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# Exercise # 5.5 (1-50)

## Evaluate Indefinite Integrals & Substitution Method.

① $\int 2(2x+4)^5 dx$, $u=2x+4$

$u = 2x+4$

$du = 2dx \rightarrow dx = \frac{1}{2} du$.

So,

$\int 2(2x+4)^5 dx$

$\int \cancel{2} u^5 \frac{1}{\cancel{2}} du$

$\int u^5 du$

$\frac{u^6}{6} + C$

$\boxed{\frac{(2x+4)^6}{6} + C}$

② $\int 7\sqrt{7x-1}\, dx$, $u = 7x-1$.

$u = 7x-1$

$du = 7dx$

$\int 7\sqrt{7x-1}\, dx$

$\int \sqrt{u}\, du$

$\int (u)^{1/2+1} du$

~~$\frac{3}{2\sqrt{(u)^{3/2}}} + C \Rightarrow \frac{1}{2\sqrt{7x-1}} + C$~~

$\frac{2}{3} u^{3/2} + C$

$\boxed{\frac{2}{3}(7x-1)^{3/2} + C}$

③ $\int 2x(x^2+5)^{-4} dx$, $u = x^2+5$

$u = x^2+5$

$du = 2x dx$

$\int 2x(x^2+5)^{-4} dx$

$\int u^{-4} du$

$-\frac{u^{-3}}{3} + C \Rightarrow \boxed{-\frac{(x^2+5)^{-3}}{3} + C}$

④ $\int \frac{4x^3}{(x^4+1)^2} dx$, $u = x^4+1$

Sol,

$u = 4x^4+1$

$du = 4x^3 dx$

$\int \frac{1}{u^2} du$

$\int u^{-2} du$

$-u^{-1} + C$

$-\frac{1}{u} + C$

$\boxed{-\frac{1}{x^4+1} + C}$

⑤ $\int (3x+2)(3x^2+4x)^4 dx$,

$u = 3x^2+4x$.

$du = 6x+4\, dx$

$du = 2(3x+2)dx$

$\frac{du}{2} = (3x+2)dx$

$\int (3x+2)(3x^2+4x)^4 dx$

$\int (\frac{1}{2} du)(u^4)$

$\frac{1}{2}\int u^4 du$

$\frac{1}{2}\left(\frac{u^5}{5}\right) + C \Rightarrow \frac{u^5}{10} + C \Rightarrow \boxed{\frac{(3x^2+4x)^5}{10} + C}$

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⑥ $\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx, \quad u = 1+\sqrt{x}$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx$

$\int u^{1/3}\, 2du$

$2\int u^{1/3}\,du$

$\cancel{2}\cdot\frac{3}{2\,\cancel{4}}u^{4/3}+c$

$\boxed{\frac{3}{2}u^{4/3}+c}$

$\boxed{\frac{3}{2}(1+\sqrt{x})^{4/3}+c}$

$u = 1-\cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}\,dt$

$2du = \sin\frac{t}{2}\,dt$

So,

⑦ $\int \sin 3x\,dx, \quad u = 3x$

$u = 3x, \quad du = 3dx \longrightarrow \frac{1}{3}du = dx$

So,

$\int \sin 3x\,dx \Rightarrow \int \frac{1}{3}\sin u\,du \Rightarrow \frac{1}{3}\int \sin u\,du.$

$\frac{1}{3}(-\cos u)+c \Rightarrow \boxed{-\frac{\cos(3x)}{3}+c}$

⑧ $\int x\sin(2x^2)\,dx, \quad u = 2x^2$

$u = 2x^2, \quad du = 4x\,dx \longrightarrow \frac{du}{4} = x\,dx.$

$\int x\sin(2x^2)\,dx \Rightarrow \frac{1}{4}\int \sin u\,du \Rightarrow \boxed{-\frac{1}{4}\cos(2x^2)+c}$

⑨ $\int \sec 2t\tan 2t\,dt, \quad u = 2t$

$u = 2t, \quad du = 2dt \rightarrow dt = \frac{1}{2}du.$

So,

$\int \sec 2t\tan 2t\,dt \Rightarrow \frac{1}{2}\int \sec u\tan u\,du.$

$\frac{1}{2}\int(\sec u)+c \Rightarrow \boxed{\frac{1}{2}\sec(2t)+c}$

⑩ $\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\,dt, \quad u = 1-\cos\frac{t}{2}$

