① $(x-y)dx + xdy = 0$

As we know that is homogeneous, Sol ⇒

$\boxed{x = vy}$

$\boxed{\frac{dx}{dy} = v\,dy + y\,dv}$

$\boxed{y = ux}$

$\boxed{dy = u\,dx + x\,du}$

Make your own neural network

this correct.

As ~~$M(x) = (x-y)$~~ $\quad x = vy$

$N(x) \quad dx = v\,dy + y\,dv$

① a

$((vy)-y)(v\,dy + y\,dv) + (vy)dy = 0$

$(vy-y)(v\,dy+y\,dv) + vy\,dy = 0$

$v^2y\,dy + vy^2dv - \cancel{vy\,dy} - y^2dv + \cancel{vy\,dy} = 0$

$v^2y\,dy + vy^2dv - y^2dv = 0$

$v^2y\,dy + y^2(v-1)dv = 0$

$y^2(1-v)dv = v^2y\,dy$

$\frac{(1-v)\,dv}{v^2} = \frac{\cancel{y}}{y^2}dy$

$\int\left(\frac{1}{v^2} - \frac{1}{v}\right)dv = \int\frac{1}{y}dy$

$\int\frac{v^{-2+1}}{-2+1}dv - \int\frac{1}{v}dv = \int\frac{1}{y}dy$

$\frac{-1}{v} - \ln|v| + c = \ln|y|$

$\therefore v = \frac{x}{y}$

$= \frac{1}{x/y} - \ln\left(\frac{x}{y}\right) + c = \ln(y)$

$-\frac{y}{x} - \ln x + \cancel{\ln y} + c = \cancel{\ln y}$

$-\frac{y}{x} - \ln x = -c$

Multiply by x.

$-y - x\ln x = -cx$

multiple by $-ve$.

$y + x\ln x = cx$

✓ $\boxed{y = -x\ln x + cx}$

② $(x+y)dx + xdy = 0$.

~~$x = vy$~~, $dx = v\,dy + y\,dv$.

~~$((vy)+y)dx\,(v\,dy+y\,dv) + x(vy)dy = 0$~~

~~$v^2y\,dy + vy^2dv + vy\,dy + y^2dv + vy\,dy = 0$~~

$y = ux$, $dy = u\,dx + x\,du$.

$(x+(ux))dx + x(u\,dx + x\,du) = 0$.

$x\,dx + ux\,dx + xu\,dx + x^2du = 0$.

$x\,dx + 2ux\,dx + x^2du = 0$

$x(1+2u)dx + x^2du = 0$

$x(1+2u)dx + x^2 du = 0$

$x(1+2u)dx = -x^2 du$

$\cancel{-(1+2u)du = \frac{x^2 du}{x}}$

$\frac{x dx}{x^2} = -\frac{du}{(1+2u)}$

$\int \frac{1}{x} dx = -\int \frac{1}{(1+2u)} du$

$\ln|x| + c = -\frac{1}{2} \ln(1+2u)$

Multiple by 2.

$2\ln x + 2c = -\ln(1+2u)$.

$e^{2\ln x} + 2c = -e^{\ln(1+2u)}$

$x^2 + c = -(1+2u)$

$\cancel{x^2 + c = 2u - 1}$

$2u - 1 = x^2 + c$

$\therefore u = \frac{y}{x}$.

$2(\frac{y}{x}) - 1 = x^2 + c$

Multiply by x.

$2y - x =$

$\int (3x^2)(\frac{1}{6x} e^{3x^2})$

$\int x e^{3x^2}$

$\frac{1}{2}\int e^{3x^3} dx$

$\frac{1}{1} du + \frac{1}{2u}$

$\frac{1}{2} \times \frac{1}{9} e^{3x^3}$

$\frac{u^{-1+1}}{2}$

$\frac{1}{2}\int (\frac{x^2}{2})(\frac{1}{6x} e^{3x^2})$

$\frac{xe^{3x^2}}{24}$

$\frac{1}{2}\int \frac{2du}{2u+1}$

$y = ux$.

$(1+2u)x dx + x^2 du = 0$.

$(1+2u)x dx = -x^2 du$

$-\int \frac{x dx}{x^2} = \int \frac{du}{2u+1}$

$-\ln x + c = \frac{1}{2} \ln(2u+1)$

Multiply by 2.

$-2\ln x + c = \ln(2u+1)$

$+e^{\ln x^{-2}} + e^c = e^{\ln(2u+1)}$

$x^{-2} \cdot c = 2u+1$.

$Cx^{-2} = 2u + 1$.

$\therefore u = \frac{y}{x}$

$Cx^{-1} = 2(\frac{y}{x}) + 1$

Multiply by x.

