\ln 2)}$

$\frac{(3^0)(\ln 3)}{(2^0)(\ln 2)}$

$\boxed{\frac{\ln 3}{\ln 2}}$

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(31) $\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2 x}$

$\lim_{x\to\infty} \frac{\ln(x+1)}{\left(\frac{\ln x}{\ln 2}\right)}$

$(\ln 2)\lim_{x\to\infty} \frac{\left(\frac{1}{x+1}\right)}{\left(\frac{1}{x}\right)}$

$(\ln 2)\lim_{x\to\infty} \frac{x}{x+1}$

$\ln 2\left(\frac{1}{1}\right)$

$\boxed{\ln 2}$.

(32) $\lim_{x\to\infty} \frac{\log_2 x}{\log_3 (x+3)}$

$\lim_{x\to\infty} \frac{\left(\frac{\ln x}{\ln 2}\right)}{\left(\frac{\ln(x+3)}{\ln 3}\right)}$

$\lim_{x\to\infty} \frac{(\ln x/\ln 2)}{\ln(x+3)/\ln 3}$

$\frac{\ln 3}{\ln 2}\lim_{x\to\infty} \frac{\ln x}{\ln(x+3)}$

$\frac{\ln 3}{\ln 2}\lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x+3}\right)}$

$\frac{\ln 3}{\ln 2}\lim_{x\to\infty} \frac{x+3}{x}$

$\boxed{\frac{\ln 3}{\ln 2}}$

(33) $\lim_{x\to 0} \frac{\ln(x^2+2x)}{\ln x}$

$\lim_{x\to 0} \frac{\left(\frac{2x+2}{x^2+2x}\right)}{(1/x)}$

$\lim_{x\to 0} \frac{2x^2+2x}{x^2+2x}$

$\lim_{x\to 0} \frac{4x+2}{2x+2}$

$\left[\frac{2}{2}\right] = \boxed{1}$.

(34) $\lim_{x\to 0} \frac{\ln(e^x-1)}{\ln x}$

$\lim_{x\to 0} \frac{\left(\frac{e^x}{e^x-1}\right)}{(1/x)}$

$\lim_{x\to 0} \frac{xe^x}{e^x-1}$

$\lim_{x\to 0} \frac{xe^x+e^x}{e^x}$

$\lim_{x\to 0} \frac{(x+1)e^x}{e^x}$

$0+1 = \boxed{1}$

(35) $\lim_{y\to 0} \frac{\sqrt{5y+25}-5}{y}$

$\lim_{y\to 0} \frac{(5y+25)^{1/2}-5}{y}$

$\lim_{y\to 0} \frac{\frac{1}{2}(5y+25)^{-1/2}(5)}{1}$

$\lim_{y\to 0} \frac{5}{2\sqrt{5y+25}}$

$\frac{5}{2\sqrt{25}} = \frac{5}{2(5)} = \boxed{\frac{1}{2}}$

(36) $\lim_{y\to 0} \frac{\sqrt{ay+a^2}-a}{y}$, $a>0$

$\lim_{y\to 0} \frac{(ay+a^2)^{1/2}-a}{y}$

$\lim_{y\to 0} \frac{\frac{1}{2}(ay+a^2)^{-1/2}(a)}{1}$

$\lim_{y\to 0} \frac{a}{2\sqrt{ay+a^2}}$

$\frac{a}{2\sqrt{a^2}} = \frac{a}{2a} = \boxed{\frac{1}{2}}$

(37) $\lim_{x\to\infty} [\ln 2x - \ln(x+1)]$

$\lim_{x\to\infty} \ln\left(\frac{2x}{x+1}\right)$

$\ln\left(\lim_{x\to\infty} \frac{2x}{x+1}\right)$

$\ln\left(\lim_{x\to\infty} \frac{2}{1}\right)$

$\boxed{\ln 2}$.

(38) $\lim_{x\to 0^+} (\ln x - \ln\sin x)$

$\lim_{x\to 0^+} \left(\ln\left(\frac{x}{\sin x}\right)\right)$

$\ln\left(\lim_{x\to 0} \frac{x}{\sin x}\right)$

$\ln\left(\lim_{x\to 0} \frac{1}{\cos x}\right) = \ln 1 = \boxed{0}$.

(39) $\lim_{x\to 0} \frac{(\ln x)^2}{\ln(\sin x)}$

$\lim_{x\to 0} \frac{2(\ln x)\left(\frac{1}{x}\right)}{\frac{\cos x}{\sin x}} \Rightarrow \lim_{x\to 0} \frac{2\ln x \sin x}{x\cos x}$

$\lim_{x\to 0} \left[\frac{2(\ln x)}{\cos x} \cdot \frac{\sin x}{x}\right] = -\infty \cdot 1 = \boxed{-\infty}$.

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(40) $\lim_{x\to 0}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)$

$\lim_{x\to 0}\frac{(3x+1)(\sin x)-x}{x\sin x}$

$\lim_{x\to 0}\frac{(3x+1)(\cos x)+(\sin x)(3)-1}{x(\cos x)+\sin x}$

$\lim_{x\to 0}\frac{(3x+1)(\cos x)+3\sin x-1}{x\cos x+\sin x}$

$\lim_{x\to 0}\frac{(3x+1)(-\sin x)+(\cos x)(3)+3\cos x}{x(-\sin x)+\cos x+\cos x}$

$\lim_{x\to 0}\frac{-(3x+1)(\sin x)+3\cos x+3\cos x}{-x\sin x+\cos x+\cos x}$

Applying limit

$\frac{0+3(1)+3(1)}{0+1+1}=\frac{6}{2}=\boxed{3}$.

