Measures of Central Tedency

Lecture#9

When to use
↳ In case of outlier/s.

3) Median :-

The median is defined as a value which divides a data set that have been ordered, into two equal parts, one part comprising of observations greater than and the other part smaller than it.

For ungrouped data;

* If no. of observations are odd;

In an arranged data, median is the middle value. i.e; $\left(\frac{n+1}{2}\right)^{th}$ value.
It is denoted by $\tilde{x}$.

Q-1:- find the median of the following values:

208, 28, 2, 37, 17, 9, 25, 32, 21, 8, 30

Sol:- Arranged data:

2, 8, 9, 17, 21, 25, 28, 30, 32, 37, 208

∴ Median = 25

(i.e; $\left(\frac{n+1}{2}\right)^{th}$ value $= \left(\frac{11+1}{2}\right)^{th}$ value $= 6^{th}$ value)

* If no. of observation are even;

In an arranged data, median is the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

Q-2:- find the median of the following values:

13, 11, 102, 58, 31, 92, 86, 29, 73, 113, 962, 61

Sol:- Arranged data;

11, 13, 29, 31, 58, 61, 73, 86, 92, 102, 113, 962

$\left(\frac{n}{2}\right)^{th} = 6^{th}$ $\left(\frac{n}{2}+1\right)^{th} = 7^{th}$

$\therefore$ Median $= \frac{61+73}{2}$

$\tilde{X} = \frac{134}{2} = 67$

Q-3:- Activity :-

Find median of the following values:

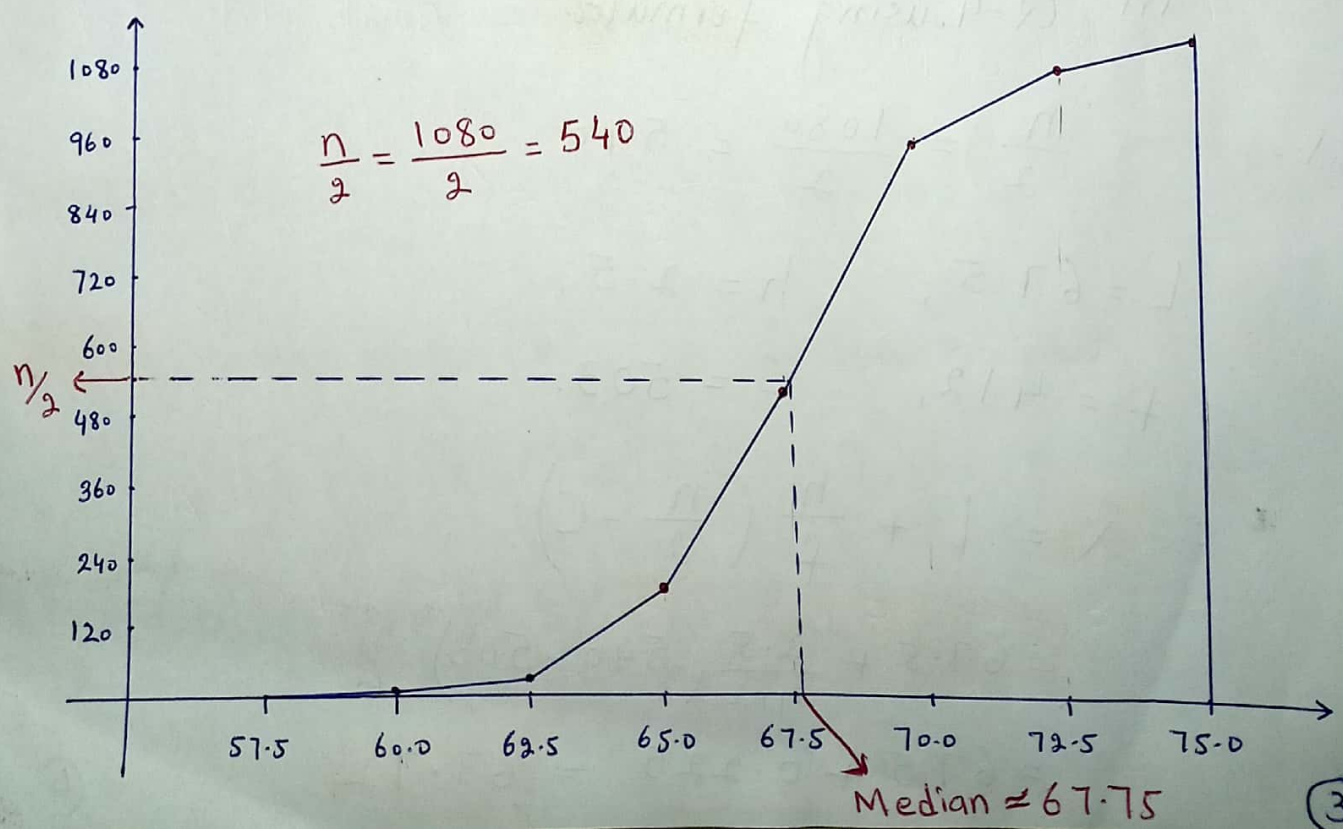
i) 101, 211, 96, 153, 202, 87, 193

ii) $15\frac{1}{5}$, 3, 9, 7, 11, 23, 20, 18, 9, 6, 13, 21

* Median using Ogive

Q-4:- Find median of the following data set:-

Height (inches)	Number f	Cumulative frequency	
57.5 - 60.0	6	6	
60.0 - 62.5	26	32	
62.5 - 65.0	190	222	
65.0 - 67.5	281	503	
67.5 - 70.0	412	915	→ median class
70.0 - 72.5	127	1042	
72.5 - 75.0	38	1080	



* Median for grouped data:

$$\tilde{X} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

where

l = lower C.B of the median class

h = class interval of the median class.

f = frequency of the median class.

$n = \Sigma f$

C = cumulative frequency of the preceeding class of the median class.

Q-5:- Find median of the data given in Q-4 using formula.

Sol:- $\frac{n}{2} = \frac{1080}{2} = 540$

$L = 67.5, \quad h = 2.5,$

$f = 412, \quad C = 503.$

$\therefore \tilde{X} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$

$= 67.5 + \frac{2.5}{412}(540 - 503)$

$= 67.5 + 0.225 = 67.72$

4) Mode :-

The mode is defined as a value which occurs most frequently in a set of data. It is denoted by $\hat{X}$.

When to use

↳ In case of qualitative data.

Q-1:- Find Mode of the following data set:

i) Ali, Hamid, Umer, Zain, Kashif, Ali, Mudassir, Zain and Ali.

Sol:- Mode = $\hat{X}$ = Ali

ii) 3, 9, 7, 6, 5, 8, 7, 7, 6, 9, 2, 7, 6.

Sol:- Mode = $\hat{X}$ = 7.

Note: There can be no mode in the data.

Q-2:- Find mode of the following data set:

7, 11, 8, 13, 21, 19, 6, 9, 16

Sol:- Mode = $\hat{X}$ = No mode

NOTE:- There can be more than one mode in the data sets.

Q-3:- Find mode of the following data set:

3, 9, 11, 6, 5, 13, 9, 6, 2, 8, 7, 9, 13, 6

Sol:- Mode = $\hat{x}$ = 6 and 9.

Q-4:- Qualitative data set

Find mode of the following data relate to sizes of shoes sold at a store during a given week.

Size of Shoes	No. of pairs.
5	2
5½	5
6	15
6½	30
7	60
7½	40
8	23
8½	11

Sol:- Mode = $\hat{x}$ = 7 size

Mode for the grouped data

$$\hat{X} = l + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times h$$

where

l = lower C.B of the modal class,

f_m = frequency of the modal class,

f_{m-1} = frequency of the preceding class of the modal class.

f_{m+1} = frequency of the following class of the modal class.

h = class-interval of the modal class.

Q-5:- Find mode of the following data set:-

Height (inches)	Number
57.5 - 60.0	6
60.0 - 62.5	26
62.5 - 65.0	190
65.0 - 67.5	281
67.5 - 70.0	412
70.0 - 72.5	127
72.5 - 75.0	38

Sol 2-

$f_m = 412$

$f_{m-1} = 281$ $\quad$ $f_{m+1} = 127$

$l = 67.5$ $\quad$ $h = 2.5$

$$\hat{x} = l + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times h$$

$$= 67.5 + \frac{412 - 281}{(412-281)+(412-127)} \times 2.5$$

$$= 67.5 + \frac{131}{131 + 285} \times 2.5$$

$$= 67.5 + 0.787$$

$$\hat{x} = 68.29$$

Find Mean of the above data set.