$Cx^{-1} = 2y + x \Rightarrow \boxed{y = \frac{C}{2x} - \frac{x}{2}}$

③ $x\,dx + (y-2x)\,dy = 0$

$x = vy$, $dx = v\,dy + y\,dv$ | $y = ux$, $dy = u\,dx + x\,du$

$x\,dx + (\cancel{y} - 2x)(u\,dx - x\,du) = 0$ with (ux) written above the cancelled y

$x\,dx + u^2x\,dx - ux^2\,du - 2xu\,dx + 2x^2\,du = 0$

$x\,dx + ux(u-2)\,dx + x^2(u-2)\,du = 0$

$x[1 + u(u-2)]\,dx + x^2(u-2)\,du = 0$

$x[1 + u^2 - 2u]\,dx + x^2(u-2)\,du = 0$

$x(u-1)^2\,dx + x^2(u-2)\,du = 0$

$$\int \frac{x\,dx}{x^2} = \int \frac{(u-2)\,du}{(u-1)^2}$$

$$\int \frac{1}{x}\,dx = \int \frac{(u-2)}{(u-1)^2}\,du$$

$$\ln x + c = \frac{u-1-1}{(u-1)^2} \Rightarrow \frac{\cancel{u-1}}{(u-1)^{\cancel{2}}} - \frac{1}{(u-1)^2}$$

$$\to \int \frac{1}{u-1}\,du - \int \frac{1}{(u-1)^2}\,du$$

$$= \ln(u-1) + \frac{1}{u-1} = \ln x + c$$

$\therefore u = y/x$

$$\ln(y/x - 1) + \frac{1}{(y/x)-1} = \ln x + c$$

$$\ln\left(\frac{y}{x} - 1\right) + \frac{1}{(y/x)-1} = \ln x + c$$

$$\ln x\left[\cancel{x}\left(\frac{y}{\cancel{x}} - 1\right)\right] + \frac{x}{y-x} = c$$

$$\ln(y-x) + \frac{x}{y-x} = c \quad \times(y-x)$$

$$\boxed{(y-x)\ln(y-x) + x = c(y-x)}$$

$\cancel{+ux(1-2u)\,dx - 2x^2(1+2u)\,du = 0}$

dx

④ $y\,dx = 2(x+y)\,dy$.

$y\dfrac{dx}{\cancel{10}} = 2x + 2y\,dy$

$\to y = ux$, $dy = u\,dx + x\,du$.

$x = vy$, $dx = v\,dy + y\,dv$.

$M(x,y) + N(x,y) = 0$.

$M(x,y) = x$

$M(tx,ty) = tx = M(x,y)$

$N(x,y) = -2(x+y)$

$= N(x,y)$.

So, $M(x,y) = N(x,y)$.

So, $\cancel{(ux)\,du = 2x + 2(ux)(u\,dx + x\,du)}$

$\cancel{ux\,dx = 2x + 2u^2x\,dx + 2ux^2\,du}$.

$ux\,dx - 2u^2x\,dx = 2x + 2ux^2\,du$.

$y\,dx - 2(x+y)\,dy = 0$.

$(ux)\,dx - 2(x + ux)(u\,dx + x\,du)$.