$\int\left(1-\cos\frac{t}{2}\right)^2\sin\frac{t}{2}\,dt \Rightarrow \int(2du)(u)^2$

$2\int u^2\,du \Rightarrow \frac{2}{3}u^3+c \Rightarrow \boxed{\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3+c}$

⑪ $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}, \quad u = 1-r^3 \rightarrow du = -3r^2\,dr$

$-3du = 9r^2\,dr, \quad$ So, $-3\int u^{-1/2}\,du$

$-6u^{1/2}+c \Rightarrow \boxed{-6\sqrt{1-r^3}+c}$

$\cot 2\theta = -2\csc^2 2\theta$

$\csc 2\theta = -2\csc 2\theta \cot 2\theta$

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(12) $\int 12(y^4+4y^2+1)^2(y^3+2y)\,dy$, $\boxed{u = y^4+4y^2+1}$

$du = 4y^3 + 8y\,dy$

$\boxed{\frac{du}{4} = y^3 + 2y\,dy}$

So, $\int 12(y^4+4y^2+1)^2(y^3+2y)\,dy$.

$3\int u\,du \Rightarrow \boxed{\frac{3}{2}u^2 + c} \rightarrow \boxed{\frac{3}{2}(y^4+4y^2+1)+c}$

(13) $\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx$, $u = x^{3/2}-1$

$u = x^{3/2}-1$

$du = \frac{3}{2}x^{1/2}dx$

$\frac{2}{3}du = \sqrt{x}\,dx$

$\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx \Rightarrow \frac{2}{3}\int \sin^2 u\,du$

$\frac{2}{3}\int \sin^2 u\,du$

$\therefore \sin^2 u = \frac{1-\cos 2\theta}{2}$

So, $\frac{2}{3}\left(\frac{u}{2} - \int\frac{\cos 2u}{2}\right)$

$\frac{2}{3}\left(\frac{u}{2} - \frac{\sin 2u}{4}\right) + c$

$\frac{2u}{3\cdot 2} - \frac{2\sin 2u}{3\cdot 4\,2} + c$

$\frac{u}{3} - \frac{1}{6}\sin 2u + c$

$\boxed{\frac{x^{3/2}-1}{3} - \frac{1}{6}\sin(2x^{3/2}-2) + c}$

(14) $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$, $u = -\frac{1}{x}$

$u = -\frac{1}{x}$, $du = \frac{1}{x^2}dx$

$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx \Rightarrow \int \cos(-u)\,du$

$+\int \cos^2 u\,du \Rightarrow \therefore \cos^2 u = \frac{1+\cos 2u}{2}$

$\Rightarrow \int \frac{1}{2} + \int \frac{\cos 2u}{2}$

$\left(\frac{u}{2} + \frac{\sin 2u}{4}\right) + c$

$\boxed{-\frac{1}{2x} + \frac{1}{4}\sin\left(-\frac{2}{x}\right) + c}$

(15) $\int \csc^2 2\theta \cot 2\theta\,d\theta$

a) $u = \cot 2\theta$

$du = -2\csc^2 2\theta\,d\theta$

$-\frac{du}{2} = \csc^2 2\theta\,d\theta$.

$-\frac{1}{2}\int u\,du \Rightarrow \boxed{-\frac{1}{4}(\cot 2\theta) + c}$

b) $u = \csc 2\theta$.

$du = -2\csc 2\theta \cot 2\theta\,d\theta$.

