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# (Exercise # 3.7) Q(1-24)

## Differentiating Implicity.

① $x^2y + xy^2 = 6$

Sol,

taking derivative on b.s

$$\frac{d}{dx}(x^2y + xy^2) = \frac{d}{dx}(6)$$

$$\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] + \left[x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x)\right] = 0$$

$$x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 = 0$$

$$(x^2 + 2xy)\frac{dy}{dx} = -y^2 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}}$$

② $x^3 + y^3 = 18xy$

Sol.

taking derivative on b.s

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(18xy)$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 18\left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right]$$

$$3x^2 + 3y^2\frac{dy}{dx} = 18x\frac{dy}{dx} + 18y$$

$$18x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 3x^2 - 18y$$

$$(18x - 3y^2)\frac{dy}{dx} = 3x^2 - 18y$$

$$\frac{dy}{dx} = \frac{\cancel{3}(x^2 - 6y)}{\cancel{3}(6x - y^2)}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2 - 6y}{6x - y^2}}$$

③ $2xy + y^2 = x + y$

Sol,

taking derivative on b.s

$$\frac{d}{dx}(2xy + y^2) = \frac{d}{dx}(x + y)$$

$$\frac{d}{dx}(2xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(x) + \frac{d}{dx}(y)$$

$$2\left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$(2x + 2y - 1)\frac{dy}{dx} = 1 - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}}$$

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④ $x^3 - xy + y^3 = 1$

Sol.

taking derivative on b.s

$$\frac{d}{dx}(x^3 - xy + y^3) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^3) - \frac{d}{dx}(xy) + \frac{d}{dx}(y^3) = 0$$

$$3x^2 - \left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$$

$$(3y^2 - x)\frac{dy}{dx} = +y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{+y - 3x^2}{3y^2 - x}}$$

⑥ $(3xy + 7)^2 = 6y$

$$\frac{d}{dx}(3xy+7)^2 = \frac{d}{dx}6y$$

$$2\left[3\left(x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] + \frac{d}{dx}(7) = 6\frac{dy}{dx}$$

$(3xy+7)$

$$2\left[3x\frac{dy}{dx} + 3y\right] + 0 = 6\frac{dy}{dx}$$

$$2(3xy+7)(3x)\frac{dy}{dx} - 6\frac{dy}{dx} = -6y(3xy+7)$$

$$\frac{dy}{dx}\left[6x(3xy+7) - 6\right] = -6y(3xy+7)$$

$$\boxed{\frac{dy}{dx} = \frac{3xy^2 + 7y}{1 - 3x^2y - 7x}}$$

⑤ $x^2(x-y)^2 = x^2 - y^2$

Sol.

taking derivative on b.s.

$$\frac{d}{dx}\left(x^2(x-y)^2\right) = \frac{d}{dx}(x^2 - y^2)$$

$$x^2\frac{d}{dx}(x-y)^2 + (x-y)^2\frac{d}{dx}x^2 = \frac{d}{dx}x^2 - \frac{d}{dx}y^2$$

$$x^2\left[2(x-y)\frac{d}{dx}(x-y)\right] + 2x(x-y)^2 = 2x - 2y\frac{dy}{dx}$$

$$x^2\left[2(x-y)\left(1 - \frac{dy}{dx}\right)\right] + 2x(x-y)^2 = 2x - 2y\frac{dy}{dx}$$

$$x^2\left[2\left(x - x\frac{dy}{dx} - y + y\frac{dy}{dx}\right)\right] + 2x(x-y)^2 = 2x - 2y\frac{dy}{dx}$$

$$2x^3 - 2x^3\frac{dy}{dx} - 2x^2y + 2x^2y\frac{dy}{dx} = 2x - 2y\frac{dy}{dx} - 2x(x-y)^2$$

$$\frac{dy}{dx}\left[-2x^2(x-y) + 2y\right] = 2x\left[1 - x(x-y) - (x-y)^2\right]$$

$$\frac{dy}{dx} = \frac{2x\left[1 - x(x-y) - (x-y)^2\right]}{-2x^2(x-y) + 2y}$$

$$\frac{dy}{dx} = \frac{\cancel{2}x\left(1 - x(x-y) - (x-y)^2\right)}{\cancel{2}\left(y - x^2(x-y)\right)}$$

$$\frac{dy}{dx} = \frac{x\left(1 - \cancel{x^2} + xy - \cancel{x^2} - y^2 + 2xy\right)}{y - x^3 + x^2y}$$

$$\boxed{\frac{dy}{dx} = \frac{x - 2x^3 + 3x^2y - xy^2}{x^2y - x^3 + y}}$$

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(7) $y^2 = \frac{x-1}{x+1}$

Sol,

$$\frac{d}{dx} y^2 = \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$2y\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$2y\frac{dy}{dx} = \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}$$

$$2y\frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{2}}{\cancel{2}y(x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y(x+1)^2}}$$

