Topic:

ABSOLUTE EXTREMA ON FINITE CLOSED INTERVAL...

Date ....

Exercise 4.1 (21 - 36)

(21) Find the absolute Maximum & Minimum values of each function on the given interval.

(21) $f(x) = \frac{2}{3}x - 5 \;, \quad -2 \le x \le 3$

Sol,

$f(x) = \frac{2}{3}x - 5$

taking derivative

$f'(x) = \frac{2}{3}\frac{d}{dx}(x) - 5\frac{d}{dx}(1)$

$f'(x) = \frac{2}{3} - 0$] - So no critical point

So, At $\boxed{x = -2}$

$f(-2) = \frac{2}{3}(-2) - 5$

$f(-2) = -\frac{4}{3} - 5$

$f(-2) = \frac{-4-15}{3}$

$\boxed{f(-2) = -19/3}$

$f(-2) = -6.33$] - Abs. min ~~max~~ at value $\boxed{x = -2}$

At $\boxed{x = 3}$

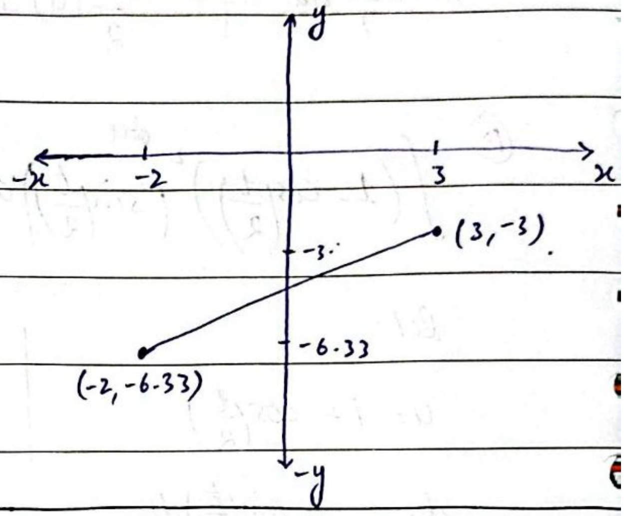
$f(3) = \frac{2}{\cancel{3}}(\cancel{3}) - 5$

$f(3) = 2 - 5$

$\boxed{f(3) = -3}$ - Abs max at value $\boxed{x = 3}$.

Graph: First Making sets.

(-2, -6.33) (3, -3)



(22) $f(x) = -x - 4, \quad -4 \le x \le 1$

Sol,

$f(x) = -x - 4$

taking derivative.

$f'(x) = -\frac{d}{dx}(x) - \frac{d}{dx}4$

$f'(x) = -1$ } No critical point

So, At $\boxed{x = -4}$

$f(-4) = -(-4) - 4$

$\boxed{f(-4) = 0}$ → Abs max at value $\boxed{x = -4}$.

At $\boxed{x = 1}$

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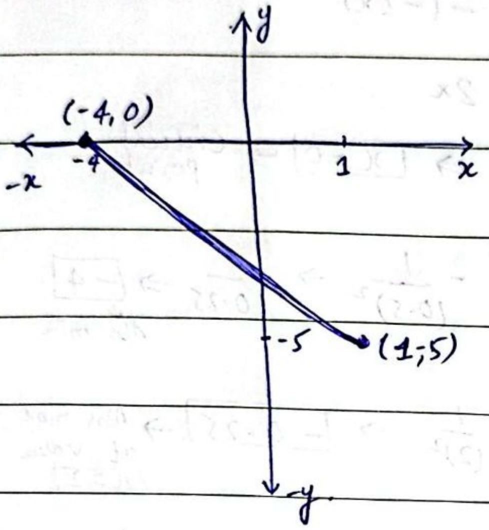
$f(1) = -\cdot(1) - 4$

