(Exercise 8.2) (1-22)

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① $\int \cos 2x\,dx$

Multiply and divide by 2.

$\frac{2}{2}\int \cos 2x\,dx$

$\frac{1}{2}\int \cos 2x \cdot 2\,dx$

So, integrating

$\frac{1}{2}(\sin 2x) + C$

$\boxed{\frac{\sin 2x}{2} + c}$

② $\int_0^{\pi} 3\sin\frac{x}{3}\,dx$

$3\int_0^{\pi} \sin\frac{x}{3}\,dx$

Dividing and Multiply with 3.

$3 \times \frac{3}{3}\int_0^{\pi} \sin\frac{x}{3}\,dx$

$9\int_0^{\pi} \sin\frac{x}{3}\cdot\frac{1}{3}\,dx$

$9\left[-\cos\frac{x}{3}\right]_0^{\pi}$

$9\left[-\cos\left(\frac{\pi}{3}\right) + \cos(0)\right]$

$9\left[-\cos(60) + 1\right]$

$9\left[-\frac{1}{2} + 1\right]$

$9\left[\frac{1}{2}\right]$

$\boxed{\frac{9}{2}}$

③ $\int \cos^3 x \sin x\,dx$

multiply & Divide by negative.

$-\int \cos^3 x(-\sin x)\,dx$

$\boxed{-\frac{1}{4}\cos^4 x + C}$

④ $\int \sin^4 2x \cos 2x\,dx$

$\frac{1}{2}\int \sin^4 2x \cos 2x \cdot 2\,dx$

$\boxed{\frac{1}{10}\sin^5 2x + C}$

⑤ $\int \sin^3 x\,dx$

$\int \sin^2 x \sin x\,dx$

$\int (1-\cos^2 x)\sin x\,dx$

$\int \overset{\sin x}{1}\,dx - \int \cos^2 x \sin x\,d$

$\boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$

⑥ $\int \cos^3 4x\,dx$

$\frac{4}{4}\int \cos^3 4x\,dx$

$\frac{1}{4}\int \cos^3 4x \cdot 4\,dx$

$\frac{1}{4}\int \cos^2 4x \cdot \cos 4x \cdot 4\,dx$

$\frac{1}{4}\int (1-\sin^2 4x)\cos 4x \cdot 4\,dx$

$\frac{1}{4}\int \cos 4x \cdot 4\,dx - \int \cos 4x \sin^2 4x\,dx$

$\frac{1}{4}\sin 4x - \frac{1}{12}\frac{\sin}{\cancel{\cos}}{}^3 4x + C$

$\boxed{\frac{1}{4}\sin 4x - \frac{1}{12}\sin^3 4x + C}$

⑦ $\int \sin^5 x\,dx$

$\int (\sin^2 x)^2 \sin x\,dx$

$\int (1-\cos^2 x)^2 \sin x\,d$

$\int (1 - 2\cos^2 x + \cos^4 x)\sin x\,dx$

$\int \sin x\,dx - \int 2\cos^2 x \sin x\,dx + \int \cos^4 x \sin x\,dx$

$\boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C}$

$\longleftrightarrow$

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⑧ $\int_0^{\pi} \sin^5 \frac{x}{2}\, dx$

$\frac{2}{2}\int_0^{\pi} \sin^5 \frac{x}{2} \cdot dx$

$2\int_0^{\pi} (\sin^2 \frac{x}{2})^2 \cdot \sin\frac{x}{2} \cdot \frac{1}{2}\, dx$

$2\int_0^{\pi} (1-\cos^2 \frac{x}{2})^2 \cdot \sin\frac{x}{2}\cdot\frac{1}{2}\, dx$

$2\int_0^{\pi} (1-2\cos^2\frac{x}{2} + \cos^4\frac{x}{2}) \sin\frac{x}{2}\cdot\frac{1}{2}\, dx$

$2\int_0^{\pi} \sin\frac{x}{2}\cdot\frac{1}{2}\, dx - 4\int_0^{\pi} \cos^2\frac{x}{2}\sin\frac{x}{2}\, dx + 2\int_0^{\pi} \cos^4\frac{x}{2}$

