# Integration Rules and Formulas

## Integration Rules – Ex. 5.4, 5.5, 7.2

| Sr. | Formulas | Examples |
|---|---|---|
| **1.** | $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$ | **Q1.** $\int x^3 dx = \frac{x^{3+1}}{3+1} + c = \frac{x^4}{4} + c$ |
| **2.** | If $n = -1$, then $\int x^{-1} dx = \int \frac{1}{x} dx = \ln\lvert x\rvert + c$ | **Q2.** $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + c$ |
| **3.** | $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$ | $\int (3x^2+1)^5 \times 6x\, dx = \frac{(3x^2+1)^6}{6} + c$ |
| **4.** | $\int \frac{f'(x)}{f(x)} dx = \ln\lvert f(x)\rvert + c$ | $\int \frac{6x}{3x^2+1} dx = \ln\lvert 3x^2+1\rvert + c$ |
| **5.** | $\int \frac{f'(x)}{[f(x)]^n} dx = \int [f(x)]^{-n} f'(x) dx = \frac{[f(x)]^{-n+1}}{-n+1} + c$ <br> $e.g., \int \frac{6x}{\sqrt{3x^2+1}} dx = \int (3x^2+1)^{\frac{-1}{2}}\, 6x\, dx = \frac{(3x^2+1)^{\frac{1}{2}}}{1/2} + c = 2\sqrt{3x^2+1} + c$ | |

## Integration as a reverse process of differentiation – Ex. 5.4, 7.3

| Sr. | Formulas | Generalization |
|---|---|---|
| **1.** | $\int \cos x\, dx = \sin x + c$ | $\int \cos(ax)\, dx = \frac{\sin(ax)}{a} + c$ |
| **2.** | $\int \sin x\, dx = -\cos x + c$ | $\int \sin(ax)\, dx = \frac{-\cos(ax)}{a} + c$ |
| **3.** | $\int \sec^2 x\, dx = \tan x + c$ | $\int \sec^2(ax)\, dx = \frac{\tan(ax)}{a} + c$ |
| **4.** | $\int \csc^2 x\, dx = -\cot x + c$ | $\int \csc^2(ax)\, dx = \frac{-\cot(ax)}{a} + c$ |
| **5.** | $\int \sec x \tan x\, dx = \sec x + c$ | $\int \sec(ax)\tan(ax)\, dx = \frac{\sec(ax)}{a} + c$ |
| **6.** | $\int \csc x \cot x\, dx = -\csc x + c$ | $\int \csc(ax)\cot(ax)\, dx = \frac{-\csc(ax)}{a} + c$ |
| **7.** | $\int e^x dx = e^x + c$ | $\int e^{ax} dx = \frac{e^{ax}}{a} + c, \quad a = \text{constant}$ |

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## Remaining trigonometric integration formulas – Ex. 5.4, 5.5

| | |
|---|---|
| **1.** | $\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln\lvert\cos x\rvert + c = \ln\lvert\sec x\rvert + c$ |
| **2.** | $\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx = \ln\lvert\sin x\rvert + c$ |
| **3.** | $\int \sec x\, dx = \ln\lvert\sec x + \tan x\rvert + c$ |
| **4.** | $\int \csc x\, dx = \ln\lvert\csc x - \cot x\rvert + c$ |
| **5.** | $\int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int (1-\cos 2x)\, dx = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$ |
| **6.** | $\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int (1+\cos 2x)\, dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$ |
| **7.** | $\int \tan^2 x\, dx = \int (\sec^2 x - 1) dx = \tan x - x + c$ |
| **8.** | $\int \cot^2 x\, dx = \int (\csc^2 x - 1) dx = -\cot x - x + c$ |

## Integration resulting in inverse trigonometric functions: Ex. 7.6

| Sr. | Formulas | Generalization |
|---|---|---|
| **1.** | $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$ | $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$ |
| **2.** | $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$ | $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ |
| **3.** | $\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}} dx = \sec^{-1} x + c$ | $\int \frac{1}{\lvert x\rvert\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$ |

## Integration by parts: Ex. 8.1

Let $u$ and $v$ be functions of $x$, i.e., $u = u(x), v = v(x)$. Then

$$\int u\, v\, dx = u \times \left(\int v\, dx\right) - \int \left(\left(\int v\, dx\right) \times u'(x) dx\right)$$

$$= u \times \text{integral of } v - \int(\text{integral of } v \times \text{derivative of } u) dx$$

**Note:** Some functions are always taken as 1st function, e.g., $\ln x$, $\sin^{-1} x$ etc.

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