(8) $x^3 = \frac{2x-y}{x+3y}$

Sol,

$$\frac{d}{dx}x^3 = \frac{d}{dx}\left(\frac{2x-y}{x+3y}\right)$$

$$3x^2 = \frac{(x+3y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x+3y)}{(x+3y)^2}$$

$$3x^2 = \frac{(x+3y)\left(2-\frac{dy}{dx}\right)-(2x-y)\left(1+3\frac{dy}{dx}\right)}{(x+3y)^2}$$

$$3x^2 = \frac{2x - x\frac{dy}{dx} + 6y - 3y\frac{dy}{dx} - \left(2x+6x\frac{dy}{dx} - y - 3y\frac{dy}{dx}\right)}{(x+3y)^2}$$

$$3x^2 = \frac{+\cancel{2x} - x\frac{dy}{dx} + 6y - \cancel{3y\frac{dy}{dx}} - \cancel{2x} - 6x\frac{dy}{dx} + y + \cancel{3y\frac{dy}{dx}}}{(x+3y)^2}$$

$$3x^2(x+3y)^2 = -6x\frac{dy}{dx} + 7y - x\frac{dy}{dx}$$

$$3x^2(x+3y)^2 = -6x\frac{dy}{dx} + 7y - x\frac{dy}{dx}$$

$$3x^2(x+3y)^2 - 7y = (-6x - x)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = -\frac{(3x^2(x+3y)^2 - 7y)}{7x}}$$

(9) $x = \tan y$

$$\frac{d}{dx}(x) = \frac{d}{dx}\tan y$$

$$1 = \sec^2 y\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sec^2 y}}$$

$$\boxed{\frac{dy}{dx} = \cos^2 y}$$

(10) $xy = \cot(xy)$

Sol,

taking derivative on b.s.

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$$\frac{d}{dx}(xy) = \frac{d}{dx}\cot(xy)$$

$$x\frac{d}{dx}y + y\frac{d}{dx}x = -\operatorname{cosec}^2(xy)\frac{d}{dx}(xy)$$

$$x\frac{dy}{dx} + y = -\operatorname{cosec}^2(xy)\left[x\frac{dy}{dx} + y\right]$$

$$x\frac{dy}{dx} + y = -x\operatorname{cosec}^2(xy)\frac{dy}{dx} - y\operatorname{cosec}^2(xy)$$

$$x\frac{dy}{dx} + x\operatorname{cosec}^2(xy)\frac{dy}{dx} = -y\operatorname{cosec}^2(xy) - y$$

$$(x + x\operatorname{cosec}^2(xy))\frac{dy}{dx} = -y(\operatorname{cosec}^2 xy + 1)$$

$$\frac{dy}{dx} = \frac{-y(\cancel{\csc^2 xy + 1})}{x(\cancel{1+\csc^2 xy})}$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x}}$$

(11) $x + \tan(xy) = 0$

Sol.

$$\frac{d}{dx}(x + \tan xy) = 0$$

$$\frac{d}{dx}(x) + \frac{d}{dx}(\tan xy) = 0$$

$$1 + \sec^2(xy)\frac{d}{dx}(xy) = 0$$

$$1 + \sec^2(xy)\left[x\frac{dy}{dx} + y\right] = 0$$

$$1 + x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy)$$

$$x\sec^2(xy)\frac{dy}{dx} = -1 - y\sec^2(xy)$$

$$\frac{dy}{dx} = \frac{-1 - y\sec^2(xy)}{x\sec^2(xy)}$$

$$\frac{dy}{dx} = \frac{-1}{x\sec^2(xy)} - \frac{y\cancel{\sec^2(xy)}}{x\cancel{\sec^2(xy)}}$$

$$\frac{dy}{dx} = -\frac{\cos^2(xy)}{x} - \frac{y}{x}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{x}\left(\cos^2(xy) + y\right)}$$

