Presentation on problem 15

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Problem Statement

Use MATLAB and the Symbolic Math Symbolic Math Toolbox to input and form LTI objects in polynomial and factored form for the following frequency functions:

$$\mathbf{a}).\mathbf{G}(\mathbf{s}) = \frac{45(s^2+37s+74)(s^3+28s^2+32s+16)}{(s+39)(s+47)(s^2+2s+100)(s^3+27s^2+18s+15)} \quad (1)$$

$$\mathbf{b}).\mathbf{G}(\mathbf{s}) = \frac{56(s+14)(s^3+49s^2+62s+53)}{(s^3+81s^2+76s+65)(s^2+88s+33)(s^2+56s+77)} \quad (2)$$

Polynomial form of G(s)

- From the expanded form of numerator and denominator, we can write polynomial form of transfer function as:

$$\frac{45s^5+2925s^4+53190s^3+147240s^2+133200s+53280}{s^7+115s^6+4499s^5+70700s^4+553692s^3+5201463s^2+3483390s+2749500}$$

Poles and zeroes of G(s)

- We can obtain the roots of a polynomial using numpy library in python3.
- Using numpy, we find roots of numerator which are the zeroes and roots of denominator which are the poles of Transfer function.
- Poles of the transfer function are:

  1)-47
  2)-39
  3)-26.34
  4)-1+9.95j
  5)-1-9.95j
  6)-0.33+0.68j
  7)-0.33-0.68j

  where j is the square root of -1

Poles and zeroes of G(s)

- Similarly, we can obtain roots of numerator also.
- Zeroes of the transfer function are:

  1)-34.88
  2)-26.83
  3)-2.12
  4)-0.59+0.5j
  5)-0.59-0.5j

  where j is the square root of -1

Factored form of G(s)

- From the obtained values of poles and zeroes, we can write factored form of the Transfer function as :

$$\frac{(s+34.88)(s+26.83)(s+2.12)(s+0.59-0.5j)(s+0.59+0.5j)}{(s+47)(s+39)(s+26.34)(s+1-9.95j)(s+1+9.95j)(s+0.33-0.68j)(s+0.33+0.68j)}$$

Polynomial form of G(s)

- From the expanded form of numerator and denominator, we can write polynomial form of transfer function as:

$$\frac{56s^4+3528s^3+41888s^2+51576s+41552}{s^7+225s^6+16778s^5+427711s^4+1093333s^3+1188715s^2+753676s+165165}$$

Poles and zeroes of G(s)

- We can obtain the roots of a polynomial using numpy library in python3.
- Using numpy, we find roots of numerator which are the zeroes and roots of denominator which are the poles of Transfer function.
- Poles of the transfer function are:
  1)-87.62
  2)-80.06
  3)-54.59
  4)-1.41
  5)-0.47+0.77j
  6)-0.47-0.77j
  7)-0.38

  where j is the square root of -1

Poles and zeroes of G(s)

- Similarly, we can obtain roots of numerator also.
- Zeroes of the transfer function are:

  1)-47.72
  2)-14
  3)-0.64+0.84j
  4)-0.64-0.84j

  where j is the square root of -1

Factored form of G(s)

- From the obtained values of poles and zeroes, we can write factored form of the Transfer function as :

$$\frac{(s+47.72)(s+14)(s+0.64-0.84j)(s+0.64+0.84j)}{(s+87.62)(s+80.06)(s+54.59)(s+1.41)(s+0.47-0.77j)(s+0.47+0.77j)(s+0.38)}$$

