ASSIGNMENT 5

EXERCISE 4.9

Q-1

Solution:

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_2 - x_3 + x_4 = 0$ $\quad \boxed{\therefore R(A) = 1}$

$x_1 = -2x_2 + x_3 - x_4$

$x_2 = r; \quad x_3 = s; \quad x_4 = t$

$x_1 = -2r + s - t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2r + s - t \\ r \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Nullity $= 4 - 1 = 3$

$\boxed{\text{Nullity} = 3}$

(b) Solution:

$$\left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\therefore R(A) = 2}$$

$$x_1 - 2x_2 + 2x_3 + 3x_4 - x_5 = 0$$
$$x_3 + 2x_4 - 2x_5 = 0$$

$$\Rightarrow x_1 = 2x_2 - 2x_3 - 3x_4 + x_5$$
$$\Rightarrow x_3 = -2x_4 + 2x_5$$

$$x_2 = r \; ; \quad x_3 = s \; ; \quad x_4 = t \; ; \quad x_5 = u$$

$$x_1 = 2r - 2s - 3t + u$$
$$x_3 = -2t + 2u$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2r - 2s - 3t + u \\ r \\ -2t + 2u \\ t \\ u \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Nullity $= 5 - 2 = 3$

Q#2

(a) Solution:

$$R \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\boxed{\therefore R(A) = 3}$$

$$x_1 + 0x_2 - 2x_3 + x_4 + 0x_5 = 0$$
$$0x_1 + x_2 + 3x_3 - x_4 - 3x_5 = 0$$
$$x_4 - x_5 = 0$$

$\cancel{x_1 = r}$; $x_3 = r$; $x_4 = s$; $x_5 = t$

$$x_1 = 2x_3 - x_4$$
$$x_2 = -3x_3 + x_4 + 3x_5$$
$$x_4 = x_5$$

$$x_1 = 2r - s$$
$$x_2 = -3r + s + 3t$$
$$x_4 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2r - s \\ -3r + s + 3t \\ r \\ t \\ \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r\begin{bmatrix} 2 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ Nullity $= 5 - 3 = 2$

(b) Solution:

$$R \begin{bmatrix} 1 & 3 & 1 & 3 & \vert & 0 \\ 0 & 1 & 1 & 0 & \vert & 0 \\ 0 & 0 & 0 & 1 & \vert & 0 \\ 0 & 0 & 0 & 0 & \vert & 0 \\ 0 & 0 & 0 & 0 & \vert & 0 \end{bmatrix}$$

$$\boxed{\therefore R(A) = 3}$$

$$x_1 + 3x_2 + x_3 + 3x_4 = 0$$
$$x_2 + x_3 = 0$$
$$x_4 = 0$$

$\cancel{x_1 = s}$; $\cancel{x_2 = s}$;

$$x_1 = -3x_2 - x_3 - 3x_4$$
$$x_2 = -x_3$$

$\cancel{x_1 =}$ $x_2 = r$; $x_3 = s$; $x_4 = t$

$$x_1 = -3r - s - 3t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3r - s - 3t \\ -s \\ 0 \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Nullity $= 4 - 3 = 1$

$$\boxed{\therefore \text{Nullity} = 1}$$

EXERCISE 4.6

Q# 1

Solution:

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ Rank(A) = 2.

~~Nulli~~ Nullity of a matrix;

$$x_1 + x_2 - x_3 = 0$$

$$x_2 = 0$$

$$\cancel{x_1} \quad x_2 = -x_1 + x_3$$

$$x_1 = r \; ; \quad x_3 = s \Rightarrow x_2 = -r + s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = r \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Nullity = 3 - 2 = 1

$\boxed{\therefore \text{Nullity} = 1}$

Q#2

Solution:

$$3x_1 + x_2 + x_3 + x_4 = 0$$

$$5x_1 - x_2 + x_3 - x_4 = 0$$

$$\underset{\sim}{R} \cancel{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} \; \underset{\sim}{R} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 1/4 & 0 & 0 \end{array}\right]$$

$\boxed{\therefore R(A) = 2}$

$$x_1 + 3x_2 + x_3 + x_4 = 0$$

$$x_2 + \frac{1}{4} x_3 = 0$$

$$x_1 = -3x_2 - x_3 - x_4$$
$$x_2 = \frac{-1}{4} x_3$$
$$x_2 = r \; ; \; x_3 = s \; ; \; x_4 = t$$
$$x_1 = -3r - s - t$$
$$x_2 = \frac{-1}{4} s$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3r - s - t \\ -\frac{1}{4}s \\ s \\ t \end{bmatrix}$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r\begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ -\frac{1}{4} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$\text{Nullity} = 4 - 2 = 2$$

Q#3

Solution:

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 10 & 0 \end{array}\right]$$
$$\therefore R(A) = 3$$
$$x_1 + 0x_2 + 5x_3 = 0$$
$$x_2 + x_3 = 0$$
$$x_3 = 0$$
$$\Rightarrow \quad x_1 = -5x_3 \quad ; \quad x_3 = 5r$$
$$x_2 = -x_3 \quad ;$$
$$x_3 = 0 \quad ;$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5r \\ -r \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = r \begin{bmatrix} -5 \\ -1 \\ 0 \end{bmatrix}$$

$\therefore$ Nullity $= 3 - 1 = 2$

Q#4

Solution:

$\cancel{x_1 - 4x}$

$$\underset{\sim}{R} \left[\begin{array}{cccc|c} 1 & -4 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\therefore R(A) = 1}$$

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$

$$x_1 = 4x_2 - 3x_3 + x_4$$

$$x_2 = r \;;\quad x_3 = s \;;\quad x_4 = t$$

$$x_1 = 4r - 3s + t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4r - 3s + t \\ r \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Nullity $= 4 - 1 = 3$