Sol 2-

X	58.75	61.25	63.75	66.25	68.75	71.25	73.75
f	6	26	190	281	412	127	38

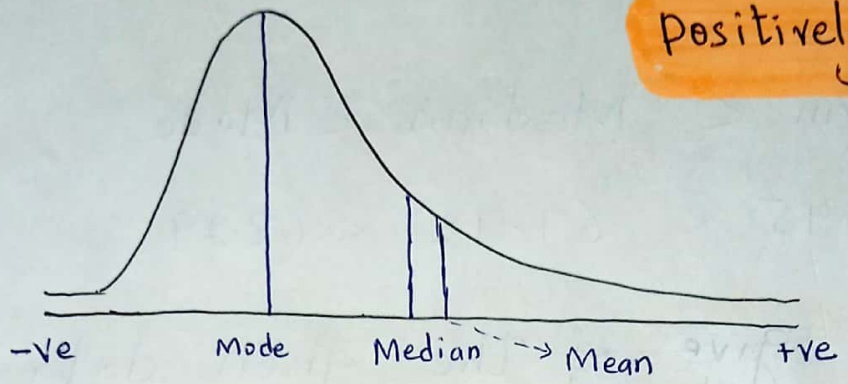
$\Sigma fX = 72850$, $\quad \Sigma f = 1080$

$\therefore \bar{X} = 67.45$

Shapes of the Graph

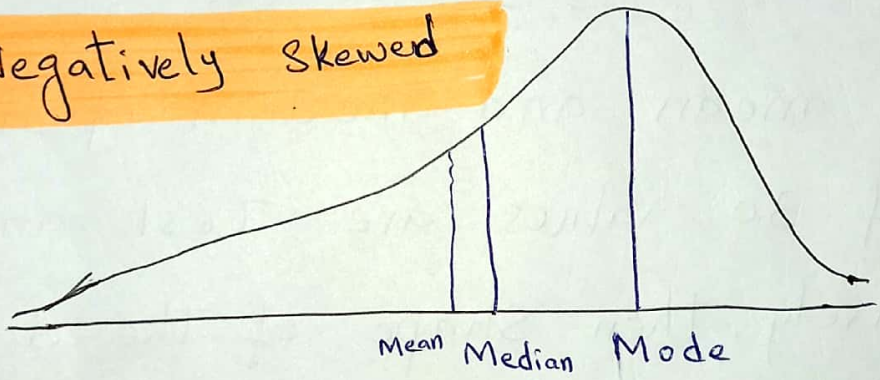
Mean > Median > Mode

Positively skewed.



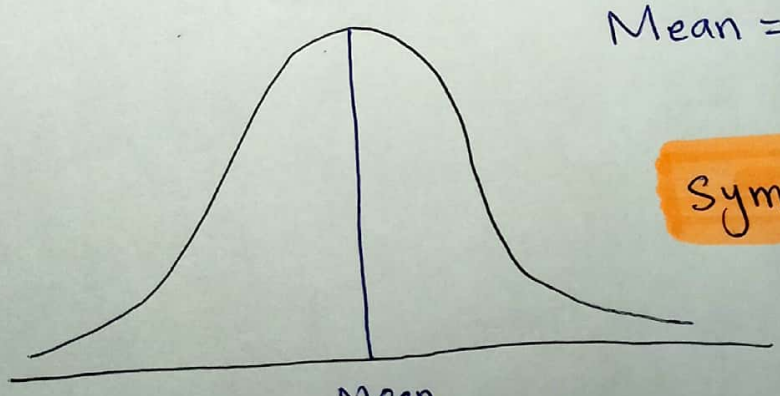
Mode > Median > Mean

Negatively skewed



Mean = Median = Mode

Symmetric



Q-1:- (As Calculated in grouped data)

Mean = 67.45 Median = 67.72

and Mode = 68.29

$\because$ Mean < Median < Mode

i.e; 67.45 < 67.72 < 68.29

Thus the Shape of the given data set is negatively Skewed.

. — .

Q-2:- If mean and median of a data set of 30 values are 7881 and 6932 respectively, then Shape of the data set is Positively Skewed.