Python code for part (a)

```
from sympy import *
s = symbols('s')

#Numerator N(s) of G(s)
N = 45*(s**2 + 37*s + 74)*(s**3 + 28*s**2 +32*s + 16)
N_e = expand(N)

#Denominator D(s) of G(s)
D = (s + 39)*(s + 47)*(s**2 + 2*s + 100)*(s**3 + 27*s**2 + 18*s + 15)
D_e = expand(D)

#Printing given transfer function G(s) and it's polynomial form
G = N/D
print("\n Given Transfer function G(s) = {}".format(G))

G_poly = expand(N)/expand(D)
print("\n Polynomial form of G(s) = {}".format(G_poly))

#Solving poles and zeroes using np.roots() method
import numpy as np

N = np.poly1d([45,2925,51390,147240,133200,53280])
zeroes = np.roots(N)
print("\n Zeroes of G(s) = ",zeroes)

D = np.poly1d([1,115,4499,70700,553692,5201463,3483390,2749500])
Poles = np.roots(D)
print("\n Poles of G(s) = ",Poles)

```

Python code for part (b)

```python
from sympy import *
s = symbols('s')

#Numerator N(s) of G(s)
N = 56*(s + 14)*(s**3 + 49*s**2 +62*s + 53)
N_e = expand(N)

#Denominator D(s) of G(s)
D = (s**3 + 81*s**2 + 76*s + 65)*(s**2 + 88*s + 33)*(s**2 + 56*s + 77)
D_e = expand(D)

#Printing given transfer function G(s) and it's polynomial form
G = N/D
print("\n Given Transfer function G(s) = {}".format(G))

G_poly = expand(N)/expand(D)
print("\n Polynomial form of G(s) = {}".format(G_poly))

#Solving poles and zeroes using np.roots() method
import numpy as np

N = np.poly1d([56,3528,41888,51576,41552])
zeroes = np.roots(N)
print("\n Zeroes of G(s) = ",zeroes)

D = np.poly1d([1,225,16778,427711,1093333,1188715,753676,165165])
Poles = np.roots(D)
print("\n Poles of G(s) = ",Poles)
```

Terminal output

```
Given Transfer function G(s) = (45*s**2 + 1665*s + 3330)*(s**3 + 28*s**2 + 32*s + 16)/((s + 39)*(s + 47)*(s**2 + 2*s + 100)*(s**3 + 27*s**2 + 18*s + 15))

Polynomial form of G(s) = (45*s**5 + 2925*s**4 + 51390*s**3 + 147240*s**2 + 133200*s + 53280)/(s**7 + 115*s**6 + 4499*s**5 + 70700*s**4 + 553692*s**3 + 5201463*s**2 + 3483390*s + 2749500)

Zeroes of G(s) =  [-34.87833935+0.j         -26.82951122+0.j
 -2.12166065+0.j          -0.58524439+0.50383255j
 -0.58524439-0.50383255j]

Poles of G(s) =  [-47.        +0.j         -39.        +0.j
-26.33820528+0.j          -1.        +9.94987437j
 -1.        -9.94987437j  -0.33089736+0.67824909j
 -0.33089736-0.67824909j]
```

Terminal output

```
Given Transfer function G(s) = (56*s + 784)*(s**3 + 49*s**2 + 62*s + 53)/((s**2 + 56*s + 77)*(s**2 + 88*s + 33)*(s**3 + 81*s**2 + 76*s + 65))

Polynomial form of G(s) = (56*s**4 + 3528*s**3 + 41888*s**2 + 51576*s + 41552)/(s**7 + 225*s**6 + 16778*s**5 + 427711*s**4 + 1093333*s**3 + 1188715*s**2 + 753676*s + 165165)

Zeroes of G(s) =  [-47.72413721+0.j         -14.        +0.j
 -0.63793139+0.83880432j  -0.63793139-0.83880432j]

Poles of G(s) =  [-87.62338822+0.j        -80.06086301+0.j        -54.5894716 +0.j
 -1.4105284 +0.j         -0.46956849+0.7690174j  -0.46956849-0.7690174j
 -0.37661178+0.j        ]
```

Github link

The python codes, terminal output pictures, tex document and the pdf file are in the Assignment 1 folder of control systems repository in the below hyper link :

Please click Here