Kalman Filter using MATLAB
4: Kalman predictor - Simulation

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Overview

1. Kalman predictor: Algorithm
2. Kalman predictor: Simulation

Kalman predictor: Algorithm

Kalman predictor: Algorithm

- Define the sets $\mathbb{A} = \{\mathbf{A}_0, \mathbf{A}_1, ..., \mathbf{A}_{N-1}\}, \mathbb{B} = \{\mathbf{B}_0, \mathbf{B}_1, ..., \mathbf{B}_{N-1}\}, \mathbb{C} = \{\mathbf{C}_0, \mathbf{C}_1, ..., \mathbf{C}_{N-1}\}, \mathbb{Q} = \{\mathbf{Q}_0, \mathbf{Q}_1, ..., \mathbf{Q}_{N-1}\}, \mathbb{R} = \{\mathbf{R}_0, \mathbf{R}_1, ..., \mathbf{R}_{N-1}\}$.

Algorithm 1 (Kalman predictor)

Require $\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{Q}, \mathbb{R}$
Initialize $\hat{\mathbf{x}}_0$ and $\mathbf{P}_0$
for $k = 0$ *to* $N - 1$ **do**
    $\mathbf{A}_k = [\mathbb{A}]_{k+1}$, $\mathbf{B}_k = [\mathbb{B}]_{k+1}$, $\mathbf{C}_k = [\mathbb{C}]_{k+1}$, $\mathbf{Q}_k = [\mathbb{Q}]_{k+1}$, $\mathbf{R}_k = [\mathbb{R}]_{k+1}$
    $\mathbf{L}_k = \mathbf{A}_k \mathbf{P}_k \mathbf{C}_k^T [\mathbf{C}_k \mathbf{P}_k \mathbf{C}_k^T + \mathbf{R}_k]^{-1}$
    Obtain $\mathbf{y}_k, \hat{\mathbf{y}}_k$ from sensor measurements and the output equation
    Obtain $\mathbf{u}_k$ from the control law (i.e. $\mathbf{u}_k = -\mathbf{K}\hat{\mathbf{x}}_k$)
    $\hat{\mathbf{x}}_{k+1} = \mathbf{A}_k \hat{\mathbf{x}}_k + \mathbf{B}_k \mathbf{u}_k + \mathbf{L}_k [\mathbf{y}_k - \hat{\mathbf{y}}_k]$
    $\mathbf{P}_{k+1} = [\mathbf{A}_k - \mathbf{L}_k \mathbf{C}_k] \mathbf{P}_k [\mathbf{A}_k - \mathbf{L}_k \mathbf{C}_k]^T + \mathbf{Q}_k + \mathbf{L}_k \mathbf{R}_k \mathbf{L}_k^T$
end for

Kalman predictor: Simulation

Kalman predictor: LTI system

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{1}$$

- Simulation parameters

$$\begin{aligned} &\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25 r_1 \\ &\mathbf{r}_2 = \mathbf{g}_2(\mathbf{0}, \mathbf{I}), \quad r_1 = g_1(0, 1). \end{aligned} \tag{2}$$

Kalman predictor: Simulation response

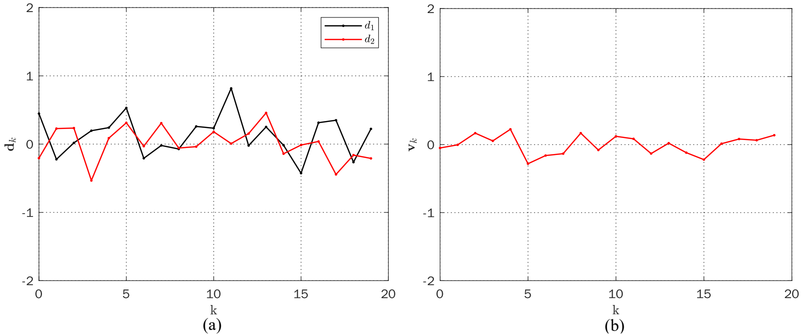

Figure 1: (a) Disturbance $\mathbf{d}_k = 0.25\mathbf{r}_2$ (b) Noise $\mathbf{v}_k = 0.25r_1$

Kalman predictor: Simulation response

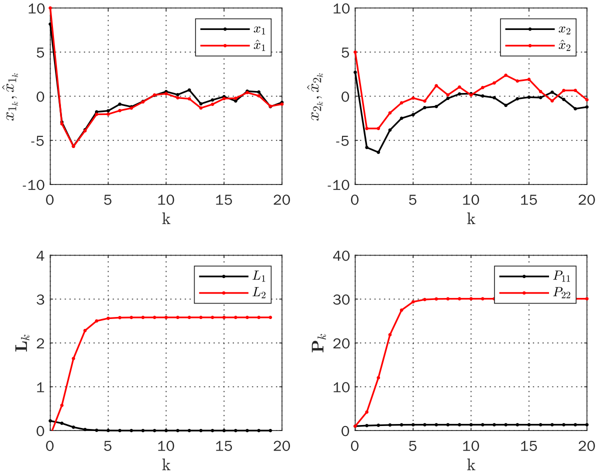

Figure 2: Kalman predictor response

Kalman predictor: LTV system (Example 1)

