Assignment 5

Task 1:
A very simple description of two competing species (N_1 and N_2) is provided by the Lotka-Volterra model

$$\frac{dN_1}{dt} = r_1 N_1 - b_1 N_1 N_2$$

$$\frac{dN_2}{dt} = r_2 N_2 - b_2 N_1 N_2$$

where $r_{1;2} > 0$ and $b_{1;2} > 0$.

1. Show that by scaling the state variables and the time, the system can be brought to the dimensionless form

$$\frac{dx}{dt} = x(1-y)$$

$$\frac{dy}{dt} = y(\rho - x)$$

Specify the formula for the r parameter.

2. Determine the fixed points (stationary solution) of the dimensionless equation and characterize they (type of fixed point). Also determine the eigenvectors of the fixed points.

3. Describe what the long-term behavior of the system looks like?

Task 2:

Consider the following model for a food chain consisting of a different number of nutrients and species that consume these nutrients. The species compete for the various nutrients.

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = N_i(\mu_i(R_1, \ldots, R_k) - m_i) \quad i = 1, \ldots, n \qquad (1)$$

$$\frac{\mathrm{d}R_j}{\mathrm{d}t} = D(S_j - R_j) - \sum_{i=1}^{n} c_{ji}\mu_i(R_1, \ldots, R_k)N_i \quad j = 1, \ldots, k \qquad (2)$$

$$\mu_i(R_1, \ldots, R_k) = \min\left(\frac{r_i R_1}{K_{1i} + R_1}, \ldots, \frac{r_i R_k}{K_{ki} + R_k}\right) \qquad (3)$$

Calculate the behavior of the system with different parameterization:

a) 3 resources and 3 types
b) 5 resources and 5 types