Note: - *Solve by Cramer's rule, as well as by Inverse method.*

Q.No:1.

$$2x+y+2z=10$$
$$x+3y+z=10$$
$$x-2y-z=-6$$

Sol:

$$2x+y+2z=10$$
$$x+3y+z=10$$
$$x-2y-z=-6$$

finding determinant $\Rightarrow$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & -2 & -1 \end{bmatrix}$$

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$\therefore R_1 - R_3.$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3 & 1 \\ 1 & -2 & -1 \end{vmatrix}$$

$\therefore R_2 - R_1$ & $R_3 - R_1.$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 0 & 0 & -2 \\ 0 & -5 & -4 \end{vmatrix}$$

$$|A| = 1(-10+0)$$

$$\boxed{|A| = -10} \quad \text{------ a)}$$

Solving by Cramer's Rule $\Rightarrow$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & -2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 10 \\ 10 \\ -6 \end{bmatrix}$$

$$x = \frac{\begin{bmatrix} 10 & 1 & 2 \\ 10 & 3 & 1 \\ -6 & -2 & -1 \end{bmatrix}}{-10}$$

$$x = \frac{10\begin{vmatrix} 3 & 1 \\ -2 & -1 \end{vmatrix} - 1\begin{vmatrix} 10 & 1 \\ -6 & -1 \end{vmatrix} + 2\begin{vmatrix} 10 & 3 \\ -6 & -2 \end{vmatrix}}{-10}$$

$$x = \frac{10(-3+2) - 1(-10+6) + 2(-20+18)}{-10}$$

$$x = \frac{-10 + \cancel{4} - \cancel{4}}{-10}$$

$$x = \frac{\cancel{-10}}{\cancel{-10}}$$

$$\boxed{x = 1}$$

$$y = \frac{\begin{bmatrix} 2 & 10 & 2 \\ 1 & 10 & 1 \\ 1 & -6 & -1 \end{bmatrix}}{-10}$$

$$y = \frac{2\begin{vmatrix} 10 & 1 \\ -6 & -1 \end{vmatrix} - 10\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & 10 \\ 1 & -6 \end{vmatrix}}{-10}$$

$$y = \frac{2(-10+6) - 10(-1-1) + 2(-6-10)}{-10}$$

$$y = \frac{-8 + 20 - 32}{-10}$$

$$y = \frac{-20}{-10}$$

$$\boxed{y = 2}$$

$$z = \frac{\begin{bmatrix} 2 & 1 & 10 \\ 1 & 3 & 10 \\ 1 & -2 & -6 \end{bmatrix}}{-10}$$

$$z = \frac{2\begin{vmatrix} 3 & 10 \\ -2 & -6 \end{vmatrix} - 1\begin{vmatrix} 1 & 10 \\ 1 & -6 \end{vmatrix} + 10\begin{vmatrix} 1 & 3 \\ 1 & -2 \end{vmatrix}}{-10}$$

$$z = \frac{2(-18+20) - 1(-6-10) + 10(-2-3)}{-10}$$

$$z = \frac{4+16-50}{-10}$$

$$z = \frac{-30}{-10}$$

$$\boxed{z = 3}$$

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Solving by Inverse Method:

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & -2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 10 \\ 10 \\ -6 \end{bmatrix}$$

minor of $a_{11} = (-1)^{1+1}(-3+2) = -1$

minor of $a_{12} = (-1)^{1+2}(-1-1) = 2$

minor of $a_{13} = (-1)^{1+3}(-2-3) = -5$

minor of $a_{21} = (-1)^{2+1}(-1+4) = -3$

minor of $a_{22} = (-1)^{2+2}(-2-2) = -4$

minor of $a_{23} = (-1)^{2+3}(-4-1) = 5$

minor of $a_{31} = (-1)^{3+1}(1-6) = -5$

minor of $a_{32} = (-1)^{3+2}(2-2) = 0$

minor of $a_{33} = (-1)^{3+3}(6-1) = 5$

Matrix of Co-factors $= \begin{bmatrix} -1 & 2 & -5 \\ -3 & -4 & 5 \\ -5 & 0 & 5 \end{bmatrix}$

$$A^t = \begin{bmatrix} -1 & -3 & -5 \\ 2 & -4 & 0 \\ -5 & 5 & 5 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{-10}\begin{bmatrix} -1 & -3 & -5 \\ 2 & -4 & 0 \\ -5 & 5 & 5 \end{bmatrix}$$

So, as we know:

$$X = A^{-1}B.$$

$$X = \frac{1}{-10}\begin{bmatrix} -1 & -3 & -5 \\ 2 & -4 & 0 \\ -5 & 5 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \\ -6 \end{bmatrix}$$

$$X = -\frac{1}{10}\begin{bmatrix} -10 - \cancel{30} + \cancel{30} \\ 20 - 40 - 0 \\ -\cancel{50} + \cancel{50} - 30 \end{bmatrix}$$

$$X = -\frac{1}{10}\begin{bmatrix} -10 \\ -20 \\ -30 \end{bmatrix}$$

$$X = \begin{bmatrix} -10/-10 \\ -20/-10 \\ -30/-10 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\boxed{x = 1}$, $\boxed{y = 2}$, $\boxed{z = 3}$

Find det (A)

$$A = \begin{bmatrix} 3 & 5 & 2 & 1 \\ -2 & -5 & -4 & 0 \\ 0 & 2 & 5 & 6 \\ 4 & 2 & 3 & 0 \end{bmatrix}$$

Sol:

$$A = \begin{bmatrix} 3 & 5 & 2 & 1 \\ -2 & -5 & -4 & 0 \\ 0 & 2 & 5 & 6 \\ 4 & 2 & 3 & 0 \end{bmatrix}$$

$\therefore$ Swapping C_4 with C_1

$$A = -\begin{bmatrix} 1 & 5 & 2 & 3 \\ 0 & -5 & -4 & -2 \\ 6 & 2 & 5 & 0 \\ 0 & 2 & 3 & 4 \end{bmatrix}$$

$\therefore R_3 - 6R_1.$

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$$|A| = -\begin{bmatrix} 1 & 5 & 2 & 3 \\ 0 & -5 & -4 & -2 \\ 0 & -28 & -7 & -18 \\ 0 & 2 & 3 & 4 \end{bmatrix}$$

$$|A| = -[1\{-5(-28+54)+4(-112+36)-2(-84+14)\}]$$

$$|A| = -[1\{-5(26)+4(-76)-2(-70)\}]$$

$$|A| = -[1\{-130-304+140\}]$$

$$|A| = -[-294]$$

$$|A| = 294$$

$$\boxed{|A| = 294}$$

THE END