(12) $x^4 + \sin y = x^3y^2$

$$\frac{d}{dx}x^4 + \frac{d}{dx}\sin y = \frac{d}{dx}(x^3y^2)$$

$$4x^3 + (+\cos y)\frac{dy}{dx} = x^3\frac{d}{dx}y^2 + y^2\frac{d}{dx}x^3 \Rightarrow 4x^3 + \cos y\frac{dy}{dx} = 2x^3y\frac{dy}{dx} + 3x^2y^2$$

$$4x^3 - 3x^2y^2 = 2x^3y\frac{dy}{dx} - \cos y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{4x^3 - 3x^2y^2}{2x^3y - \cos y}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2(4x - 3y^2)}{2x^3y - \cos y}}$$

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(13) $y \sin\left(\frac{1}{y}\right) = 1 - xy$

Sol,

$$\frac{d}{dx} y \sin\left(\frac{1}{y}\right) = \frac{d}{dx}(1) - \frac{d}{dx}(xy)$$

$$y \frac{d}{dx} \sin\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) \frac{d}{dx} y = 0 - \left[x \frac{d}{dx}(y) + y \frac{d}{dx}(x)\right]$$

$(-y^{-2} dy/dx)$

$$y \left(\cos\left(\frac{1}{y}\right)\right) + \sin\left(\frac{1}{y}\right) \frac{dy}{dx} = -x \frac{dy}{dx} - y$$

$$\frac{-\cancel{y} \cos\left(\frac{1}{y}\right)}{y^{\cancel{2}}} \frac{dy}{dx} + \sin\left(\frac{1}{y}\right) \frac{dy}{dx} = -x \frac{dy}{dx} - y$$

$$\frac{-\cos\left(\frac{1}{y}\right)}{y} \frac{dy}{dx} + \sin\left(\frac{1}{y}\right) \frac{dy}{dx} + x \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} \left[-\frac{1}{y} \cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) + x\right] = -y$$

$$\frac{dy}{dx} = \frac{-y}{-\frac{1}{y} \cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) + x}$$

$$\boxed{\frac{dy}{dx} = \frac{-y^2}{-\cos\left(\frac{1}{y}\right) + y \sin\left(\frac{1}{y}\right) + xy}}$$

(14) $x \cos(2x + 3y) = y \sin x$

Sol.

$$\frac{d}{dx} x \cos(2x + 3y) = \frac{d}{dx} y \sin x$$

$$x \frac{d}{dx} \cos(2x+3y) + \cos(2x+3y) \frac{d}{dx}(x) =$$

$$y \frac{d}{dx} \sin x + \sin x \frac{d}{dx} y.$$

$$x(-\sin(2x+3y) \frac{d}{dx}(2x+3y) + \cos(2x+3y)$$

$$= y \cos x + \sin x \frac{dy}{dx}.$$

$$-x \sin(2x+3y)\left(2 + 3 \frac{dy}{dx}\right) + \cos(2x+3y)$$

$$= y \cos x + \sin x \frac{dy}{dx}$$

$$-2x \sin(2x + 3y) - 3x \sin(2x+3y) \frac{dy}{dx}$$

$$+ \cos(2x+3) = y \cos x + \sin x \frac{dy}{dx}$$

$$\frac{dy}{dx}(-3x \sin(2x+3y) - \sin x) =$$

$$y \cos x - \cos(2x+3y) + 2x \sin(2x+3y)$$

$$\frac{dy}{dx} = \frac{y \cos x - \cos(2x+3y) + 2x \sin(2x+3y)}{-3x \sin(2x+3y) - \sin x}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos(2x+3y) - 2x \sin(2x+3y) - y \cos x}{3x \sin(2x+3y) + \sin x}}$$

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Finding Double Derivatives:

(15) $\theta^{1/2}+r^{1/2}=1$

Sol.