$= ux\,dx - 2xu\,dx - 2x^2\,du - 2u^2x\,dx - 2ux^2\,du = 0$

$- ux\,dx - 2x^2\,du - 2u^2x\,dx - 2ux^2\,du = 0$

$$uxdx - 2xudu - 2x^2du - 2u^2xdx - 2ux^2du = 0$$

$$-2uxdx - 2u^2xdx - 2x^2du - 2ux^2du = 0$$

$$x(-u - 2u^2)dx = 2x^2(1+u)du$$

$$\frac{\cancel{x}dx}{2x^{\cancel{2}}} = \frac{u+1}{(-u-2u^2)}du$$

$$\int \frac{1}{2}x^{-1}dx = \int \frac{u+1}{-u-2u^2}du$$

$$\frac{1}{2}\ln|x| = \int\left(\frac{A}{u} + \frac{B}{2u+1}\right)du$$

$$A(2u+1) + B(u) = u+1$$

$$2Au + Bu + A = u+1 \longrightarrow u(2A+B) + \underline{A} = u+1$$

$$2A + B = 1 \qquad \boxed{A = 1}$$

$$2 + B = 1$$

$$B = 1-2$$

$$\boxed{B = -1}$$

So, $\frac{1}{2}\ln(x) = \int\left(\frac{1}{u} + \frac{-1}{2u+1}\right)du$

$$\frac{1}{2}\ln(x) = \int\left(\frac{1}{2u+1} - \frac{1}{u}\right)du$$

$$\frac{1}{2}\ln(x) = \int\frac{1}{2u+1}du - \int\frac{1}{u}du$$

$$\frac{1}{2}\ln(x) = \quad v = 2u+1$$

$$dv = 2du$$

$$\frac{1}{2}dv = du$$

$$\frac{1}{\cancel{2}}\ln(x) = \frac{1}{\cancel{2}}\ln(2u+1) - \ln(u) + c$$

$$e^{\ln(x)} = e^{\ln(2u+1)} - e^{\ln(u)} + e^c$$

$$x = 2u+1 - uc$$

$$x = 2(y/x) + 1 - (y/x)c$$

$$\therefore u = y/x$$

$$\times x \qquad x^2 = 2y + \cancel{x} - yc$$

$$\boxed{x = 2y - yc}$$

$$y\,dx = 2(x+y)dy$$

$$x = vy$$

$$dx = v\,dy + y\,dv$$

$$y(vdy + ydv) = 2(vy+y)dy$$

$$vydy + y^2dv - 2vydy - 2ydy = 0$$

$$-vydy + y^2dv - 2ydy = 0$$

$$y^2dv - 2ydy = vydy$$

$$y^2dv = 2ydy + vydy$$

$$y^2dv = y(2+v)dy$$

$$\frac{dv}{(v+2)} = \frac{y}{y^2}dy$$

$$\int\frac{1}{(v+2)}dv = \int\frac{1}{y}dy$$

$$\ln(v+2) = \ln y + c$$

$$\therefore v = \frac{x}{y}$$

$$\cancel{\ln\left(\frac{x}{y}+2\right) = \ln y + c}$$

$$v + 2 = yC$$

$$y \times \frac{x}{y} + 2xy = yC \times y$$

$$\boxed{x + 2y = y^2c}$$

⑤ $(y^2+yx)dx - x^2dy = 0$

$(y^2+yx)dx - x^2dy = 0$

$y = ux \quad , \quad dy = udx + xdu$

$((ux)^2 + (ux)x)dx - x^2(udx + xdu) = 0$

$(u^2x^2 + ux^2)dx - x^2udx + x^3du = 0$

$u^2x^2dx + \cancel{ux^2dx} - \cancel{x^2udx} + x^3du = 0$

$u^2x^2dx = -x^3du$

$\frac{\cancel{x^2}dx}{x^3} = -\frac{du}{u^2}$

$\int \frac{1}{x}dx = -\int \frac{1}{u^2}du$

$\ln|x| + C = -u^{-1}$

$\ln|x| + C = -\frac{1}{u}$ $\quad \because u = y/x$

$\ln|x| + C = -\frac{x}{y}$

$\frac{1}{\ln|x| + C} = \frac{y}{x}$

$\boxed{y = \frac{x}{\ln|x| + C}}$

⑥ $(y^2+yx)dx + x^2dy = 0$

$(y^2+yx)dx + x^2dy = 0$

$y = ux, \quad dy = udx + xdu$

$(u^2x^2 + ux^2)dx + x^2(udx + xdu) = 0$

$u^2x^2dx + ux^2dx + x^2udx + x^3du = 0$

$x^2(u^2dx + udx + udx + xdu) = 0$

$u^2dx + 2udx + xdu = 0$

$u(u+2)dx + xdu = 0$

$u(u+2)dx = -xdu$

$\frac{dx}{x} = -\frac{du}{u(u+2)}$

$\int \frac{1}{x}dx = -\int \frac{1}{u(u+2)}du$

$\ln(x) + C = -\frac{1}{2}\ln(u) - \frac{1}{2}\ln(u+2) \rightarrow \times 2$

$2\ln|x| + C = -\ln u - \ln(u+2)$

$e^{\ln x^2 + C} = \ln\left(\frac{u}{u+2}\right)$

$C/x^2 = \frac{u}{u+2}$

$\because u = y/x$

$\frac{C}{x^2} = \frac{y/x}{y/x + 2}$

$\frac{c}{x^2} = \frac{y}{y+2x}$

$cy + 2cx = yx^2$

$\cancel{y = \frac{cy + 2x}{x^2}}$

$cy - yx^2 = 2cx$

$y(x^2 - c) = 2cx$

$\boxed{y = \frac{2cx}{(x^2 - c)}}$ Ans:

⑦ $\frac{dy}{dx} = \frac{y-x}{y+x}$

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$\left(u + x\frac{du}{dx}\right) = \frac{(ux) - x}{(ux) + x}$

$x\frac{du}{dx} = \frac{(ux) - x - (u(ux) + x)}{(ux) + x}$

$x\frac{du}{dx} = \frac{\cancel{ux} - x - u^2x - \cancel{ux}}{ux + x}$

$x\frac{du}{dx} = -\frac{u^2x - x}{ux + x}$

$x\frac{du}{dx} = -\frac{\cancel{x}(u^2+1)}{\cancel{x}(u+1)}$

$x\frac{du}{dx} = -\frac{(u^2+1)}{(u+1)}$

$\int \frac{(u+1)}{(u^2+1)}du = -\int \frac{dx}{x}$

$\int \frac{u}{(u^2+1)} + \frac{1}{(u^2+1)}du = -\ln x + C$

$\frac{1}{2}\ln(u^2+1) + \tan^{-1}u = -\ln x + C$

$\cancel{2}\frac{1}{\cancel{2}}\ln\left(\frac{y^2}{x^2}+1\right) + 2\tan^{-1}\left(\frac{y}{x}\right) = -2\ln x + C$

$\ln\left(\left(\frac{y}{x}\right)^2+1\right) + 2\tan^{-1}\left(\frac{y}{x}\right) = -2\ln x + C$

$\ln\left(\frac{y^2+x^2}{\cancel{x^2}}\right) + \cancel{\ln x^2} + 2\tan^{-1}\left(\frac{y}{x}\right) = C$

$\boxed{\ln(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = C}$

CS CamScanner

(12) (8) $\frac{dy}{dx} = \frac{x+3y}{3x+y}$.

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$.

