Lecture

"Expected Values"

$$E(X) = \Sigma x P(x)$$

$$\therefore \quad E(X^2) = \Sigma x^2 P(x)$$

or $$E(2X+1) = \Sigma (2x+1) P(x)$$

where $$E(2X+1) = 2E(X)+1$$

or $$E(3X^2-1) = \Sigma (3x^2-1) P(x)$$

where $$E(3X^2-1) = 3E(X^2)-1$$

Q-1:- Find $E(X^2)$ for the following data set:

X	$P(x)$	X^2	$X^2 P(X)$
0	0.41	0	0
1	0.37	1	0.37
2	0.16	4	0.64
3	0.05	9	0.45
4	0.01	16	0.16
			1.62

$$\therefore E(X^2) = \Sigma X^2 P(X) = 1.62$$

Q-2: Find $E(2X+1)$ of the following:

X	$P(X)$	$2X+1$	$(2X+1)P(X)$
0	$27/64$	1	$27/64$
1	$27/64$	3	$81/64$
2	$9/64$	5	$45/64$
3	$1/64$	7	$7/64$
			$160/64$

Method I:

$$E(2X+1) = \Sigma (2X+1) P(X)$$

$$= \frac{160}{64} = 2.5$$

Method II:

$$E(2X+1) = 2E(X) + 1$$

$$= 2 \times 0.75 + 1$$

$$= 1.5 + 1$$

$$= 2.5$$

"Variance and Standard deviation of Discrete prob. distributions"

$$\text{Variance}(X) = E(X^2) - [E(X)]^2$$

$$= \sum x^2 P(x) - \left[\sum x P(x)\right]^2$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

5.11

Q-3: The U.S. Census Bureau collects demographics concerning the no. of people in families per household. Assume the distribution of the number of people per household is shown in the following table:

x	$P(x)$
2	0.27
3	0.25
4	0.28
5	0.13
6	0.04
7	0.03

a) Calculate the expected no. of people in families per household in the United States.

b) Compute the variance and standard deviation of the no. of people in families per household.

Solution

x	$P(x)$	$x\,P(x)$	x^2	$x^2 P(x)$
2	0.27	0.54	4	1.08
3	0.25	0.75	9	2.25
4	0.28	1.12	16	4.48
5	0.13	0.65	25	3.25
6	0.04	0.24	36	1.44
7	0.03	0.21	49	1.47
	1	3.51		13.97

i) $\because E(X) = \Sigma x\, p(x)$

$= 3.51$

$\therefore$ expected no. of people in families per household $= 3.51$

ii) Variance $(X) = ?$

$$\text{Variance} = E(x^2) - [E(x)]^2$$
$$= \Sigma x^2 P(x) - [\Sigma x p(x)]^2$$
$$= 13.97 - 3.51^2$$
$$= 1.65$$

& $S.D(X) = \sqrt{Var(X)} = \sqrt{1.65} = 1.28$

"Linear relation ships"

Let $$y = a + bX$$

then $$\text{Mean}(Y) = a + b\ \text{mean}(X)$$

i.e; $$E(Y) = a + b\ E(X)$$

and

$$\text{Variance}(Y) = \text{Variance}(a + bX)$$

$$= \text{Variance}(a) + \text{Variance}(bX)$$

$$= 0 + b^2\ \text{Variance}(X)$$

$$\therefore\ \text{Var}(Y) = b^2\ \text{Var}(X)$$

and Thus

$$S.D(Y) = b \times S.D(X).$$