So, $-\frac{1}{2}\int u\,du \Rightarrow -\frac{1}{4}(\csc 2\theta) + c$

$\boxed{-\frac{1}{4}(\csc 2\theta) + c}$

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(16) $\int \frac{dx}{\sqrt{5x+8}}$

a) $u = 5x+8$

$du = 5dx$

$\frac{du}{5} = dx$

$\int \frac{dx}{\sqrt{5x+8}}$

$\frac{1}{5}\int \frac{1}{\sqrt{u}} du$

$\frac{2}{5}(u)^{1/2} + c$

$\boxed{\frac{2}{5}(5x+8)^{1/2} + c}$

b) $u = \sqrt{5x+8}$

$du = \frac{1}{2}(5x+8)^{-1/2}(5)\,dx$

$\frac{2}{5}du = \frac{dx}{\sqrt{5x+8}}$

so,

$\frac{2}{5}\int du \Rightarrow \frac{2}{5}u + c$

$\boxed{\frac{2}{5}(\sqrt{5x+8}) + c}$

(17) $\int \sqrt{3-2s}\, ds$

let:

$u = 3-2s$, $du = -2ds$

$-\frac{1}{2}\int \sqrt{u}\, du$

$(-\frac{1}{2})(\frac{2}{3}u^{3/2} + c)$

$\cancel{-\frac{1}{4\sqrt{u}}} \cancel{4}/c$

$\boxed{-\frac{1}{3\cancel{4}(3-2s)^{-3/2}} + c}$

(18) $\int \frac{1}{\sqrt{5s+4}} ds$

let:

$u = 5s+4$

$du = 5ds$

$ds = \frac{du}{5}$

$\frac{1}{5}\int \frac{1}{\sqrt{u}} du$

$\frac{2}{5}(u^{1/2} + c)$

$\boxed{\frac{2}{5}(\sqrt{5s+4}) + c}$

(19) $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

$u = 1-\theta^2$

$du = -2\theta d\theta$

$\theta\, d\theta = -\frac{du}{2}$

$-\frac{1}{2}\int \sqrt[4]{u}\, du$

$-\frac{1}{2}\int (u)^{1/4} du$

$-\frac{1}{2}\left(\frac{4}{5}u^{5/4}\right) + c$

$\boxed{-\frac{2}{5}(1-\theta^2)^{5/4} + c}$

(20) $\int 3y\sqrt{7-3y^2}\, dy$

let:

$u = 7-3y^2$

$du = -6y\, dy$

$-\frac{du}{2} = 3y\, dy$

$-\frac{1}{2}\int (u)^{1/2} du$

$-\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)(u^{3/2}) + c$

$\boxed{-\frac{1}{3}u^{3/2} + c}$

(21) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

let:

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$2du = \frac{dx}{\sqrt{x}}$

so,

$-2\frac{1}{u} + c$

$2\int \frac{1}{u^2} du \Rightarrow 2\int u^{-2} du \Rightarrow -2(u^{-1}) + c \Rightarrow \boxed{\frac{-2}{1+\sqrt{x}} + c}$

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(22) $\int \cos(3z+4)\,dz$

let,

$u = 3z+4$

$du = 3\,dz$

$\frac{du}{3} = dz$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin u + c$

$\boxed{\frac{1}{3}\sin(3z+4)+c}$

(23) $\int \sec^2(3x+2)\,dx$

let,

$u = 3x+2$

$du = 3\,dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\int \sec^2 u\,du.$

$\frac{1}{3}(\tan u) + c$

$\boxed{\frac{1}{3}\tan(3x+2)+c}$

(24) $\int \tan^2 x \sec^2 x\,dx$

let

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^2\,du \Rightarrow \frac{u^3}{3} + c$

$\boxed{\frac{1}{3}(\tan x)^3 + c}$

(25) $\int \sin^5\frac{x}{3}\cos\frac{x}{3}\,dx$

let:

$u = \sin(\frac{x}{3})$

$du = \frac{1}{3}\cos\frac{x}{3}\,dx$

$3\,du = \cos\frac{x}{3}\,dx$

So,

$\int \sin^5\frac{x}{3}\cos\frac{x}{3}\,dx$

$\int u^5 \cdot 3\,du$

$3\int u^5\,du$

$\frac{u^6}{2} + c$

$\boxed{\frac{1}{2}\left(\sin^6\left(\frac{x}{3}\right)\right)^6 + c}$

(26) $\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\,dx$

let

$u = \tan\frac{x}{2}$

$2\,du = \sec^2\frac{x}{2}\,dx$

$2\int u^7\,du$

$\frac{\cancel{2}u^8}{\cancel{8}4} + c$

$\frac{\tan^8(\frac{x}{2})}{84} + c$

$\boxed{\frac{1}{4}\tan^8\left(\frac{x}{2}\right) + c}$

(27) $\int r^2\left(\frac{r^3}{18}-1\right)^5 dr$

let:

$u = \frac{r^3}{18} - 1$

$du = \frac{r^2}{6}\,dr$

$6\,du = r^2\,dr$

$6\int u^5\,du$

$\cancel{6}/\cancel{6}u^6 + c$

$\boxed{B\left(\frac{r^3}{18}-1\right)^6 + c}$

(28) $\int r^4\left(7-\frac{r^5}{10}\right)^3 dr$

let, $u = 7 - \frac{r^5}{10}$

$du = -\frac{r^4}{2}\,dr$

$-2\,du = r^4\,dr$

$-2\int u^3\,du$

$-\frac{1}{2}(u^4) + c$

$\boxed{-\frac{1}{2}\left(7-\frac{r^5}{10}\right)^4 + c}$

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(29) $\int x^{1/2} \sin(x^{3/2}+1)\,dx$

let, $u = x^{3/2}+1$

$du = \frac{3}{2}x^{1/2}dx$

$\frac{2}{3}du = x^{1/2}dx$

$\frac{2}{3}\int \sin u\,du$

$-\frac{2}{3}\cos u + c$

$\boxed{-\frac{2}{3}\cos(x^{3/2}+1)+c}$

(30) $\int \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv$

let, $u = \csc\left(\frac{v-\pi}{2}\right)$

$du = -\frac{1}{2}\csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv$

$-2du = \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv$

$-2\int du \Rightarrow -2u + c \Rightarrow$

$\boxed{-2\left(\csc\left(\frac{v-\pi}{2}\right)\right)+c}$

(31) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt$

let, $u = \cos(2t+1)$

$du = \cancel{2dt}\ -2\sin(2t+1)\,dt$

$-\frac{du}{2} = dt\sin(2t+1)$

$\frac{1}{2}\int \cancel{\frac{\sin u}{\cos u}}\,du$

$-\frac{1}{2}\int \frac{1}{u^2} \Rightarrow \frac{1}{2u}+c$

$\boxed{\frac{1}{2(\cos(2t+1))}+c}$

(32) $\int \frac{\sec z\tan z}{\sqrt{\sec z}}dz$

let, $u = \sec z$

$du = \sec z\tan z\,dz$

$\int \frac{1}{\sqrt{u}}du$

$\int u^{-1/2}du$

$2u^{1/2}+c$

$\boxed{2(\sqrt{\sec z})+c}$

(33) $\int \frac{1}{t^2}\cos\left(\frac{1}{t}-1\right)dt$.

let, $u = \frac{1}{t}-1$

$du = -t^{-2}dt$

$-du = \frac{1}{t^2}dt$

$-\int \cos u\,du$

$-\sin u + c$

$\boxed{-\sin\left(\frac{1}{t}-1\right)+c}$

(34) $\int \frac{1}{\sqrt{t}}\cos(\sqrt{t}+3)\,dt$

let, $u = \sqrt{t}+3$ , $du = \frac{1}{2\sqrt{t}}dt \rightarrow 2du = \frac{dt}{\sqrt{t}}$

So,

$\int \frac{1}{\sqrt{t}}\cos(\sqrt{t}+3)\,dt \Rightarrow 2\int \cos u\,du \Rightarrow 2\sin u + c \Rightarrow \boxed{2\sin(\sqrt{t}+3)+c}$

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(35) $\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\, d\theta$

$u = \sin\left(\frac{1}{\theta}\right)$

$du = \left(\cos\frac{1}{\theta}\right)\left(\frac{1}{\theta^2}\right) d\theta$

$-du = \frac{1}{\theta^2} \cos\frac{1}{\theta}\, d\theta$

$\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\, d\theta$

$-\int \cancel{\sin}\, u\, du$

$-\frac{u^2}{2} + c$

$\boxed{-\frac{1}{2}\left(\sin\left(\frac{1}{\theta}\right)\right)^2 + c}$

(36) $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\, d\theta$.

