Kalman filter using MATLAB
8: Kalman smoother - Simulation

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Overview

1. Kalman smoother: Algorithm

2. Kalman smoother: Simulation

Kalman smoother: Algorithm

Kalman smoother: Algorithm

Algorithm 1 (Kalman smoother)

1: Compute and store $\hat{\mathbf{x}}_{k|k-1}$, $\hat{\mathbf{x}}_{k|k}$, $\mathbf{P}_{k|k-1}$, $\mathbf{P}_{k|k}$ for $k = 1, 2, ..., N$ using Kalman filter algorithm.
2: **for** $k = N - 1$ *to* 0 **do**
3: $\quad \mathbf{A}_k = [\mathbb{A}]_{k+1}$
4: $\quad \mathbf{L}_{s_k} = \mathbf{P}_{k|k}\mathbf{A}_k^T\mathbf{P}_{k+1|k}^{-1}$
5: $\quad \hat{\mathbf{x}}_{k|N} = \hat{\mathbf{x}}_{k|k} + \mathbf{L}_{s_k}[\hat{\mathbf{x}}_{k+1|N} - \hat{\mathbf{x}}_{k+1|k}]$
6: $\quad \mathbf{P}_{k|N} = \mathbf{P}_{k|k} + \mathbf{L}_{s_k}[\mathbf{P}_{k+1|N} - \mathbf{P}_{k+1|k}]\mathbf{L}_{s_k}^T$
7: **end for**

Kalman smoother: Simulation

Kalman smoother: LTI system

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{1}$$

- Simulation parameters

$$\begin{aligned} &\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25 r_1 \\ &\mathbf{r}_2 = \mathbf{g}_2(\mathbf{0}, \mathbf{I}), \quad r_1 = g_1(0, 1). \end{aligned} \tag{2}$$