$\left(u + x\frac{du}{dx}\right) = \frac{x+3(ux)}{3x+(ux)}$

$u + x\frac{du}{dx} = \frac{\cancel{x}(1+3u)}{\cancel{x}(3+u)}$

$x\frac{du}{dx} = \frac{1+3u}{3+u} - u$

$x\frac{du}{dx} = \frac{1+\cancel{3u}-\cancel{3u}-u^2}{3+u}$

$x\frac{du}{dx} = \left(\frac{1-u^2}{3+u}\right)$

$\int\left(\frac{3+u}{1-u^2}\right)du = \int\frac{dx}{x}$

$\int\left(\frac{2}{1-u} + \frac{1}{1+u}\right)du = \int\frac{dx}{x}$

$-2\ln(1-u) + \ln(1+u) = \ln|x| + C$

$\ln\left(\frac{1+u}{(1-u)^2}\right) = \ln|x| + C$

$\therefore u = y/x$

$\ln\left(\frac{(1+y/x)}{(1-y/x)^2}\right) = \ln|x| + C$

$e^{\ln}\left(\frac{x+y}{x} \times \left(\frac{x}{x-y}\right)^2\right) = e^{\ln|x|+C}$

$\frac{\cancel{x}(x+y)}{(x-y)^2} = Cx$

$\boxed{(x-y)^2 = C(x+y)}$

(9) $-y\,dx + (x + \sqrt{xy})\,dy = 0$.

$x = vy$

$dx = v\,dy + y\,dv$.

$-y(v\,dy + y\,dv) + (vy + \sqrt{(vy)y})\,dy = 0$

$-vy\,dy - y^2dv + (vy + \sqrt{vy^2})\,dy = 0$

$-\cancel{vy\,dy} - y^2dv + \cancel{vy\,dy} + y\sqrt{v}\,dy = 0$

$-y^2dv + y\sqrt{v}\,dy = 0$

$\frac{y\,dy}{y^2} = \frac{dv}{\sqrt{v}}$

$\int\frac{1}{y}dy = \int\frac{1}{\sqrt{v}}dv$

$\ln|y| + C = 2\sqrt{v}$

$(\ln|y| + C)^2 = \left(2\sqrt{\frac{x}{y}}\right)^2$

$(\ln|y| + C)^2 = 4\left(\frac{x}{y}\right)$

$\boxed{4x = y(\ln|y| + C)^2}$

(10) $x\frac{dy}{dx} = y + \sqrt{x^2-y^2}$, $\boxed{x>0}$

$-\frac{1}{2}+1$, $\frac{-1+2}{2} = \frac{1}{2}$.

$x\frac{dy}{dx} = y + \sqrt{x^2-y^2}$

$x\frac{dy}{dx} - y - \sqrt{x^2-y^2} = 0$

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$x\left(u + x\frac{du}{dx}\right) = (ux) - \sqrt{x^2-(ux)^2}$

$xu + x^2\frac{du}{dx} = ux - \sqrt{x^2-u^2x^2}$

$\cancel{xu} + x^2\frac{du}{dx} = \cancel{ux} - x^2\sqrt{1-u^2}$

$x^2\frac{du}{dx} = -x\sqrt{1-u^2}$

$\frac{du}{\sqrt{1-u^2}} = -\frac{\cancel{x^2}\,dx}{\cancel{x\,x}}$

$\int\frac{1}{\sqrt{1-u^2}}du = -\int\frac{1}{x}dx$

$\sin^{-1}(u) = -\ln(x) + C$

$\boxed{\sin^{-1}(y/x) = -\ln(x) + C}$

(11) $xy^2\frac{dy}{dx} = y^3 - x^3$, $y(1) = 2$

$\therefore y = ux$

$\therefore \frac{dy}{dx} = u + x\frac{du}{dx}$

$x(ux)^2\left(u + x\frac{du}{dx}\right) = (ux)^3 - x^3$

$\cancel{u^3x^3} + x^4u^2\frac{du}{dx} = \cancel{u^3x^3} - x^3$

$x^4u^2\frac{du}{dx} = -x^3$

$u^2du = -\frac{x^3}{x^4}dx$

$\int u^2du = -\int\frac{1}{x}dx$.

$\frac{1}{3}u^3 = -\ln|x| + C$

$\frac{1}{3}\left(\frac{y}{x}\right)^3 = -\ln|x| + C$

$y^3/3x^3 = -\ln|x| + C$

$y^3 = -3x^3\ln|x| + 3x^3C$ — (i)

$\therefore y = 2, x = 1$

$(2)^3 = -3(1)^3\ln(1) + 3(1)^3C$

$8 = 0 + 3C$ in eq-(i)

$\boxed{C = 8/3}$.

$y^3 = -3x^3\ln|x| + \cancel{3}x^3(8/\cancel{3})$

$\boxed{y^3 = -3x^3\ln|x| + 8x^3}$

(12) $(x^2+2y^2)\frac{dx}{dy} = xy$, $y(-1)=1$ ⇒ $\boxed{2x^4 = y^2+x^2}$ ⇒ $\boxed{y^2 = 2x^4 - x^2}$

$(x^2+2y^2)\frac{dx}{dy} = xy$;

$x = vy$. $y = ux$

$\frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{dy}{dx} = u + x\frac{du}{dx}$

