Kalman Filter using MATLAB
6: Kalman Filter - Simulation

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Overview

1. Kalman filter: Algorithm

2. Kalman filter: Simulation

Kalman filter: Algorithm

Kalman filter: Algorithm

- Define the sets $\mathbb{A} = \{\mathbf{A}_0, \mathbf{A}_1, ..., \mathbf{A}_{N-1}\}, \mathbb{B} = \{\mathbf{B}_0, \mathbf{B}_1, ..., \mathbf{B}_{N-1}\}, \mathbb{C} = \{\mathbf{C}_1, \mathbf{C}_2, ..., \mathbf{C}_N\}, \mathbb{Q} = \{\mathbf{Q}_0, \mathbf{Q}_1, ..., \mathbf{Q}_{N-1}\}, \mathbb{R} = \{\mathbf{R}_1, \mathbf{R}_2, ..., \mathbf{R}_N\}$.

Algorithm 1 (Kalman filter)

Require $\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{Q}, \mathbb{R}$

Initialize $\hat{\mathbf{x}}_{0|0}$ and $\mathbf{P}_{0|0}$

for $k = 1$ *to* N **do**

$\quad \mathbf{A}_{k-1} = [\mathbb{A}]_k,\ \mathbf{B}_{k-1} = [\mathbb{B}]_k, \mathbf{C}_k = [\mathbb{C}]_k,\ \mathbf{Q}_{k-1} = [\mathbb{Q}]_k,\ \mathbf{R}_k = [\mathbb{R}]_k$

$\quad \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1}$

$\quad \mathbf{P}_{k|k-1} = \mathbf{A}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1}$

$\quad \mathbf{L}_k = \mathbf{P}_{k|k-1}\mathbf{C}_k^T[\mathbf{C}_k\mathbf{P}_{k|k-1}\mathbf{C}_k^T + \mathbf{R}_k]^{-1}$

$\quad$ Obtain $\mathbf{y}_k, \hat{\mathbf{y}}_{k|k-1}$ from sensor measurements and the output equation

$\quad \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{L}_k[\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}]$

$\quad \mathbf{P}_{k|k} = [\mathbf{I} - \mathbf{L}_k\mathbf{C}_k]\mathbf{P}_{k|k-1}[\mathbf{I} - \mathbf{L}_k\mathbf{C}_k]^T + \mathbf{L}_k\mathbf{R}_k\mathbf{L}_k^T$

end for

Kalman filter: Simulation

Kalman fliter: LTI system

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{1}$$

- Simulation parameters

$$\begin{gathered} \mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ \mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25 r_1 \\ \mathbf{r}_2 = \mathbf{g}_2(\mathbf{0}, \mathbf{I}), \quad r_1 = g_1(0,1). \end{gathered} \tag{2}$$