Kalman smoother: Simulation response

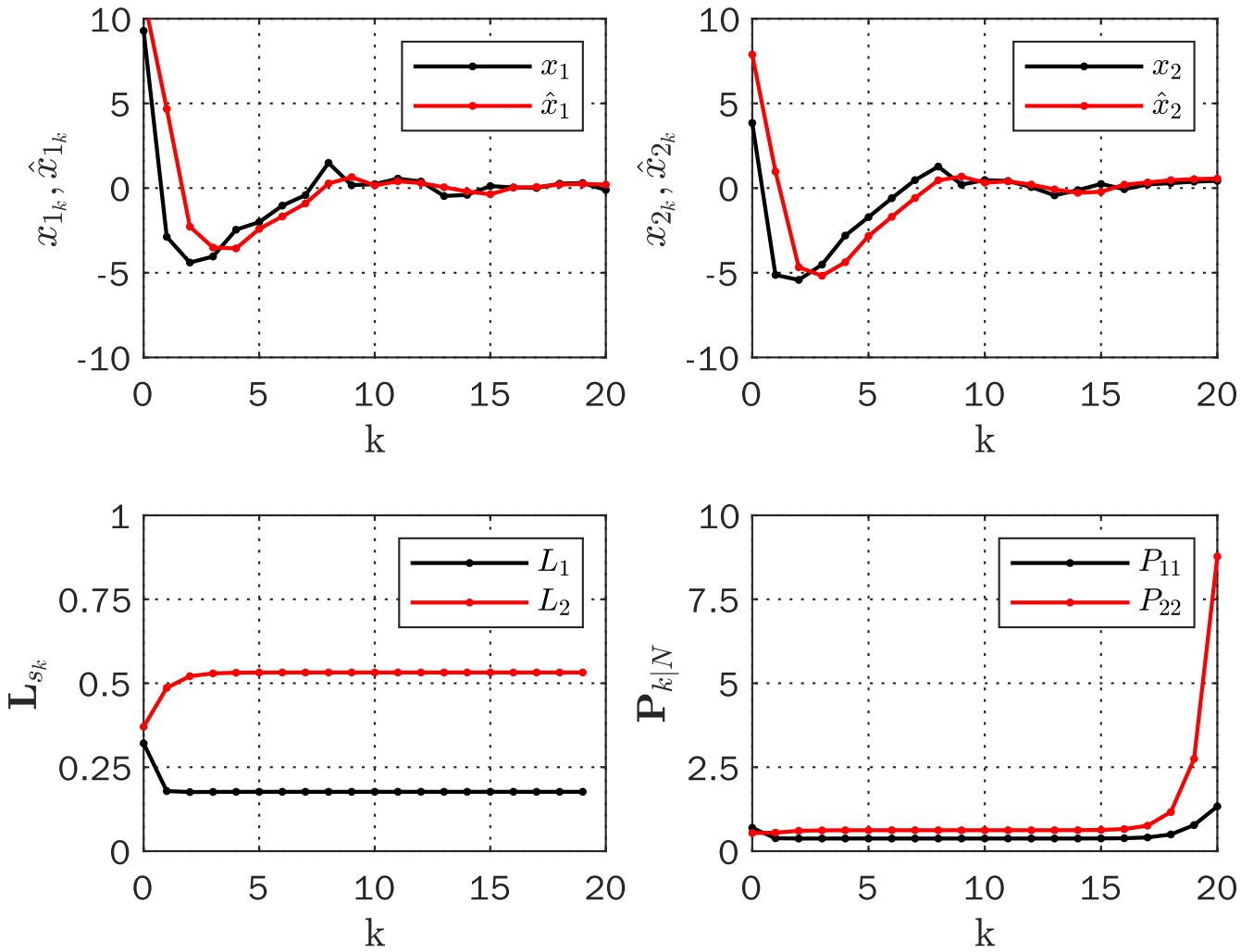

Figure 1: Kalman smoother response - LTI system

Kalman smoother: Simulation response

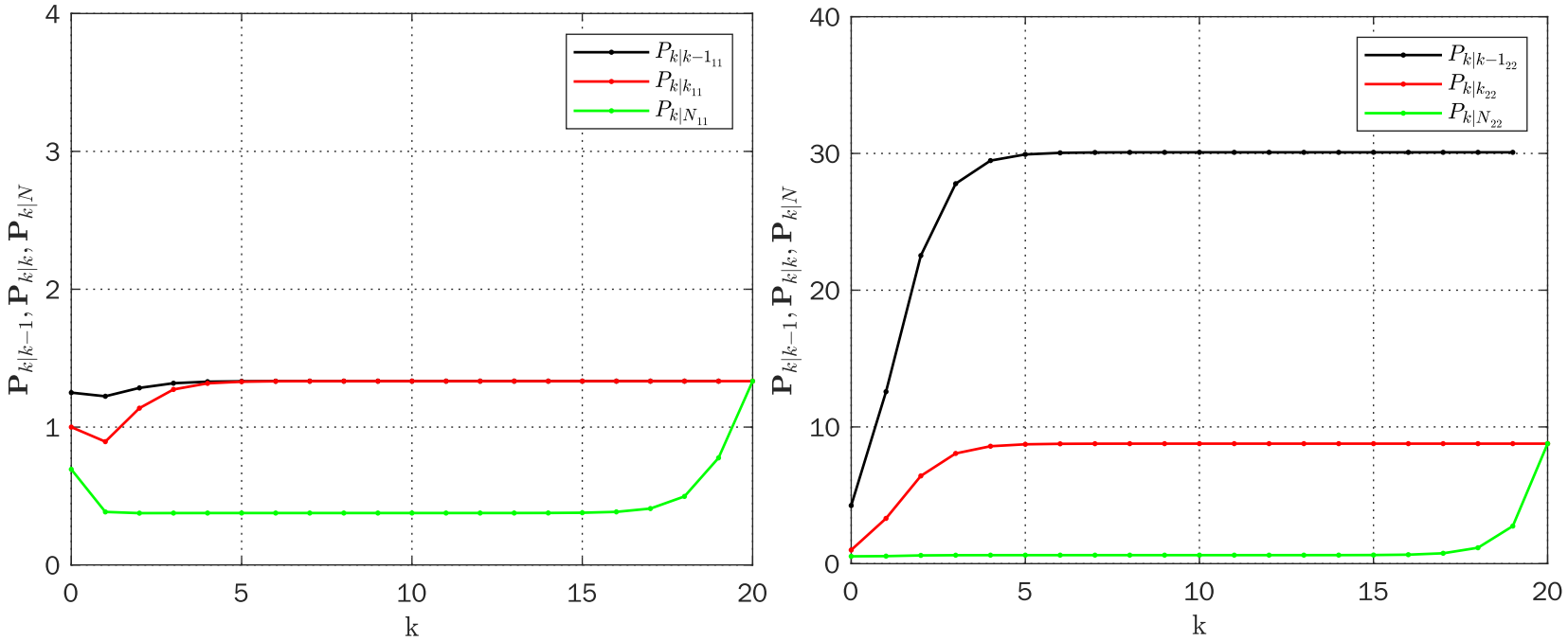

Figure 2: Riccati matrix elements

Kalman smoother: LTV system (Example 1)