$\boxed{f(1) = -5} \rightarrow$ Abs min at value $\boxed{x=1}$

Graph,

$(-4,0), (1,-5)$



$\boxed{f(-1) = 0} \rightarrow$ Abs min at $\boxed{x=-1}$

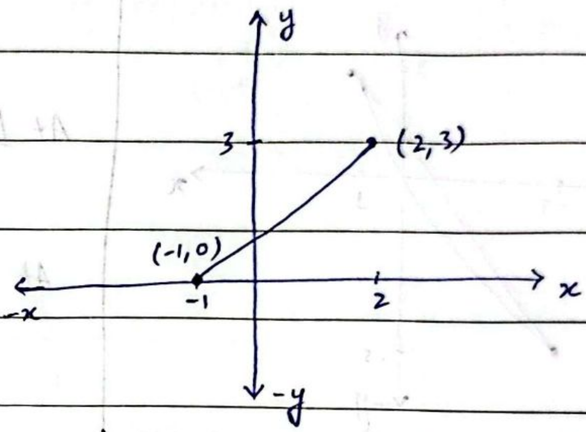
At $\boxed{x=2}$

$f(2) = (2)^2 - 1$

$f(2) = 4 - 1$

$\boxed{f(2) = 3} \rightarrow$ Abs max at value $\boxed{x=2}$

Graph: $(-1,0), (2,3)$



(23) $f(x) = x^2 - 1, \quad -1 \le x \le 2$

Sol,

$f(x) = x^2 - 1$

taking derivative

$f'(x) = \frac{d}{dx} x^2 - \frac{d}{dx}(1)$

$f'(x) = 2x$

$2x = 0$

$\boxed{x=0}$ is critical point

At $\boxed{x=-1}$.

$f(-1) = (-1)^2 - 1$

(24) $f(x) = 4 - x^2, \quad -3 \le x \le 1$

Sol,

$f(x) = 4 - x^2$

taking derivatives.

$f'(x) = \frac{d}{dx}(4) - \frac{d}{dx}(x)^2$

$f'(x) = -2x$

$-2x = 0$

$\boxed{x=0} \rightarrow$ critical point.

At $\boxed{x=-3}$

$f(-3) = 4 - (-3)^2$

$f(-3) = 4 - 9$

$\boxed{f(-3) = -5} \rightarrow$ Abs min at value $\boxed{x=-3}$

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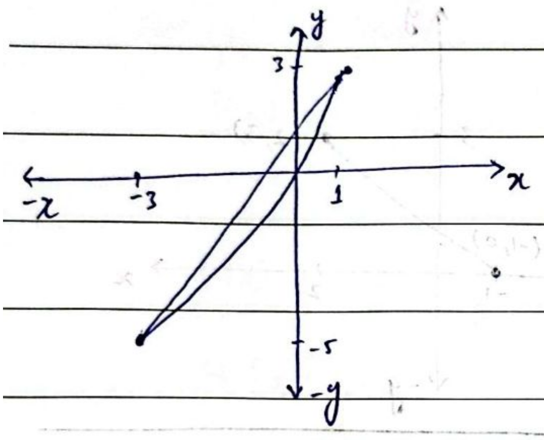
At $\boxed{x=1}$

$f(1) = 4-(1)^2$

$\boxed{f(1) = 3} \rightarrow$ Abs max at value $\boxed{x=1}$.

Graph:

$(-3,-5), (1,3)$



(25) $F(x) = -\frac{1}{x^2}$, $0.5 \le x \le 2$

Sol,

$F(x) = -x^{-2}$

taking derivative

$f'(x) = -(-2x)$

$F(x) = 2x$

$2x = 0 \Rightarrow \boxed{x=0} \rightarrow$ Critical point.

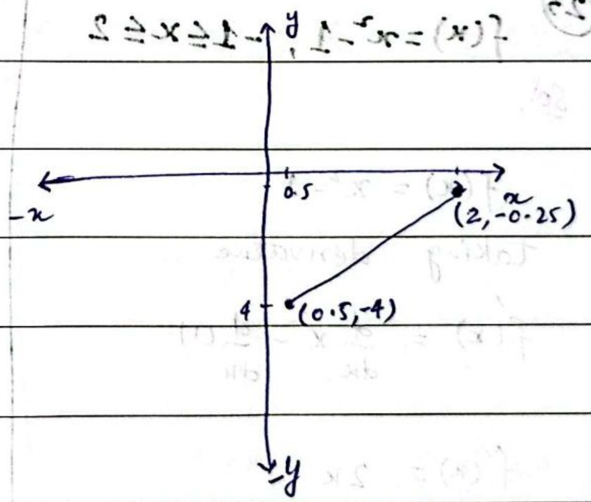
At $\boxed{x=0.5}$,

$f(0.5) = -\frac{1}{(0.5)^2} \Rightarrow -\frac{1}{0.25} \Rightarrow \boxed{-4}$ Abs min

At $x=2$

$f(2) = -\frac{1}{(2)^2} \Rightarrow \boxed{-0.25} \rightarrow$ Abs max at value $\boxed{x=2}$.