$\left[-2\cos(\frac{x}{2}) + \frac{4}{3}\cos^3(\frac{x}{2}) - \frac{2}{5}\cos^5(\frac{x}{2})\right]_0^{\pi}$

$[0] - \left[-2\cos(0) + \frac{4}{3}\cos^3(0) - \frac{2}{5}\cos^5(0)\right]$

$2 - \frac{4}{3} + \frac{2}{5} \Rightarrow \frac{30 - 20 + 6}{15} = \boxed{\frac{16}{15}}$

⑨ $\int \cos^3 x\, dx$

$\int (\cos^2 x)\cos x\, dx$

$\int (1-\sin^2 x)\cos x\, dx$

$\int \cos x\, dx - \int \sin^2 x \cos x\, dx$

$\boxed{\sin x - \frac{1}{3}\sin^3 x + c}$

⑩ $\int_0^{\pi/6} 3\cos^5 3x\, dx$

$\int_0^{\pi/6} (\cos^2 3x)^2 \cos 3x \cdot 3dx$

$\int_0^{\pi/6} (1-\sin^2 3x)^2 \cos 3x \cdot 3dx$

$\int_0^{\pi/6} (1-2\sin^2 3x + \sin^4 3x)\cos 3x \cdot 3dx$

$\int_0^{\pi/6} \cos 3x \cdot 3dx - 2\int_0^{\pi/6} \sin^2 3x \cos 3x$

$+ \int_0^{\pi/6} \sin^4 3x$

$= \left[\sin(3x) - \frac{2}{3}\sin^3(3x) + \frac{1}{5}\sin^5(3x)\right]_0^{\pi/6}$

$= \left[\cancel{\frac{\sqrt{3}}{2} - \frac{2}{3}\frac{\sqrt{3}}{2} + \frac{1}{5}\frac{\sqrt{3}}{2}}\right] - 0$

$= \left[1 - \frac{2}{3} + \frac{1}{5}\right] \Rightarrow \boxed{\frac{8}{15}}$

⑪ $\int \sin^3 x \cos^3 x\, dx$

$\int \sin^3 x \cos^2 x \cos x\, dx$

$\int \sin^3 x (1-\sin^2 x)\cos x\, dx$

$\int \sin^3 x \cos x\, dx - \int \sin^5 x \cos x\, dx$

$(a-b)^3 = \ldots$ Date ....

$$\boxed{\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C}$$

(12) $\int \cos^3 2x \sin^5 2x\,dx$

$\frac{1}{2}\int \cos^3 2x \sin^5 2x \cdot 2dx$

$\frac{1}{2}\int \cos^2 2x \cdot \cos 2x \cdot \sin^5 2x\, 2dx$

$\frac{1}{2}\int (1-\sin^2 2x) \cdot \cos 2x \cdot \sin^5 2x \cdot 2dx$

$\frac{1}{2}\int \sin^5 2x \cos 2x\; 2dx - \frac{1}{2}\int \sin^7 2x \cos 2x\, 2dx$

$$\boxed{\frac{1}{12}\sin^6 2x - \frac{1}{16}\sin^8 2x + C}$$

(13) $\int \cos^2 2x\,dx$

$\int \frac{1+\cos 2x}{2}\,dx$

$\frac{1}{2}\int (1+\cos 2x)\,dx$

$\frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\,dx$

$$\boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$$

(14) $\int_0^{\pi/2} \sin^2 x\,dx$

$\int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx$

$\frac{1}{2}\int_0^{\pi/2} (1-\cos 2x)\,dx$

$\frac{1}{2}\int_0^{\pi/2} dx - \frac{1}{2}\int_0^{\pi/2} \cos 2x\,dx$

$\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\pi/2}$

$\left[\frac{\pi}{4} - 0\right] - 0 \Rightarrow \boxed{\frac{\pi}{4}}$

(15) $\int_0^{\pi/2} \sin^7 y\,dy \Rightarrow \int_0^{\pi/2} \sin^6 y \sin y\,dy$

$\int_0^{\pi/2} (\sin^2 y)^3 \sin y\,dy \Rightarrow \int_0^{\pi/2} (1-\cos^2 y)^3 \sin y\,dy$