- LTV system

$$\begin{aligned}\mathbf{A}_k &= \mathbf{A} + (-1)^k 0.5\mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-1)^k 0.1\mathbf{B} \\ \mathbf{C}_k &= \mathbf{C}\end{aligned} \tag{3}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{4}$$

- Simulation parameters

$$\begin{aligned}&\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1\end{aligned} \tag{5}$$

Kalman smoother: Simulation response

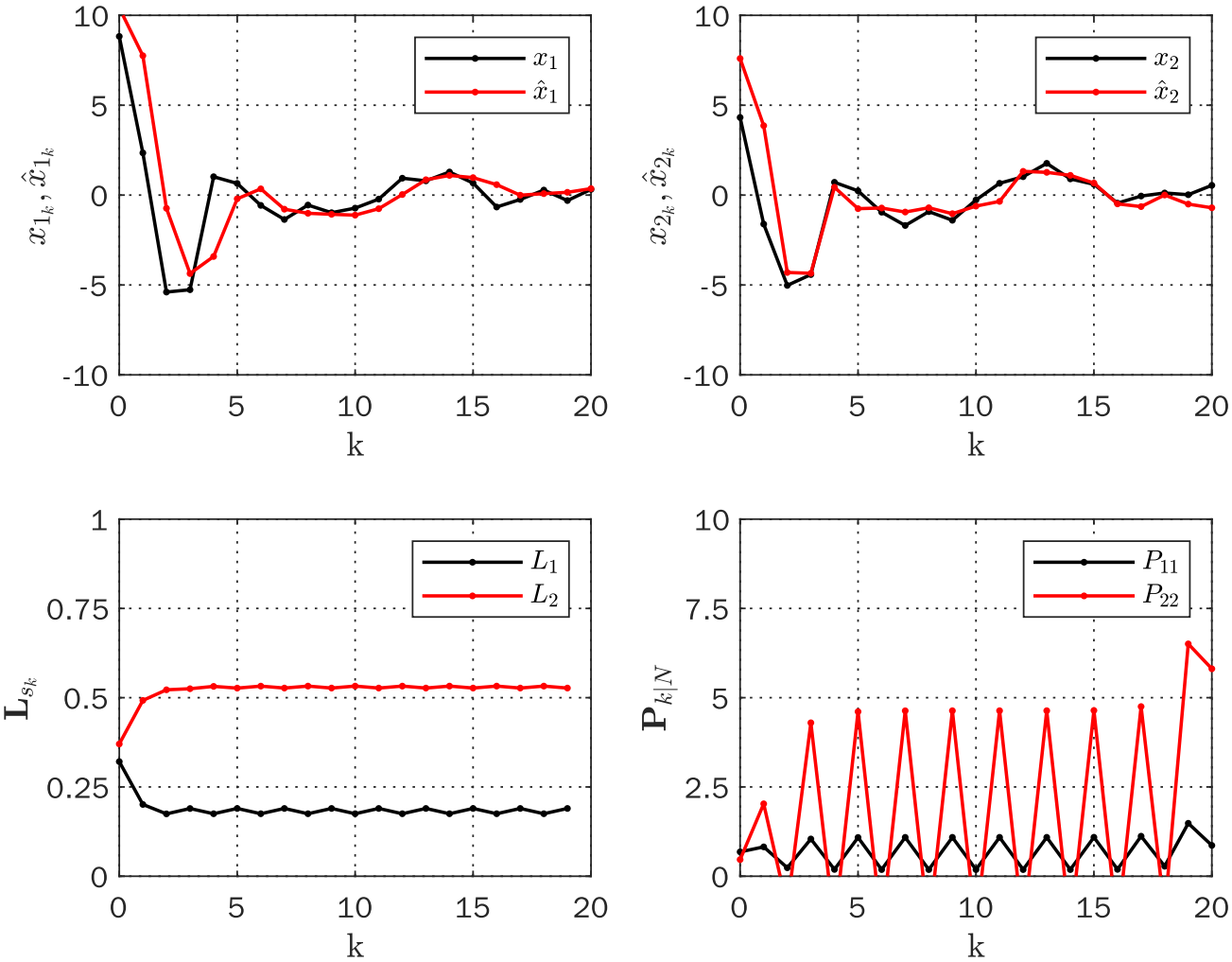

Figure 3: Kalman smoother response - LTV system 1

Kalman smoother: LTV system (Example 2)

- LTV system

$$\begin{aligned}\mathbf{A}_k &= \mathbf{A} + (-0.75)^k \mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-0.5)^k \mathbf{B} \\ \mathbf{C}_k &= \mathbf{C}\end{aligned} \tag{6}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{7}$$