Graph: $(\frac{1}{2}, -4)$, $(2, -0.25)$



(26) $F(x) = -\frac{1}{x}$, $-2 \le x \le -1$

Sol:

$F(x) = -\frac{1}{x}$

$F(x) = -x^{-1}$

$F'(x) = x^{-2}$

$F'(x) = \frac{1}{x^2}$

However $x=0$ is not critical point since zero is not in Domain.

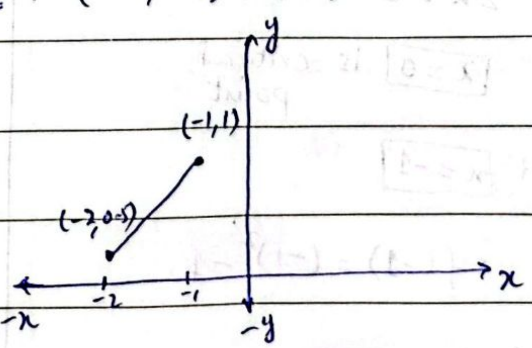
At $\boxed{x=-2}$

$f(-2) = +\frac{1}{2} = \boxed{0.5} \rightarrow$ Abs min

At $\boxed{x=-1}$.

$f(-1) = \boxed{1} \rightarrow$ Abs max.

Graph: $(-2, 0.5)$, $(-1, 1)$



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(27) $h(x) = \sqrt[3]{x}, \quad -1 \le x \le 8$

$h(x) = (x)^{1/3}$

$h'(x) = \frac{1}{3}x^{-2/3}$

$\frac{1}{3}x^{-2/3} = 0$

$\boxed{x=0} \rightarrow$ Critical point.

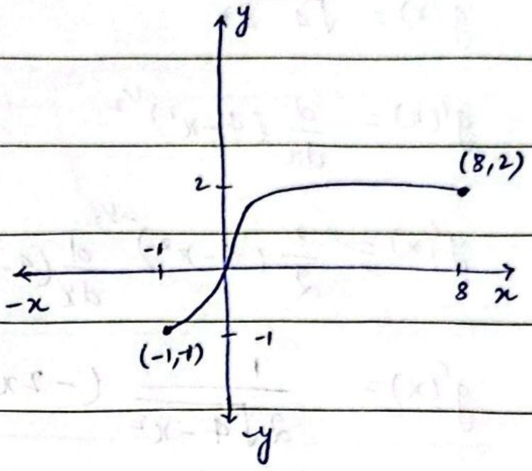
At $\boxed{x=-1}$

$f(-1) = \sqrt[3]{-1} = \boxed{-1} \rightarrow$ Abs. min.

At $\boxed{x=8}$

$f(8) = \sqrt[3]{8} = (2^3)^{1/3} = \boxed{2} \rightarrow$ Abs max

Graph: $(-1,-1), (8,2)$



(28) $h(x) = -3x^{2/3}, \quad -1 \le x \le 1$

$h(x) = -3x^{2/3}$

$h'(x) = -3\frac{d}{dx}x^{2/3}$

$h'(x) = -\cancel{3}\left(\frac{2}{\cancel{3}}\right)x^{2/3-1}$

$h'(x) = -2x^{-1/3}$

$\frac{-2}{x^{1/3}} = 0$

$\boxed{x=0} \rightarrow$ Critical point

At $\boxed{x=-1}$

$h(-1) = -3(-1)^{2/3} = \boxed{-3}$

At $\boxed{x=1}$

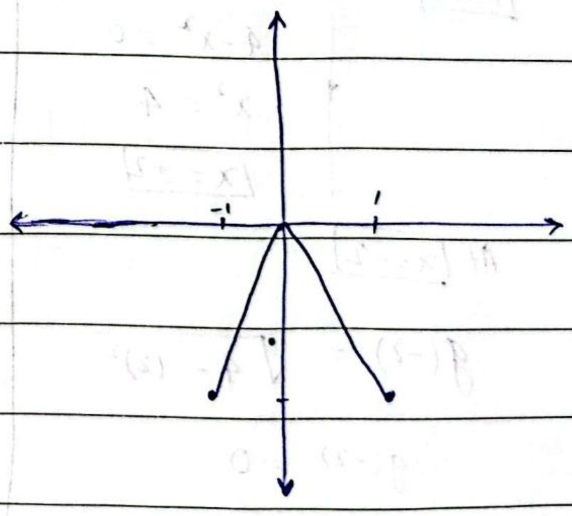
$h(1) = -3(1)^{2/3} = \boxed{-3}$

Abs max $= \boxed{0}$

Abs min $= \boxed{-3}$

Graph:

$(-1,-3), (1,-3)$



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(29) $g(x) = \sqrt{4-x^2}\ , \ -2 \leq x \leq 1$

$g(x) = \sqrt{4-x^2}$

$g'(x) = \frac{d}{dx}(4-x^2)^{1/2}$

$g'(x) = \frac{1}{2}(4-x^2)^{-1/2}\frac{d}{dx}(4-x^2)$

$g'(x) = \frac{1}{2\sqrt{4-x^2}}(-2x)$

$g'(x) = -\frac{\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}$

$g'(x) = \frac{-x}{\sqrt{4-x^2}}$

Critical point at $x=-2$ & $x=0$ but not at $x=2$ because 2 is not in domain.

$\boxed{x=0}$

$\sqrt{4-x^2} = 0$

$4-x^2 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$

At $\boxed{x=-2}$

$g(-2) = \sqrt{4-(2)^2}$

$g(-2) = 0$

At $\boxed{x=1}$

$g(1) = \boxed{\sqrt{3}} \rightarrow$ Abs min

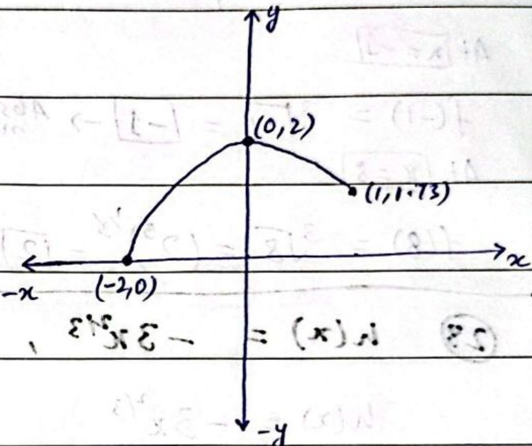
At $x=0$

$g(0) = \sqrt{4-0}$

$\boxed{g(0) = 2} \rightarrow$ Abs max at value $\boxed{x=0}$

Graph:

$(0,2), (-2,0), (1, 1.73)$



(30) $g(x) = -\sqrt{5-x^2}, -\sqrt{5} \leq x \leq 0$

$g'(x) = -\frac{d}{dx}(5-x^2)^{1/2}$

$g'(x) = -\frac{1}{2}(5-x^2)^{-1/2}\frac{d}{dx}(5-x^2)$

$g'(x) = \cancel{-}\frac{1}{\cancel{2}\sqrt{5-x^2}}(\cancel{+2}x)$

$g'(x) = \frac{x}{\sqrt{5-x^2}}$

Critical point $x=0$ and $x=-\sqrt{5}$ but not at $x=\sqrt{5}$ because $\sqrt{5}$ is not in the Domain.

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At $x = -\sqrt{5}$

$$g(-\sqrt{5}) = -\sqrt{5-(-\sqrt{5})^2}$$

$$g(-\sqrt{5}) = -\sqrt{5-5}$$

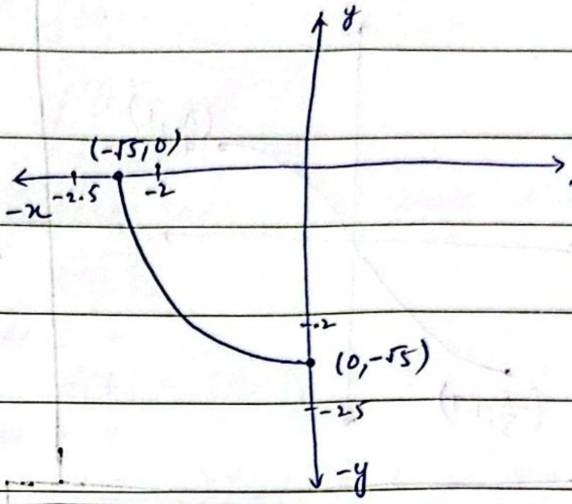
$$\boxed{g(-\sqrt{5}) = 0} \rightarrow \text{Abs max}$$