$x^2 + 2(ux)^2 = x(ux)\left(u + x\frac{du}{dx}\right)$

$x^2 + 2u^2x^2 = ux^2\left(u + x\frac{du}{dx}\right)$

$x^2 + 2u^2x^2 = u^2x^2 + ux^3\frac{du}{dx}$

$x^2 + u^2x^2 = ux^3\frac{du}{dx}$

$x^2(1+u^2) = ux^3\frac{du}{dx}$

$\int\frac{x^2\,dx}{x^3} = \int\frac{u}{(1+u^2)}\,du$

$\ln|x| + C = \frac{1}{2}\ln(u^2+1)$

$\times 2$ $e^{2\ln|x| + C} = e^{\ln(u^2+1)}$

$Cx^2 = u^2+1$

$\therefore (u = y/x)$

$Cx^2 = \left(\frac{y}{x}\right)^2 + 1$

$Cx^2 = \frac{y^2}{x^2} + 1$

$Cx^2 \times x^2 = \frac{y^2+x^2}{x^2} \times x^2$

$\boxed{Cx^4 = y^2 + x^2}$ — i)

$\therefore y = 1$, $x = -1$

$C(-1)^4 = (1)^2 + (-1)^2$

$C = 1+1$ ⇒ $\boxed{C = 2}$

(13) $(x + ye^{y/x})dx - xe^{y/x}dy = 0$ $y(1) = 0$.

$(x + ye^{y/x})dx - xe^{y/x}dy = 0$

$y = ux$, $dy = u\,dx + x\,du$.

$(x + (ux)e^{ux/x})dx - xe^{ux/x}(u\,dx + x\,du) = 0$.

$(x + uxe^u)dx - xe^u(u\,dx + x\,du) = 0$.

$x\,dx + \cancel{uxe^u dx} - \cancel{uxe^u dx} \mp x^2e^u du = 0$

$x\,dx \mp x^2e^u du = 0$

$x\,dx = +x^2e^u du$

$\frac{x\,dx}{x^2} = +e^u du$

$\int\frac{1}{x}dx = +\int e^u du$

$\ln|x| + C = e^u$.

$\boxed{e^{y/x} = \ln|x| + C}$

$y = 0$, $x = 1$

$e^{0/1} = \ln|1| + C$

$\boxed{C = 1}$.

$\boxed{e^{y/x} = \ln|x| + 1}$

(14) $y\,dx + x(\ln x - \ln y - 1)\,dy = 0$; $y(1) = e$

$x = vy$, $dx = v\,dy + y\,dv$.

$y(v\,dy + y\,dv) + (vy)(\ln(vy) - \ln y - 1)\,dy = 0$.

$vy\,dy + y^2dv + vy(\ln v + \cancel{\ln y} - \cancel{\ln y} - 1)\,dy = 0$

$\cancel{vy\,dy} + y^2dv + vy\ln v\,dy - \cancel{vy\,dy} = 0$

$y^2dv + vy\ln v\,dy = 0$

$-\frac{y\,dy}{y^2} = +\frac{dv}{v\ln v}$

$-\int\frac{1}{y}dy = +\int\frac{1}{v\ln v}dv$

$\therefore u = v$, $\frac{dv}{v} = du$.

$-\ln y + C = \ln u$

$\ln u + \ln y = C$

$\ln v + \ln y = C$

$\therefore v = x/y$

$\ln vy = C$

$y\ln\left(\frac{x}{y}\right) = C$

$\boxed{y(\ln x - \ln y) = C}$ — i)

$\therefore y = e$, $(x = 1)$

$e(\ln(1) - \ln(e)) = C$

$C = -e\ln e$ $\therefore e\ln e = e$

$\boxed{C = -e}$

$\boxed{y(\ln x - \ln y) = -e}$

(15) $x\dfrac{dy}{dx}+y=\dfrac{1}{y^2}$

Divide by x.

$$\frac{dy}{dx}+\frac{y}{x}=\frac{1}{xy^2}$$

As we know;

Multiply by y^2

$$y^2\frac{dy}{dx}+\frac{y^3}{x}=\frac{1}{x}$$

$$u=y^3;\ \frac{du}{dy}=3y^2,\ \frac{dy}{dx}=\frac{1}{3y^2}\frac{du}{dx}$$

$$y^2\left[\frac{1}{3y^2}\frac{du}{dx}\right]+\frac{u}{x}=\frac{1}{x}$$

$$\frac{1}{3}\frac{du}{dx}+\frac{u}{x}=\frac{1}{x}$$

Multiple by 3.

$$\frac{du}{dx}+\frac{3u}{x}=\frac{3}{x}$$

$$\frac{du}{dx}=\frac{3}{x}-\frac{3u}{x}$$

$$\frac{du}{dx}=\frac{3(1-u)}{x}$$

$$\int\frac{du}{1-u}=\int\frac{3}{x}dx$$

$$-\ln|1-u|=3\ln x+c$$

$$\ln\left(\frac{1}{1-u}\right)=\ln x^3+c$$

$$e^{\ln\left(\frac{1}{1-u}\right)}=e^{\ln x^3+c}$$

$$\frac{1}{1-u}=Cx^3$$

$\therefore u=y^3$.

$$\frac{1}{1-y^3}=Cx^3$$

$$\frac{1}{Cx^3}=1-y^3$$

$$y^3=1-\frac{1}{Cx^3}$$

$$\boxed{y^3=1+Cx^{-3}}$$

(16) $\dfrac{dy}{dx}-y=e^xy^2$

Sol.