$\int \frac{\cos\sqrt{\theta}}{\sin\sqrt{\theta}} \cdot \frac{1}{\sin\sqrt{\theta}} \cdot \frac{1}{\sqrt{\theta}}\, d\theta$

$\therefore \cot\sqrt{\theta} = \frac{\cos\sqrt{\theta}}{\sin\sqrt{\theta}}, \quad \csc\sqrt{\theta} = \frac{1}{\sin\sqrt{\theta}}$

$\int \frac{\cot\sqrt{\theta}\csc\sqrt{\theta}}{\sqrt{\theta}} \cdot d\theta$

let: $u = \cancel{\cot}\csc\sqrt{\theta}$

$du = (-\csc\sqrt{\theta}\cot\sqrt{\theta})\left(\frac{1}{2\sqrt{\theta}}\right) d\theta$

$-2du = \left(\frac{1}{\sqrt{\theta}} \csc\sqrt{\theta}\cot\sqrt{\theta}\right) d\theta$

$-2\int du \Rightarrow -2u + c \Rightarrow \boxed{-2(\csc\sqrt{\theta}) + c}$

$\boxed{\frac{-2}{\sin\sqrt{\theta}} + c}$

(37) $\int t^3 (1+t^4)^3\, dt$

let:

$u = 1 + t^4$

$du = 4t^3\, dt$

$\frac{du}{4} = t^3\, dt$

$\frac{1}{4}\int u^3\, du$

$\frac{u^4}{4\times4} + c$

$\boxed{\frac{(1+t^4)^4}{16} + c}$

(38) $\int \sqrt{\frac{x-1}{x^5}}\, dx \Rightarrow \int \sqrt{\frac{1}{x^4} - \frac{1}{x^5}}\, dx$

$\int \frac{1}{x^2}\sqrt{1 - \frac{1}{x}}$ So, $u = 1 - \frac{1}{x}$

$du = \frac{1}{x^2}\, dx$

$\int \sqrt{u}\, du \Rightarrow \frac{2}{3} u^{3/2} + c \Rightarrow \boxed{\frac{2}{3}\left(1 + \frac{1}{x}\right)^{3/2} + c}$

(39) $\int \frac{1}{x^2}\sqrt{2 - \frac{1}{x}}\, dx \Rightarrow u = 2 - \frac{1}{x}$

$du = \frac{1}{x^2}\, dx$

$\int \sqrt{u}\, du \Rightarrow \frac{2}{3} u^{3/2} + c$

$\boxed{\frac{2}{3}\left(2 - \frac{1}{x}\right)^{3/2} + c}$

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(40) $\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx$

$\int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$

$u = 1-\frac{1}{x^2}$

$du = \frac{2}{x^3}\,dx$

$\frac{1}{2}du = \frac{1}{x^3}dx$

$\frac{1}{2}\int u^{1/2}du$

$\frac{1}{3}u^{3/2}+c$

$\boxed{\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}+c}$

(41) $\int\sqrt{\frac{x^3-3}{x^{11}}}\,dx \Rightarrow \int\sqrt{\frac{x^3-3}{x^9\cdot x^3}}$

So,

$\int\sqrt{\frac{\cancel{x^3}}{\cancel{x^3}}-\frac{3}{x^9}} = \int\sqrt{1-\frac{3}{x^9}}$

Let $u = 1-\frac{3}{x^3}$ $du = \frac{9}{x^4}dx = \frac{1}{9}du = \frac{1}{x^4}dx$

$\int\sqrt{\frac{x^3-3}{x^{11}}}\,dx \rightarrow \int\frac{1}{x^4}\sqrt{\frac{x^3-3}{x^3}}\,dx$

$\int\frac{1}{x^4}\sqrt{1-\frac{3}{x^3}}\,dx \Rightarrow \frac{1}{9}\int(u)^{1/2}du$

$\frac{1}{9}\left(\frac{2}{3}u^{3/2}+c\right) \Rightarrow \boxed{\frac{2}{27}\left(1-\frac{3}{x^3}\right)+c}$