- LTV system

$$\begin{aligned} \mathbf{A}_k &= \mathbf{A} + (-1)^k 0.5\mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-1)^k 0.1\mathbf{B} \\ \mathbf{C}_k &= \mathbf{C} \end{aligned} \tag{3}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \tag{4}$$

- Simulation parameters

$$\begin{aligned} &\mathbf{P}_0 = \mathbf{I}_2, \quad \mathbf{Q} = \mathbf{I}_2, \quad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix} 2.73 & -2.75 \end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix} 10 & 5 \end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1 \end{aligned} \tag{5}$$

Kalman predictor: Simulation response

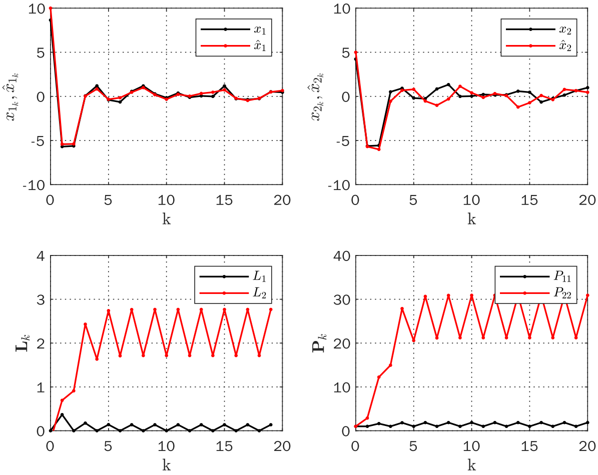

Figure 3: Kalman predictor response

Kalman predictor: LTV system (Example 2)

- LTV system

$$\begin{aligned}\mathbf{A}_k &= \mathbf{A} + (-0.75)^k\mathbf{I} \\ \mathbf{B}_k &= \mathbf{B} + (-0.5)^k\mathbf{B} \\ \mathbf{C}_k &= \mathbf{C}\end{aligned} \tag{6}$$

- System parameters

$$\mathbf{A} = \begin{bmatrix}0.5 & 0 \\ -1 & 1.5\end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix}0.5 \\ 0.1\end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix}1 & 0.5\end{bmatrix} \tag{7}$$

- Simulation parameters

$$\begin{aligned}&\mathbf{P}_0 = \mathbf{I}_2, \qquad \mathbf{Q} = \mathbf{I}_2, \qquad \mathbf{R} = 1 \\ &N = 20, \quad \mathbf{K} = \begin{bmatrix}2.73 & -2.75\end{bmatrix}, \quad \hat{\mathbf{x}}_0 = \begin{bmatrix}10 & 5\end{bmatrix}^T \\ &\mathbf{x}_0 = \hat{\mathbf{x}}_0 + 2.5\mathbf{r}_2, \quad \mathbf{d}_k = 0.25\mathbf{r}_2, \quad \mathbf{v}_k = 0.25\mathbf{r}_1\end{aligned} \tag{8}$$

Kalman predictor: Simulation response

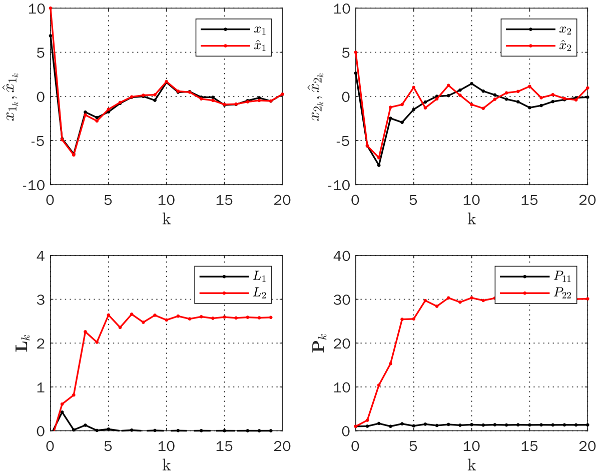

Figure 4: Kalman predictor response

Remarks

- For the LTI system example the gain matrix $\mathbf{L}_k$ and Riccati matrix $\mathbf{P}_k$ converges to some fixed matrices, say $\mathbf{L}, \mathbf{P}$ as k increases.
- For LTV systems the convergence of $\mathbf{L}_k, \mathbf{P}_k$ depends on the convergence of $\mathbf{A}_k, \mathbf{B}_k$.
- The Kalman predictor estimates the state accurately for both LTI and LTV systems.

Thank you