$$\frac{dy}{dx}-y=e^xy^2$$

Divide by y^2

$$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}=e^x$$

$$u=\frac{1}{y},\ \frac{dy}{dx}=-y^2\frac{du}{dx}$$

$$\cancel{\frac{du}{dy}=-\frac{1}{y^2}}$$

$$\frac{1}{y^2}\left[-y^2\frac{du}{dx}\right]-u=e^x$$

$$-\frac{du}{dx}-u=e^x$$

$$\frac{du}{dx}+u=-e^x$$

$$I.F\Rightarrow e^{\int dx}=e^x$$

$$e^x\left[\frac{du}{dx}+u\right]=e^x(-e^x)$$

$$\int\frac{d(e^xu)}{dx}=-\int e^{2x}$$

$$e^xu=-\frac{1}{2}e^{2x}+C$$

Divide by e^x

$$\frac{e^xu}{e^x}=-\frac{1}{2}\frac{e^{2x}}{e^x}+\frac{C}{e^x}$$

$$u=-\frac{1}{2}+Ce^{-x}$$

$\therefore\left(u=\dfrac{1}{y}\right)$

$$\frac{1}{y}=-\frac{1}{2}+Ce^{-x}$$

$$\boxed{y=\frac{1}{-\frac{1}{2}+Ce^{-x}}}$$

(17) $\dfrac{dy}{dx}=y(xy^3-1)$

$$\frac{dy}{dx}=xy^4-y$$

$$\frac{dy}{dx}+y=xy^4$$

divide by y^4

$$\frac{1}{y^4}\frac{dy}{dx}+\frac{1}{y^3}=x$$

$$u=\frac{1}{y^3},\ \frac{du}{dy}=-\frac{3}{y^4}$$

$$\frac{dy}{dx}=-\frac{y^4}{3}\frac{du}{dx}$$

$$\frac{1}{y^4}\left[-\frac{y^4}{3}\frac{du}{dx}\right]+u=x$$

$$-\frac{1}{3}\frac{du}{dx}+u=x$$

Multiply by.

$$\frac{du}{dx}=(x-u)3$$

$$\frac{du}{dx}=+3x-3u$$

$$\frac{du}{dx}-3u=3x$$

$$e^{\int-3dx}=\boxed{e^{-3x}}$$

$$e^{-3x}\left[\frac{du}{dx}-3u\right]=(3x)e^{-3x}$$

$$\int\frac{d(ue^{-3x})}{dx}=-\int 3xe^{-3x}$$

$$ue^{-3x}=xe^{-3x}+\frac{1}{3}e^{-3x}+C$$

Divide by e^{-3x}

$$u=x+\frac{1}{3}+Ce^{+3x}$$

$$\therefore\boxed{u=\frac{1}{y^3}}$$

$$\frac{1}{y^3}=x+\frac{1}{3}+Ce^{3x}$$

$$\boxed{y^{-3}=x+\frac{1}{3}+Ce^{3x}}$$

Exercise # 2.5

(18) $x\frac{dy}{dx} - (1+x)y = xy^2$

divide by x.

$$\frac{dy}{dx} - \left(\frac{1+x}{x}\right)y = y^2$$

$$\frac{dy}{dx} - \left(\frac{1}{x}+1\right)y = y^2$$

divide by y^2

$$\frac{1}{y^2}\frac{dy}{dx} - \left(\frac{1}{x}+1\right)\left(\frac{1}{y}\right) = 1 \quad \text{—i)}$$

$$u = \frac{1}{y}, \quad \frac{du}{dy} = -\frac{1}{y^2}$$

$$\frac{dy}{dx} = -y^2\frac{du}{dx}$$

$$\frac{1}{\cancel{y^2}}\left(-\cancel{y^2}\frac{du}{dx}\right) - \left(\frac{1}{x}+1\right)u = 1$$

$$-\frac{du}{dx} - \frac{u}{x} - u = 1$$

$$\frac{du}{dx} + \frac{u}{x} + u = -1$$

$$\frac{du}{dx} + \left(\frac{1}{x}+1\right)u = -1$$

I.F. $e^{\int(1+\frac{1}{x})dx} = e^{x+\ln x} = \boxed{xe^x}$

$$xe^x\left[\frac{du}{dx}\left(\frac{1}{x}+1\right)u\right] = -xe^x$$

$$\int\frac{d}{dx}(uxe^x) = -\int xe^x dx$$

$$uxe^x = -xe^x + e^x + c$$

divide by xe^x

$$\therefore \frac{uxe^x}{xe^x} = \frac{-xe^x}{xe^x} + \frac{e^x}{xe^x} + \frac{c}{xe^x}$$

$$u = -1 + \frac{1}{x} + Cx^{-1}e^{-x}$$

$$\frac{1}{y} = -1 + \frac{1}{x} + Cx^{-1}e^{-x}$$

$$\boxed{y^{-1} = -1 + \frac{1}{x} + Cx^{-1}e^{-x}}$$

(19) $t^2\frac{dy}{dt} + ty^2 = ty$

divide by t^2

$$\frac{dy}{dt} + \frac{y^2}{t^2} = \frac{y}{t}$$

Divide by y^2

$$\frac{1}{y^2}\frac{dy}{dt} + \frac{1}{t^2} = \frac{1}{ty} \quad \text{—i)}$$

$$\cancel{u = \frac{1}{t^2}, \quad \frac{du}{dt} = \frac{2}{y^3}}$$

$$\frac{1}{y^2}\frac{dy}{dt} - \frac{1}{t}y = -\frac{1}{t^2}$$

$$u = \frac{1}{y}, \quad \frac{du}{dy} = -\frac{1}{y^2}, \quad \frac{dy}{dt} = -y^2\frac{du}{dt}$$

