Problem #2

Let's first define the Lagrange function

$$\mathcal{L} = f(x) + \lambda^T h(x)$$
$$\mathcal{L} = x_1 x_2 + \lambda(x_1 + x_2 - 1)$$

The partial derivation of the Lagrange function in terms of **x** and λ

$$D_{x_1}\mathcal{L} = \frac{\partial \mathcal{L}}{\partial x_1} = x_2 + \lambda = 0$$
$$D_{x_2}\mathcal{L} = \frac{\partial \mathcal{L}}{\partial x_2} = x_1 + \lambda = 0$$
$$D_{\lambda}\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \lambda} = x_1 + x_2 - 1 = 0$$

By expressing the first and second equations in terms of x_1 and x_2 respectively, and inserting it into the third equation, we have

$$-\lambda - \lambda - 1 = 0$$
$$\lambda = -0.5.$$

Thus, $x_1 = x_2 = 0.5$.

We can plot the function on MATLAB or google and we can see the curve continues to go up and down, and the point where the gradient will be zero at is at $x_1 = x_2 = 0.5$ for $\lambda = -0.5$, but this is neither the maxima nor the minima of the function.