Kalman Filter using MATLAB
10: Further topics

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Overview

Advanced topics in Kalman Filter

Extended Kalman Filter (EKF)

- In EKF we extend the Kalman filter to estimate the states of a nonlinear system.
- Consider the nonlinear system:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{d}_k \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \end{aligned} \tag{1}$$

- The Kalman filter is designed for (1) using the linearized model:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k + \mathbf{d}_k \\ \mathbf{y}_k &= \mathbf{C}_k \mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{2}$$

  where

$$\mathbf{A}_k = \frac{\partial \mathbf{f}}{\partial \mathbf{x}_k}, \qquad \mathbf{B}_k = \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} \qquad \mathbf{C}_k = \frac{\partial \mathbf{h}}{\partial \mathbf{x}_k}. \tag{3}$$

Unscented Kalman Filter (UKF)

- UKF is also used to estimate the states of nonlinear systems.
- In UKF an unscented transformation is used to update the expectation and variance, instead of the linearized model.
- The UKF algorithm consists of the following steps
  1. Select the σ points $\mathbf{x}_{k-1_1}, ..., \mathbf{x}_{k-1_{2n+1}}$ near the estimate of $\mathbf{x}_{k-1}$.
  2. Use the nonlinear model (1) to compute the transformed σ points: $\mathbf{x}_{k_1}, ..., \mathbf{x}_{k_{2n+1}}$.
  3. Use the transformed σ points to compute an estimate of the expectation and variance of $\mathbf{x}_k$.

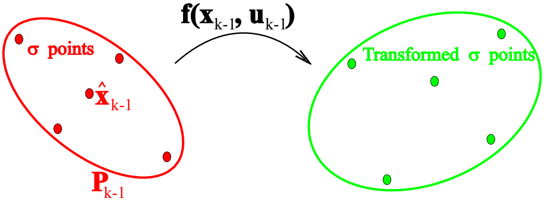

Figure 1: Unscented transformation

Linear Quadratic Guassian (LQG) Control

- LQG consists of a Linear Quadratic Regulator (LQR) controller and a Kalman filter as the state estimator.
- In LQG, the control law becomes

$$\mathbf{u}_k = -\mathbf{K}_k \hat{\mathbf{x}}_k \tag{4}$$

where $\mathbf{K}_k$ is the feedback gain computed using LQR and $\hat{\mathbf{x}}_k$ is the state estimate computed using Kalman filter, i.e. $\hat{\mathbf{x}}_k = \mathbf{x}_{k|k}$.

Applications of Kalman Filter

Applications of Kalman Filter

1. **Mechanical systems**: Kalman filter is extensively used for state estimation of mechanical systems such as robots, automobiles and aerospace vehicles.
2. **Electrical systems**: Kalman filter has many applications in electrical systems such as electrical machines and drives, power converters, power systems, etc.
3. **Medical science**: Kalman filters are also used for estimating the states of many biological systems.
4. **Economics**: Kalman estimators are used in business and economics for prediction and forecasting the trends.

References

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Thank you

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