$$\frac{1}{\cancel{y^2}}\left(-\cancel{y^2}\frac{du}{dt}\right) - \frac{ut}{\cancel{}} = -\frac{1}{t^2}$$

$$-\frac{du}{dt} - u\frac{1}{t} = -\frac{1}{t^2}$$

$$\frac{du}{dt} + u\frac{1}{t} = +\frac{1}{t^2}$$

$$\frac{du}{dt} + \frac{1}{t}u = +\frac{1}{t^2}$$

$$I.F = e^{\int \frac{1}{t} dt} = e^{\ln t} = \boxed{t}.$$

$$t\left[\frac{du}{dt} + \frac{1}{t}u\right] = \left(\frac{1}{t^2}\right)t$$

$$\frac{d(tu)}{dt} = \frac{1}{t}$$

$$\int d(tu) = \int \frac{1}{t}\,dt$$

$$tu = \ln t + c$$

$$e^{tu} = tc$$

$$Ct = e^{tu}$$

$$\therefore u = \frac{1}{y}$$

$$Ct = e^{t\left(\frac{1}{y}\right)}$$

$$\boxed{Ct = e^{t/y}}$$

(20) $$3(1+t^2)\frac{dy}{dt} = 2ty(y^3-1)$$

$$3(1+t^2)\frac{dy}{dt} = 2ty^4 - 2ty$$

divide by y^4.

$$\frac{3(1+t^2)}{y^4}\frac{dy}{dt} = 2t - \frac{2t}{y^3}$$

$$\frac{3(1+t^2)}{y^4}\frac{dy}{dt} + \frac{2t}{y^3} = 2t$$

$$u = \frac{1}{y^3}, \quad \frac{du}{dt} = -\frac{3}{y^4}\frac{dy}{dt}, \quad \frac{dy}{dt} = -\frac{y^4}{3}\frac{du}{dt}$$

$$\frac{3(1+t^2)}{y^4}\left(-\frac{y^4}{3}\frac{du}{dt}\right) + 2ut = 2t$$

$$-(1+t^2)\frac{du}{dt} + 2tu = 2t$$

divide by $-(1+t^2)$

$$\frac{du}{dt} - \frac{2tu}{(1+t^2)} = -\frac{2t}{1+t^2}$$

$$\frac{du}{dt} - \frac{2t}{1+t^2}u = -\frac{2t}{1+t^2}$$

$$I.F = e^{-\int \frac{2t}{1+t^2}} \Rightarrow e^{-\ln(1+t^2)} = \boxed{\frac{1}{1+t^2}}$$

$$\frac{1}{1+t^2}\left[\frac{du}{dt} - \frac{2t}{1+t^2}u\right] = -\left[\frac{2t}{1+t^2}\right]\left[\frac{1}{1+t^2}\right]$$

$$\frac{d}{dt}\left[\frac{1}{1+t^2}u\right] = \frac{-2t}{(1+t^2)^2}$$

$$\int d\left(\frac{1}{1+t^2}u\right) = -\int \frac{2t}{(1+t^2)^2}\,dt$$

$$1+t^2 \times \frac{1}{1+t^2}u = 1+t^2 \times \frac{1}{1+t^2} + C \times 1+t^2$$

$$u = 1 + C(1+t^2)$$

$$\therefore u = \frac{1}{y^3}$$

$$\frac{1}{y^3} = 1 + C(1+t^2)$$

$$y^{-3} = 1 + C(1+t^2)$$

$$\boxed{y^3 = \frac{1}{1+C(1+t^2)}}$$

(21) $x^2\frac{dy}{dx} - 2xy = 3y^4, \quad y(1) = \frac{1}{2}.$

divide by x^2

$$\frac{dy}{dx} - \frac{2xy}{x^2} = \frac{3y^4}{x^2}$$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{3y^4}{x^2}$$

divide by y^4.

$$\frac{1}{y^4}\frac{dy}{dx} - \frac{2}{xy^3} = \frac{3}{x^2}$$

$$u = \frac{1}{y^3}, \quad \frac{du}{dy} = \frac{-3}{y^4}$$

$$\frac{dy}{dx} = -\frac{y^4}{3}\frac{du}{dx}$$

$$\frac{1}{\cancel{y^4}}\left(-\frac{\cancel{y^4}}{3}\frac{du}{dx}\right] - \frac{2}{x}u = \frac{3}{x^2}$$

$$-\frac{1}{3}\frac{du}{dx} - \frac{2u}{x} = \frac{3}{x^2}$$

$x \longrightarrow -3$

$$\frac{du}{dx} + \frac{6u}{x} = \frac{9}{x^2}$$

I.F. $e^{\int \frac{6}{x}dx} \Rightarrow e^{6\ln x} = \boxed{x^6}.$

$$x^6\left[\frac{du}{dx} + \frac{6u}{x}\right] = -\left(\frac{9}{\cancel{x^2}}\right)x^6$$