At $x=0$

$$g(0) = -\sqrt{5-(0)^2}$$

$$\boxed{g(0) = -\sqrt{5}} \rightarrow \text{Abs min.}$$

Graph: $(-\sqrt{5}, 0), (0, -\sqrt{5})$



(31) $f(\theta) = \sin\theta, \; -\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$

Sol,

$$f'(\theta) = \cos\theta$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2} \text{ (critical point)}$$

But $-\frac{\pi}{2}$ is not because it is not in domain.

At $\boxed{x = -\frac{\pi}{2}}$

$$f\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right)$$

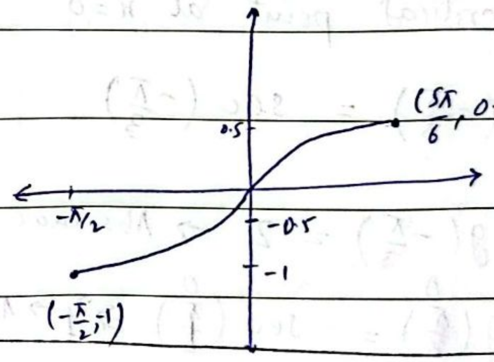
$$\boxed{f\left(-\frac{\pi}{2}\right) = -1} \rightarrow \text{Abs min}$$

At $x = \frac{5\pi}{6}$

$$f\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right)$$

$$\boxed{f\left(\frac{5\pi}{6}\right) = \frac{1}{2}} \rightarrow \text{Abs max}$$

Graph: $\left(-\frac{\pi}{2}, -1\right), \left(\frac{5\pi}{6}, 0.5\right)$



(32) $f(\theta) = \tan\theta, \; -\frac{\pi}{3} \le \theta \le \frac{\pi}{4}$

$$f'(\theta) = \sec^2\theta$$

$\rightarrow$ No critical point.

At $\boxed{x = -\frac{\pi}{3}}$

$$f\left(-\frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right) = \boxed{-\sqrt{3}} \rightarrow \text{Abs min.}$$

At $\boxed{x = \pi/4}$

$$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = \boxed{1} \rightarrow \text{Abs max.}$$

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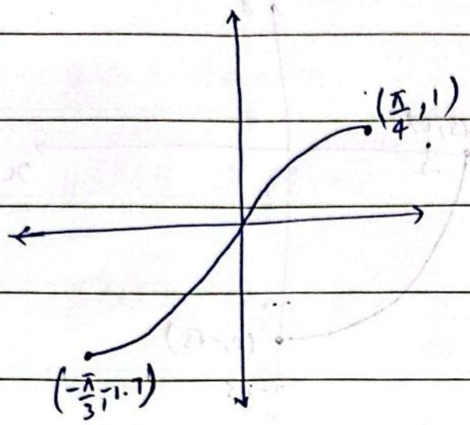
Graph: $\left(-\frac{\pi}{3}, -1.7\right), \left(\frac{\pi}{4}, 1\right)$



(33) $g(x) = \csc x, \quad \frac{\pi}{3} \le x \le \frac{2\pi}{3}$

Sol,

$g'(x) = -\csc x \cot x$

critical point at $x = \frac{\pi}{2}$

At $\left(x = \frac{\pi}{3}\right)$

$g\left(\frac{\pi}{3}\right) = \csc\left(\frac{\pi}{3}\right)$

$g\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} \rightarrow$ Abs max.