$\int_0^{\pi/2} \cancel{\sin y\,dy - 3\int_0^{\pi/2} \cos^2 y \sin y\,dy +}$

$\int_0^{\pi/2} \left((1)^3 - 3(1)^2(\cos^2 y) + 3(1)(\cos^2 y)^2 - (\cos^2 y)^3\right) \sin y\,dy$

$\int_0^{\pi/2} (1 - 3\cos^2 y + 3\cos^4 y - \cos^6 y) \sin y\,dy$

$\int_0^{\pi/2} \sin y - 3\int_0^{\pi/2} \cos^2 y \sin y\,dy + 3\int_0^{\pi/2} \cos^4 y \sin y\,dy - \int_0^{\pi/2} \cos^6 y \sin y\,dy$

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$$\left[-\cos y+\cos^3 y \mp \frac{3}{5}\cos^5 y+\frac{1}{7}\cos^7 y\right]_0^{\pi/2}$$

$$[0]-\left[-1+1\mp\frac{3}{5}+\frac{1}{7}\right] \Rightarrow 1\mp 1+\frac{3}{5}\mp\frac{1}{7} \Rightarrow \frac{\cancel{35}\mp\cancel{35}+21\mp 5}{35} = \boxed{16/35}$$

(16) $\int 7\cos^7 t\,dt$

$$7\int \cos^6 t \cos t\,dt$$

$$7\int (\cos^2 t)^3 \cos t\,dt$$

$$7\int (1-\sin^2 t)^3 \cos t\,dt$$

$$7\left[\int (1-3\sin^2 t+3\sin^4 t-\sin^6 t)\cos t\,dt\right]$$

$$7\left[\int \cos t\,dt-3\int \sin^2 t\cos t\,dt+3\int \sin^4 t\cos t\,dt-\int \sin^6 t\cos t\,dt\right]$$

$$7\left[+\sin t \mp \cancel{3}\sin^3 t+\frac{3}{5}\sin^5 t\mp\frac{1}{7}\sin^7 t\right]+C$$

$$\boxed{+7\sin t\mp 7\sin^3 t+\frac{21}{5}\sin^5 t\mp\sin^7 t+C}$$

(17) $\int_0^{\pi} 8\sin^4 x\,dx$

$$8\left[\int_0^{\pi}(\sin^2 x)^2\,dx\right]$$

$$8\left[\int_0^{\pi}(1-\cos^2 x)^2\,dx\right]$$

$$8\left[\int_0^{\pi}(1-2\cos^2 x+\cos^4 x)\,dx\right]$$

(17) $\int_0^{\pi} 8\sin^4 x\,dx$

$$8\int_0^{\pi}\left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$\frac{2\cancel{8}}{\cancel{4}}\int_0^{\pi}(1-\cos 2x)^2\,dx$$

$$2\int_0^{\pi}(1-2\cos 2x+\cos^2 2x)\,dx$$

$$2\int_0^{\pi}dx-2\int_0^{\pi}\cos 2x\cdot 2\,dx+2\int_0^{\pi}\left(\frac{1+\cos 4x}{2}\right)dx$$

$$\left[2x-2\sin 2x\right]_0^{\pi}+\int_0^{\pi}(1+\cos 4x)\,dx$$

$$\left[2x-2\sin 2x\right]_0^{\pi}+\int_0^{\pi}dx+\int_0^{\pi}\cos 4x\,dx$$

$$2\pi+\left[x+\frac{1}{2}\sin 4x\right]_0^{\pi} \Rightarrow 2\pi+\pi$$

$$=\boxed{3\pi}$$

(18) $\int 8\cos^4 2\pi x\,dx$

$8\int \left(\cancel{1-\sin}\,\frac{1+\cos 4\pi x}{2}\right)^2 dx$

$2\int (1+\cos 4\pi x)^2 dx$

$2\int 1+2\cos 4\pi x+\cos^2 4\pi x)\,dx$

$2\int dx + 4\int \cos 4\pi x\,dx + 2\int\left(\frac{1+\cos 8\pi x}{2}\right)dx$

$3\int dx + 4\int \cos 4\pi x\,dx + \int \cos 8\pi x\,dx$

$3x + \frac{\cancel{4}}{\cancel{4}\pi}\sin 4\pi x + \frac{1}{8\pi}\sin 8\pi x + C$