(42) $\int\sqrt{\frac{x^4}{x^3-1}}\,dx$

$u = x^3-1$

$du = 3x^2dx$

$\frac{du}{3} = x^2dx$

$\int\sqrt{\frac{x^4}{x^3-1}}\,dx$

$\int\frac{x^2}{\sqrt{x^3-1}}\,dx$

$\frac{1}{3}\int\frac{1}{\sqrt{u}}du$

$\frac{1}{3}\int(u)^{-1/2}du$

$\frac{1}{3}\left(2u^{1/2}\right)+c$

$\boxed{\frac{2}{3}(x^3-1)+c}$

(43) $\int x(x-1)^{10}dx$

$u = x-1$

$du = dx$

$x = u+1$

$\int u+1(u)^{10}du$

$\int u^{11}+u^{10}du$

$\int u^{11}du+\int u^{10}du$

$\frac{u^{12}}{12}+\frac{u^{11}}{11}+c$

$\boxed{\frac{(x-1)^{12}}{12}+\frac{(x-1)^{11}}{11}+c}$

(44) $\int x\sqrt{4-x}\,dx$

$u = 4-x$

$x = u+4$

$du = -dx$

$-\int(u+4)(u)^{1/2}du$

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$\int (4-u)(-u^{1/2})\,du$

$\int (u^{3/2} - 4u^{1/2})\,du$

$\int u^{3/2}du - 4\int u^{1/2}du$

$\frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(4-x)^{5/2} - \frac{8}{3}(4-x)^{3/2} + C}$

(45) $\int (x+1)^2(1-x)^5\,dx$

$u = 1-x$

$du = -dx$

$x = 1-u$

$\int (1-u+1)^2(u)^5\,du$

$\int (2-u)^2(-u)^5\,du$

$\int (+u^7 - 4u^5 + 4u^6)\,du$

$\int u^7du - 4\int u^5du + 4\int u^6du$

$\frac{1}{8}u^8 - \frac{2}{3}u^6 + \frac{4}{7}u^7 + C$

$\boxed{\frac{(1-x)^8}{8} - \frac{2(1-x)^6}{3} + \frac{4(1-x)^7}{7} + C}$

(46) $\int (x+5)(x-5)^{1/3}\,dx$

$u = x-5$

$du = dx$

$x = u+5$

$\int (u+5+5)(u)^{1/3}\,du$

$\int (u+20)(u^{1/3})\,du$

$\int (u^{4/3} + 10u^{1/3})\,du$

$\int u^{4/3}du + 10\int u^{1/3}du$

$\frac{3}{7}u^{7/3} + \frac{3}{40}u^{4/3} + C$

$\boxed{\frac{3}{7}(x-5)^{7/3} + \frac{3}{40}(x-5)^{4/3} + C}$

(47) $\int x^3\sqrt{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$x^2 = u-1$

$x\,dx = \frac{1}{2}du$

$\int x^2\sqrt{x^2+1}\;x\,dx$

$\int (u-1)(u^{1/2})\,du$

$\int u^{3/2}du - \int u^{1/2}du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x^2+1)^{5/2} - \frac{2}{3}(x^2+1)^{3/2} + C}$

(48) $\int 3x^5\sqrt{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2dx$

$x^3 = u-1$

$\int 3x^3 \cdot x^2\sqrt{x^3+1}\,dx$

$\int (u-1)(u^{1/2})\,du$

$\int u^{3/2}du - \int u^{1/2}du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x^3+1)^{5/2} - \frac{2}{3}(x^3+1)^{3/2} + C}$

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(49) $\int \frac{x}{(x^2-4)^3} dx$

$u = x^2 - 4$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{1}{u^3} du$

$\frac{1}{2}\int u^{-3} du$

$-\frac{1}{4u^2} + C$

$\boxed{-\frac{1}{4(x^2-4)^2} + C}$

(50) $\int \frac{x}{(x-4)^3} dx$

$u = x - 4$

$du = dx$

$x = u + 4$

$\int \frac{u+4}{u^3} du$

$\int u^{-2} + \frac{4}{u^3}$

$\int u^{-2} du + 4\int u^{-3} du$

$-\frac{1}{u} - \frac{2}{u^2} + C$

$\boxed{-\frac{1}{x-4} - \frac{2}{(x-4)^2} + C}$