Q#5

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R(A) = 1$

$x_1 - 3x_2 + x_3 = 0$

$x_1 = +3x_2 - x_3$

$x_2 = r \;;\; x_3 = s$

$x_1 = 3r - s \Rightarrow \cancel{x_1 = 3r - s}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3r - s \\ r \\ s \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = r\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Nullity $= 3 - 1 = 2$

Q#6

Solution:

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore R(A) = 2$

$x_1 + x_2 + x_3 = 0$

$x_2 + 5x_3 = 0$

$\cancel{x_1 = -x_2 - x_3}$

$x_2 = -x_1 - x_3$

$x_2 = -5x_3$

$x_1 = \lambda$; $x_3 = S$;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \lambda \\ -\lambda - S \\ S \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + S \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Nullity $= 3 - 2 = 1$

$\boxed{\therefore \text{Nullity} = 1}$. Ans

01-01-2024

ASSIGNMENT 5

EXERCISE 4.9

Q-1

(a)

<u>Solution:</u>

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$$

By $R_2 - 2R_1$; $R_3 - 3R_1$; $R_4 - 4R_1$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of matrix is 1.

i.e. $\text{Rank}(A) = 1$.

Q-2 (b) $\longrightarrow$ Last page.

<u>Solution:</u>

$$A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & 4 \end{bmatrix}$$

By $R_2 + 3R_1$; $R_3 - 2R_1$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 5 & 10 & -10 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

The rank of matrix is 3

i.e. $\text{Rank}(A) = 3$.

Q#2

(a)

Solution:

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$$

By $R_3 + 2R_1$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ 0 & -1 & -3 & 1 & 3 \\ 0 & +1 & 3 & 0 & -4 \end{bmatrix}$$

Multiply R_2 with "-1";

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & -1 & -3 \\ 0 & -1 & -3 & 1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$$

By $R_3 + R_2$; By $R_4 - R_2$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

The rank of matrix is 3.

$\therefore$ Rank $(A) = 3$.

(b)

Solution:

$$A = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & -2 \end{bmatrix}$$

By $R_3 + 3R_1$; $R_4 - 3R_1$; $R_5 - 2R_1$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 9 & 9 & 8 \\ 0 & -5 & -5 & -8 \\ 0 & -6 & -6 & -8 \end{bmatrix}$$

By $R_3 - 9R_2$; $R_4 + 5R_2$; $R_5 + 6R_2$;

$$\underset{\sim}{R} \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

~~By $R_3 + R_2$~~; $\frac{1}{8}R_3$

$$\begin{pmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 0 & -8 \end{pmatrix}$$

$R_4 + 8R_3$, $R_5 + 8R_3$

$$\sim \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of matrix is 3.

$\therefore$ Rank(A) = 3.

Q-1

(b) Solution:

$$A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & 4 \end{bmatrix}$$

By $R_2 + 3R_1$; $R_3 - 2R_1$;

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 5 & 10 & -10 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

By $R_2 \longleftrightarrow R_1$;

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 5 & 10 & -10 \end{bmatrix}$$

By $R_3 - 5R_2$;

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of matrix is 2.

$\therefore$ Rank(A) = 2.

EXERCISE 4.6

Q# 1

Solution:

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + 0x_2 + x_3 = 0$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right]$$

By $R_2 + 2R_1$; $R_3 + R_1$;

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

By $R_3 - R_2$;

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

~~The rank of matrix is 2.~~

$\therefore$ ~~$R(A) = 2$~~

By $R_1 - R_2$;

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 1$; $x_2 = 0$; $x_3 = 1$

$(x_1, x_2, x_3) = (\cancel{0}1, 0, 1)$

Q#2

Solution:

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

$$\sim \left[\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right]$$

By $C_1 \leftrightarrow C_2$;

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 0 \\ -1 & 5 & 1 & -1 & 0 \end{array}\right]$$

By $R_2 + R_1$,

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 0 \\ 0 & 8 & 2 & 0 & 0 \end{array}\right]$$

By $\frac{1}{8}R_2$, $\qquad 1-3\left(\frac{1}{4}\right)$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{4} & 0 & 0 \end{array}\right] \qquad 1-\frac{3}{4} \qquad \frac{1}{4}$$

By $R_1 - 3R_2$;

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 1 & 0 \\ 0 & 1 & \frac{1}{4} & 0 & 0 \end{array}\right]$$

By $\cancel{R_1 \leftrightarrow R_2}$; $\cancel{C}$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{array}\right]$$

$\therefore x_1 = \frac{-1}{4}s$; $x_2 = \frac{-1}{4}s - t$, $x_3 = s$; $x_4 = t$

$$(x_1, x_2, x_3, x_4) = \left(\frac{-1}{4}s, \frac{-1}{4}s, -t, s, t\right)$$

$$v_1 = \left(\frac{-1}{4}, \frac{-1}{4}, 1, 0\right) \ \& \ v_2 = (0, -1, 0, 1)$$

Solution: **Q#43**

$$2x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 0x_2 + 5x_3 = 0$$
$$0x_1 + x_2 + x_3 = 0$$

$$R \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

By $R_1 \longleftrightarrow R_2$;

$$R \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$1-2(0)$
$1-0$
$3-2(5)$
$3-10$

By $R_2 - 2R_1$;

$1-(-7)$

$$R \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

By $R_3 - R_2$;

$$R \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right]$$

By $\frac{1}{8} R_3$;

$$R \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

By $R_2 + 7R_3$; $R_1 - 5R_3$;

$$R \left[\begin{array}{ccc|c} 1 & 0 & 60 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(x_1, x_2, x_3) = (0, 0, 0)$.