(41) $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$

$\lim_{x\to 1}\left(\frac{\ln x-(x-1)}{\ln x(x-1)}\right)$

$\lim_{x\to 1}\frac{\left(\frac{1}{x}-1\right)}{\ln x+(x-1)\left(\frac{1}{x}\right)}=\lim_{x\to 1}\left(\frac{1-x}{x\ln x+x-1}\right)$

$\lim_{x\to 1}\left(\frac{1-x}{x\ln x+x-1}\right)$

$\lim_{x\to 1}\left(\frac{-1}{\ln(x+1)+1}\right)$

$\frac{-1}{(0+1)+1}=\boxed{-\frac{1}{2}}$.

(42) $\lim_{x\to\theta}(\operatorname{cosec}x-\cot x+\cos x)$

$\lim_{x\to\theta}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}+\cos x\right)$

$\lim_{x\to 0}\left(\frac{1-\cos x+\cos x\sin x}{\sin x}\right)$

$\lim_{x\to 0}\left(\frac{\sin x+\cos x(\cos x)+\sin x(-\sin x)}{\cos x}\right)$

$\lim_{x\to 0}\left(\frac{\sin x+\cos^2 x-\sin^2 x}{\cos x}\right)$

$\frac{0+1+0}{1}=\boxed{1}$.

(43) $\lim_{\theta\to 0}\frac{\cos\theta-1}{e^\theta-\theta-1}$

$\lim_{\theta\to 0}\frac{-\sin\theta}{e^\theta(\cos\theta)(1)-1}$

$\lim_{\theta\to 0}\frac{-\cos\theta}{e^\theta}\Rightarrow\frac{-1}{1}=\boxed{-1}$.

(44) $\lim_{h\to 0}\frac{e^h-(1+h)}{h^2}$

$\lim_{h\to 0}\frac{e^h(1)(1)-(1)}{2h}$

$\lim_{h\to 0}\frac{e^h}{2}=\boxed{\frac{1}{2}}$

(45) $\lim_{t\to\infty}\frac{e^t+t^2}{e^t-t}$

$\lim_{t\to\infty}\frac{e^t+2t}{e^t-1}$

$\lim_{t\to\infty}\frac{e^t+2}{e^t}$

$\lim_{t\to\infty}\frac{e^t}{e^t}\Rightarrow\boxed{1}$

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(46) $\lim_{x\to\infty} x^2 e^{-x}$

$\lim_{x\to\infty} \frac{x^2}{e^x}$

$\lim_{x\to\infty} \frac{2x}{e^x}$

$\lim_{x\to\infty} \frac{2}{e^x}$

$\frac{2}{\infty} = 0$ $\left[\because \frac{1}{\infty} = 0\right]$

(47) $\lim_{x\to 0} \frac{x - \sin x}{x \tan x}$

$\lim_{x\to 0} \frac{1 - \cos x}{x(\sec^2 x) + \tan x}$

$\lim_{x\to 0} \frac{\sin x}{2x\sec^2 x \tan x + 2\sec^2 x}$

$\frac{0}{2} = 0$

(48) $\lim_{x\to 0} \frac{(e^x - 1)^2}{x \sin x}$

$\lim_{x\to 0} \frac{2(e^x - 1)e^x}{x\cos x + \sin x}$

$\lim_{x\to 0} \frac{2e^{2x} + 2e^x}{x\cos x + \sin x}$

$\lim_{x\to 0} \frac{4e^{2x} - 2e^x}{-x\sin x + 2\cos x}$

$\frac{2}{2} = 1$

(49) $\lim_{\theta\to 0} \frac{\theta - \sin\theta\cos\theta}{\tan\theta - \theta}$

$\lim_{\theta\to 0} \frac{1 + \sin\theta(\sin\theta) + \cos\theta(-\cos\theta)}{\sec^2\theta - 1}$

$\lim_{\theta\to 0} \frac{-\sin^2\theta - \cos^2\theta}{\sec^2\theta - 1}$

$\lim_{\theta\to 0} \frac{2\sin^2\theta}{\tan^2\theta} \Rightarrow$

$\lim_{\theta\to 0} 2\cos^2\theta \Rightarrow 2$

(50) $\lim_{x\to 0} \frac{\sin 3x - 3x + x^2}{\sin x \sin 2x}$

$\lim_{x\to 0} \frac{3\cos 3x - 3 + 2x}{\cos x \cdot 2\sin x\cos x + \sin 2x \cos x}$

$2\sin x \cos 2x$

$\lim_{x\to 0} \frac{-3\sin 3x(3) + 2}{(2\sin x)(-\sin 2x \cdot 2) + 2\cos 2x(\cos x) + \sin 2x(-\sin x) + (\cos x)(\cos 2x)}$

$\lim_{x\to 0}$ Applying limit.

$\frac{0+2}{0+1+0+2} = \frac{1}{2}$