At $\left(x = \frac{2\pi}{3}\right)$

$g\left(\frac{2\pi}{3}\right) = \csc\left(\frac{2\pi}{3}\right)$

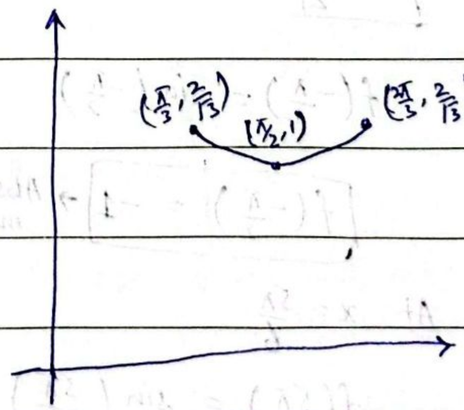
$g\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \rightarrow$ Abs max

At $\left(x = \frac{\pi}{2}\right)$

$g\left(\frac{\pi}{2}\right) = \csc\left(\frac{\pi}{2}\right)$

$g\left(\frac{\pi}{2}\right) = 1 \rightarrow$ Abs min

Graph: $\left(\frac{2\pi}{3}, \frac{2}{\sqrt{3}}\right), \left(\frac{\pi}{2}, 1\right), \left(\frac{\pi}{3}, \frac{2}{\sqrt{3}}\right)$



(34) $g(x) = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{6}$

$g'(x) = \sec x \tan x$

critical point at $x = 0$

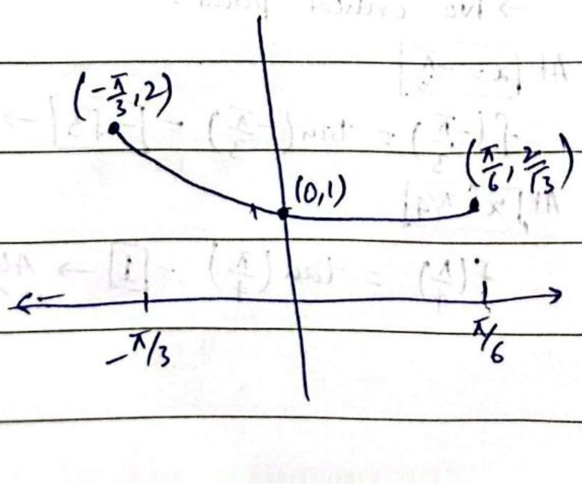
$g\left(-\frac{\pi}{3}\right) = \sec\left(-\frac{\pi}{3}\right)$

$g\left(-\frac{\pi}{3}\right) = 2 \rightarrow$ Abs max

$g(0) = \sec(0) \Rightarrow 1 \rightarrow$ Abs min.

$g\left(\frac{\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right) \Rightarrow \frac{2}{\sqrt{3}}$

Graph: $\left(-\frac{\pi}{3}, 2\right), \left(\frac{\pi}{6}, \frac{2}{\sqrt{3}}\right), (0, 1)$



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(35) $f(t) = 2-|t|\ , -1 \le t \le 3$

$f(t) = 2-|t|$

$f(t) = 2-\sqrt{t^2}$

$f(t) = 2-(t^2)^{1/2}$

$f'(t) = -\frac{1}{2}(t^2)^{-1/2}\frac{d}{dt}(t^2)$

$f'(t) = \frac{-1}{\cancel{2}\sqrt{t^2}}(\cancel{2}t)$

$f'(t) = \frac{-t}{\sqrt{t^2}} = \frac{-t}{|t|}$

Critical point $\boxed{t=0}$

At $t=-1$.

$f(-1) = 2-|-1|$

$\boxed{f(-1) = 1}$

At $t=3$

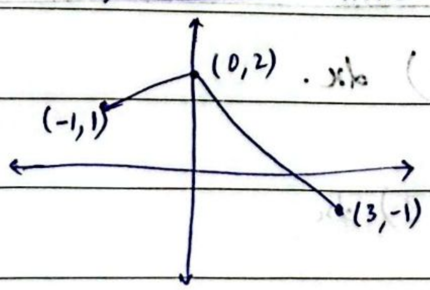
$f(3) = 2-|3|$

$\boxed{f(3) = -1} \rightarrow$ Abs min

At $t=0$

$f(0) = 2-|0|$

$\boxed{f(0) = 2} \rightarrow$ Abs max.



(36) $f(t) = |t-5|\ , 4 \le t \le 7$

$f(t) = ((t-5)^2)^{1/2}$

$f'(t) = \frac{t-5}{|t-5|}$

Critical point at ~~t-5~~0 $t-5=0$

$\boxed{t=5}$

At $t=4$

$f(4) = |4-5| = 1$

At $t=7$

$f(7) = |7-5| = 2 \rightarrow$ Abs max.

At $t=5$

$f(5) = |5-5| = 0 \rightarrow$ Abs min

Graph: $(4,1), (7,2), (5,0)$