Kalman filter: Simulation response

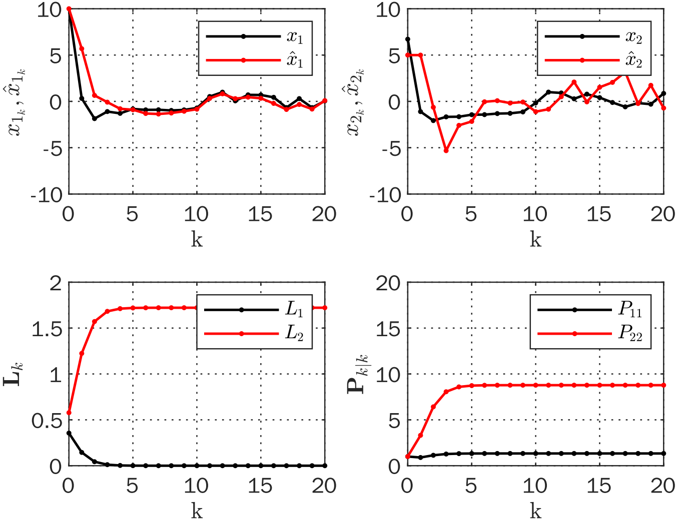

Figure 1: Kalman filter response - LTI system

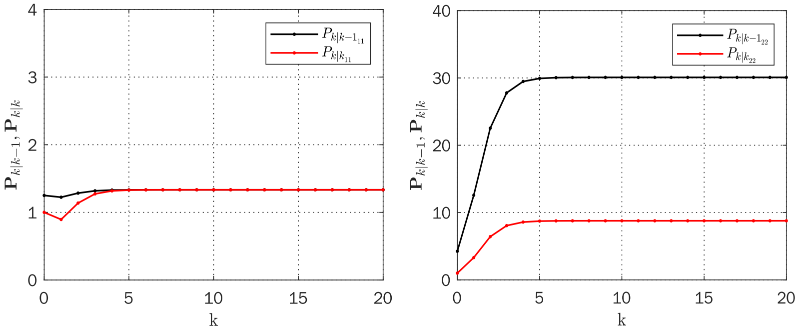

Figure 2: Riccati matrix elements

Kalman filter: LTV system (Example 1)

- LTV system

$$\begin{aligned} \mathbf{A}_k &= \mathbf{A} + (-1)^k 0.5\mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-1)^k 0.1\mathbf{B} \\ \mathbf{C}_k &= \mathbf{C} \end{aligned} \tag{3}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{4}$$

- Simulation parameters

$$\begin{aligned} &\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1 \end{aligned} \tag{5}$$

Kalman filter: Simulation response

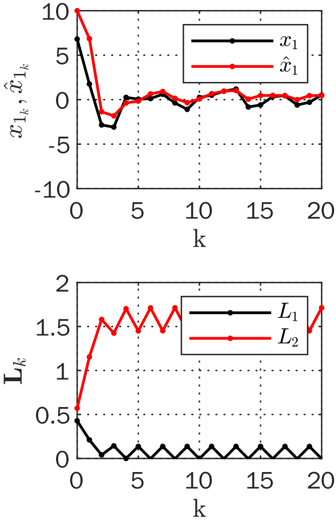

Figure 3: Kalman filter response - LTV system 1

Kalman filter: LTV system (Example 2)

- LTV system

$$\begin{aligned}\mathbf{A}_k &= \mathbf{A} + (-0.75)^k\mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-0.5)^k\mathbf{B} \\ \mathbf{C}_k &= \mathbf{C}\end{aligned} \tag{6}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{7}$$

- Simulation parameters

$$\begin{aligned}&\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1\end{aligned} \tag{8}$$

Kalman filter: Simulation response

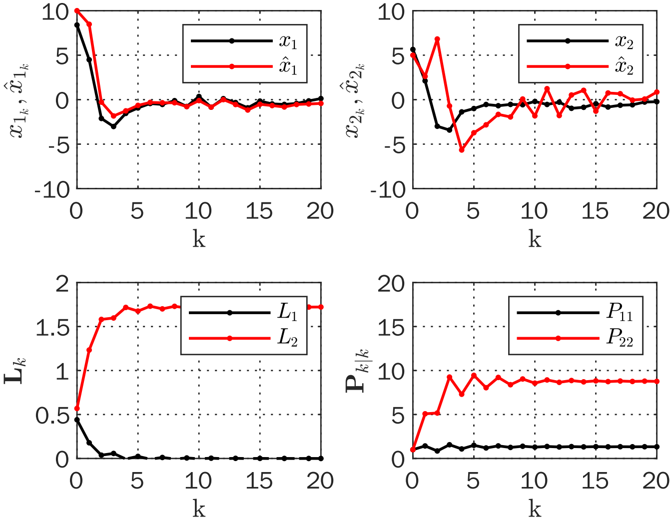

Figure 4: Kalman filter response - LTV system 2

Remarks

- For the LTI system example the gain matrix $\mathbf{L}_k$ and Riccati matrix $\mathbf{P}_{k|k}$ converges to some fixed matrices, say $\mathbf{L}, \mathbf{P}$ as k increases.
- For LTV systems the convergence of $\mathbf{L}_k, \mathbf{P}_{k|k}$ depends on the convergence of $\mathbf{A}_k, \mathbf{B}_k$.
- The Kalman filter gives the estimate of the state with lesser variance than Kalman predictor.

Thank you