"Discrete Probability Distributions"

* Random Variables:

A random variable is a function that associates a real number with each element in the Sample Space.

Q-1:- If four coins are tossed. Make a distribution of head occurance.

Sol:-

No. of head (X)	$P(X) = {}^4C_x / 2^4$
0	$1/16$
1	$4/16$
2	$6/16$
3	$4/16$
4	$1/16$
	1

$\because \Sigma P(X) = 1$. hence proved a complete distribution.

Q-2:- A Shipment of 20 Similar laptop Computers to a retail outlet contains 3 that are defective. If a School makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

Sol:- Let $X \longrightarrow$ defective items chosen

$$\therefore \quad P(X) = \frac{{}^{3}C_{x} \; {}^{17}C_{2-x}}{{}^{20}C_{2}} \quad \text{for } x = 0, 1, 2.$$

Probability Distribution:

X	P(X)
0	${}^{3}C_{0} \; {}^{17}C_{2} \div {}^{20}C_{2} = 136/190.$
1	${}^{3}C_{1} \; {}^{17}C_{1} \div {}^{20}C_{2} = 51/190$
2	${}^{3}C_{2} \; {}^{17}C_{0} \div {}^{20}C_{2} = 3/190$

$$\frac{190}{190} = 1$$

"Expected values"

$$E(X) = \sum x\,P(x)$$

where $E(X) = \text{Mean}$.

Q-1:- For the probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, given in following table:

x	0	1	2	3	4
$P(x)$	0.41	0.37	0.16	0.05	0.01

find the average no. of imperfections per 10 meters of this fabric.

Sol:-

X	$P(x)$	$x\,P(x)$
0	0.41	0
1	0.37	0.37
2	0.16	0.32
3	0.05	0.15
4	0.01	0.04
		0.88

$\therefore$ Average $= E(X) = \sum x\,P(x)$

$= 0.88$

Q-2: Find the Mean of X for the following discrete random variable:

$$P(X) = {}^3C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \qquad x = 0, 1, 2, 3.$$

Sol:-

x	$P(x)$		$xP(x)$
0	${}^3C_0 \times \left(\frac{1}{4}\right)^0 \times \left(\frac{3}{4}\right)^3$	$= 27/64$	0
1	${}^3C_1 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^2$	$= 27/64$	$27/64$
2	${}^3C_2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^1$	$= 9/64$	$18/64$
3	${}^3C_3 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{1}{4}\right)^0$	$= 1/64$	$3/64$
		1	$48/64$

$$\therefore E(X) = \sum X P(X)$$

$$= \frac{48}{64} = 0.75$$

Lecture

"Expected Values"

$$E(X) = \sum x P(x)$$

$$\therefore \quad E(X^2) = \sum x^2 P(x)$$

or $$E(2X+1) = \sum (2X+1) P(x)$$

where $E(2X+1) = 2E(X)+1$

or $$E(3X^2-1) = \sum (3X^2-1) P(x)$$

where $E(3X^2-1) = 3E(X^2)-1$

Q-1:- Find $E(X^2)$ for the following data set:

X	$P(x)$	X^2	$X^2 P(X)$
0	0.41	0	0
1	0.37	1	0.37
2	0.16	4	0.64
3	0.05	9	0.45
4	0.01	16	0.16
			1.62

$$\therefore E(X^2) = \sum X^2 P(X) = 1.62$$

Q-2: Find $E(2X+1)$ of the following:

X	$P(X)$	$2X+1$	$(2X+1)P(X)$
0	$27/64$	1	$27/64$
1	$27/64$	3	$81/64$
2	$9/64$	5	$45/64$
3	$1/64$	7	$7/64$
			$160/64$

Method I:

$$E(2X+1) = \Sigma (2X+1) P(X)$$

$$= \frac{160}{64} = 2.5$$

Method II:

$$E(2X+1) = 2E(X) + 1$$

$$= 2 \times 0.75 + 1$$

$$= 1.5 + 1$$

$$= 2.5$$

"Variance and Standard deviation of Discrete prob. distributions"

$$\text{Variance}(X) = E(X^2) - [E(X)]^2$$

$$= \sum x^2 P(x) - \left[\sum x P(x)\right]^2$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

S.11

Q-3: The U.S. Census Bureau collects demographics Concerning the no. of people in families per household. Assume the distribution of the number of people per household is shown in the following table:

x	P(x)
2	0.27
3	0.25
4	0.28
5	0.13
6	0.04
7	0.03

a) Calculate the expected no. of people in families per household in the United States.

b) Compute the variance and Standard deviation of the no. of people in families per household.

Solution

x	$P(x)$	$x\,P(x)$	x^2	$x^2\,P(x)$
2	0.27	0.54	4	1.08
3	0.25	0.75	9	2.25
4	0.28	1.12	16	4.48
5	0.13	0.65	25	3.25
6	0.04	0.24	36	1.44
7	0.03	0.21	49	1.47
	1	3.51		13.97

i) $\because E(X) = \Sigma x\,p(x)$

$= 3.51$

$\therefore$ expected no. of people in families per household $= 3.51$

ii) Variance $(X) = ?$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \Sigma x^2 P(x) - [\Sigma x P(x)]^2$$

$$= 13.97 - 3.51^2$$

$$= 1.65$$

& $S.D(X) = \sqrt{Var(X)} = \sqrt{1.65} = 1.28$

"Linear relationships"

Let $y = a + bX$

then

$$\text{Mean}(Y) = a + b\ \text{mean}(X)$$

i.e; $E(Y) = a + b\,E(X)$

and

$$\text{Variance}(Y) = \text{Variance}(a + bX)$$

$$= \text{Variance}(a) + \text{Variance}(bX)$$

$$= 0 + b^2\ \text{Variance}(X)$$

$\therefore\ \text{Var}(Y) = b^2\ \text{Var}(X)$

and Thus

$$S.D(Y) = b \times S.D(X).$$