$\boxed{3x + \frac{1}{\pi}\sin 4\pi x + \frac{1}{8\pi}\sin 8\pi x + C}$

(19) $\int 16\sin^2 x\cos^2 x\,dx$

$16\int\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx$

$4\int (1-\cos^2 2x)\,dx$

$4\int dx - 4\int\left(\frac{1-\cos 4x}{2}\right)dx$

~~$4x - 2\int dx - 2\int \cos 4x\,dx$~~

$4x - 2x - \frac{1}{2}\sin 4x + C$

~~$2x - \frac{1}{2}\sin 4x + C$~~

$\therefore \sin 4x = 2\sin 2x\cos 2x$

~~$2x - \sin 2x\cos 2x + C$~~

~~$\therefore \sin 2x = 2\sin x\cos x$~~

$\therefore \cos 2x = 2\cos^2 x - 1$

$2x - 2\sin x\cos x(2\cos^2 x - 1) + C$

$\boxed{2x - 4\sin x\cos^3 x + 2\sin x\cos x + C}$

(20) $\int_0^{\pi} 8\sin^4 y\cos^2 y\,dy.$

$8\int_0^{\pi} (\sin^2 y)^2\left(\frac{1+\cos 2y}{2}\right)dy$

$8\int_0^{\pi}\left(\frac{1-\cos 2y}{2}\right)^2\left(\frac{1+\cos 2y}{2}\right)dy$

$\frac{\cancel{8}}{\cancel{8}}\int_0^{\pi}(1-\cos 2y)^2(1+\cos^2 y)\,dy$

$\int_0^{\pi}(1-2\cos 2y+\cos^2 2y)(1+\cos^2 y)\,dy$

~~$\int_0^{\pi}(1+\cos^2 y - 2\cos^2 y - 2\cos 2y\cos^2 y + \cos^2 2y + \cos^4 2y$~~

$\int_0^{\pi} dy - \int_0^{\pi}\cos 2y\,dy - \int_0^{\pi}\cos^2 2y\,dy +$

$\int_0^{\pi}\cos^3 2y\,dy$

$\left[y - \frac{1}{2}\sin 2y\right]_0^{\pi} - \int_0^{\pi}\left(\frac{1-\cos 4y}{2}\right)dy$

$+\int_0^{\pi}(1-\sin^2 2y)\cos 2y\,dy\,dy$

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$$\pi - \frac{1}{2}\int_0^{\pi} dy - \frac{1}{2}\int_0^{\pi} \cos 4y\,dy + \int_0^{\pi} \cos 2y\,dy - \int_0^{\pi} \sin^2 2y \cos 2y\,dy.$$

$$\pi + \left[-\frac{1}{2}y - \frac{1}{8}\sin 4y + \frac{1}{2}\sin 2y - \frac{1}{2}\cdot\frac{\sin^3 2y}{3} \right]_0^{\pi}$$

$$\pi - \frac{\pi}{2} \Rightarrow \boxed{\frac{\pi}{2}}.$$

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(21) $\int 8\cos^3 2\theta \sin 2\theta\, d\theta$

$8\int \cos^3 2\theta \sin 2\theta\, d\theta$

$\cancel{8}\left(-\frac{1}{\cancel{2}}\right)\frac{\cos^4 2\theta}{\cancel{4}} + c$

$\boxed{-\cos^4 2\theta + c}$

(22) $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\, d\theta$

$\int_0^{\pi/2} \sin^2 2\theta \cos^2 2\theta \cos 2\theta\, d\theta$

$\int_0^{\pi/2} \sin^2 2\theta (1-\sin^2 2\theta) \cos 2\theta\, d\theta.$

$\int_0^{\pi/2} \sin^2 2\theta \cos 2\theta\, d\theta - \int \sin^4 2\theta \cos 2\theta\, d\theta$

$\left[\frac{1}{6}\sin^3 2\theta - \frac{1}{10}\sin^5 2\theta\right]_0^{\pi/2}$

$[0-0] = \boxed{0}.$