$$\frac{d}{dx}(x^6u) = -9x^4$$

$$\int d(x^6u) = -\int 9x^4\,dx$$

$$x^6u = -\frac{9}{5}x^5 + C$$

$$x^6u = -\frac{9}{5}x^5 + c$$

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$$\therefore u = \frac{1}{y^3}$$

$$x^6\left(\frac{1}{y^3}\right) = -\frac{9}{5}x^5 + c \quad \text{—i)}$$

$(y = 1/2), (x = 1)$

$$(1)^6\left(\frac{1}{(1/2)^3}\right) = -\frac{9}{5}(1)^5 + c$$

$$8 = -\frac{9}{5} + c$$

$$C = 8 + \frac{9}{5}$$

$$\boxed{C = \frac{49}{5}}$$

in eq—i)

$$x^6\left(\frac{1}{y^3}\right) = -\frac{9}{5}x^5 + \frac{49}{5}$$

Divide by x^6

$$\frac{1}{y^3} = -\frac{9}{5}x^{-1} + \frac{49}{5}x^{-6}$$

$$\boxed{y^{-3} = -\frac{9}{5x} + \frac{49}{5x^6}}$$

(22) $y^{1/2}\frac{dy}{dx} + y^{3/2} = 1, \quad y(0) = 4.$

$$u = y^{3/2}, \quad \frac{du}{dy} = \frac{3}{2}y^{1/2}$$

$$\frac{dy}{dx} = \frac{2}{3y^{1/2}}\frac{du}{dx}$$

$$\cancel{y^{1/2}}\left[\frac{2}{3\cancel{y^{1/2}}}\frac{du}{dx}\right] + u = 1$$

$$\frac{2}{3}\frac{du}{dx} + u = 1$$

$$\frac{du}{dx} + \frac{3}{2}u = \frac{3}{2}$$

I.F.

$$e^{\int \frac{3}{2} dx} \Rightarrow \boxed{e^{3/2 x}}$$

$$\left[\frac{du}{dx} + \frac{3}{2}u\right] e^{3/2x} = \frac{3}{2} e^{3/2x}$$

$$\int d(e^{3/2x} u) = \frac{3}{2}\int e^{3/2x} dx$$

$$u e^{3/2x} = \frac{\cancel{3}}{\cancel{2}} \frac{e^{3/2x}}{\cancel{3/2}} + C$$

$$u e^{3/2x} = e^{3/2x} + C$$

Divide by $e^{3/2x}$

$$u = 1 + Ce^{-3/2x}$$

$\therefore u = y^{3/2}$

i) $\boxed{y^{3/2} = 1 + Ce^{-3/2x}}$

As given,

$y = 4, \; x = 0$

$$(4)^{3/2} = 1 + Ce^{-3/2(0)}$$

$$8 = 1 + C$$

$$\boxed{C = 7}$$

in eq-i)

$$\boxed{y^{3/2} = 1 + 7e^{-3/2x}}$$

(23) $\frac{dy}{dx} = (x+y+1)^2$

$u = x+y+1, \quad \frac{dy}{dx} = \frac{du}{dx} - 1$

$$\frac{du}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{du}{dx} - 1 = (u)^2$$

$$\frac{du}{dx} = u^2 + 1$$

$$du = u^2+1 \; dx$$

$$\int \frac{1}{u^2+1} du = \int dx$$

$$\tan^{-1}(u) = x + C$$

$$\tan^{-1}(x+y+1) = x + C$$

Multiply by tan.

$$\cancel{\tan}\left(\frac{1}{\cancel{\tan}}(x+y+1)\right) = \tan(x+C)$$

$$x+y+1 = \tan(x+C)$$

$$\boxed{y = \tan(x+C) - x - 1}$$

(24) $\frac{dy}{dx} = \frac{1-x-y}{x+y}$

$$\frac{dy}{dx} = \frac{1-(x+y)}{(x+y)}$$

let,

$u = (x+y), \quad \frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} = 1 + \frac{dy}{dx},$

$$\frac{du}{dx} - 1 = \frac{1-u}{u}$$

$$\frac{du}{dx} = \frac{1-u}{u} + 1$$

$$\frac{du}{dx} = \frac{1-\cancel{u}+\cancel{u}}{u}$$

$$\frac{du}{dx} = \frac{1}{u}$$

$$\int u\,du = \int dx$$

$$\frac{1}{2}u^2 = x + C$$

$\therefore x + y = u$

$$\frac{1}{2}(x+y)^2 = x + C$$

$$\boxed{\frac{1}{2}(x+y)^2 = x + C}$$

(25) $\frac{dy}{dx} = \tan^2(x+y)$

$u = x+y, \quad \frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = \tan^2(u)$

$\frac{du}{dx} = 1 + \tan^2 u$

$\frac{du}{dx} = \sec^2 u$

$\int \cos^2 u \, du = \int dx$

$\left(\frac{1+\cos 2u}{2}\right) du = dx$

$\frac{1}{2} \cancel{du}\, u + \frac{1}{4} \sin 2u = x + C$

Multiply by 4.

$2u + \sin 2u = 4x + C$

$(u = x+y)$

$2(x+y) + \sin 2(x+y) = 4x + C$

$\underline{2x} + 2y + \sin 2(x+y) - \underline{4x} = C$

$\boxed{2y - 2x + \sin 2(x+y) = C}$