$$\frac{d}{d\theta}(\theta^{1/2}+r^{1/2})=\frac{d}{d\theta}(1)$$

$$\frac{d}{d\theta}\theta^{1/2}+\frac{d}{d\theta}r^{1/2}=0$$

$$\frac{1}{2\sqrt{\theta}}+\frac{1}{2}r^{-1/2}\frac{dr}{d\theta}=0$$

$$\frac{1}{2\sqrt{r}}\frac{dr}{d\theta}=-\frac{1}{2\sqrt{\theta}}$$

$$\frac{dr}{d\theta}=-\frac{1}{\cancel{2}\sqrt{\theta}}\times\cancel{2}\sqrt{r}$$

$$\boxed{\frac{dr}{d\theta}=-\frac{\sqrt{r}}{\sqrt{\theta}}}$$

(16) $r-2\sqrt{\theta}=\frac{3}{2}\theta^{2/3}+\frac{4}{3}\theta^{3/4}$

Sol.

$$\frac{d}{d\theta}r-2\frac{d}{d\theta}\theta^{1/2}=\frac{3}{2}\frac{d}{d\theta}\theta^{2/3}+\frac{4}{3}\frac{d}{d\theta}\theta^{3/4}$$

$$\frac{dr}{d\theta}-\cancel{2}\left(\frac{1}{\cancel{2}\sqrt{\theta}}\right)=\frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{2}}{\cancel{3}}\theta^{-1/3}\right)+\frac{\cancel{4}}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{4}}\theta^{-1/4}\right)$$

$$\frac{dr}{d\theta}-\frac{1}{\sqrt{\theta}}=\theta^{-1/3}+\theta^{-1/4}$$

$$\boxed{\frac{dr}{d\theta}=\theta^{-1/3}+\theta^{-1/4}+\theta^{-1/2}}$$

(17) $\sin(r\theta)=\frac{1}{2}$

$$\frac{d}{d\theta}(\sin r\theta)=\frac{d}{d\theta}\frac{1}{2}$$

$$\cos(r\theta)\frac{d}{d\theta}(r\theta)=0$$

$$\cos(r\theta)\left[r\frac{d\theta}{d\theta}\theta+\theta\cdot\frac{dr}{d\theta}\right]=0$$

$$\cos(r\theta)\left[r+\theta\frac{dr}{d\theta}\right]=0$$

$$r\cos(r\theta)+\theta\cos(r\theta)\frac{dr}{d\theta}=0$$

$$\frac{dr}{d\theta}=-\frac{r\cancel{\cos(r\theta)}}{\theta\cancel{\cos(r\theta)}}\Rightarrow\boxed{\frac{dr}{d\theta}=-\frac{r}{\theta}}$$

(18) $\cos r+\cot\theta=r\theta$

~~LHS~~: L.H.S,

$$\frac{d}{d\theta}\cos r+\frac{d}{d\theta}\cot\theta=$$

$$-\sin r\frac{dr}{d\theta}+(-\csc^2\theta)$$

$$-\sin r\frac{dr}{d\theta}-\csc^2\theta$$

R.H.S

$$r+\theta\frac{dr}{d\theta}$$

...

Finding Double Derivatives:

$$-\sin r\frac{dr}{d\theta} - \csc^2\theta = r + \theta\frac{dr}{d\theta}$$

$$(-\sin r - \theta)\frac{dr}{d\theta} = r + \csc^2\theta$$

$$\frac{dr}{d\theta} = \frac{r+\csc^2\theta}{(-\sin r - \theta)}$$

$$\boxed{\frac{dr}{d\theta} = -\frac{r+\csc^2\theta}{\sin r + \theta}}$$

(19). $x^2 + y^2 = 1$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}x}{\cancel{2}y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = -\left[\frac{y - x\frac{dy}{dx}}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = -\left[\frac{y - x(-\frac{x}{y})}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3} \quad \therefore x^2 = 1 - y^2$$

$$\frac{d^2y}{dx^2} = \frac{-\cancel{y^2} - 1 + \cancel{y^2}}{y^3}$$

$$\boxed{y'' = -1/y^3}$$

(20) $x^{2/3} + y^{2/3} = 1$

$$\frac{d}{dx}x^{2/3} + \frac{d}{dx}y^{2/3} = \frac{d}{dx}(1)$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}}{\cancel{3}}x^{-1/3} \Big/ \cancel{2/3}\, y^{-1/3}$$

$$\frac{dy}{dx} = -(y/x)^{1/3}$$

$$y'' = -\frac{x^{1/3}\, d/dx\, y^{1/3} - y^{1/3}\, d/dx\, x^{1/3}}{(x^{1/3})^2}$$