- Simulation parameters

$$\begin{aligned}&\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1\end{aligned} \tag{8}$$

Kalman smoother: Simulation response

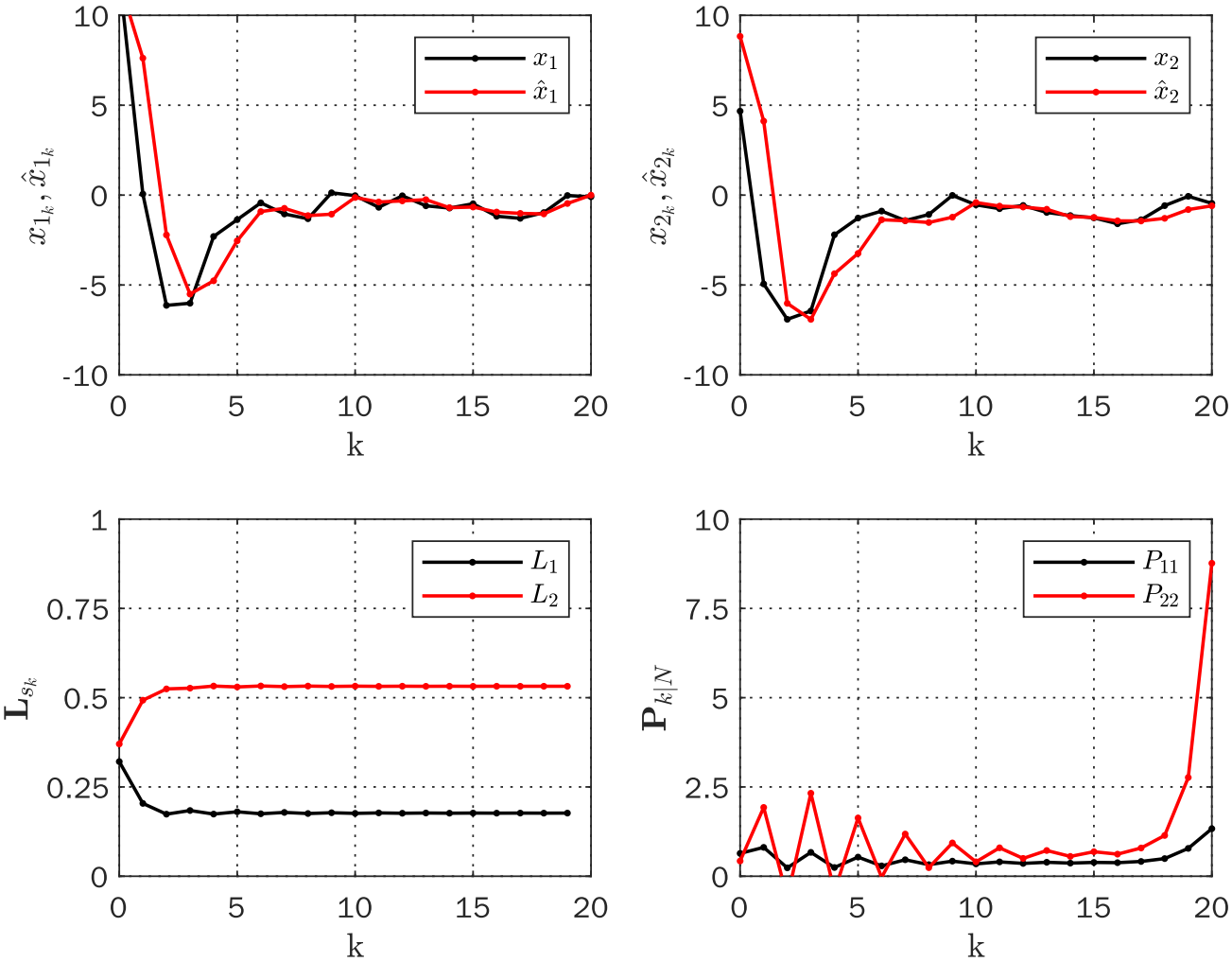

Figure 4: Kalman smoother response - LTV system 2

Remarks

- For the LTI system example the gain matrix $\mathbf{L}_{s_k}$ and Riccati matrix $\mathbf{P}_{k|N}$ converges to some fixed matrices, say $\mathbf{L}, \mathbf{P}$.
- For LTV systems the convergence of $\mathbf{L}_{s_k}$ depends on the convergence of $\mathbf{A}_k, \mathbf{B}_k$.
- The Kalman smoother gives the estimate of the state with lesser variance than Kalman predictor and filter.

Thank you