$$y'' = \frac{1}{3}x^{-2/3}y^{-1/3} + \frac{1}{3}y^{1/3}x^{-4/3}$$

$$\boxed{y'' = \frac{y^{1/3}}{3x^{4/3}} + \frac{1}{3y^{1/3}x^{2/3}}}$$

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(21) $y^2 = x^2 + 2x$

$$\frac{d}{dx}y^2 = \frac{d}{dx}x^2 + \frac{d}{dx}2x$$

$$2y\frac{dy}{dx} = 2x + 2$$

$$\frac{dy}{dx} = \frac{\cancel{2}(x+1)}{\cancel{2}y}$$

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$y'' = \frac{y(1) + (x+1)\left(\frac{dy}{dx}\right)}{y^2}$$

$$y'' = \frac{y + x\frac{dy}{dx} + \frac{dy}{dx}}{y^2}$$

$$y'' = \frac{y + x\left(\frac{x+1}{y}\right) + \left(\frac{x+1}{y}\right)}{y^2}$$

$$y'' = \frac{y^2 + x^2 + x + x + 1}{y^3}$$

$$\boxed{y'' = \frac{y^2 + (x+1)^2}{y^3}}$$

(22) $y^2 - 2x = 1 - 2y.$

$$\frac{d}{dx}y^2 - 2\frac{d}{dx}x = \frac{d}{dx}(1) - 2\frac{d}{dx}y$$

$$2y\frac{dy}{dx} - 2 = 0 - 2\frac{dy}{dx}$$

$$(2y+2)\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{\cancel{2}}{\cancel{2}(y+1)}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y+1}}$$

$$y'' = \frac{(y+1)(0) - (1)\left(\frac{d}{dx}y + \frac{d}{dx}(1)\right)}{(y+1)^2}$$

$$y'' = \frac{0 - \frac{dy}{dx}}{(y+1)^2}$$

$$y'' = \frac{\frac{-1}{y+1}}{(y+1)^2}$$

$$\boxed{y'' = \frac{-1}{(y+1)^3}}$$

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(23) $2\sqrt{y} = x - y$

$$2\frac{d}{dx}y^{1/2} = \frac{d}{dx}x - \frac{d}{dx}y$$

$$\cancel{2}\left(\frac{1}{\cancel{2}}y^{-1/2}\frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$$

$$\left(\frac{1}{\sqrt{y}} + 1\right)\frac{dy}{dx} = 1$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y^{-1/2}+1}}$$

$$y'' = \frac{(y^{-1/2}+1)(0) - (1)\left(\frac{d}{dx}y^{-1/2} + 0\right)}{(y^{-1/2}+1)^2}$$

$$y'' = \frac{0 - \left(-\frac{1}{2}y^{-3/2}\frac{dy}{dx}\right)}{(y^{-1/2}+1)^2}$$

$$y'' = \frac{-\left(-\frac{1}{2}y^{-3/2}\left(\frac{1}{y^{-1/2}+1}\right)\right)}{(y^{-1/2}+1)^2}$$

$$y'' = \frac{y^{-3/2}}{2(y^{-1/2}+1)^3}$$

$$\boxed{y'' = \frac{1}{2y^{3/2}(y^{-1/2}+1)^3}}$$

(24) $xy + y^2 = 1$

$$x\frac{d}{dx}y + y\frac{d}{dx}x + \frac{d}{dx}y^2 = \frac{d}{dx}(1)$$

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -y$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x+2y}}$$

$$y'' = -\left[\frac{(x+2y)\frac{dy}{dx} - y\left(1 + 2\frac{dy}{dx}\right)}{(x+2y)^2}\right]$$

$$y'' = \left[\frac{-\cancel{(x+2y)}\left(-\frac{y}{\cancel{x+2y}}\right) + y + 2y\left(-\frac{y}{x+2y}\right)}{(x+2y)^2}\right]$$

$$y'' = \frac{y + y \mp \frac{2y^2}{x+2y}}{(x+2y)^2}$$

$$y'' = \frac{2xy + 4y^2 + x \cdot 2y^2}{(x+2y)^3}$$

$$y'' = \frac{2xy + 4y^2 - 2y^2}{(x+2y)^3}$$

$$\boxed{y'' = \frac{2xy - 2y^2}{(x+2y)^3}}$$

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